# On Efficient Partnership Dissolution under Ex Post Individual Rationality*

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**Abstract**

This paper studies efficient partnership dissolution with ex post participation constraints in a setting with interdependent valuations. We derive a sufficient condition that ensures the existence of an efficient dissolution mechanism that satisfies Bayesian incentive compatibility, ex post budget balancedness, and ex post individual rationality. For equal-share partnerships, we show that our sufficient condition is satisfied for any symmetric type distribution whenever the interdependence in valuations is non-positive. This result improves former existence results demonstrating that the stronger requirement of ex post individual rationality does not always rule out efficiency. We also discuss a situation with ex post quitting rights. *Journal of Economic Literature* Classification Numbers: D02, D40, D44, D82, C72.

KEYWORDS: mechanism design; efficient trade; ex post individual rationality; Groves mechanism; quitting right; interdependent valuation.

# 1 Introduction

Many business projects involve partnerships such as joint ventures and strategic alliances. A partnership comes to an end, for example when the project (e.g., development of a new product or technology) has been completed, or simply when the partners have conflicting opinions about future management of their business. Efficient dissolution of a partnership consists in allocating the partnership's asset (e.g., the developed product/technology or the company itself) to the partner with the highest valuation, in exchange for monetary compensations. Cramton *et al.* (1987, CGK henceforth) first consider the problem of efficient partnership dissolution in a symmetric model with independent private values. CGK show that while efficient dissolution is impossible when the initial ownership of the partnership is extreme as in the buyer-seller situation (Myerson and Satterthwaite (1983)), it is always (i.e., for all type distributions) possible when the partnership is equally shared among the agents. In the present paper, focusing on equal-share partnerships, we study the possibility of efficient dissolution in a symmetric *interdependent* valuation setting as in the subsequent contribution by Fieseler *et al.* (2003, FKM henceforth). The distinguishing feature of this paper is that, in contrast to CGK and FKM where individual rationality (or participation) constraints are required to be fulfilled at the *interim* stage, we impose the stronger requirement of *ex post* individual rationality.

Interdependence in valuations naturally arises in many situations, e.g., where each agent is responsible for a different part of the project and thus receives a different piece of private information which also affects the others' valuations of the entire project. In an environment where the private and common value components are additively separable, FKM show that when the interdependence is positive (i.e., valuations are increasing in the other agents' signals), efficient dissolution is not always possible even for the equal-share case, while it becomes easier when the interdependence is negative (i.e., valuations are decreasing in the other agents' signals). In the case of negative interdependence, efficiency is easier to achieve as winning and losing are each a blessing: winning reveals that the other agents' signals are lower than one's own which contributes to raising the winner's valuation, and a symmetric argument applies to losing. In the case of positive interdependence, conversely, winning and losing are bad news, and winner's and loser's curses make efficiency more difficult to achieve.

Both CGK and FKM consider efficient mechanisms that satisfy *interim* individual rationality (IIR) as well as Bayesian (i.e., interim) incentive compatibility (IC) and (ex post) budget balancedness (BB). Our point of departure in the present paper is that it is desirable to have a mechanism such that no agent regrets his participation *ex post*, and thus we look for efficient mechanisms that satisfy *ex post* individual rationality (EPIR) along with IC and BB. Given the result of FKM, we mainly restrict our attention to the case where interdependence in valuations is non-positive (i.e., valuations are private or negatively interdependent).[1] For this case, we show that efficient dissolution of an equal-share partnership is always possible even with EPIR. This demonstrates, for the case of equal-share partnerships, that whenever efficient dissolution is always possible with IIR, one can safely replace IIR with EPIR incurring no loss in efficiency as well as IC and BB. The proof is done by construction of a mechanism that satisfies the desired properties.

EPIR mechanisms are also considered by Gresik (1991a, 1991b), Makowski and Mezzetti (1994), and Kosmopoulou (1999). Gresik (1991a, 1991b) considers EPIR and Bayesian IC bilateral trading mechanisms that maximize *ex ante* expected gains from trade. In a general setting with independent private valuations, Makowski and Mezzetti (1994) provide characterizations of ex post efficient, IIR, ex post BB, Bayesian IC mechanisms and ex post efficient, EPIR, ex ante BB, dominant strategy IC mechanisms, while Kosmopoulou (1999) shows a payoff equivalence result between these two classes of mechanisms in a restricted environment. Different from these papers, our approach concerns Bayesian IC mechanisms that satisfy *ex post* efficiency, EPIR, and *ex post* BB.

While we motivate our study of EPIR mechanisms by requirement that a desirable mechanism should be ex post regret-free in participation, one may consider a situation in which agents are allowed to quit or veto the mechanism ex post in any event. Compte and Jehiel (2006, 2007) study mechanism design with ex post quitting/veto rights in a bargaining problem. Noting that with quitting rights *off* as well as on equilibrium, the IC constraints are also modified,[2] they show that inefficiencies are inevitable in their bargaining model (even with correlations in types). We examine the modified IC constraints in our environment, and show that our mechanism always dissolves the partnership efficiently even with quitting rights when the degree of negative interdependence is large.

Related papers, other than FKM, that consider partnership dissolution with interdependent valuations include Kittsteiner (2003), Morgan (2004), Jehiel and Pauzner (2006), and Chien (2007) among others. Kittsteiner (2003) studies the $k$-double auction (and that with interim veto) in the case of positively interdependent valuations and derives equilibrium bidding strategies. He demonstrates that when allowing for interim veto, inefficiencies may occur, and the $k$-double auction may not maximize ex ante expected

[1] While valuations may be assumed to be positively interdependent in standard cases, e.g., when the information is about quality (anyone prefers high quality), they may well be negatively interdependent in other cases, e.g., when the agents have opposite characteristics in that they derive utility from mutually exclusive properties of the asset, i.e., "if information about the increased likelihood of property A (which yields relatively more utility for partner $i$) means that property B (which yields relatively more utility for partner $j$) becomes less likely" (FKM, Footnote 6).

[2] See also Matthews and Postlewaite (1989) and Forges (1999) for similar considerations.

gains from trade. In a pure common value setting, Morgan (2004) is concerned with dissolution mechanisms that lead to fair outcomes in which the agents obtain equal ex post payoffs. He examines the fairness properties of several simple mechanisms. Jehiel and Pauzner (2006) consider a one-sided incomplete information setting, where only one agent has private information, with interdependent valuations. They show that in some cases there is no mechanism that efficiently dissolves the partnership, and the second-best outcome can be achieved when one agent has the full ownership, as opposed to the symmetric settings of CGK and FKM. Chien (2007) studies second-best dissolution mechanisms and provides a characterization of incentive efficiency.

The paper is organized as follows. Section 2 describes our partnership dissolution problem. Section 3 derives our main sufficient condition for existence. Positive results are provided for two-agent partnerships in Section 4 and for $n$-agent partnerships in Section 5. Section 6.3 considers quitting rights.

## 2 Setup

In this section, we describe our problem of partnership dissolution, where we mostly follow the setup of FKM. There are one asset, and $n$ risk-neutral agents indexed by $i \in N = \{1, \ldots, n\}$, where $n \geq 2$. Each agent $i$ initially owns a share $\alpha_i$ of the asset ($0 \leq \alpha_i \leq 1$ and $\sum_{i \in N} \alpha_i = 1$). Each agent $i$ has private information represented by type $\theta_i$. We will denote $\theta = (\theta_1, \ldots, \theta_n)$ and $\theta_{-i} = (\theta_1, \ldots, \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n)$. Agents' types are statistically independent. The type $\theta_i$ is distributed according to a commonly known distribution $F_i$ with support $\Theta_i = [\underline{\theta}_i, \overline{\theta}_i]$ and positive continuous density $f_i$. We denote $\Theta = \prod_{i \in N} \Theta_i$.

Agent $i$'s valuation for the entire asset is given by a function $v_i(\theta_i, \theta_{-i})$, where the arguments are always ordered by the agents' indices: $v_i(\theta_i, \theta_{-i}) = v_i(\theta_1, \ldots, \theta_n)$. The function $v_i(\theta_i, \theta_{-i})$ is assumed to be strictly increasing in $\theta_i$, and continuously differentiable. We further assume the single crossing property:

$$v_{i,i}(\theta) > v_{i,j}(\theta)$$

for all $i, j \neq i$ and $\theta \in \Theta$, where $v_{i,k} = \partial v_i / \partial \theta_k$. The ex post utility of agent $i$ with valuation $v_i$, share $s_i$, and money $m_i$ is given by $v_i s_i + m_i$.

In a direct revelation mechanism, or simply mechanism, each agent $i$ simultaneously reports his own type $\theta_i$, and then receives a share $s_i(\theta)$ of the asset and a monetary transfer $t_i(\theta)$. More precisely, a mechanism is a pair $(s, t)$ of (measurable) functions $s \colon \Theta \to [0, 1]^n$ such that $\sum_{i \in N} s_i(\theta) = 1$ (an *assignment rule*) and $t \colon \Theta \to \mathbb{R}^n$ (a *transfer rule*). Given a mechanism $(s, t)$, the interim utility of agent $i$ with type $\theta_i$, when he reports $\hat{\theta}_i$ while the other agents report their types $\theta_{-i}$ truthfully, is given by

$$U_i(\theta_i, \hat{\theta}_i) = E_{\theta_{-i}}[v_i(\theta_i, \theta_{-i}) s_i(\hat{\theta}_i, \theta_{-i})] + E_{\theta_{-i}}[t_i(\hat{\theta}_i, \theta_{-i})],$$

where $E_{\theta_{-i}}[\cdot]$ is the expectation operator with respect to $\theta_{-i}$. We denote $U_i(\theta_i) = U_i(\theta_i, \theta_i)$.

A mechanism $(s,t)$ is *interim incentive compatible* (IC) if truth-telling constitutes a Bayesian Nash equilibrium in the incomplete information game induced by $(s,t)$, i.e., for all $i \in N$,

$$U_i(\theta_i) \geq U_i(\theta_i, \hat{\theta}_i) \tag{IC}$$

for all $\theta_i, \hat{\theta}_i \in \Theta_i$. It is *ex post budget balanced* (BB) if the monetary transfers sum to zero for each realization of the types, i.e., $\sum_{i \in N} t_i(\theta) = 0$ for all $\theta \in \Theta$. It is *ex post efficient* (EF) if it allocates the asset to an agent with the highest valuation for each realization, i.e., for all $i \in N$ and all $\theta \in \Theta$, $s_i(\theta) > 0 \Rightarrow i \in \arg\max_j v_j(\theta)$. It is sufficient to consider the efficient assignment rule $s^*$ defined by

$$s_i^*(\theta) = \begin{cases} 1 & \text{if } i = m(\theta), \\ 0 & \text{if } i \neq m(\theta), \end{cases} \tag{2.1}$$

where $m(\theta) = \max(\arg\max_j v_j(\theta))$.[3]

In the present study, we are interested in mechanisms that satisfy *no ex post regret of participation*, or *ex post individual rationality*, as a desideratum additional to the above three, while much work in the literature, including that of CGK and FKM, is concerned with *interim* individual rationality. Let $u_i(\theta)$ be agent $i$'s ex post utility under truth-telling:

$$u_i(\theta) = v_i(\theta) s_i(\theta) + t_i(\theta),$$

and $u_i^0(\theta)$ the outside option to agent $i$: $u_i^0(\theta) = \alpha_i v_i(\theta)$. The mechanism $(s,t)$ is *ex post individually rational* (EPIR) if for any realization of types, no agent regrets his participating in the mechanism even after observing the realized value of his initial share, i.e., for all $i \in N$,

$$u_i(\theta) \geq u_i^0(\theta) \tag{EPIR}$$

for all $\theta \in \Theta$; $(s,t)$ is *interim individually rational* (IIR) if given his type, but before he learns the other agent's type, each agent prefers to participate in the mechanism, i.e., for all $i \in N$,

$$U_i(\theta_i) \geq E_{\theta_{-i}}\left[u_i^0(\theta_i, \theta_{-i})\right] \tag{IIR}$$

for all $\theta_i \in \Theta_i$. Clearly, EPIR implies IIR, but not vice versa.

