Performance Pay, Efficiency Wages and Unemployment Fluctuations [1]

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January 2009

Abstract

This paper argues that introducing worker shirking problems into otherwise standard search models helps generate larger fluctuations of employment and smaller fluctuations of wages, than previously thought in the efficiency wage literature. Unlike in the standard efficiency wage models, I incorporate performance way into the model, in which compensation consists of two parts: base wage and performance pay. Two different wage settings are examined. In setup I, performance pay is assumed zero and the base wage level is chosen unilaterally by the employer. This case is similar to the standard shirking model of efficiency wages. In setup II, both the base wage level, which must be greater than some given level and the non-negative bonus pay level are chosen by the employer. I find that the size of workers' rent is constant under setup I, but can be countercyclical under setup II, suggesting the promising role of performance pay in delivering larger fluctuations of unemployment.

Key Words: The business cycle; Employment fluctuations; Shirking; Search and matching; workers' match rent

JEL Classifications: E24; E32; J41; J63

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[1] All errors are my own. Financial support from the University of Chicago and the John Olin Foundation is gratefully acknowledged. The updated version of the paper is posted at http://nori.tawara.googlepages.com.

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1 Introduction

This paper argues that introducing worker effort choices into the Mortensen-Pissarides (MP) search-matching model (Mortensen and Pissarides 1994; Pissarides 1985) makes the model-generated unemployment rate more volatile than previously thought in the literature. Some previous work has demonstrated that introducing shirking models of efficiency wages (Shapiro and Stiglitz 1984) into aggregate business cycle models does not help generating "rigid" wages (Gomme 1999, e.g.). On the other, some authors have argued that introducing mechanisms of making wages "rigid" into the MP model may help to generate large fluctuations of unemployment and vacancies as observed in data (Shimer 2004, 2005; Hall 2005). These two sub-literature seem to imply that introducing shirking models of efficiency wages into the MP model may not help in delivering large fluctuations of vacancies and unemployment as observed in data.

I introduce the performance pay contingent on the worker effort level into the model. Upon meeting, the firm chooses the non-negative performance pay and the base salary level, which must be greater than some exogenously given lower bound. As the labor market tightness increases, or the unemployment rate decreases, the performance pay needs to be increased. The workers' gain from employment is decreasing in the labor market tightness, and is increasing in the performance pay level. I show that the workers' gain from employment is decreasing in the labor market tightness, i.e., countercyclical in an economy with cyclical shocks to labor productivity. This is contrasted to that the worker's rent is acyclic, i.e., independent of the labor market tightness when performance pay is prohibited as in the standard efficiency wage model. This finding suggests that introducing performance pay into the MP model appears promising to deliver larger fluctuations of the equilibrium unemployment rate, in light of the finding by Brugemann and Moscarini (2007). Brugemman and Moscarini argues that countercyclical workers' rent is important for the MP model to deliver larger fluctuations of unemployment.

Literature

I am aware of two papers that combine the MP model with the shirking model of efficiency wages a la Shapiro and Stiglitz. Rocheteau (2001) examines steady state of such model, in which wages are determined by maximizing the Nash product so that the no-shirking conditions are satisfied. Costain and Jansen (2006) examines a search matching model with aggregate shocks to productivity in which wage determination is similar to Rocheteau (2001). In both papers, the no shirking conditions, which say that employees' surplus has to be sufficiently large to motivate employees to exert efforts, are binding at some aggregate states and are slack at other aggregate states. In this paper, I examine a variant of search matching model with aggregate shocks to labor productivity in which the no-shirking conditions hold with equality at all aggregate states by assuming that firms unilaterally determine wages by taking into account the no-shirking conditions.

A number of papers have assessed matching models of frictional labor markets. Hall (2003, 2005), Shimer (2005) and others argue that a matching models of Diamond, Mortensen and Pissarides, cannot deliver fluctuations of vacancies and unemployment of an empirically plausible magnitude when one feeds productivity shocks of the empirical size into the model. Hagedorn and Manovskii (2008) criticize the calibration practices undertaken by these authors, by arguing that important parameters such as workers' bargaining power and job-seekers leisure value, are not calibrated properly by matching models' predictions to data. My paper is related to their work, in the sense that I offer an alternative way to put disciplines to calibration exercises by using the binding no-shirking conditions.

The idea that introducing performance pay into the MP model makes the model-generated unemployment more volatile may be related to Alexopoulos (2004), who develops the real business cycle model with worker shirking problems, in which a shirking employee is punished not by dismissal but by nonpayment of the deferred wage. Unlike Alexopoulos, both dismissal and nonpayment of performance pay are punishment for shirking workers in my model.

The structure of the paper goes as follows. The next section describes an environment of the matching model with work incentive problems. Section 3 examines steady state equilibrium. Section 4 examines the model with aggregate shocks to worker productivity. Section 5 concludes.

2 Model Setup

Time is discrete: $t \in \{1, 2, 3, \cdots\}$. The beginning of *period* t is called *date* $t-1$, and its end *date* t.

