Strategy-Proof and Anonymous Rule in Queueing Problems: A Relationship between Equity and Efficiency

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Abstract

In this paper, we consider a relationship between equity and efficiency in queueing problems. We show that under strategy-proofness, anonymity in welfare implies queue-efficiency. Furthermore, we also give a characterization of the equally distributed pairwise pivotal rule, as the only rule that satisfies strategy-proofness, anonymity in welfare and budget-balance.

keywords: Queueing Problems, Strategy-Proofness, Anonymity in welfare, Efficiency

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1 Introduction

We consider the situations as follows. There are one service (banking, postal, phone, web server, etc.) and money. Agents want to use the service, but no two agents can be served simultaneously. Each agent's serving time is the same, and normalized to unity. Each agent has a constant unit waiting cost which may differ among agents, and his waiting cost is the product of his waiting times and his unit waiting cost. Although agents must wait their turns, they have high values for use enough to do so. To compensate the waiting agents, monetary transfers are possible. The *queueing problems* are concerned with what the orders and the monetary transfers we select for each unit waiting costs of agents.[1] A *rule* is formulated as a function assigning a pair of queue and monetary transfers to agents' unit waiting costs.

In the most previous literature analyzing the queueing problems, efficiency has been focused on mainly. A rule is *queue-efficient* if the total waiting cost among agents is minimized. While queue-efficiency concerns efficiency for queues, *budget-balance* (the sum of transfers is zero) does efficiency for transfers. Queue-efficiency combined with budget-balance is equivalent to *Pareto-efficiency.*

The literature are divided into two groups; one has the cooperative game approach and another has the non-cooperative game approach. In the cooperative game approach, Maniquet (2003) has constructed a transferable utility game from the queueing problem and defined the rule which assigns the Shapley value to the agents' final utility. He has characterized the rule with Pareto-efficiency and auxiliary axioms. Chun (2006a) has constructed a different transferable utility game, and defined and characterized the Shapley value rule corresponding to the game. As literature that have considered the rules using other cooperative game solutions, Katta and Sethuraman (mimeo) and Kar et al. (mimeo) have analyzed the core rule and the prenucleous rule, respectively.

In the non-cooperative game approach, Dolan (1978) has provided a rule that satisfies queue-efficiency and *strategy-proofness.* Strategy-proofness is the condition that it is a dominant strategy for any agent to report his true cost, i.e., it prevents agents from manipulating strategically. While Dolan's (1978) rule does not satisfy budget-balance, the *equally distributed pairwise pivotal rule*[2] proposed by Suijs (1996) satisfies not only strategy-proofness and queue-efficiency but also budget-balance (that is, Pareto-efficiency).[3]

[1]In concluding remarks, we will give other problems which have the same structure as the queueing problems.

[2]This name has been given by Kayı and Ramaekers (mimeo).

[3]Mitra and Mutuswami (mimeo) has characterized the class of rules that satisfy

Moreover, this rule is the only rule that satisfies Pareto-efficiency, strategy-proofness, and equal treatment of equals in welfare [Kayı and Ramaekers (mimeo)]. Equal treatment of equals in welfare requires that the rule assign an allocation for which the welfare levels of agents are equal, as long as their unit waiting costs are the same.

In both approaches, the authors have analyzed the efficient rules mainly. However, agents may not care about anything but equity. In such environments, an equitable rule is desirable from agents' point of view, although efficiency is a goal of the whole society. *Anonymity in welfare* is a condition of equity[4] in the sense that the names of the agents do not matter in the rule from the viewpoint of welfare level. Anonymity in welfare requires that when the unit waiting costs of two agents are switched, their welfare under the rule be also switched. In this paper, we consider a relationship between equity and efficiency, and show that under strategy-proofness, anonymity in welfare implies queue-efficiency. Therefore, a rule automatically achieves the whole society's goal, as long as it pursues agents' concern. Furthermore, we also give another characterization of the equally distributed pairwise pivotal rule, as the only rule that satisfies strategy-proofness, anonymity in welfare and budget-balance.

In an auction model, the Vickrey allocation rule is the only rule that satisfies strategy-proofness, anonymity in welfare, and auxiliary axioms [Serizawa (2006)]. Although queueing model can also be considered as an auction model, there is a crucial difference between Serizawa (2006) and ours. While Serizawa (2006) has researched a homogeneous goods model, queueing model corresponds to a heterogeneous goods model. Thus, both results are independent.