We say that the partnership is *EPIR-dissolvable* (*IIR-dissolvable*, resp.) if there exists an IC, EF, and BB mechanism that is also EPIR (IIR, resp.).

[3]Our analysis is not affected by this particular choice of a tie-breaking rule.

## 3 A Sufficient Condition for Existence

Let us recall the revenue equivalence result of FKM.

**Revenue Equivalence** (FKM)**.** *Let $s^*$ be the EF assignment rule. Then, $(s^*, t)$ is IC if and only if for all $i \in N$,*

$$U_i(\theta_i) = U_i(\underline{\theta}_i) + \int_{\underline{\theta}}^{\theta_i} E_{\theta_{-i}}[v_{i,i}(x, \theta_{-i}) s_i^*(x, \theta_{-i})]\, dx \tag{3.1}$$

*for all $\theta_i \in \Theta_i$.*

Following FKM, for each $i$ let $\theta_i^*(\theta_{-i}) \in \Theta_i$ be defined by

$$v_i(\theta_i^*(\theta_{-i}), \theta_{-i}) = \max_{j \neq i} v_j(\theta_i^*(\theta_{-i}), \theta_{-i})$$

if the equation has a solution, and arbitrarily if not. Let $t^{\mathrm{G}}$ denote the *generalized Groves mechanism* defined by

$$t_i^{\mathrm{G}}(\theta) = \begin{cases} 0 & \text{if } i = m(\theta), \\ v_i(\theta_i^*(\theta_{-i}), \theta_{-i}) & \text{if } i \neq m(\theta). \end{cases} \tag{3.2}$$

Observe that $(s^*, t^{\mathrm{G}})$ is IC, and in fact, ex post IC (truth-telling is an ex post equilibrium). Due to the Revenue Equivalence, $(s^*, t)$ is IC if and only if $t$ yields, up to constant, the same interim expected transfer as the generalized Groves mechanism $t^{\mathrm{G}}$. Therefore, $(s^*, t)$ is IC if and only if there exist functions $k_i$, $i \in N$, such that

$$t_i(\theta) = t_i^{\mathrm{G}}(\theta) - k_i(\theta)$$

and

$$E_{\theta_{-i}}[k_i(\theta_i, \theta_{-i})] = E_{\theta_{-i}}[k_i(\theta_i', \theta_{-i})] \tag{3.3}$$

for all $\theta_i, \theta_i' \in \Theta_i$.

The other properties, BB and EPIR, are also rewritten in terms of the $k_i$ functions as above. Denote by $b^{\mathrm{G}}(\theta)$ the *budget deficit* generated by the generalized Groves mechanism $t^{\mathrm{G}}$:

$$b^{\mathrm{G}}(\theta) = \sum_{i \in N} t_i^{\mathrm{G}}(\theta).$$

Then, $t$ satisfies BB if and only if

$$\sum_{i \in N} k_i(\theta) = b^{\mathrm{G}}(\theta) \tag{3.4}$$

for all $\theta \in \Theta$. Let $u_i^{\mathrm{G}}(\theta)$ denote the ex post utility of agent $i$ under $(s^*, t^{\mathrm{G}})$:

$$u_i^{\mathrm{G}}(\theta) = v_i(\theta) s_i^*(\theta) + t_i^{\mathrm{G}}(\theta).$$

Then, $t$ satisfies EPIR if and only if for all $i \in N$, $u_i^{\mathrm{G}}(\theta) - k_i(\theta) \geq u_i^0(\theta)$ for all $\theta \in \Theta$, or equivalently,

$$\inf_{\theta \in \Theta} \{u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - k_i(\theta)\} \geq 0. \tag{3.5}$$

In summary, the partnership is EPIR-dissolvable if and only if there exist functions $k_1, \dots, k_n$ that satisfy the conditions (3.3), (3.4), and (3.5).

We focus on a specific form of $k_i$ functions. Specifically, our approach is to set

$$k_i(\theta) = b_i(\theta) - E_{\theta_{-i}}[b_i(\theta)] + \frac{1}{n-1} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)] + C_i$$

for some functions $b_i$ that satisfy

$$\sum_{i \in N} b_i(\theta) = b^{\mathrm{G}}(\theta)$$

and constants $C_i$ with $\sum_{i \in N} C_i = 0$. It is immediate to verify that these $k_i$ functions satisfy the IC condition (3.3) and the BB condition (3.4). The resulting transfer rule $t = t^{\mathrm{G}} - k$ is then written as

$$t_i(\theta) = t_i^{\mathrm{G}}(\theta) - b_i(\theta) + E_{\theta_{-i}}[b_i(\theta)] - \frac{1}{n-1} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)] - C_i. \tag{3.6}$$

This can be given the following interpretation. The starting point is the Groves transfer rule $t_i^{\mathrm{G}}$, which generates a budget deficit $b^{\mathrm{G}}$. Functions $b_i$ are considered as defining a *burden sharing rule* of the budget deficit $b^{\mathrm{G}}$, where $b_i(\theta)$ is the burden borne by agent $i$. The term $E_{\theta_{-i}}[b_i(\theta)]$ is added to give the agent the right incentives to report the truth, while the other two terms, which are independent of $\theta_i$, are to keep the budget balance unaffected.

It remains to determine a condition under which the EPIR condition (3.5) is satisfied. The following result offers a sufficient condition for EPIR-dissolution in terms of burden sharing functions $b_i$.

**Theorem 1.** *If there exist functions $b_1, \dots, b_n$ such that $\sum_{i \in N} b_i(\theta) = \sum_{i \in N} t_i^{\mathrm{G}}(\theta)$ for all $\theta$ and*

$$\sum_{i \in N} \inf_{\theta \in \Theta} \left\{ u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - b_i(\theta) + E_{\theta_{-i}}[b_i(\theta)] - \frac{1}{(n-1)} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)] \right\} \geq 0, \tag{3.7}$$

*then the partnership is EPIR-dissolvable.*

*Proof.* Suppose that the condition (3.7) is satisfied with functions $b_i$ where $\sum_{i \in N} b_i(\theta) = \sum_{i \in N} t_i^{\mathrm{G}}(\theta)$, and let the transfer rule $t$ be as in (3.6). By construction, $(s^*, t)$ satisfies EF and IC. It satisfies BB if and only if $\sum_{i \in N} C_i = 0$.

Now, for each $i \in N$, define

$$C_i^* = \inf_{\theta \in \Theta} \left\{ u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - b_i(\theta) + E_{\theta_{-i}}[b_i(\theta)] - \frac{1}{(n-1)} \sum_{j \neq i} E_{\theta_{-j}}[b_j(\theta)] \right\}.$$

EPIR is thus satisfied if and only if $C_i^* \leq C_i$. Therefore, BB and EPIR are simultaneously satisfied if and only if $\sum_{i \in N} C_i^* \geq 0$, which completes the proof. ▌

*Remark* 3.1. Under our assumption that the types are independently distributed, the sufficiency result remains valid for general quasi-linear utilities (with certain regularity conditions). The revenue equivalence result holds in such an environment and a generalized Groves mechanism is available (see Bergemann and Välimäki (2002)).

*Remark* 3.2. This class of transfer rules defined by (3.6) contains the *expected externality* (or AGV) *mechanism* $t^{\mathrm{E}}$ as a special case. To see this, set $b_i(\theta) = t_i^{\mathrm{G}}(\theta)$ (and $C_i = 0$), and then we have

$$t_i^{\mathrm{E}}(\theta) = E_{\theta_{-i}}[t_i^{\mathrm{G}}(\theta)] - \frac{1}{n-1} \sum_{j \neq i} E_{\theta_{-j}}[t_j^{\mathrm{G}}(\theta)]. \tag{3.8}$$

Another natural transfer rule is induced by the *equal burden sharing*:

$$b_i(\theta) = \frac{1}{n} b^{\mathrm{G}}(\theta). \tag{3.9}$$

This rule will be employed in the analysis in Section 4.

From now on, we restrict our attention to the symmetric and separable environment as in FKM: we assume that for all $i \in N$, $[\underline{\theta}_i, \overline{\theta}_i] = [\underline{\theta}, \overline{\theta}]$, $F_i = F$, and

$$v_i(\theta) = g(\theta_i) + \sum_{j \neq i} h(\theta_j),$$

where $g$ and $h$ are continuously differentiable and satisfy $g' > 0$ and $g' > h'$. Under this assumption, $v_i(\theta) \geq v_j(\theta)$ if and only if $\theta_i \geq \theta_j$, so that $m(\theta) = \max(\arg\max_j v_j(\theta)) = \max(\arg\max_j \theta_j)$. Then, the generalized Groves mechanism becomes

$$t_i^{\mathrm{G}}(\theta) = \begin{cases} 0 & \text{if } i = m(\theta), \\ g\big(\theta_{m(\theta)}\big) + \sum_{j \neq i} h(\theta_j) & \text{if } i \neq m(\theta), \end{cases} \tag{3.10}$$

and thus its budget deficit is

$$\begin{aligned} b^{\mathrm{G}}(\theta) &= (n-1) g\big(\theta_{m(\theta)}\big) + \sum_{i \neq m(\theta)} \sum_{j \neq i} h(\theta_j) \\ &= (n-1) g\big(\theta_{m(\theta)}\big) + h\big(\theta_{m(\theta)}\big) + (n-2) \sum_{j \in N} h(\theta_j). \end{aligned} \tag{3.11}$$

In this environment, FKM obtain the following results for partnership dissolution with IIR.

**Fact 0** (FKM)**.** (1) *If $h' > 0$, then the equal-share partnership is not IIR-dissolvable for some distribution function $F$.*

(2) *If $h' \leq 0$, then the equal-share partnership is IIR-dissolvable for any distribution function $F$.*

A trivial corollary to Fact 0 is that if $h' > 0$, then the equal-share partnership is not EPIR-dissolvable for some distribution function $F$. In the following sections, we consider whether the equal-share partnership is always (i.e., for all distribution functions) EPIR-dissolvable when $h' \leq 0$.

## 4 Symmetric Two-Agent Partnerships

In this section, we consider EPIR-dissolution of two-agent equal-share partnerships: $n = 2$ and $\alpha_1 = \alpha_2 = 1/2$ so that

$$u_i^0(\theta) = \frac{1}{2}v_i(\theta).$$

For $i = 1, 2$ we write $-i$ for the agent $j \neq i$, and denote $\theta^1 = \theta_{m(\theta)}$ and $\theta^2 = \theta_{-m(\theta)}$. In this case, the generalized Groves mechanism and its budget deficit are given by

$$t_i^{\mathrm{G}}(\theta) = \begin{cases} 0 & \text{if } i = m(\theta), \\ v(\theta^1) & \text{if } i \neq m(\theta) \end{cases} \tag{4.1}$$

and $b^{\mathrm{G}}(\theta) = v(\theta^1)$, respectively, where we denote

$$v(x) = g(x) + h(x).$$

Our main question here is whether the two-agent equal-share partnership is EPIR-dissolvable for any type distribution $F$. Given Fact 0, we restrict our attention to the case of non-positive interdependence, i.e., $h' \leq 0$. For this case, we show that the answer to our question is the affirmative.