Agents and Preferences. This economy is populated by a continuum of risk-neutral workers and firms, both of whom discount future payoffs at the common discount factor $\beta \in [0, 1)$. The preference of each worker is given by

$$\mathbb{E}\sum_{t=1}^{\infty}\beta^t[c_t + (1-e_t)z]$$

where c_t is consumption, $e_t \in \{0, 1\}$ the fraction of time devoted to work, which is also called the employee's effort level. Consumption c is the wage when the worker is employed and it is the unemployment benefit $b \geq 0$ when unemployed. The preference of each entrepreneur (or firm) is given by:

$$\mathbb{E}\sum_{t=1}^{\infty}\beta^t[\pi_t - \mathbb{I}_v k]$$

where π_t is the profit, and $\mathbb{I}_v$ is the indicator function for the event that the firm is vacant. Parameter $k > 0$ is a flow cost of recruiting.

Production Technology. A pair of a worker and firm during employment in tenure $t \in \{1, 2, 3, \cdots\}$ produces $e_t \cdot y$, in which $e_t \in \{0, 1\}$ is the worker's effort level, and $y \in \mathbb{R}$ is match quality. Each match separates with probability $s \in [0, 1]$ each period for exogenous reasons.

Search Technology. Let $\theta \equiv v/u$ be the vacancy-unemployment ratio, where v is a measure of vacant firms and u unemployed job seekers. Each unemployed worker contacts a vacant firm with probability $p(\theta)$ per period, where $p'(\theta) > 0$. Each vacant firm contacts a job-seeker with probability $q(\theta)$ per period, where $q'(\theta) < 0$.

Upon Meeting. Upon meeting, the firm offers the fixed salary $w \geq \overline{w} \geq 0$ and performance pay $p \geq 0$. Then, the worker decides whether to accept the offer or not. If the worker turns the offer down, then the worker and firm return to the labor market and search for a partner again, in which case payoff U accrues to workers and 0 to firms. When they choose to match, the worker chooses the effort level $e \in \{0, 1\}$. At this stage, the effort level is not observed by the firm. Then, the payment of the fixed salary $w \geq 0$ is made. After this, the effort level is revealed to the firm[3]. Then, the firm chooses whether or not to pay performance pay $p \geq 0$. I assume that firm can commit to the payment of performance pay contingent on the observed effort level[4]. At the end of each employment period, a flow payoff of $w_t + e_t \cdot p_t + (1 - e_t)z$ accrues to the worker, and $y \cdot e_t - w_t - e_t \cdot p_t$ to the firm. Note that the worker's effort level is observed by the firm *after* the payment of fixed salary w is made. Assume that the performance pay p can be contingent on the effort level in an enforceable way. Match quality y is observable and verifiable. It remains the same during the entire employment duration.

Free Entry of Vacancies. Each vacant firm incurs recruiting cost $c > 0$ per period in finding a worker. Entry into labor markets is free.

[3] This sequence of events is the same as the setup of Gomme (1999).

[4] Hence, there are no problems examined by Levin (2003) and MacLeod and Malcomson (1998),

Determination of Payment: Base wage and performance pay. I assume that a compensation vector of a fixed salary of $w \geq \underline{w} \geq 0$ and a performance payment of $p \geq 0$ is chosen unilaterally by the employer upon meeting. To focus on the interesting cases, I consider two different restrictions on the space of compensation levels (w, p). First, I consider the case in which $p = 0$ and $w \geq 0$ is chosen by the firm. This case corresponds to the shirking model of efficiency wages a la Shapiro and Stiglitz (1984). Second, I consider the case in which $w \geq \underline{w} \geq 0$ and $p \geq 0$ is chosen by the employer. The lower bound assumption on the salary level, $w \geq \underline{w}$, is seen capturing one part of the reality. Moreover, as will be demonstrated below, the model with this kind of setup will deliver both qualitatively and quantitatively different property of equilibrium.

3 Steady State Equilibrium

Asset Equations. Now, I write down a bunch of asset equations in equilibrium, in which every employee exerts high effort. In writing down the condition in which employees do not shirk, I consider the employee's choices of whether to exert high effort.

The value of unemployment U satisfies the following equations:

$$U = z + b + \beta f(\theta)W + \beta[1 - f(\theta)]U \tag{1}$$

where b is the amount of unemployment benefits per period, W the value of employment for each employee. The value of employment for each employee, exerting high effort, W must satisfy the following asset equation:

$$W = w + p + (1 - s)\beta W + s\beta U \tag{2}$$

where w is the fixed base wage level, and p is the performance pay per period. The value of each filled job J must satisfy the following equation:

$$J = y - w - p + (1 - s)\beta J \tag{3}$$

Generalized No-shirking Conditions. Recall that monitoring employees' effort is perfect in the sense that the employee's effort level is observed with no noise, after the fixed salary payment is made. When the worker chooses high effort (e=1), the employment continues unless the match receives exogenous separation shocks. When the worker chooses low effort (e=0), the employment is terminated. The employee chooses high effort (e=1) if $w + p + \beta(1 - s)W + \beta sU \geq w + z + \beta U$, which is rewritten as:

$$\beta(1 - s)(W - U) + p \geq z \tag{4}$$

We call this inequality the *generalized no-shirking condition*, or *GNSC* in what follows. The reason why I put *generalized* is that it is generalized version of the no-shirking condition (NSC) of the shirking model of efficiency wages (Shapiro and Stiglitz, e.g.). Observe that when performance pay p is zero in GNSC, the GNSC becomes NSC, i.e., the no-shirking condition in the standard shirking model of efficiency wages.