In Section 2, we set up the model and state the results. In Section 3, we provide proofs. Finally, in Section 4 we make some concluding remarks.

2 The model and the result

Let $N = \{1, \ldots, n\}$ be the set of *agents*. An n-tuple $\sigma = (\sigma_1, \ldots, \sigma_n) \in N^n$ is a *queue* if $\sigma_i \neq \sigma_j$ for all $i, j \in N$ with $i \neq j$. Each agent $i \in N$ is assigned a *position* σ_i in a queue σ and a *monetary transfer* $t_i \in \mathbb{R}$. Each agent $i \in N$ has *quasi-linear preferences* on $X \equiv N \times \mathbb{R}$, i.e., if agent i's *unit waiting cost* is $c_i \in \mathbb{R}_+$, then his preference is represented by $u_i(\sigma_i, t_i; c_i) = -(\sigma_i - 1)c_i + t_i$.

strategy-proofness and Pareto-efficiency.

[4]Envy-freeness is also well-known as a condition of equity. Envy-freeness requires that no agent should end up with a higher utility by consuming what any other agent consumes. Chun (2006b) and Kayı and Ramaekers (mimeo) have analyzed the envy-free rules.

A list $c \equiv (c_i)_{i \in N} \in \mathbb{R}^n_+$ is a *cost profile.* Let $\mathcal{C} \equiv \mathbb{R}^n_+$ be the set of cost profiles. Let $Z \equiv \{(\sigma, t) \equiv (\sigma_i, t_i)_{i \in N} \in X^n$: (a) σ is a queue and (b) $\sum_{i \in N} t_i \leq 0\}$ be the set of *feasible allocations.* A *rule* is a function f from $\mathcal{C}$ to Z.

We consider rules satisfying the following conditions.

Definition 1 A rule f is *strategy-proof* if for all $c \in \mathcal{C}$, all $i \in N$ and all c'_i, $u_i(f_i(c); c_i) \geq u_i(f_i(c'_i, c_{-i}); c_i)$.

This is an incentive compatible condition that it is a dominant strategy for any agent to report his true unit waiting cost.

Next, we introduce two conditions of equity.

Definition 2 A rule f is *anonymous in welfare* if for all $c \in \mathcal{C}$ and all $i, j \in N$, $u_i(f_i(c); c_i) = u_i(f_j(c'_i, c'_j, c_{-\{i,j\}}); c_i)$, where $c'_i = c_j$ and $c'_j = c_i$.

This condition says that when the unit waiting costs of two agents are switched, their welfare under the rule are also switched.[5] Under this condition, the names of the agents do not matter from the viewpoint of welfare level.

Definition 3 A rule f satisfies *equal treatment of equals in welfare* if for all $c \in \mathcal{C}$ and all $i, j \in N$ with $c_i = c_j$, $u_i(f_i(c); c_i) = u_j(f_j(c); c_j)$.

This condition says that the rule assigns allocations for which the welfare levels of agents are equal, as long as their unit waiting costs are the same. Note that anonymity in welfare implies equal treatment of equals in welfare.

Finally, we introduce conditions of efficiency.

Definition 4 A rule f is *Pareto-efficient* if for all $c \in \mathcal{C}$, there is no $z \in Z$ such that $u_i(z_i; c_i) \geq u_i(f_i(c); c_i)$ for all $i \in N$ and $u_j(z_j; c_j) > u_j(f_j(c); c_j)$ for some $j \in N$.

Pareto-efficiency is decomposable into two conditions of efficiency: *queue-efficiency* (efficiency for queues) and *budget balance* (efficiency for transfers).

A queue σ is an *efficient queue for c* if $\sum_{i \in N}(\sigma_i - 1)c_i \leq \sum_{i \in N}(\sigma'_i - 1)c_i$ for all queue σ'. Let $Q^*(c)$ be the set of efficient queues for c. By definition,

[5] In divisible object models, such as exchange [Barberà and Jackson (1995)] or allotment [Sprumont (1991)] models, anonymity requires that two agents' outcome are switched when their preferences are switched. However, in indivisible object models, such as auction, queueing or public decision models, any rule cannot satisfy this type of anonymity. Thus, we need to impose the condition in terms of welfare instead of outcome [as Moulin (1986) and Serizawa (2006)]. To distinguish these two conditions, we call the condition used in this paper anonymity in welfare.

$\sigma \in Q^*(c)$ means that, under σ, the agents are served in decreasing order with respect to c, i.e., $\sigma \in Q^*(c)$ if and only if for each $i, j \in N$ with $i \neq j$, $c_i \geq c_j$ whenever $\sigma_i < \sigma_j$.