**Theorem 2** (Two-Agent Case)**.** *Assume $h' \leq 0$. Then, the equal-share partnership is EPIR-dissolvable for any distribution function $F$.*

*Proof.* See Appendix A.1. ∎

The proof consists in finding burden sharing functions $b_i$ as in Theorem 1. We show in Appendix A.1 that the equal burden sharing

$$b_i(\theta) = \frac{1}{2}b^{\mathrm{G}}(\theta)$$

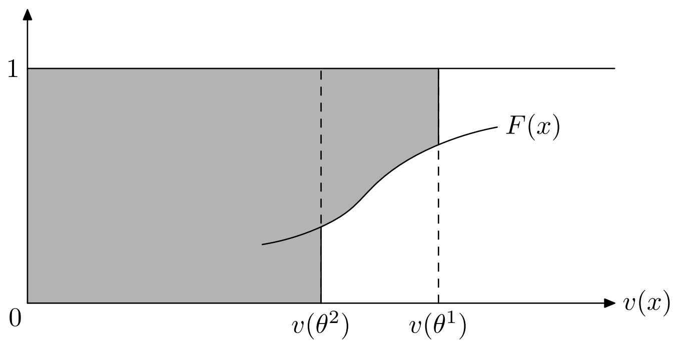


Figure 1: Pricing rule $p^*(\theta^1, \theta^2)$

indeed satisfies the condition (3.7) with equality. The implementing mechanism (obtained by (3.6)) is then given by

$$t_i^*(\theta) = \begin{cases} -\dfrac{1}{2}\left[v(\theta^1) - \displaystyle\int_{\theta^2}^{\theta^1} F(x)\,dv(x)\right] & \text{if } i = m(\theta), \\ \dfrac{1}{2}\left[v(\theta^1) - \displaystyle\int_{\theta^2}^{\theta^1} F(x)\,dv(x)\right] & \text{if } i \neq m(\theta). \end{cases} \tag{4.2}$$

(Recall that it satisfies IC as well as BB by construction.)

Figure 1 illustrates the EPIR graphically (assuming $v > 0$ and $v' > 0$). The shaded area in the figure depicts the bracketed term in (4.2), which may be interpreted as the *price* of the asset (per unit) which we denote $p^*(\theta^1, \theta^2)$ as a function of $\theta^1$ and $\theta^2$:

$$p^*(\theta^1, \theta^2) = v(\theta^1) - \int_{\theta^2}^{\theta^1} F(x)\,dv(x), \tag{4.3}$$

where the winner pays $(1/2)p^*(\theta^1, \theta^2)$ to the loser for the 1/2 units of the asset the loser owns. The figure immediately shows that $v(\theta^2) \leq p^*(\theta^1, \theta^2) \leq v(\theta^1)$. Since if $h' \leq 0$, then

$$\begin{aligned} v(\theta^2) &= g(\theta^2) + h(\theta^2) \geq g(\theta^2) + h(\theta^1) = v_{-m(\theta)}(\theta), \\ v(\theta^1) &= g(\theta^1) + h(\theta^1) \leq g(\theta^1) + h(\theta^2) = v_{m(\theta)}(\theta), \end{aligned}$$

it follows that $v_{-m(\theta)}(\theta) \leq p^*(\theta^1, \theta^2) \leq v_{m(\theta)}(\theta)$, which implies EPIR.

To explore its properties, let us compare our transfer rule $t^*$ with the expected externality mechanism $t^{\mathrm{E}}$ defined by (3.8), which is written in the present environment as

$$t_i^{\mathrm{E}}(\theta) = \int_{\theta_i}^{\theta_{-i}} v(x)\,dF(x). \tag{4.4}$$

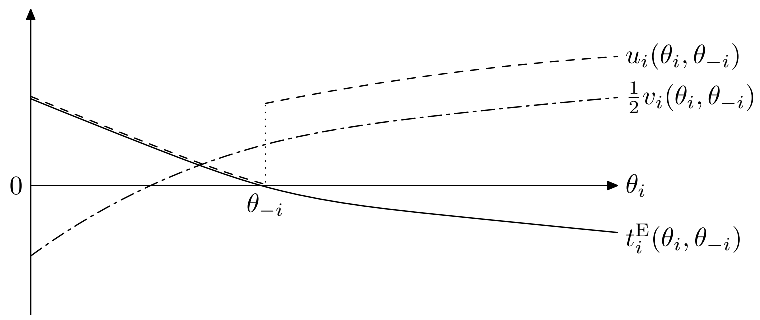


Figure 2: Mechanism $(s^*, t^{\mathrm{E}})$

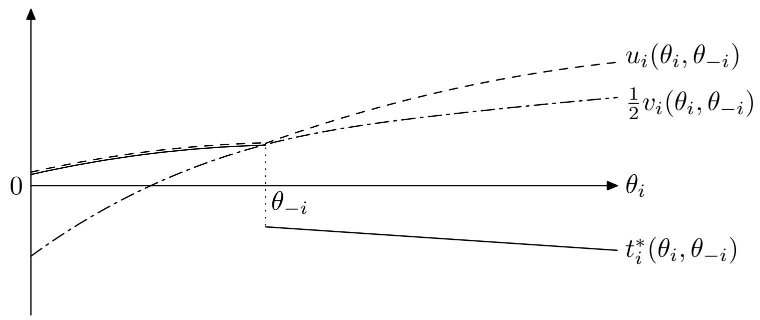


Figure 3: Mechanism $(s^*, t^*)$

By construction, $(s^*, t^{\mathrm{E}})$ is IC and BB, and FKM show that, for equal-share partnerships, it satisfies IIR for any $F$ whenever $h' \leq 0$. Observe that the function $t_i^{\mathrm{E}}$ is continuous in $\theta$ and assigns zero transfer when $\theta_1 = \theta_2$. Figures 2 depicts, for a fixed value of $\theta_{-i}$, typical behavior of the transfer $t_i^{\mathrm{E}}$ that agent $i$ receives as well as his ex post payoff $u_i$ and outside option $(1/2)v_i$ (under truth-telling) for the FKM mechanism $(s^*, t^{\mathrm{E}})$ as functions of $\theta_i$. When $\theta_i$ is smaller than but very close to $\theta_{-i}$, agent $i$ loses his share in receipt of a transfer almost equal to zero, and thus ends up regretting his participation in the mechanism. Therefore, the expected externality mechanism $t^{\mathrm{E}}$ does not satisfy EPIR in general. In contrast, our mechanism $(s^*, t^*)$ is constructed so that EPIR is satisfied also for such $\theta$'s, as shown in Figure 3, where the transfer function $t_i^*$ exhibits discontinuity around $\theta_i = \theta_{-i}$.

*Remark* 4.1. A recent paper by Athanassoglou *et al.* (2008) also studies EPIR in two-agent partnership dissolution with *private* valuations. While in the main part of the paper they consider agents whose objective is to minimize maximum regret, they also provide some results in the standard Bayesian framework. First, they show that the "binary search mechanism"

EPIR-dissolves the partnership if the ownership shares are equal and the type distribution is uniform, but may not otherwise. (The binary search mechanism proceeds as follows: Suppose $[\underline{\theta}, \overline{\theta}] = [0, 1]$, and let the agents' bids be $\theta_1 = 0.\theta_1^1\theta_1^2\theta_1^3\cdots$ and $\theta_2 = 0.\theta_2^1\theta_2^2\theta_2^3\cdots$ in binary notation, where $\theta_i^k = 0, 1$. Then, the agent with the higher bid receives the asset, and if $\theta_1^\ell = \theta_2^\ell = \theta^\ell$ for all $\ell < k$ and $\theta_1^k \neq \theta_2^k$, then the *price* $p(\theta_1, \theta_2)$ of the asset is set $\sum_{\ell=1}^{k-1} 2^\ell + 1/2^k$.) Second, they show that if the shares are unequal, then in any efficient, IC, and BB mechanism such that the pricing rule $p(\theta)$ $(= -t_{m(\theta)}/r_{-m(\theta)}$, where $r_i$ is agent $i$'s initial share and $m(\theta) = \arg\max_j \theta_j)$ satisfies the anonymity condition (i.e., $p(\theta)$ does not depend on the identity of the agents), $p(\theta)$ must be a step function (such as the binary search mechanism).

## 5 Symmetric $n$-Agent Partnerships

In this section, we show that our positive result in the previous section on the two-agent equal-share partnership extends to the $n$-agent equal-share case, where

$$u_i^0(\theta) = \frac{1}{n} v_i(\theta).$$

We note that this extension is nontrivial, since for $n \geq 3$, the equal burden sharing $b_i(\theta) = (1/n)b^{\mathrm{G}}(\theta)$ no longer satisfies our sufficient condition (3.7) in Theorem 1. (Recall that the Groves budget deficit $b^{\mathrm{G}}$ is now given by (3.11).) Nevertheless, we can show the following positive result.

**Theorem 3** ($n$-Agent Case)**.** *Assume $h' \leq 0$. Then, the equal-share partnership is EPIR-dissolvable for any distribution function $F$.*

*Proof.* See Appendix. ▮

To prove this result, we set the functions $b_i$ in Theorem 1 to be

$$b_i(\theta) = \frac{1}{n} b^{\mathrm{G}}(\theta) + \begin{cases} -\dfrac{n-1}{n} \displaystyle\int_{\theta^2}^{\theta^1} F(x)\, dg(x) & \text{if } i = m(\theta), \\ \dfrac{1}{n} \displaystyle\int_{\theta^2}^{\theta^1} F(x)\, dg(x) & \text{if } i \neq m(\theta), \end{cases} \tag{5.1}$$

where $\theta^1 = \theta_{m(\theta)}$ and $\theta^2 = \max_{j \neq m(\theta)} \theta_j$. We verify that the sufficient condition in Theorem 1 is satisfied with this choice of $b_i$ for any type distribution $F$, provided that $h' \leq 0$. The implementing mechanism obtained by (3.6) is then written as

$$t_i^*(\theta) = \begin{cases} -\dfrac{n-1}{n}\left[g(\theta^1) - \displaystyle\int_{\theta^2}^{\theta^1} F(x)\,dg(x)\right] \\ \dfrac{1}{n}\left[g(\theta^1) - \displaystyle\int_{\theta^2}^{\theta^1} F(x)\,dg(x)\right] \\ \quad - \dfrac{1}{n}h(\theta^1) + \dfrac{1}{n-1}\displaystyle\sum_{j\neq i} h(\theta_j) + \dfrac{1}{n(n-1)}\sum_{j\neq i}\int_{\theta_j}^{\theta_i} F^{n-1}(x)\,dh(x) \\ \quad - \begin{cases} \displaystyle\sum_{j\neq i} h(\theta_j) & \text{if } i = m(\theta) \\ 0 & \text{if } i \neq m(\theta). \end{cases} \end{cases} \tag{5.2}$$

When $n = 2$, the equal burden sharing and the one defined by (5.1) lead to the very same transfer rule; i.e., the transfer rule (5.2) reduces to (4.2).