Wage Determination. When the worker and firm choose to match, the firm chooses a wage level of $w \geq \underline{w}$, and the performance pay level $p \geq 0$ so that the generalized no-shirking condition is satisfied.

Free Entry of Vacancies. Each vacant firm incurs recruiting cost $c > 0$ per period in finding a worker. Entry into the labor market is free. This implies the following equation:

$$\frac{c}{\beta q(\theta)} = J \tag{5}$$

Now, we can define a steady state equilibrium of a search model with workers' shirking problems.

Definition 1. *A steady state equilibrium of a search matching model with workers' shirking problems is defined as a list $\{U, W, J, w, p, \theta, u\}$ such that: (i) U, W,* and *J satisfy equations (1), (2) and (3);* (ii) θ satisfies equation (6); (iii) w and p maximize J subject to GNSC; and (iv) u satisfies the steady state accounting: $(1-u)s = uf(\theta)$.

The definition given above is very standard except the part (iii), where two wage components are chosen by the firm so that they satisfy the GNSC.

3.1 Case of zero performance pay: Efficiency wages

I examine both analytical and quantitative properties of the equilibrium of the model described above, in which performance pay is zero. This case corresponds to the case of the standard shirking model of efficiency wages a la Shapiro and Stiglitz. I am aware of two papers that introduce the shirking model of efficiency wages into the MP model: Rocheteau (2001) and Costain and Jansen (2006). How this section differs from these two papers is described in Introduction.

Proposition 1. *Consider the model developed above in which $p = 0$. The workers' rent from match $W - U$ is independent of the vacancy unemployment ratio in equilibrium.*

Proof. Since the workers' gain from employment $W - U$ equals $\frac{w-z-b}{1-\beta[1-s-f(\theta)]}$, it is increasing in base salary $w \geq 0$. Given this, the cost-minimizing firm chooses the fixed base wage $w \geq 0$ so that the NSC binds, implying that $W - U = \frac{z}{\beta(1-s)}$. ||

Since the NSC is binding at the firm's cost-minimizing solution, the solution will be $W - U = \frac{z}{\beta(1-s)}$, $\quad J = G - \frac{z}{\beta(1-s)}$, where the match surplus G is defined by $G \equiv W + J - U$. By using equations (2) and (3), one can show that

$$G = \frac{y - (1-\beta)U}{1 - \beta(1-s)} \tag{6}$$

A steady state equilibrium of the model defined above, in which performance pay p is zero, is characterized by a pair (θ, U) satisfying the following two equations:

$$U = z + b + \beta f(\theta)\frac{z}{\beta(1-s)} + \beta U \tag{7}$$

$$\frac{c}{\beta q(\theta)} = G - \frac{z}{\beta(1-s)} \tag{8}$$

where the match surplus G is defined above. Although my interest in this paper is aggregate fluctuations of labor market aggregates under aggregate shocks, it is helpful to consider, first, the steady state of the model.

Calibration in steady state. My strategy for quantitative exercises below is to calibrate parameters in steady state and then quantify model generated moments using those parameter values. One remarkable thing is that one of important parameters, z, can be calibrated with some disciplines: Given the observed value of θ, and the vacancy cost k, the binding no-shirking conditions offer the value of non-working time, z [5]. In steady state without aggregate shocks, we have:

$$\frac{c}{\beta q(\theta)} = G - \frac{z}{\beta(1-s)}$$

and

$$G = y - z - b + \beta(1-s)G - f(\theta)\frac{z}{(1-s)}$$

These two imply that:

[5] Hall and Shimer assumes that $z = 0$. Hagedorn and Manovskii calibrate $z + b = 0.96$.

$$\frac{c}{\beta q(\theta)} + \frac{z}{\beta(1-s)} = \frac{y - z - b - f(\theta)\frac{z}{1-s}}{1 - \beta(1-s)} \tag{9}$$

Let $f(\theta) = \mu\theta^{1-\alpha}$ and $q(\theta) = \mu\theta^{-\alpha}$. I choose the model period to be two weeks. I set $\beta = \exp[\log(1/1.05)/26] = 0.9981$, so that it is consistent with the annual interest rate 0.05. Shimer (2005) estimates that the monthly job finding rate from 1951 to 2003 on average is 0.45. The corresponding bi-weekly job-finding probability is $f = 0.2584$. The average unemployment rate from 1951 to 2003 is 0.0567. This, along with the steady state accounting, implies that the bi-weekly job separation rate is $s = uf/(1-u) = 0.0567 * 0.2584/(1-0.0567) = 0.0155$. I normalize that $y = 1$. Hall (2005) found that the average value of the vacancy unemployment ratio during December 2000 to December 2002 was 0.539 according to JOLTS. But this number may be too small since the economy was in downturns during that period. Hence I use $\theta = 0.6$. This implies that $q = f/\theta = 0.2584/0.6 = 0.4307$. The replacement ratio in the U.S. is roughly 0.4. Not all the unemployed qualified for receiving the unemployment benefit are not receiving the benefit. So I choose $b = 0.3$, instead of 0.4, as has been common in the early literature.