Definition 5 A rule $f = (\sigma, t)$ is *queue-efficient* if for all $c \in \mathcal{C}$, $\sigma(c) \in Q^*(c)$.

Definition 6 A rule $f = (\sigma, t)$ is *budget-balanced* if for all $c \in \mathcal{C}$, $\sum_{i \in N} t_i(c) = 0$.

The following theorem states the relationship between equity and efficiency.

Theorem 1 If $f = (\sigma, t)$ is *strategy-proof* and *anonymous in welfare*, then it is *queue-efficient*.

The proof is in the next section.

Note that strategy-proofness and equal treatment of equals in welfare do not imply queue-efficiency. For example, let $f = (\sigma, t)$ be such that for all $c \in \mathcal{C}$, $\sigma_i(c) = i$ and $t_i(c) = -\sum_{j \neq i}(j-1)c_j$. One can easily check that this rule satisfies strategy-proofness and equal treatment of equals but does not satisfy queue-efficiency.[6] Thus, anonymity in welfare is crucial in theorem 1.

Kayı and Ramaekers (2007) have characterized the rules satisfying strategy-proofness, Pareto-efficiency, and equal treatment of equals in welfare. They have shown that the only rule satisfying the three axioms is the *equally distributed pairwise pivotal rule* defined as follows.

Definition 7 A rule $f = (\sigma, t)$ is the *equally distributed pairwise pivotal rule* if for each $c \in \mathcal{C}$, $\sigma(c) \in Q^*(c)$, and for each agent $i \in N$,

$$t_i(c) = -\sum_{j \neq i} \sum_{\ell \in \{i,j\} \cap F_i(\sigma(c))} c_\ell + \frac{1}{n-2} \sum_{j \neq i} \sum_{k \neq i,j} \sum_{\ell \in \{j,k\} \cap F_j(\sigma(c))} c_\ell,$$

where $F_i(\sigma(c))$ is the set of followers of agent i under $\sigma(c)$.

We explain this rule with an example used by Kayı and Ramekers (2007).

Example 1 Let $N = \{1, 2, 3, 4\}$ and $c \in \mathcal{C}$ be such that $c_1 > c_2 > c_3 > c_4$. First, agents are served in decreasing order with repect to c, that is $\sigma(c) = (1, 2, 3, 4)$. Next, consider each pair of agents, and make each agent

[6]This example does not satisfy budget-balance. It is an open question whether or not strategy-proofness, equal treatment of equals in welfare, and budget-balance imply queue-efficiency. In an auction model, Ando et al. (2008) have discussed a problem which corresponds to it. However, they also do not have a general result.

in the pair pay the cost that the agent imposes on another agent, that is, the predecessor pays the unit waiting cost of the follower and the follower pays nothing. Then, distribute the sum of these two payments equally among the others. The final monetary transfer is the sum of all transfers for each possible pair. The following table shows how payments are calculated.

	1	2	3	4
12	$-c_2$	0	$\frac{c_2}{2}$	$\frac{c_2}{2}$
13	$-c_3$	$\frac{c_3}{2}$	0	$\frac{c_3}{2}$
14	$-c_4$	$\frac{c_4}{2}$	$\frac{c_4}{2}$	0
23	$\frac{c_3}{2}$	$-c_3$	0	$\frac{c_3}{2}$
24	$\frac{c_4}{2}$	$-c_4$	$\frac{c_4}{2}$	0
34	$\frac{c_4}{2}$	$\frac{c_4}{2}$	$-c_4$	0
sum	$-c_2 - \frac{c_3}{2}$	$-\frac{c_3}{2}$	$\frac{c_2}{2}$	$\frac{c_2}{2} + c_3$

One can check that the equally distributed pairwise pivotal rule satisfies anonymity in welfare. Furthermore, since anonymity in welfare implies equal treatment of equals in welfare, we have the following corollary.

Corollary 1 A rule f is *strategy-proof*, *anonymous in welfare* and *budget-balanced* if and only if it is the *equally distributed pairwise pivotal rule.*

3 Proof

At first, we give some definitions (see Figure 1). Given $c \in \mathcal{C}$, let

$$\begin{aligned} K^1(c) &\equiv \{i \in N : c_i > \min_{j \in N} c_j\}, \\ K^2(c) &\equiv \{i \in K^1(c) : c_i > \min_{j \in K^1(c)} c_j\}, \\ K^3(c) &\equiv \{i \in K^2(c) : c_i > \min_{j \in K^2(c)} c_j\}, \text{ and so on.} \end{aligned}$$

That is, $K^1(c)$ is the set of agents whose unit waiting costs are not minimal in N, $K^2(c)$ is the set of agents in $K^1(c)$ whose unit waiting costs are not minimal in $K^1(c)$, and so on.