When $n \geq 3$, the mechanism that the equal burden sharing induces through (3.6) differs from (5.2), and the former does not satisfy EPIR: it is satisfied for the winner (the type-$\theta^1$ agent) and the loser with the highest bid (the type-$\theta^2$ agent), but not for the other losers with lower bids. To see this, let us consider for simplicity the case of $h \equiv 0$, in which case $b^{\mathrm{G}}(\theta) = (n-1)g(\theta^1)$. If we set $b_j(\theta) = (1/n)b^{\mathrm{G}}(\theta)$, each expectation term $E_{\theta_{-j}}[b_j(\theta)]$ in (3.6) is increasing in $\theta_j$, since

$$E_{\theta_{-j}}\left[g(\theta^1)\right] = \int_{\underline{\theta}}^{\theta_j} F^{n-1}(x)\,dg(x) + \int_{\underline{\theta}}^{\overline{\theta}} g(x)\,dF^{n-1}(x).$$

Thus, the term $\{1/(n-1)\}\sum_{j\neq i} E_{\theta_{-j}}[b_j(\theta)]$ in (3.6), subtracted to conform to BB, is larger for an agent with a lower type $\theta_i$, and can in fact become so large that EPIR is violated when $\theta_i$ is close to the lowest bid but $F^{n-1}(\theta_i)$ is close to one. The adjusting term in (5.1) is added to $(1/n)b^{\mathrm{G}}(\theta)$ to rule out this possibility, thereby making the resulting mechanism fulfill EPIR.

*Remark* 5.1. In the *private* valuation case where $g(\theta_i) = \theta_i$ and $h \equiv 0$, our mechanism coincides with the mechanism proposed by Fujinaka (2006), who considers Bayesian implementation of envy-free allocation of a single indivisible goods with private values. An allocation rule in that context is a mechanism (as described in Section 2) that is deterministic (i.e., for all $\theta \in \Theta$, $s_i(\theta) = 1$ for some $i \in N$). IC, EF, and BB are defined as previously. An allocation rule $(s, t)$ is *envy-free* if every agent prefers his own allocation to that of any other agent for any realization of types, i.e., for all $i, j \in N$,

$$u_i(\theta) \geq s_j(\theta)v_i(\theta) + t_j(\theta) \tag{5.3}$$

for all $\theta \in [\underline{\theta}, \overline{\theta}]^n$. Notice that, for deterministic mechanisms, envy-freeness implies EF. Fujinaka (2006) shows that his proposed mechanism satisfies envy-freeness (*a fortiori* EF), IC, and BB. We note that envy-freeness, together with BB, implies EPIR for the equal-share partnership. To see this, take the summation of (5.3) with respect to $j$. Since $\sum_{j=1}^{n} s_j(\theta) = 1$ and $\sum_{j=1}^{n} t_j(\theta) = 0$, we have $nu_i(\theta) \geq v_i(\theta)$, or $u_i(\theta) \geq (1/n)v_i(\theta)$.

# 6 Discussion

## 6.1 Asymmetric Distributions and Ownership Shares

In this subsection, we allow for asymmetries in type distributions and ownership shares; for simplicity we focus on the two-agent case. Agent $i$'s type distribution is now denoted by $F_i$ with support $[\underline{\theta}_i, \overline{\theta}_i]$, and $i$'s ownership share by $\alpha_i \geq 0$, where $\alpha_1 + \alpha_2 = 1$. Assume that the interval

$$I = [\underline{\theta}_1, \overline{\theta}_1] \cap [\underline{\theta}_2, \overline{\theta}_2]$$

has a nonempty interior, or otherwise ex post implementation is obviously possible. We keep the valuation function symmetric so that $v_i(\theta_i, \theta_{-i}) = g(\theta_i) + h(\theta_{-i})$, and assume that $h' \leq 0$ and $v' = g' + h' > 0$.

We apply our Theorem 1 by setting

$$b_i(\theta) = \lambda_i b^{\mathrm{G}}(\theta) \ (= \lambda_i v(\theta^1)), \tag{6.1}$$

where $\lambda_i \geq 0$ ($\lambda_1 + \lambda_2 = 1$) is a *constant* which does not depend on $\theta$. Let

$$\begin{aligned} C_i(\theta_i, \theta_{-i}) &= u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - \lambda_i b_i^{\mathrm{G}}(\theta) + \lambda_i B_i^{\mathrm{G}}(\theta_i) - \lambda_{-i} B_{-i}^{\mathrm{G}}(\theta_{-i}) \\ &= \big(g(\theta^1) + h(\theta_{-i})\big) - \alpha_i\big(g(\theta_i) + h(\theta_{-i})\big) - \lambda_i\big(g(\theta^1) + h(\theta^1)\big) \\ &\quad + \big(\lambda_i B_i^{\mathrm{G}}(\theta_i) - \lambda_{-i} B_{-i}^{\mathrm{G}}(\theta_{-i})\big) \\ &= \big\{(1-\lambda_i)g(\theta^1) - \alpha_i g(\theta_i)\big\} + \big\{-\lambda_i h(\theta^1) + (1-\alpha_i)h(\theta_{-i})\big\} \\ &\quad + \big(\lambda_i B_i^{\mathrm{G}}(\theta_i) - \lambda_{-i} B_{-i}^{\mathrm{G}}(\theta_{-i})\big), \end{aligned} \tag{6.2}$$

where

$$\begin{aligned} B_i^{\mathrm{G}}(x) &= E_{\theta_{-i}}[b^{\mathrm{G}}(\theta) | \theta_i = x] \\ &= \int_{\underline{\theta}_{-i}}^{x} F_{-i}(z)\, dv(z) + \int_{\underline{\theta}_{-i}}^{\overline{\theta}_{-i}} v(z)\, dF_{-i}(z) \end{aligned}$$

defined for all $x \in \mathbb{R}$.

We would like to find a condition under which $\min_{\theta \in \Theta} C_1(\theta) + \min_{\theta' \in \Theta} C_2(\theta') \geq 0$ holds.[4] A necessary condition for this is

$$C_1(x, x) + C_2(y, y) \geq 0 \tag{6.3}$$

[4] In fact, $C_i$ is a continuous function and thus attains its minimum on the compact domain $\Theta = [\underline{\theta}_1, \overline{\theta}_1] \times [\underline{\theta}_2, \overline{\theta}_2]$.

for all $x, y \in I$, where

$$
\begin{aligned}
&C_1(x,x) + C_2(y,y) \\
&\quad = (1-\lambda_1-\alpha_1)\big(g(x)+h(x)\big) + \big(\lambda_1 B_1^{\mathrm{G}}(x) - \lambda_2 B_2^{\mathrm{G}}(\theta_2)\big) \\
&\qquad + (1-\lambda_2-\alpha_2)\big(g(y)+h(y)\big) + \big(\lambda_2 B_2^{\mathrm{G}}(\theta_2) - \lambda_1 B_1^{\mathrm{G}}(\theta_1)\big) \\
&\quad = (1-\lambda_1-\alpha_1)\big(v(x)-V(x)\big) + \lambda_1 \int_x^y F_1(z)\,dv(z) - \lambda_2 \int_x^y F_2(z)\,dv(z) \\
&\quad = \int_x^y \big[\big(\lambda_1 F_1(z) - \lambda_2 F_2(z)\big) - (1-\lambda_1-\alpha_1)\big]\,dv(z).
\end{aligned}
$$

If inequality (6.3) holds for all $x, y \in I$, then the integrand in the last line above must be identically zero on $I$. Thus, for $\inf_{\theta\in\Theta} C_1(\theta) + \inf_{\theta'\in\Theta} C_2(\theta') \geq 0$, it is necessary that $\lambda_1 F_1 - \lambda_2 F_2$ be constant on $I$ and equal to $1-\lambda_1-\alpha_1$. In this case, we have $C_1(x,x)+C_2(y,y) = 0$ for all $x, y \in I$.

We can show that this is indeed sufficient as well.

**Proposition 4.** *Suppose $h' \leq 0$ and $g' + h' > 0$. Then,* $\min_{\theta\in\Theta} C_1(\theta) + \min_{\theta'\in\Theta} C_2(\theta') \geq 0$ *if and only if* $\lambda_1 F_1 - \lambda_2 F_2$ *is identically equal to* $1-\lambda_1-\alpha_1$ *on $I$.*

*Proof.* It suffices to show the "if" part. For this, it is sufficient to show that $C_i$ attains its minimum on the set $\{(\theta_1,\theta_2) \mid \theta_1 = \theta_2 \in I\}$. Then, as already observed, if $\lambda_1 F_1 - \lambda_2 F_2$ is identically equal to $1-\lambda_1-\alpha_1$ on $I$, then $C_1(x,x)+C_2(y,y) = 0$ for all $x, y \in I$, implying that $\min_{\theta\in\Theta} C_1(\theta) + \min_{\theta'\in\Theta} C_2(\theta') = 0$.

The partial derivatives of $C_i$ are computed as follows:

$$
\frac{\partial C_i}{\partial \theta_i}(\theta_i,\theta_{-i}) = \begin{cases} (1-\alpha_i)g'(\theta_i) - \lambda_i\big(1-F_{-i}(\theta_i)\big)v'(\theta_i) & \text{if } \theta_i > \theta_{-i}, \\ -\alpha_i g'(\theta_i) + \lambda_i F_{-i}(\theta_i) v'(\theta_i) & \text{if } \theta_i < \theta_{-i}, \end{cases}
$$

$$
\frac{\partial C_i}{\partial \theta_{-i}}(\theta_i,\theta_{-i}) = \begin{cases} (1-\alpha_i)h'(\theta_{-i}) - \lambda_{-i}F_i(\theta_{-i})v'(\theta_{-i}) & \text{if } \theta_i > \theta_{-i}, \\ -\alpha_i h'(\theta_{-i}) + \big(1-\lambda_i-\lambda_{-i}F_i(\theta_{-i})\big)v'(\theta_{-i}) & \text{if } \theta_i < \theta_{-i}. \end{cases}
$$

By the assumption that $h' \leq 0$ and $v' = g'+h' > 0$, we have $\partial C_i/\partial\theta_{-i} \leq 0$ when $\theta_i > \theta_{-i}$, and $\partial C_i/\partial\theta_{-i} \geq 0$ when $\theta_i < \theta_{-i}$. Moreover, if $\theta_i > \overline{\theta}_{-i}$, then

$$
\frac{\partial C_i}{\partial \theta_i}(\theta_i, \overline{\theta}_{-i}) \geq 0,
$$

while if $\theta_i < \underline{\theta}_{-i}$, then

$$
\frac{\partial C_i}{\partial \theta_i}(\theta_i, \underline{\theta}_{-i}) \leq 0.
$$

These inequalities show that the minimum of $C_i$ is attained on $\{(\theta_1,\theta_2) \mid \theta_1 = \theta_2 \in I\}$. ▮

Note that the condition in the above result is satisfied only in very restrictive cases. To illustrate this, let us consider the following example.