The values of two parameters c and z remain to be chosen. The binding no-shirking condition gives z as a function of c, which I denote as $c(z)$, with some abuse of notation. Note that the flow wage w is given by:

$$\frac{w - b - z}{1 - \beta[1 - s - f(\theta)]} = \frac{z}{\beta(1-s)} \tag{10}$$

Empirically, the value of w is very close to y, which is set to 0.97 for the following reason. Interpret the matching model above as the one with capital, following Pissarides (2000). Then, labor productivity y in my matching model can be interpreted as output minus capital income. The ratio of dividends to the GDP for the U.S. economy from 1946 to 2000 is around 0.02. I assume that $y - w$ in my matching model roughly corresponds to the dividends payouts. This means that $(y - w)/(y + rk) = 0.02$, where $y = 1$ by normalization. Since capital income rk is about 0.3 of output $y+rk = 1+rk$, rk should be 3/7, and output 10/7. Hence, we have $(1-w)/(1+3/7) = 0.02$, implying that $w \simeq 0.97$. [6] I choose the value of z so that the model generated w through equation (10) is 0.97. The value of $z = 0.5234$ does this job. Remarkably, this value is close to the value of non-working time calculated by Hall (2006). It is also slightly below the value calibrated by Hagedorn and Manovskii. Once the value of z is obtained, one can compute the value of c satisfying equation (9). The value of $c = 0.7424$ does this job. This value is greater than the value of vacancy costs calibrated by Hagedorn and Manovskii (2008).

The values of matching function parameters remain to be chosen. Shimer (2005) estimates $\alpha = 0.72$. Mortensen and Nagypal (2007) argue that α should be smaller than 0.72. Many authors use a value of 0.5. I choose $\alpha = 0.5$. I choose the value of μ so that the implied $f = \mu\theta^{0.5}$ is equal to 0.45. $\mu = 0.2584/0.6^{0.5} = 0.3336$.

TABLE 1: Exogenously given parameters

parameter	interpretation	value	calibration strategy
β	discount factor	0.9981	biweekly frequency
y	labor productivity	1	normalization
α	matching function parameter	0.5	Petrongolo and Pissarides
b	unemployment benefit	0.25	Hall and Milgrom

Notes: "Exogenous" means that parameter values are selected without using explict mapping between models and data.

The parameter values exogenously given in Table 1 are the same regardless of the models setup.

[6] This number is close to the implied wage productivity ratio in standard DMP models. For instance, in Shimer's (2005) calibration, the implied w/y is 0.973.

TABLE 2: Calibrated Parameters: Case of zero performance pay

parameter	interpretation	value	calibration strategy
z	the value of non-working (leisure)	0.5624	$w = 0.97$
s	match separation prob.	0.0155	$u = 0.0567$ and $f = 0.2584$
μ	matching function parameter	0.3336	$f = 0.2584$ and $\theta = 0.6$
c	vacancy cost	0.7424	$\theta = 0.6$

Notes: "Calibrated" here means that the parameter values are selected with explicit reference to moment matching between the model and data.

The calibrated values of parameters s and μ in Table 2 will be also used in the calibration of the model with positive performance pay below. However, the values of parameters z and c in the same table will be reselected in the calibration under positive performance pay below.

Now I do some 'numerical' comparative statics in steady state before quantifying the model with aggregate shocks below. For $y = 1$, the model delivers $\theta = 0.6$, and $w = 0.97$. For $y = 1.01$, the model delivers $\theta = 0.6698$, and $w = 0.9783$. For $y = 0.99$, the model delivers $\theta = 0.5346$, and $w = 0.9617$. The elasticity of θ with respect to y is roughly 11.27. The value of this implied elasticities are larger than 7.56, which is the regression coefficient of data of the deviations of $\log\theta$ from its HP filtered trend on data of deviations of log productivity from its HP trend. The elasticity of wages with respect to productivity is 0.8557. This number is way greater than 0.449, which Hagedorn and Manovskii argue to be the empirical elasticity of wages with respect to productivity. However, Pissarides (2007) recently argues that the relevant empirical elasticity of wages with respect to productivity should be close to 1.

TABLE 3: "Comparative Statics" in Steady State: Case of zero performance pay

y	0.99	1	1.01
θ	0.5346	0.6	0.6698
w	0.9617	0.97	0.9783

TABLE 4: Data versus Implied Productivity Elasticities
The case of zero performance pay

	$\eta_{\theta,y}$	$\eta_{w,y}$
Model, efficiency wages	11.27	0.856
Data	7.56 (U.S., 1951-2003)	0.449^a (BLS, 1951-2004)
Model (Shimer)	1.75	0.964
Model (H and M)	21.72	0.449

Note: η_{xy} denotes the elasticity of variable x with respect to variable y, where all variables are reported in logs as deviations from an HP trend with a smoothing parameter of either 1600 or 10^5. The superscript a indicates a smoothing parameter of 1600. Data for η_{xy} is the regression coefficient of variable x on variable y. The value of $\eta_{\theta,y}$ for Model (Shimer) is what is implied by his TABLE 3 ($\eta_{\theta,y} = \frac{\rho(\theta,y)\sigma(\theta)}{\sigma(y)} = \frac{0.999*0.035}{0.02} = 1.75$).