Proof of Theorem. Suppose that $f = (\sigma, t)$ is strategy-proof and anonymous in welfare. We will show that for all $c \in \mathcal{C}, \sigma(c) \in Q^*(c)$, that is, the agents are served decreasing order with respect to c. First, we show that the agents in $K^1(c)$ are served before the agents in $N \backslash K^1(c)$.

Claim. *For all* $c \in \mathcal{C}$ *and for all* $i \in K^1(c)$, $\sigma_i(c) \leq \# K^1(c)$.
We prove by induction on the cardinality of $K^1(c)$ as follows.

(A) For all $c \in \mathcal{C}$ with $\#K^1(c) = 0$, and for all $i \in K^1(c)$, $\sigma_i(c) \leq 0$.

(B) If for all $c \in \mathcal{C}$ with $\#K^1(c) = k_1 - 1$, and for all $i \in K^1(c)$, $\sigma_i(c) \leq k_1 - 1$, then for all $c \in \mathcal{C}$ with $\#K^1(c) = k_1$, and for all $i \in K^1(c)$, $\sigma_i(c) \leq k_1$.

Since $\#K^1(c) = 0$ is equivalent to $K^1(c) = \emptyset$, (A) is obviously true. We have only to show (B).

Let k_1 be such that $1 \leq k_1 < n$, and suppose that for all $c \in \mathcal{C}$ with $\#K^1(c) = k_1 - 1$, and for all $i \in K^1(c)$ $\sigma_i(c) \leq k_1 - 1$. Let $c = (c_1, \ldots, c_n) \in \mathcal{C}$ be such that $\#K^1(c) = k_1$ and let $i \in K^1(c)$ be given. Let $c'_i \equiv \min_{j \in N} c_j$ and $c' \equiv (c'_i, c_{-i})$. Since $\#K^1(c') = k_1 - 1$, the hypothesis of induction implies that $\sigma_h(c') \leq k_1 - 1$ for all $h \in K^1(c')$. Thus, there exists $j \in N \backslash K^1(c')$ such that $\sigma_j(c') = k_1$. We divide the argument into two cases.

Case 1. $j = i$.
Strategy-proofness implies that

$$\begin{aligned} -(\sigma_i(c) - 1)c_i + t_i(c) &\geq -(k_1 - 1)c_i + t_i(c'_i) \text{ and} \\ -(k_1 - 1)c'_i + t_i(c'_i) &\geq -(\sigma_i(c) - 1)c'_i + t_i(c). \end{aligned}$$

These two inequalities then imply that

$$(\sigma_i(c) - k_1)(c'_i - c_i) \geq 0.$$

Since $c'_i - c_i < 0$, we must have $\sigma_i(c) - k_1 \leq 0$. Therefore, $\sigma_i(c) \leq k_1$.

Case 2. $j \neq i$ (see Figure 2).
Let $c'_j \equiv c_i$ and $c'' \equiv (c'_i, c'_j, c_{-\{i,j\}})$. Then it holds by anonymity in welfare that

$$u_i(f_i(c); c_i) = u_i(f_j(c''); c_i) \text{ and } u_i(f_i(c'); c_i) = u_i(f_j(c'); c_i),$$

that is,

$$\begin{aligned} -(\sigma_i(c) - 1)c_i + t_i(c) &= -(\sigma_j(c'') - 1)c_i + t_j(c'') \text{ and} \\ -(k_1 - 1)c'_i + t_j(c') &= -(\sigma_i(c') - 1)c'_i + t_i(c'). \end{aligned}$$

They are equivalent to

$$\begin{aligned} t_i(c) &= (\sigma_i(c) - \sigma_j(c''))c_i + t_j(c'') \text{ and} && (1) \\ t_j(c') &= (k_1 - \sigma_i(c'))c'_i + t_i(c'). && (2) \end{aligned}$$