**Example 6.1.** Let $[\underline{\theta}_1, \overline{\theta}_1] = [0, 1]$ and $[\underline{\theta}_2, \overline{\theta}_2] = [a, 1+a]$, where $0 \leq a < 1$. Consider the uniform distributions $F_1(x) = x$ on $[0, 1]$ and $F_2(x) = x - a$ on $[a, 1+a]$. Then, the condition in Proposition 4 holds for some $\lambda_1, \lambda_2$ if and only if the ownership shares $(\alpha_1, \alpha_2)$ satisfy $\alpha_1 = (1-a)/2$, where $\lambda_1 = \lambda_2 = 1/2$.

Recall, however, that the condition $\min_{\theta \in \Theta} C_1(\theta) + \min_{\theta' \in \Theta} C_2(\theta') \geq 0$ for the functions $C_i$ defined by (6.2) is only a sufficient condition for EPIR-dissolution of the partnership. What Proposition 4 shows is that the mechanism that assigns the Groves budget deficit to each agent with a constant weight independent of $\theta$ as in (6.1) achieves EPIR-dissolution only in restrictive cases, and hence, in order to accommodate asymmetries one has to assign weights that vary depending on $\theta$, possibly through the distribution functions $F_i$. Resulting mechanisms will necessarily be complicated, and exploring more general conditions for EPIR-dissolution of asymmetric partnerships is left for future research.

### 6.2 Positive Interdependence

In this subsection, we consider the possibility of EPIR-dissolution when the interdependence is positive, i.e., $h' > 0$. With positive interdependence, we know from FKM that even for the equal-share, EPIR-dissolution for *all* type distributions is not possible. We here address the question whether it is possible for *some* distributions. We show for $n = 2$ that the answer is positive under an additional assumption, that $h'$ vanishes for the lowest and the highest type.

**Assumption 6.1.** $h'(\underline{\theta}) = h'(\overline{\theta}) = 0$ and $|h''(\underline{\theta})|, |h''(\overline{\theta})| < \infty$.

**Proposition 5.** *Under Assumption 6.1, the equal-share partnership is EPIR-dissolvable for some distribution function $F$.*

The proof proceeds as follows. Consider again the equal burden sharing $b_i(\theta) = (1/2)b^{\mathrm{G}}(\theta)$, and examine the condition (3.7) in Theorem 1. Then as in the proof of Theorem 2 (see (A.1) and (A.2) in Appendix A.1), the condition (3.7) holds if and only if for all $x \in [\underline{\theta}, \overline{\theta}]$,

$$F(x)g'(x) - (1 - F(x))h'(x) \geq 0,$$
$$(1 - F(x))g'(x) - F(x)h'(x) \geq 0,$$

or

$$\frac{h'(x)}{g'(x) + h'(x)} \leq F(x) \leq \frac{g'(x)}{g'(x) + h'(x)}. \tag{6.4}$$

Since

$$\frac{h'(x)}{g'(x) + h'(x)} < \frac{1}{2} < \frac{g'(x)}{g'(x) + h'(x)}$$

by the assumption that $h' < g'$, one can show that if Assumption 6.1 holds, then there exists a distribution function $F$, which must be increasing satisfying $F(\underline{\theta}) = 0$ and $F(\overline{\theta}) = 1$, such that (6.4) holds true.

Note that in order for EF to hold, it is necessary that the asset should (not, resp.) be given to the agent with the highest type $\overline{\theta}$ (the lowest type $\underline{\theta}$. The assumption $h'(\underline{\theta}) = h'(\overline{\theta}) = 0$ in effect prevents the highest and the lowest types from suffering the winner's and the loser's curses.

**Example 6.2.** Let $[\underline{\theta}, \overline{\theta}] = [0, 1]$, $g(x) = x$, and $h(x) = x^2/2 - x^3/3$. If $F(x) = x$, then (6.4) holds, so that the equal-share partnership is EPIR-dissolvable.

It has be noted that Assumption 6.1 is only a sufficient condition for EPIR-dissolution for some $F$. For a function $h$ that does not satisfy Assumption 6.1, there exists no distribution function $F$ that satisfies the inequality (6.4), but it does not mean that EPIR-dissolution is impossible: it only says that the equal burden sharing does not lead to EPIR-dissolution in that case. Here again, in order to allow for the general case, one has to consider non-constant burden weights that vary according to $\theta$.

### 6.3 Ex Post Quitting Right

While we motivated our study of EPIR mechanisms by our desire that a mechanism be free from ex post regret of participation, one may imagine a situation in which agents actually reserve the right to quit the mechanism after observing the outcome. Compte and Jehiel (2006, 2007) consider a bargaining model with (ex post) *quitting rights* or *veto*, in which agents may enjoy their outside option *on and off* the equilibrium paths. Note, in contrast, that EPIR is imposed only on the equilibrium path (i.e., at the truth-telling outcome). See also Matthews and Postlewaite (1989) and Forges (1999) for similar considerations. In this section, we examine the performance of the mechanism $(s^*, t^*)$ when we allow for quitting rights.

Introducing ex post quitting rights implies requiring EPIR. It also modifies the IC constraints, as each agent may assert the quitting right after he makes a false report, thus affecting the incentives to deviate. To formulate the modified IC constraints, let

$$u_i(\theta_i, \hat{\theta}_i, \theta_{-i}) = v_i(\theta_i, \theta_{-i}) s_i(\hat{\theta}_i, \theta_{-i}) + t_i(\hat{\theta}_i, \theta_{-i})$$

and

$$U_i^*(\theta_i, \hat{\theta}_i) = E_{\theta_{-i}} \left[ \max \left\{ u_i(\theta_i, \hat{\theta}_i, \theta_{-i}), \frac{1}{2} v_i(\theta_i, \theta_{-i}) \right\} \right],$$

and denote $U_i^*(\theta_i) = U_i^*(\theta_i, \theta_i)$. The max operator in $U_i^*(\theta_i, \hat{\theta}_i)$, the expected utility of agent $i$ with type $\theta_i$ when he reports $\hat{\theta}_i$, represents the assumption that the agent can take the outside option $(1/2)v_i(\theta_i, \theta_{-i})$ whenever it is larger than his ex post utility $u_i(\theta_i, \hat{\theta}_i, \theta_{-i})$. A mechanism $(s, t)$ satisfies *interim incentive compatibility with ex post quitting rights*, or *interim incentive compatibility starred* (IC*), if for all $i \in N$,

$$U_i^*(\theta_i) \geq U_i^*(\theta_i, \hat{\theta}_i) \tag{IC*}$$

for all $\hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$. We say that the partnership is *dissolvable with quitting rights* if there exists a mechanism that satisfies EF, BB, IC*, and EPIR.

Clearly, IC* is a stronger condition than IC. We demonstrate that efficient dissolution is always possible even with IC* when the degree of negative dependence is large enough that $g' + h' \leq 0$.

**Theorem 6.** *The equal-share partnership is dissolvable with quitting rights for any distribution function $F$ if $g' + h' \leq 0$.*

For this result, it is sufficient to show the following. The proof is given in Appendix A.2.

**Proposition 7.** *The mechanism $(s^*, t^*)$ defined by* (2.1) *and* (4.2) *satisfies IC* for any distribution $F$ if and only if $g' + h' \leq 0$.*

To gain the intuition behind the result, consider the borderline case where $g' + h' \equiv 0$. In this case, we may assume without loss of generality that $g(x) + h(x) = 0$ for all $x \in [\underline{\theta}, \overline{\theta}]$, so that $t_i^*(\theta_i, \theta_{-i}) = 0$ for all $\theta_i, \theta_{-i} \in [\underline{\theta}, \overline{\theta}]$, i.e., the mechanism is such that the agent with a higher report receives the entire asset with no monetary transfer. Now consider agent $i$ with type $\theta_i$, and suppose that his report $\hat{\theta}_i$ overstates his type $\theta_i$ (i.e., $\hat{\theta}_i > \theta_i$). Define

$$\Delta(\theta_{-i}) = \frac{1}{2} v(\theta_i, \theta_{-i}) - u_i(\theta_i, \hat{\theta}_i, \theta_{-i}),$$

which is the "ex post regret" of agent $i$ when agent $-i$ truthfully reports $\theta_{-i}$ (here we assume $\theta_{-i} \neq \theta_i, \hat{\theta}_i$). Agent $i$ has a (strict) incentive to exercise the quitting right ex post if and only if $\Delta(\theta_{-i}) > 0$. In the current case, $\Delta(\theta_{-i}) > 0$ if and only if $\theta_{-i} \in (\theta_i, \hat{\theta}_i)$ as in Figure 4, which depicts the graphs of the ex post utility $u_i(\theta_i, \hat{\theta}_i, \theta_{-i})$ and the outside option $(1/2)v_i(\theta_i, \theta_{-i})$ as functions of $\theta_{-i}$. If $\theta_i < \theta_{-i} < \hat{\theta}_i$, agent $i$ receives the asset and obtains ex post utility $u_i(\theta_i, \hat{\theta}_i, \theta_{-i}) = v_i(\theta_i, \theta_{-i}) = g(\theta_i) + h(\theta_{-i}) < g(\theta_i) + h(\theta_i) = 0$, which is smaller than his outside option $(1/2)v_i(\theta_i, \theta_{-i})$. But the outside option $(1/2)v_i(\theta_i, \theta_{-i})$ $(< 0)$ that agent $i$ will take when $\theta_{-i} \in (\theta_i, \hat{\theta}_i)$ is smaller than the ex post utility $u_i(\theta_i, \hat{\theta}_i, \theta_{-i})$ $(= 0)$ that he would have obtained if he reported his type truthfully. If $\theta_{-i} < \theta_i$ or $\theta_{-i} > \hat{\theta}_i$, the outcome is no different from the one under truth telling. After all, the agent has no incentive to overstate his type even with ex post quitting rights (a symmetric argument applies to understatements). In fact, we have

$$U_i^*(\theta_i, \hat{\theta}_i) - U_i^*(\theta_i) = \frac{1}{2}\big(U_i(\theta_i, \hat{\theta}_i) - U_i(\theta_i)\big),$$

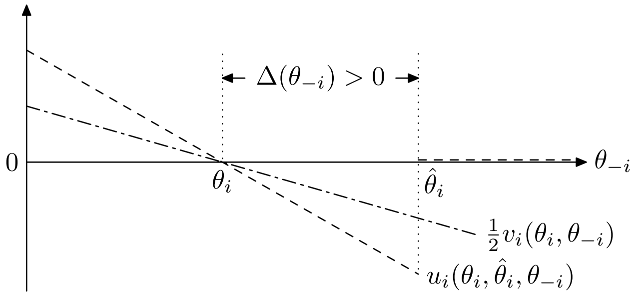


Figure 4: Case of $g' + h' \equiv 0$

which is negative by IC. When $g'+h' < 0$, the set of $\theta_{-i}$'s such that $\Delta(\theta_{-i}) > 0$ becomes smaller, which in effect makes false reports less preferable than in the case of $g' + h' \equiv 0$. In this case, it holds that $U_i^*(\theta_i, \hat{\theta}_i) - U_i^*(\theta_i) < (1/2)(U_i(\theta_i, \hat{\theta}_i) - U_i(\theta_i))$.