My calibrated search model with efficiency wages delivers a sufficiently large size of productivity elasticity of the v-u ratio. It delivers a productivity elasticity of wages of 0.856, which is way larger than aggregate data (0.449), which Hagedorn and Manovskii (2008) estimate. If the goal of introducing shirking problems of efficiency wages into a search matching model were to deliver sticky wages whose elasticity with respect to productivity is 0.499, as in some previous papers such as Gomme(1999), then such attempt would be a failure, although it reduces the elasticity slightly compared with the benchmark calibration of Shimer(2005). However, Pissarides (2007) has recently argued that what is relevant for assessing a search matching model is behavior of wages in only *new* matches. He argues that a plausible empirical elasticity of wages in new matches with respect to productivity is close to one. In this respect, the productivity elasticity of wages generated by my calibrated model is not that different from what Pissarides (2007) considers empirically plausible.

Comparison to Shimer and Hagedorn and Manovskii. The key to understanding my calibration is that the binding NSCs offers tight disciplines in calibrating the parameter of the non-working (leisure) time. Comparison of calibration procedures across three different papers (Shimer 2005; Hagedorn and Manovskii 2008; this paper) may help in understanding my point. Start with a standard DMP model with Nash bargaining wage determination. There are two nontrivial key parameters that must be calibrated: the sum of the leisure value and the unemployment benefit level, and employees' bargaining power. Shimer implicitly assumes that the leisure value is zero and invokes the empirical replacement ratio of 0.4 to assume that the unemployment benefit is 0.4. He invokes Hosios' (Hosios 1990) condition to posit that employees' bargaining power is equal to the elasticity of matches with respect to unemployment , which he estimates to be 0.72 by regressing the measured job finding probability on the measured vacancy unemployment ratio. Hagedorn and Manovskii calibrated these two parameters by explicitly matching the model's implications to data. They choose the value of the two parameters so that the model generates the empirically observed productivity elasticity of aggregate wages (0.499) and the profit-output ratio (0.03). As argued above, there appears to be controversies about what the relevant empirical moment of productivity elasticity of wages is. It may be desirable not to use productivity elasticity of wages as a target moment in calibration. My matching model with efficiency wages does not contain a parameter of employees' bargaining power, since wages are determined by the binding no-shirking condition. Therefore only the data of profit-output ratio can identify the value of non-working.

3.2 Case of positive performance pay

The subsection considers the case in which both the non-negative bonus pay $p \geq 0$ and the base salary level $w \geq \underline{w} > 0$ are chosen by the employer when the worker and firm meet. The model in this section is similar to Alexopoulps (2004), who examines the setup in which the firm punishes the shirking employee by not paying the deferred wage, and employment continues. My model in the section differs from Alexopoulos in that employment is terminated when the worker shirks in my model, while it is not in Alexopoulos. I argue that my setup is more consistent with the profit-maximizing firm, since the firm can reduce total payment by letting employment terminated when the worker shirks. The insight in this section is also related to Yang (2008), who examines the selection of contract forms by the size of turnover costs.

Proposition 2. Consider the model in which both performance pay and base wage are chosen by an employer in each period. Assume that the lower bound of the base salary is greater than the unemployment benefit, i.e., $\underline{w} > b$. In equilibrium, the workers' surplus is decreasing in labor productivity y.

Proof of Proposition 2.

The workers' gain from employment is computed as:

$$W - U = \frac{w + p - z - b}{1 - \beta[1 - s - f(\theta)]}$$

Consider the case in which $w = \underline{w}$, and $p = 0$. There are two cases: (A) $\underline{w} \geq z + b$; and (B) $\underline{w} < z + b$.

Consider, first, the case (A) in which $\underline{w} \geq z + b$. In this case, the worker's participation constraint is satisfied: $W - U = \frac{\underline{w}-z-b}{1-\beta[1-s-f(\theta)]} \geq 0$. If the GNSC is satisfied for $w = \underline{w}$ and $p = 0$, i.e., $\frac{z}{\beta(1-s)} \leq \frac{\underline{w}-z-b}{1-\beta[1-s-f(\theta)]}$, then $w = \underline{w}$ and $p = 0$ is the firm's optimal choice. So, the workers' match surplus is $\frac{\underline{w}-z-b}{1-\beta[1-s-f(\theta)]}$, which is decreasing in the market tightness θ. Unless the

GNSC is satisfied for $w = \underline{w}$ and $p = 0$, either performance pay p or the fixed salary w needs to increase. The key is that the amount of a payment increase to satisfy the GNSC is smaller for the performance pay p than for the fixed salary w[7]. Hence, the cost-minimizing firm chooses to increase the amount of performance pay p until the GNSC binds. Hence, at the firm's optimum, the GNSC is binding, which gives the amount of performance pay: $p = z - \frac{\beta(1-s)(\underline{w}-b)}{1+\beta f(\theta)}$, which is increasing in θ, since $\underline{w} > b$ by assumption. Since the binding GNSC implies $z = p + \beta(1-s)(W-U)$, the workers' match surplus $W - U$ is decreasing in θ.

Next, consider the case (B) in which $\underline{w} < z + b$. Consider the payment of $w = \underline{w}$, and $p = 0$. In this case, the workers' participation constraint is not satisfied. The sum of $w + p$ must increase until $W = U$. In light of the GNSC and the firm's cost minimization, increasing performance pay p is more *efficient*. So, increase performance pay p until the worker's participation constraint binds, $W = U$, which implies $p = z + b - \underline{w}$, which is positive by assumption. The implied NSC of $z \leq p$ is violated since $\underline{w} > b$. The performance pay p has to increase until the GNSC holds with equality: $z = \beta(1-s)(W-U)+p$, where $W - U = \frac{\underline{w}+p-z-b}{1-\beta[1-s-f(\theta)]}$. This binding GNSC gives the performance pay, which is the same as computed above. Therefore, in this case, too, the performance pay is increasing in θ, and the workers' gain from employment is decreasing in θ. Hence, in every case considered, the workers' gain from employment is decreasing in θ. The free entry equation implies that θ is increasing in aggregate productivity y, this completes the proof of Proposition 2. ||

The result that the workers' gain from employment is decreasing in aggregate productivity y is contrasted with Proposition 1, saying that when performance pay is prohibited as in the traditional shirking model of efficiency wages (Shapiro and Stiglitz), workers' gains from employment are

[7] Yang (2008) examines the similar mechanism under the relational incentive contracts in different contexts.

independent of the v-u ratio. This result appears promising in light of Brugemann and Moscarini (2007), who demonstrate analytically that countercyclical workers' rents, in stead of its acyclicality, are important for search models to deliver large fluctuations of vacancies and unemployment.

The reason why I consider the case of $\underline{w} > b$ is that the case of $\underline{w} = b$ is not interesting. To see this, suppose $\underline{w} = b$. Consider the payment of $w = b(= \underline{w})$ and $p = 0$. The workers' participation constraint is not satisfied. Increase performance pay p until the workers' participation constraint holds with equality, which implies $p = z$. The GNSC is weakly satisfied. Recall that $z > 0$ by assumption. Hence, in this case, the workers' gains from employment is zero, i.e., $W - U = 0$, and the the value of a filled job is positive, since $y - p - w = y - z - b > 0$ by assumption. In this case, employers captures all of the match rents.

Assume $z > b$ in what follows. In the calibration and simulation exercises below, I consider a case of $\underline{w} = z + b > b$, which falls into the case (A) of Proposition 1 above.

Calibration in steady state.

Consider the case $\underline{w} = z + b > b$. This case corresponds to the case A of Proposition 2 above. The optimal choice of performance pay is $p = z - \frac{\beta(1-s)z}{1+\beta f(\theta)}$ and the base wage is $w = z + b$. The worker's rent from match is $W - U = \frac{z}{1+\beta f(\theta)}$, which is decreasing in θ. Let

$$G = \frac{y - (1-\beta)U}{1 - \beta(1-s)} \tag{11}$$

The steady state equilibrium in this case is given by (θ, U) satisfying the following two equations:

$$\begin{aligned} U &= z + b + \beta f(\theta)\frac{z}{1+\beta f(\theta)} + \beta U && (12) \\ \frac{c}{\beta q(\theta)} &= G - \frac{z}{1+\beta f(\theta)} && (13) \end{aligned}$$

I choose the values of parameters z and c in two different ways.

CASE A. First, I choose the value of non-working z so that the total payment

$$w + p = 2z + b - \frac{\beta(1-s)z}{1+\beta f(\theta)}$$

is targeted to 0.97 when labor productivity y is normalized to 1. Given z, the value of c is calibrated using the free entry equation. The values of $z = 0.5907$ and $c = 0.7424$ do this job.

CASE B. Second, to make the comparison with the case of zero performance pay clear, the same parameter value of non-working $z = 0.5624$ as in the case of zero performance pay is used. Given this value of z, the free entry equation gives $c = 1.5967$. In thise case, the implied total wage pay $w + p$ equals 0.9355.

With the calibrated model at hand, the numerical comparative statics around the steady state is undertaken here.

TABLE 5: "Comparative Statics" in Steady State: Case of positive performance pay

y	0.99	1	1.01
θ (Case A)	0.5089	0.6	0.7018
$w + p$ (Case A)	0.9624	0.97	0.9776
θ (Case B)	0.5255	0.6	0.6808
$w + p$ (Case B)	0.9296	0.9355	0.9413

Notes: y denotes labor productivity, θ denotes the vacancy unemployment ratio, $w + p$ denotes total payment. What Case A and Case B mean is explained in the main text.

TABLE 6: Data versus Implied Productivity Elasticities

The case of positive performance pay

	$\eta_{\theta,y}$	$\eta_{w,y}$
Model, performance pay, Case A	16.07	0.7835
Model, performance pay, Case B	12.95	0.6254
Model, efficiency wages	11.27	0.8557
Data	7.56 (U.S., 1951-2003)	0.449^a (BLS, 1951-2004)
Model (Shimer)	1.75	0.964
Model (H and M)	21.72	0.449

Note: η_{xy} denotes the elasticity of variable x with respect to variable y, where all variables are reported in logs as deviations from an HP trend with a smoothing parameter of either 1600 or 10^5. The superscript a indicates a smoothing parameter of 1600. Data for η_{xy} is the regression coefficient of variable x on variable y. The value of $\eta_{\theta,y}$ for Model (Shimer) is what is implied by his TABLE 3 ($\eta_{\theta,y} = \frac{\rho(\theta,y)\sigma(\theta)}{\sigma(y)} = \frac{0.999*0.035}{0.02} = 1.75$).