When $g'+h' > 0$, on the other hand, the set of $\theta_{-i}$'s such that $\Delta(\theta_{-i}) > 0$ exceeds the interval $(\theta_i, \hat{\theta}_i)$. Indeed, consider $\theta_{-i}$ slightly smaller than $\theta$. Then, agent $i$ has to make a considerable amount of monetary transfer, according to $t_i^*(\hat{\theta}_i, \theta_i)$, compared to his valuation $v_i(\theta_i, \theta_{-i})$, in which case he exercises the quitting right, thereby enjoying a discrete marginal gain. If such $\theta_{-i}$'s are assigned sufficiently larger probability densities than those in $(\theta_i, \hat{\theta}_i)$, the marginal gain that results from quitting can give a significant impact on $U_i^*(\theta_i, \hat{\theta}_i)$, violating IC*.

**Example 6.3.** Suppose that $g(x) = x$ and $h(x) = -\gamma x$, so that the valuation function $v_i$ is given by

$$v_i(\theta) = \theta_i - \gamma\theta_{-i},$$

where $\gamma \geq 0$, and thus $v(x) = v_i(x, x) = (1 - \gamma)x$. By Proposition 7 in Appendix A.2, the necessary and sufficient condition for our mechanism $(s^*, t^*)$ satisfies IC* for *any* type distribution is that $1 - \gamma \leq 0$, or $\gamma \geq 1$.

Now, we fix a type distribution $F$, and examine the condition for $\gamma$ under which IC* is satisfied for this given distribution $F$. Specifically, let $[\underline{\theta}, \overline{\theta}] = [0, 1]$, and $F$ be the uniform distribution on $[0, 1]$: $F(x) = x$. Then the transfer function $t^*$ is written as

$$t_i^*(\theta) = \begin{cases} -\dfrac{1}{2}(1-\gamma)\left[\theta_i - \dfrac{1}{2}\left\{(\theta_i)^2 - (\theta_{-i})^2\right\}\right] & \text{if } \theta_i > \theta_{-i}, \\ \dfrac{1}{2}(1-\gamma)\left[\theta_{-i} - \dfrac{1}{2}\left\{(\theta_{-i})^2 - (\theta_i)^2\right\}\right] & \text{if } \theta_i < \theta_{-i}. \end{cases} \tag{6.5}$$

In this case, we can show that the mechanism $(s^*, t^*)$ satisfies IC* if and only if $1 - 2\gamma \leq 0$, or $\gamma \geq 1/2$. The proof is given in Appendix A.3.

# Appendix

## A.1 Proof of Theorem 2

Recall that in the symmetric two-agent environment in consideration,

$$u_i^{\mathrm{G}}(\theta) = v_i(\theta) + t_i^{\mathrm{G}}(\theta) = \begin{cases} v_i(\theta) = g(\theta_i) + h(\theta_{-i}) & \text{if } \theta_i > \theta_{-i}, \\ v(\theta^1) = g(\theta_{-i}) + h(\theta_{-i}) & \text{if } \theta_i < \theta_{-i}, \end{cases}$$

where $v(x) = g(x) + h(x)$.

*Proof of Theorem 2.* We show that $b_i(\theta) = (1/2)b^{\mathrm{G}}(\theta) = (1/2)v(\theta^1)$ satisfies the condition (3.7) in Theorem 1. For each $i \in N$, let

$$C_i(\theta) = u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - \frac{1}{2}v(\theta^1) + E_{\theta_{-i}}\left[\frac{1}{2}v(\theta^1)\right] - E_{\theta_i}\left[\frac{1}{2}v(\theta^1)\right].$$

It suffices to show that $C_i(\theta) \geq 0$ for all $\theta \in \Theta$. Verify that

$$u_i^{\mathrm{G}}(\theta) - u_i^0(\theta) - \frac{1}{2}v(\theta^1) = \begin{cases} \frac{1}{2}\big(h(\theta_{-i}) - h(\theta_i)\big) & \text{if } \theta_i > \theta_{-i}, \\ \frac{1}{2}\big(g(\theta_{-i}) - g(\theta_i)\big) & \text{if } \theta_i < \theta_{-i}, \end{cases}$$

and

$$\begin{aligned} E_{\theta_{-i}}\left[v(\theta^1)\right] &= v(\theta_i)F(\theta_i) + \int_{\theta_i}^{\overline{\theta}} v(x)\, dF(x) \\ &= \int_{\underline{\theta}}^{\theta_i} F(x)\, dv(x) + \int_{\underline{\theta}}^{\overline{\theta}} v(x)\, dF(x). \end{aligned}$$

When $\theta_1 = \theta_2$, it is clear that $C_i(\theta) = 0$.

If $\theta_i > \theta_{-i}$, then

$$\begin{aligned} C_i(\theta) &= \frac{1}{2}\big(h(\theta_{-i}) - h(\theta_i)\big) + \frac{1}{2}\int_{\theta_{-i}}^{\theta_i} F(x)\, dv(x) \\ &= \frac{1}{2}\int_{\theta_{-i}}^{\theta_i} F(x)\, dg(x) + \frac{1}{2}\int_{\theta_{-i}}^{\theta_i} \big(1 - F(x)\big)\, d(-h)(x) \geq 0, \end{aligned} \tag{A.1}$$

while if $\theta_i < \theta_{-i}$, then

$$\begin{aligned} C_i(\theta) &= \frac{1}{2}\big(g(\theta_{-i}) - g(\theta_i)\big) + \frac{1}{2}\int_{\theta_{-i}}^{\theta_i} F(x)\, dv(x) \\ &= \frac{1}{2}\int_{\theta_i}^{\theta_{-i}} \big(1 - F(x)\big)\, dg(x) + \frac{1}{2}\int_{\theta_i}^{\theta_{-i}} F(x)\, d(-h)(x) \geq 0, \end{aligned} \tag{A.2}$$

as the integral terms are all non-negative by the assumption that $h' \leq 0$. ∎

## A.2 Proof of Proposition 7

It suffices to show that the mechanism $(s^*, t^*)$ defined by (2.1) and (4.2) satisfies IC* if and only if $g' + h' \leq 0$.

Fix any agent $i \in N$ and his type $\theta_i \in [\underline{\theta}, \overline{\theta}]$, and suppose that agent $i$ reports $\hat{\theta}_i$, while agent $-i$ truthfully reports his type $\theta_{-i}$. Define the "ex post regret" under $(s^*, t^*)$,

$$\Delta(\theta_{-i}) = \frac{1}{2} v_i(\theta_i, \theta_{-i}) - u_i(\theta_i, \hat{\theta}_i, \theta_{-i}), \tag{A.3}$$

as a function of agent $-i$'s type $\theta_{-i}$. Then, we have

$$U_i^*(\theta_i, \hat{\theta}_i) = U_i(\theta_i, \hat{\theta}_i) + E_{\theta_{-i}} \left[ \Delta(\theta_{-i}) \mathbf{1}_{\{\Delta(\theta_{-i}) \geq 0\}} \right].$$

Notice that

$$\Delta(\theta_{-i}) = \begin{cases} -\frac{1}{2} v_i(\theta_i, \theta_{-i}) - t_i^*(\hat{\theta}_i, \theta_{-i}) & \text{if } \hat{\theta}_i > \theta_{-i}, \\ \frac{1}{2} v_i(\theta_i, \theta_{-i}) - t_i^*(\hat{\theta}_i, \theta_{-i}) & \text{if } \hat{\theta}_i < \theta_{-i}. \end{cases} \tag{A.4}$$

**Lemma A.1.** *If $\hat{\theta}_i > \theta_i$, then $\Delta(\theta_{-i}) < 0$ for all $\theta_{-i} > \hat{\theta}_i$, while if $\hat{\theta}_i < \theta_i$, then $\Delta(\theta_{-i}) < 0$ for all $\theta_{-i} < \hat{\theta}_i$.*

*Proof.* Consider the former case where $\hat{\theta}_i > \theta_i$. Suppose that $\theta_{-i} > \hat{\theta}_i$, so that player $-i$ obtains the entire asset. Then, we have

$$\begin{aligned} u_i(\theta_i, \hat{\theta}_i, \theta_{-i}) &= t_i^*(\hat{\theta}_i, \theta_{-i}) \\ &\geq \frac{1}{2} v_i(\hat{\theta}_i, \theta_{-i}) \\ &> \frac{1}{2} v_i(\theta_i, \theta_{-i}), \end{aligned}$$

where the first inequality follows from the fact that $(s^*, t^*)$ satisfies EPIR, while the second inequality follows from the assumption that $v_i(\theta_i, \theta_{-i})$ is strictly increasing in $\theta_i$.

Consider then the latter case where $\hat{\theta}_i < \theta_i$. Suppose that $\theta_{-i} < \hat{\theta}_i$, so that player $i$ obtains the asset. Then we have

$$\begin{aligned} u_i(\theta_i, \hat{\theta}_i, \theta_{-i}) &= v_i(\theta_i, \theta_{-i}) + t_i^*(\hat{\theta}_i, \theta_{-i}) \\ &\geq v_i(\theta_i, \theta_{-i}) - \frac{1}{2} v_i(\hat{\theta}_i, \theta_{-i}) \\ &> \frac{1}{2} v_i(\theta_i, \theta_{-i}), \end{aligned}$$

where the first inequality follows from EPIR, $v_i(\hat{\theta}_i, \theta_{-i}) + t_i^*(\hat{\theta}_i, \theta_{-i}) \geq (1/2) v_i(\hat{\theta}_i, \theta_{-i})$, while the second from the assumption that $v_i(\theta_i, \theta_{-i})$ is strictly increasing in $\theta_i$. ∎

**Lemma A.2.** *For each* $\hat{\theta}_i \neq \theta_i$*, there exists* $\beta(\hat{\theta}_i) \in [\underline{\theta}, \overline{\theta}]$ *such that if* $\hat{\theta}_i > \theta_i$*, then* $\beta(\hat{\theta}_i) < \hat{\theta}_i$ *and*

$$\{\theta_{-i} \neq \hat{\theta}_i \mid \Delta(\theta_{-i}) > 0\} = \Big(\beta(\hat{\theta}_i), \hat{\theta}_i\Big),$$

*while if* $\hat{\theta}_i < \theta_i$*, then* $\beta(\hat{\theta}_i) > \hat{\theta}_i$ *and*

$$\{\theta_{-i} \neq \hat{\theta}_i \mid \Delta(\theta_{-i}) > 0\} = \Big(\hat{\theta}_i, \beta(\hat{\theta}_i)\Big).$$