4 Aggregate Shocks to Labor Productivity [Incomplete]

This section adds aggregate shocks to productivity to the model developed above. Let there be N possible aggregate productivity levels. Labor productivity in state $i \in \{1, 2, \cdots, N\}$ is denoted by y_i, where $y_1 < y_2 < ... < y_N$. I assume that aggregate state follows a N-state Markov chain with transition matrix Π_y, whose ith row and jth column element π_{ij} indicates the probability that next period's state is j when the current state is i.

The value of being unemployed when the aggregate productivity is $y \in \{y_1, y_2, ..., y_N\}$, which is denoted by U_y, satisfies:

$$U_y = z + b + \beta f(\theta_y)\mathbb{E}_y(W_{y'} - U_{y'}) + \beta\mathbb{E}_y U_{y'} \tag{14}$$

where z is the flow value of leisure, b is the unemployment benefit, and $\mathbb{E}_y$ is the expectation operator conditional on that the current aggregate productivity is y. Note that the value of employment for a worker and the vacancy unemployment ratio also depend on aggregate productivity y. y' in the equation indicates the next period's aggregate productivity. The value of employment for a worker when aggregate productivity is y, which I denote as W_y, satisfies:

$$W_y = w_y + p_y - \beta s \mathbb{E}_y (W_{y'} - U_{y'}) + \beta \mathbb{E}_y W_{y'} \tag{15}$$

where it is assumed that the amount of the leisure time for each non-shirking employee is zero, w_y and p_y are a base wage and performance pay when productivity is y. The value of a filled job in state y, which is denoted by J_y, satisfies:

$$J_y = y - w_y - p_y + (1-s)\beta \mathbb{E}_y J_{y'} \tag{16}$$

Free entry of vacant jobs in state y implies that

$$c = \beta q(\theta_y) \mathbb{E}_y J_{y'} \tag{17}$$

where $c > 0$ is a flow cost of posting a vacant job. The generalized no-shirking condition in state y is:

$$\beta(1-s)\mathbb{E}_y (W_{y'} - U_{y'}) + p_y \geq z \tag{18}$$

since the employee prefers not to shirk in state y if:

$$w_y + p_y + \beta(1-s)\mathbb{E}_y W_{y'} + \beta s \mathbb{E}_y U_{y'} \geq w_y + z + \beta \mathbb{E}_y U_{y'}$$

Let the match surplus in state y be defined by $G_y \equiv W_y + J_y - U_y$.

Recall that the base wage and performance pay are unilaterally determined by the employer in each employment period after the aggregate productivity level is revealed.[8].

Quantitative Exercises:

The stochastic process for labor productivity is described by the AR(1) process:

$$\log y_{t+1} = \rho \log y_t + \epsilon_{t+1} \tag{19}$$

where $\rho \in (0,1)$ and ϵ_{t+1} is a draw from $N(0, \sigma_\epsilon^2)$. I follow Hagedorn and Manovskii (2008) in choosing $\rho = 0.9809$ and $\sigma_\epsilon = 0.0046$. The mean of y is normalized to 1. I approximate the AR(1) process by a discrete 7-state Markov chain, following Tauchen (1986). The grid of log productivity is $\log y \in \{-0.0709, -0.0473, -0.0236, 0, 0.0236, 0.0473, 0.0709\}$. This means that $y \in \{0.9316, 0.9538, 0.9767, 1, 1.0239, 1.0484, 1.0735\}$. The transition matrix Π is given by:

$$\Pi = \begin{bmatrix} 0.9886 & 0.0114 & 0 & 0 & 0 & 0 & 0 \\ 0.0028 & 0.9884 & 0.0088 & 0 & 0 & 0 & 0 \\ 0 & 0.0038 & 0.9895 & 0.0067 & 0 & 0 & 0 \\ 0 & 0 & 0.0051 & 0.9898 & 0.0051 & 0 & 0 \\ 0 & 0 & 0 & 0.0067 & 0.9895 & 0.0038 & 0 \\ 0 & 0 & 0 & 0 & 0.0088 & 0.9884 & 0.0028 \\ 0 & 0 & 0 & 0 & 0 & 0.0114 & 0.9886 \end{bmatrix}$$

The standard deviation of the simulated data of $\log y$, using the transition matrix above, is 0.03. The standard deviation of log productivity, relative to its HP filtered trend, for the U.S. quarterly data is 0.02 when the smoothing parameter is 10^5 (Shimer 2005, 1951-2003), and 0.013 when the smoothing parameter is 1600 (Hagedorn and Manovskii 2008, 1951Q1-2004Q4). Note that the standard deviation in biweekly frequency should be larger than in

[8] This assumption is not empirically implausible in light of recent work of Hagedorn and Manovskii (2008). They calibrate the parameter of employees' bargaining power to be close to 0.06.

quarterly frequency.

4.1 Case of zero performance pay: Efficiency wages

Consider the case in which performance pay p_y equals zero in every aggregate state y. Then, one obtains that:

$$G_y = y - z - b + \beta(1-s)\mathbb{E}_y G_{y'} - f(\theta_y)\frac{z}{(1-s)} \tag{20}$$

This equation can be solved numerically for G_y. Using G_y, the free entry equation in state y is rewritten as:

$$\frac{c}{\beta q(\theta_y)} = \mathbb{E}_y G_{y'} - \frac{z}{\beta(1-s)} \tag{21}$$

The value of parameters are the same as in steady state. The surplus vector G_y, the vacancy unemployment ratio vector θ_y, and the wage vector w_y are computed as:

$$\begin{aligned}
\theta &= (0.22, 0.3216, 0.4483, 0.6, 0.778, 0.9836, 1.2136)' \\
G &= (1.5761, 1.7957, 2.0248, 2.2596, 2.5, 2.7455, 2.9917)' \\
w &= (0.9158, 0.9332, 0.9514, 0.97, 0.9891, 1.0085, 1.028)'
\end{aligned}$$

The simulations are undertaken as follows. Using the transition matrix Π above, I simulate the Markov chain with 7-state. In order to generate 212 "quarterly" data points, corresponding to quarterly data from 1951 through 2003, I need to generate 1272 "bi-weekly" data points, since my model works at biweekly frequency. To do so, for each simulation, I generate 2000 data points, and throw away the first 728 data points to eliminate the dependence on the initial value chosen. Then, I aggregate the generated "biweekly-frequency" data to quarterly frequency, resulting in 212 "quarterly" generated data points. Then I detrend the log of the model-generated "quarterly" data using an HP filter with the smoothing parameter 1600[9]. Then I compute the standard deviation of such model-generated data. I repeat this 10,000 times. The result is summarized by Table 7.

TABLE 7: Data versus Model-generated Moments: Case of zero performance pay

	$\sigma(\theta)/\sigma(y)$	$\sigma(u)/\sigma(y)$	$\sigma(v)/\sigma(y)$	Proj(w|y)	smoothing parameter
Data (Shimer)	19.1(7.56)	9.5 (-3.9)	10.1 (3.67)	—	10^5
Data (H and M)	19.9 (7.83)	9.6 (-2.9)	10.7 (4.9)	0.499	1600
Model, efficiency wages	12.26	5.671	6.071	0.829	1600
Model (Shimer)	1.75	0.45	1.35	0.964	10^5
Model (H and M)	22.46	11.15	13.0	0.449	1600

Note: $\sigma(x)$ denotes the standard deviation of variable $x \in \{\theta, y, u, v\}$, where all variables are reported in logs as deviations from an HP trend with smoothing parameter mentioned in the Table. For data of $\sigma(\theta)/\sigma(y)$, I also report the regression coefficient of variable θ on variable y, which is 7.56. Proj(w|y) is the regression coefficient of variable w on variable y.

My calibrated search model with efficiency wages delivers a sufficiently large fluctuations of v-u ratios, the unemployment rate and vacancies, compared with data. TABLE 4 shows that the size of fluctuations of variable θ, u and v, measured by the standard deviation, is greater than the empirical regression coefficient of each variable on productivity (the numbers in parentheses). They are also not way below the empirical standard deviations. Turning to the wage fluctuations, my calibrated model with efficiency wages delivers a standard deviation of wages in relative to productivity of

[9] This number of the smoothing parameter is standard in the RBC literature when detrending quarterly data.

0.83, which is way larger than data (0.449), which Hagedorn and Manovskii (2008) estimate. If the goal of introducing shirking problems of efficiency wages into a search matching model were to deliver sticky wages whose elasticity with respect to productivity is 0.499, as in some previous papers such as Gomme(1999), then such attempt would be a failure, since it reduces the wage fluctuations only slightly compared with the benchmark calibration of Shimer(2005).

However, as Pissarides (2007) has recently raised the issue, what is relevant for assessing a search matching model may be behavior of wages in only *new* matches. He argues that a plausible empirical elasticity of wages in new matches with respect to productivity is close to one. In this respect, the productivity elasticity of wages generated by my calibrated model is not that different from what Pissarides (2007) considers empirically plausible.

4.2 Case of positive performance pay

Assume that $\underline{w} = z + b$. Consider the case in which a firm can choose performance pay p_y as well as base wage w_y in aggregate state y. Then, one obtains that:

$$G_y = y - z - b + \beta(1-s)\mathbb{E}_y G_{y'} - f(\theta_y)\frac{z-b}{1+\beta f(\theta_y)} \tag{22}$$

This equation can be solved numerically for G_y. Using G_y, the free entry equation in state y is rewritten as:

$$\frac{k}{\beta q(\theta_y)} = \mathbb{E}_y G_{y'} - \frac{z-b}{1+\beta f(\theta_y)} \tag{23}$$

The surplus vector G_y, the vacancy unemployment ratio vector θ_y, and the wage vector w_y are as follows. In CASE A ($z = 0.603$ and $c = 0.7596$),

$$\begin{aligned}
\theta &= (0.0431, 0.1420, 0.3217, 0.6014, 1.0045, 1.5554, 2.2488)' \\
G &= (0.7528, 1.1262, 1.5473, 2.0098, 2.5149, 3.0625, 3.6296)' \\
w + p &= (0.9275, 0.9415, 0.9556, 0.9694, 0.9828, 0.9956, 1.0074)'
\end{aligned}$$

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