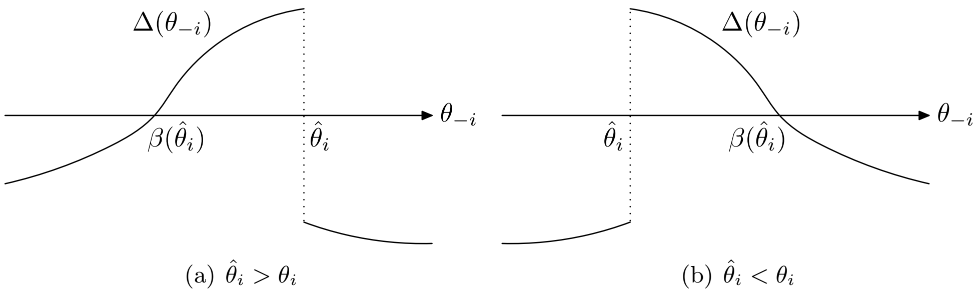


Figure 5: Ex post regret $\Delta(\theta_{-i})$

*Proof.* Consider first the case where $\hat{\theta}_i > \theta_i$. By Lemma A.1, $\Delta(\theta_{-i}) < 0$ for all $\theta_{-i} > \hat{\theta}_i$, and thus we consider $\theta_{-i} < \hat{\theta}_i$, where player $i$ obtains the entire asset. Since $\Delta(\theta_{-i})$ is continuous (in fact differentiable) on $[\underline{\theta}, \hat{\theta}_i)$, it is sufficient to show that $\Delta(\theta_{-i})$ is strictly increasing on $[\underline{\theta}, \hat{\theta}_i)$ and that $\lim_{\theta_{-i} \nearrow \hat{\theta}_i} \Delta(\theta_{-i}) > 0$. Indeed, recalling that

$$\begin{aligned}\Delta(\theta_{-i}) &= -\frac{1}{2}v_i(\theta_i, \theta_{-i}) - t_i^*(\hat{\theta}_i, \theta_{-i}) \\ &= -\frac{1}{2}v_i(\theta_i, \theta_{-i}) + \frac{1}{2}\left[v(\hat{\theta}_i) - \int_{\theta_{-i}}^{\hat{\theta}_i} F(x)\, dv(x)\right],\end{aligned}$$

we have

$$\begin{aligned}\Delta'(\theta_{-i}) &= -\frac{1}{2}h'(\theta_{-i}) + \frac{1}{2}F(\theta_{-i})\big(g'(\theta_{-i}) + h'(\theta_{-i})\big) \\ &= \frac{1}{2}F(\theta_{-i})g'(\theta_{-i}) - \frac{1}{2}\big(1 - F(\theta_{-i})\big)h'(\theta_{-i}) > 0 \qquad \text{(A.5)}\end{aligned}$$

for all $\theta_{-i} \in (\underline{\theta}, \hat{\theta}_i)$, where the inequality follows from the assumption that $g' > 0$ and $h' \leq 0$. Since $\lim_{\theta_{-i} \nearrow \hat{\theta}_i} t_i^*(\hat{\theta}_i, \theta_{-i}) = -(1/2)v_i(\hat{\theta}_i, \hat{\theta}_i)$, we also have

$$\lim_{\theta_{-i} \nearrow \hat{\theta}_i} \Delta(\theta_{-i}) = -\frac{1}{2}v_i(\theta_i, \hat{\theta}_i) + \frac{1}{2}v_i(\hat{\theta}_i, \hat{\theta}_i) > 0,$$

where the inequality follows from the assumption that $v_i(\theta_i, \theta_{-i})$ is strictly increasing in $\theta_i$. Thus, defining $\beta(\hat{\theta}_i)$ as follows gives the first expression in the lemma:

$$\beta(\hat{\theta}_i) = \begin{cases} \underline{\theta} & \text{if } \Delta(\theta_{-i}) > 0 \text{ for all } \theta_{-i} \in [\underline{\theta}, \hat{\theta}_i), \\ \theta_{-i} \text{ satisfying } \Delta(\theta_{-i}) = 0 & \text{otherwise.} \end{cases}$$

For the other the case where $\hat{\theta}_i < \theta_i$, the similar argument shows that $\Delta(\theta_{-i})$ is strictly decreasing on $(\hat{\theta}_i, \overline{\theta}]$ and that $\lim_{\theta_{-i} \searrow \hat{\theta}_i} \Delta(\theta_{-i}) > 0$. Thus, define $\beta(\hat{\theta}_i)$ as follows:

$$\beta(\hat{\theta}_i) = \begin{cases} \overline{\theta} & \text{if } \Delta(\theta_{-i}) > 0 \text{ for all } \theta_{-i} \in (\hat{\theta}_i, \overline{\theta}], \\ \theta_{-i} \text{ satisfying } \Delta(\theta_{-i}) = 0 & \text{otherwise.} \end{cases}$$

This completes the proof. ▮

**Lemma A.3.** *Let $\beta(\hat{\theta}_i)$ be as in Lemma A.2. Then, for all $i \in N$ and all $\theta_i, \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$,*

$$\begin{cases} \beta(\hat{\theta}_i) \geq \theta_i & \text{if } \hat{\theta}_i > \theta_i, \\ \beta(\hat{\theta}_i) \leq \theta_i & \text{if } \hat{\theta}_i < \theta_i \end{cases}$$

*if and only if $g' + h' \leq 0$.*

*Proof.* It suffices to examine the sign of $\Delta(\theta_{-i})$ at $\theta_{-i} = \theta_i$. If $\theta_i < \hat{\theta}_i$, then

$$\begin{aligned} \Delta(\theta_i) &= -\frac{1}{2} v_i(\theta_i, \theta_i) - t_i^*(\hat{\theta}_i, \theta_i) \\ &= -\frac{1}{2} v_i(\theta_i, \theta_i) + \frac{1}{2} \left[ v(\hat{\theta}_i) - \int_{\theta_i}^{\hat{\theta}_i} F(x)\, dv(x) \right] \\ &= \frac{1}{2} \int_{\theta_i}^{\hat{\theta}_i} \big(1 - F(x)\big)\, dv(x), \end{aligned} \tag{A.6}$$

while if $\hat{\theta}_i < \theta_i$, then

$$\begin{aligned} \Delta(\theta_i) &= \frac{1}{2} v_i(\theta_i, \theta_i) - t_i^*(\hat{\theta}_i, \theta_i) \\ &= \frac{1}{2} v_i(\theta_i, \theta_i) - \frac{1}{2} \left[ v(\theta_i) - \int_{\hat{\theta}_i}^{\theta_i} F(x)\, dv(x) \right] \\ &= \frac{1}{2} \int_{\hat{\theta}_i}^{\theta_i} F(x)\, dv(x). \end{aligned}$$

It follows that $\Delta(\theta_i) \leq 0$ for all $\theta_i \neq \hat{\theta}_i$ in both cases if and only if $v' = g' + h' \leq 0$ (since $0 < F(x) < 1$ for all $x \in (\underline{\theta}, \overline{\theta})$). But, by Lemma A.2, for all $\theta_i < \hat{\theta}_i$ ($\theta_i > \hat{\theta}_i$, resp.), $\Delta(\theta_i) \leq 0$ if and only if $\beta(\hat{\theta}_i) \geq \theta_i$ ($\beta(\hat{\theta}_i) \leq \theta_i$, resp.). ▮

*Proof of Proposition 7.* "If" part: Suppose that $g' + h' \leq 0$. We want to show that for each $i \in N$, $U_i^*(\theta_i) \geq U_i^*(\theta_i, \hat{\theta}_i)$ for all $\theta_i, \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$. We show this only for the case where $\theta_i < \hat{\theta}_i$. In this case, we have $\beta(\hat{\theta}_i) < \hat{\theta}_i$ as in Lemma A.2, and

$$U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) = \big(U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i)\big) - \int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \Delta(y)\, dF(y). \tag{A.7}$$

Recalling (A.4), we have

$$\begin{aligned}\Delta(y) &= -\frac{1}{2}\big(g(\theta_i) + h(y)\big) + \frac{1}{2}\left(v(\hat{\theta}_i) - \int_y^{\hat{\theta}_i} F(x)\, dv(x)\right) \\ &= \frac{1}{2}\big(g(y) - g(\theta_i)\big) + \frac{1}{2}\int_y^{\hat{\theta}_i} \big(1 - F(x)\big)\, dv(x),\end{aligned}$$

and therefore,

$$\begin{aligned}&\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \Delta(y)\, dF(y) \\ &\quad= \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i} \big(g(y) - g(\theta_i)\big)\, dF(y) - \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\theta_i} \big(g(y) - g(\theta_i)\big)\, dF(y) \\ &\qquad+ \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \int_y^{\hat{\theta}_i} \big(1 - F(x)\big)\, dv(x)\, dF(y) \\ &\quad= \frac{1}{2}\big(U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i)\big) - \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\theta_i} \big(g(y) - g(\theta_i)\big)\, dF(y) \\ &\qquad+ \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \int_y^{\hat{\theta}_i} \big(1 - F(x)\big)\, dv(x)\, dF(y),\end{aligned}$$

where in the last equality we used the formula

$$U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i) = \int_{\theta_i}^{\hat{\theta}_i} \big(g(y) - g(\theta_i)\big)\, dF(y),$$

which follows from the Revenue Equivalence. Hence,

$$\begin{aligned}\text{(A.7)} &= \frac{1}{2}\big(U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i)\big) + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)} \big(g(y) - g(\theta_i)\big)\, dF(y) \\ &\quad+ \frac{1}{2}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \int_y^{\hat{\theta}_i} \big(1 - F(x)\big)\, d(-v)(x)\, dF(y).\end{aligned} \tag{A.8}$$

Here, the first term is non-negative by IC, and so are the other two since $g$ is increasing and $v' = g' + h' \leq 0$ by assumption. Thus, we have $U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) \geq 0$ as desired.

"Only if" part: Suppose that $g' + h' > 0$. We want to find a type distribution and types $\theta_i, \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$ for which $U_i^*(\theta_i) < U_i^*(\theta_i, \hat{\theta}_i)$. For ease of notation, we let $[\underline{\theta}, \overline{\theta}] = [0, 1]$.

We first give a heuristic argument to outline the formal proof that follows. Let the type distribution $F$ be given by

$$F(x) = \begin{cases} 0 & \text{if } 0 \leq x < \dfrac{1}{4}, \\ \dfrac{1}{2} & \text{if } \dfrac{1}{4} \leq x < \dfrac{3}{4}, \\ 1 & \text{if } \dfrac{3}{4} \leq x \leq 1, \end{cases}$$

which violates the full-support assumption, and set $\theta_i = 1/4$ and $\hat{\theta}_i = 1/2$. Then we have

$$U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i) = \int_{\theta_i}^{\hat{\theta}_i} \big(g(y) - g(\theta_i)\big)\, dF(y) = 0,$$

while, since $\beta(\hat{\theta}_i) < \theta_i$ $(< \hat{\theta}_i)$ by Lemma A.3,

$$\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i} \Delta(y)\, dF(y) = \Delta(\theta_i) F(\theta_i) > 0.$$

Thus, from (A.7) we have $U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) < 0$.

Now let us consider the following sequence of distribution functions $(F_n)_{n=1,2,\ldots}$ with full support on $[0, 1]$:

$$F_n(x) = \begin{cases} \dfrac{4n+3}{2n+3} x & \text{if } 0 \leq x < \dfrac{1}{4}, \\ \dfrac{2^{4n+1}}{2n+3}\Big(x - \dfrac{1}{2}\Big)^{2n+1} + \dfrac{1}{2n+3}\Big(x - \dfrac{1}{2}\Big) + \dfrac{1}{2} & \text{if } \dfrac{1}{4} \leq x < \dfrac{3}{4}, \\ \dfrac{4n+3}{2n+3}(x - 1) + 1 & \text{if } \dfrac{3}{4} \leq x \leq 1. \end{cases}$$

The function $F_n$ is continuously differentiable on $[0, 1]$, and satisfies $F_n(0) = 0$, $F_n(1/2) = 1/2$, and $F_n(1) = 1$. Note that for $x \in [1/4, 3/4]$, $F_n(x) \to 1/2$ as $n \to \infty$. For each $F_n$, let $\Delta_n$ and $\beta_n$ be as in (A.3) and Lemma A.2, respectively. Set $\theta_i = 1/4$ and $\hat{\theta}_i = 1/2$, where $\beta_n(\hat{\theta}_i) < \theta_i$ by Lemma A.3. We first have

$$\begin{aligned} U_i(\theta_i) - U_i(\theta_i, \hat{\theta}_i) &= \int_{\theta_i}^{\hat{\theta}_i} \big(g(y) - g(\theta_i)\big)\, dF_n(y) \\ &< \big(g(\hat{\theta}_i) - g(\theta_i)\big)\big(F_n(\hat{\theta}_i) - F_n(\theta_i)\big) \to 0 \text{ as } n \to \infty. \end{aligned}$$

On the other hand, by (A.5), $\Delta'_n(x)$ is bounded from above uniformly for $n$ and $x \in [0, \theta_i]$, and by (A.6), $\Delta_n(\theta_i)$ is bounded from zero uniformly for $n$, as

$$\begin{aligned}\Delta_n(\theta_i) &= \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i} \big(1 - F_n(x)\big)\, dv(x) \\ &\geq \frac{1}{2}\big(v(\hat{\theta}_i) - v(\theta_i)\big)\big(1 - F_n(\hat{\theta}_i)\big) = \frac{1}{4}\big(v(\hat{\theta}_i) - v(\theta_i)\big) > 0.\end{aligned}$$

It follows that we can take a $\delta > 0$ with $\theta_i - \delta \geq \underline{\theta}$ and a $D > 0$ such that for all $n$, $\Delta_n(x) > D$ for all $x \in [\theta_i - \delta, \theta_i]$. Hence, we have

$$\begin{aligned}\int_{\beta_n(\hat{\theta}_i)}^{\hat{\theta}_i} \Delta_n(y)\, dF_n(y) &> \int_{\theta_i-\delta}^{\theta_i} \Delta_n(y)\, dF_n(y) \\ &> D\delta \frac{4n+3}{2n+3} > D\delta > 0\end{aligned}$$

for all $n$. Thus, from (A.7) we have $U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) < 0$ for sufficiently large $n$. ▌

## A.3 Proof for Example 6.3

We show the following.

**Proposition A.4.** *Let $[\underline{\theta}, \overline{\theta}] = [0, 1]$, and $F$ be the uniform distribution on $[0, 1]$. Assume that $g(x) = x$ and $h(x) = -\gamma x$, where $\gamma \geq 0$. Then, $(s^*, t^*)$ satisfies IC* if and only if $\gamma \geq 1/2$.*

*Proof.* We have already shown in Proposition 7 that $(s^*, t^*)$ satisfies IC* if $g' + h' = 1 - \gamma \leq 0$. It is therefore sufficient to consider only the case where $1 - \gamma > 0$. In this case, by Lemma A.3, $\theta_i$ falls between $\beta(\hat{\theta}_i)$ and $\hat{\theta}_i$.

"If" part: Assume that $1/2 \leq \gamma$ ($< 1$), or ($0 <$) $1 - \gamma \leq 1/2$. We want to show that $U_i^*(\theta_i) \geq U_i^*(\theta_i, \hat{\theta}_i)$ for all $\theta_i, \hat{\theta}_i \in [\underline{\theta}, \overline{\theta}]$. We show this only for the case where $\theta_i < \hat{\theta}_i$. In this case, we have $\beta(\hat{\theta}_i) < \hat{\theta}_i$ as in Lemma A.2. By (A.8),

$$\begin{aligned}&U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) \\ &\quad = \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i} (y - \theta_i)\, dy + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)} (y - \theta_i)\, dy \\ &\qquad - \frac{1}{2}(1-\gamma)\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i}\int_y^{\hat{\theta}_i} (1 - x)\, dx\, dy \\ &\quad \geq \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i} (y - \theta_i)\, dy + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)} (y - \theta_i)\, dy - \frac{1}{4}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i}\int_y^{\hat{\theta}_i} dx\, dy\end{aligned}$$

$$= \frac{1}{4}\big(\hat{\theta}_i - \theta_i\big)^2 + \frac{1}{4}\big(\theta_i - \beta(\hat{\theta}_i)\big)^2 - \frac{1}{8}\big(\hat{\theta}_i - \beta(\hat{\theta}_i)\big)^2$$
$$= \frac{1}{8}\big(\hat{\theta}_i + \beta(\hat{\theta}_i) - 2\theta_i\big)^2 \geq 0,$$

as desired.

"Only if" part: Assume that $\gamma < 1/2$. We want to find $\theta_i, \hat{\theta}_i \in [0,1]$ for which $U_i^*(\theta_i) < U_i^*(\theta_i, \hat{\theta}_i)$. Take a small number $\delta > 0$ such that $\delta < 1 - 2\gamma$. Note that $(1+\delta)/2 < 1 - \gamma$. Then take a large number $A > 1$ so that $1/(2A-2)^2 < \delta$, and let $B = 2A - 1$ (and hence $1/(B-1)^2 < \delta$). Finally let $\varepsilon > 0$ be a positive number, which will be taken to be sufficiently small. Set $\theta_i = A\varepsilon$ and $\hat{\theta}_i = B\varepsilon$. When $\theta_{-i} = \varepsilon$ $(< \hat{\theta}_i)$,

$$\begin{aligned}
\Delta(\varepsilon) &= -\frac{1}{2} v_i(\theta_i, \theta_{-i}) - t_i^*(\hat{\theta}_i, \theta_{-i}) \\
&= -\frac{1}{2}(A\varepsilon - \gamma\varepsilon) + \frac{1}{2}(1-\gamma)\left[B\varepsilon - \frac{1}{2}\left\{(B\varepsilon)^2 - \varepsilon^2\right\}\right] \\
&= \frac{\varepsilon}{2}\left[-(A-\gamma) + (1-\gamma)\left\{B - \frac{1}{2}(B^2-1)\varepsilon\right\}\right].
\end{aligned}$$

We claim that for sufficiently small $\varepsilon$, $\Delta(\varepsilon) > 0$. Indeed, as $\varepsilon \to 0$, the bracketed term in the last line goes to $-(A-\gamma) + (1-\gamma)B = -(B-1)\gamma - (A-\gamma) > -(B-1)(1/2) - (A - 1/2) = 0$. It therefore follows from Lemma A.2 that $\beta(B\varepsilon) < \varepsilon$ for sufficiently small $\varepsilon$.

By (A.8),

$$\begin{aligned}
&U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) \\
&\quad = \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i} (y - \theta_i)\, dy + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)} (y - \theta_i)\, dy \\
&\qquad - \frac{1}{2}(1-\gamma)\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i}\int_y^{\hat{\theta}_i} (1 - x)\, dx\, dy \\
&\quad < \frac{1}{2}\int_{\theta_i}^{\hat{\theta}_i} (y - \theta_i)\, dy + \frac{1}{2}\int_{\theta_i}^{\beta(\hat{\theta}_i)} (y - \theta_i)\, dy \\
&\qquad - \frac{1}{2}\frac{1+\delta}{2}\int_{\beta(\hat{\theta}_i)}^{\hat{\theta}_i}\int_y^{\hat{\theta}_i} (1 - \hat{\theta}_i)\, dx\, dy \\
&\quad = \frac{1}{8}\big(\hat{\theta}_i + \beta(\hat{\theta}_i) - 2\theta_i\big)^2 - \frac{1}{8}\big\{(1+\delta)(1-\hat{\theta}_i) - 1\big\}\big(\hat{\theta}_i - \beta(\hat{\theta}_i)\big)^2 \\
&\quad = \frac{1}{8}\big(-\varepsilon + \beta(B\varepsilon)\big)^2 - \frac{1}{8}\big\{(1+\delta)(1-B\varepsilon) - 1\big\}\big(B\varepsilon - \beta(B\varepsilon)\big)^2 \\
&\quad < \frac{1}{8}\varepsilon^2 - \frac{1}{8}\big\{(1+\delta)(1-B\varepsilon) - 1\big\}(B\varepsilon - \varepsilon)^2 \\
&\quad = \frac{1}{8}\varepsilon^2\left[1 - \big\{(1+\delta)(1-B\varepsilon) - 1\big\}(B-1)^2\right],
\end{aligned}$$

where the second inequality follows from $0 \le \beta(B\varepsilon) < \varepsilon$. We claim that for sufficiently small $\varepsilon$, $U_i^*(\theta_i) - U_i^*(\theta_i, \hat{\theta}_i) < 0$. Indeed, as $\varepsilon \to 0$, the bracketed term in the last line goes to $1 - \delta(B-1)^2$, which is negative by the choice of $\delta$ and $B$. ▮

## References


Athanassoglou, S., S. J. Brams, and J. Sethuraman (2008). "Minimizing Regret when Dissolving a Partnership," mimeo.

Bergemann, D. and J. Välimäki (2002). "Information Acquisition and Efficient Mechanism Design," *Econometrica* **70**, 1007-1033.

Chien, H.-K. (2007). "Incentive Efficient Mechanisms for Partnership Dissolution," mimeo.

Compte, O. and P. Jehiel (2006). "Veto Constraint in Mechanism Design: Inefficiency with Correlated Types," forthcoming in *American Economic Journal: Microeconomics.*

Compte, O. and P. Jehiel (2007). "On Quitting Rights in Mechanism Design," *American Economic Review Papers and Proceedings* **97**, 137-141.

Cramton, P., R. Gibbons, and P. Klemperer (1987). "Dissolving a Partnership Efficiently," *Econometrica* **55**, 615-632.

Fieseler, K., T. Kittsteiner, and B. Moldovanu (2003). "Partnerships, Lemons, and Efficient Trade," *Journal of Economic Theory* **113**, 223-234.

Forges, F. (1999). "Ex Post Individually Rational Trading Mechanisms," in A. Alkan, C. Aliprantis, and N. Yannelis, eds., *Current Trends in Economics: Theory and Applications*, Springer-Verlag, Berlin.

Fujinaka, Y. (2006). "Efficiency, Envy-Freeness, and Bayesian Incentive Compatibility in Economies with One Indivisible Good and Money," Working Paper Series No.203, Kobe University.

Gresik, T. A. (1991a). "Ex Ante Efficient, Ex Post Individually Rational Trade," *Journal of Economic Theory* **54**, 131-145.

Gresik, T. A. (1991b). "Ex Ante Incentive Efficient Trading Mechanisms without the Private Valuation Restriction," *Journal of Economic Theory* **55**, 41-63.

Jehiel, P. and A. Pauzner (2006). "Partnership Dissolution with Interdependent Values," *RAND Journal of Economics* **37**, 1-22.

KITTSTEINER, T. (2003). "Partnerships and Double Auctions with Interdependent Valuations," *Games and Economic Behavior* **44**, 54-76.

KOSMOPOULOU, G. (1999). "Payoff Equivalence between Bayesian and Ex Post Individually Rational Dominant Strategy Mechanisms," *Economic Theory* **13**, 229-237.

MAKOWSKI, L. AND C. MEZZETTI (1994). "Bayesian and Weakly Robust First Best Mechanisms: Characterizations," *Journal of Economic Theory* **64**, 500-519.

MATTHEWS, S. A. AND A. POSTLEWAITE (1989). "Pre-play Communication in Two-Person Sealed-Bid Double Auctions," *Journal of Economic Theory* **48**, 238-263.

MORGAN, J. (2004). "Dissolving a Partnership (Un)fairly," *Economic Theory* **23**, 909-923.

MYERSON, R. B. AND M. A. SATTERTHWAITE (1983). "Efficient Mechanisms for Bilateral Trading," *Journal of Economic Theory* **29**, 265-281.