By strategy-proofness, we also have

$$-(\sigma_j(c'') - 1)c'_j + t_j(c'') \geq -(k_1 - 1)c'_j + t_j(c').$$

This is equivalent to

$$\begin{aligned} t_j(c'') &\geq (\sigma_j(c'') - k_1)c'_j + t_j(c'), \\ &= (\sigma_j(c'') - k_1)c'_j + (k_1 - \sigma_i(c'))c'_i + t_i(c'), \end{aligned} \tag{3}$$

where equality comes from (2). Strategy-proofness also implies that

$$-(\sigma_i(c') - 1)c'_i + t_i(c') \geq -(\sigma_i(c) - 1)c'_i + t_i(c).$$

This is equivalent to

$$\begin{aligned} 0 &\geq (\sigma_i(c') - \sigma_i(c))c'_i + t_i(c) - t_i(c'), \\ &= (\sigma_i(c') - \sigma_i(c))c'_i + [(\sigma_i(c) - \sigma_j(c''))c_i + t_j(c'')] - t_i(c'), \\ &\geq (\sigma_i(c') - \sigma_i(c))c'_i + (\sigma_i(c) - \sigma_j(c''))c_i \\ &\quad + [(\sigma_j(c'') - k_1)c'_j + (k_1 - \sigma_i(c'))c'_i + t_i(c')] - t_i(c') \\ &= (\sigma_i(c) - k_1)(c_i - c'_i), \end{aligned}$$

where the first equality comes from (1), the second inequality comes from (3), and the second equality comes from $c'_j = c_i$. Since $c_i - c'_i > 0$, we must have $\sigma_i(c) - k_1 \leq 0$. Therefore, $\sigma_i(c) \leq k_1$. □

By the claim, each agent in $K^1(c)$ is served before the agents in $N \backslash K^1(c)$, in other words, each agent in $N \backslash K^1(c)$ is served after the agents in $K^1(c)$. Since the unit waiting costs of the agents belonging to $N \backslash K^1(c)$ are the same, the orders of them are arbitrary for queue-efficiency. Thus, the remaining problem to show queue-efficiency is that for all $c \in \mathcal{C}$, the agents in $K^1(c)$ are served decreasing order with respect to c.

By the same technique as the claim, we can show that for all $c \in \mathcal{C}$, each agent in $K^2(c)$ is served before the agents in $K^1(c) \backslash K^2(c)$, in other words, each agent in $K^1(c) \backslash K^2(c)$ is served after the agents in $K^2(c)$ (and also before the agents in $N \backslash K^1(c)$). Since the unit waiting costs of the agents belonging to $K^1(c) \backslash K^2(c)$ are the same, the orders of them are arbitrary for queue-efficiency. Thus, the remaining problem to show queue-efficiency is that for all $c \in \mathcal{C}$, the agents in $K^2(c)$ are served decreasing order with respect to c.

By continuing the same argument repeatedly at most n times, we can show that for all $c \in \mathcal{C}, \sigma(c) \in Q^*(c)$. Q.E.D.

4 Concluding Remarks

In this paper, we have analyzed a relationship between equity and efficiency, and have shown that strategy-proofness and anonymity in welfare imply queue-efficiency. Agents are usually more concerned with equity than efficiency, while efficiency is the whole society's goal. From our result, we can say that a rule automatically achieves the whole society's goal, as long as it pursues agents' concern. Furthermore, we have also given another characterization of the equally distributed pairwise pivotal rule, as the only rule that satisfies strategy-proofness, anonymity in welfare, and budget-balance. By this result, the equally distributed pairwise pivotal rule is the best rule from not only an efficient point of view, but also an equitable one.

In this paper, we have researched the queueing models with one service and the same serving time. Recently, several authors have analyzed extended models; Mitra (2005) and Chun and Heo (2007) have analyzed models with multiple services and the same serving time, Mitra (2002), Chun (2004), and Mishra and Rangarajan (2005) have analyzed models with one service and different serving times. In these researches, efficiency has been focused on mainly. It is a further research to analyze rules focused on equity mainly in extended models.

There are many problems that have the same structure as the queueing problems. Parking area problems and seat reservation problems are examples.

- **Parking area problems**: There are month-to-month parking spaces which are in-line as viewed from the exit. Each agent wants to contract one and only one parking space. The more distant from the exit, the more costs agents have (imagine the cost as the distance or necessary time from the parking space to the exit). How should we determine the agents' parking space and the monetary transfers among them?

- **Seat reservation models**: There is an event (concert, movie, sports, etc.), and there are seats which are in-line as viewed from the stage. Each agent wants to reserve one and only one seat. The more distant from the stage, more costs agents have (imagine the cost as the disutility). How should we determine the agents' seats and the monetary transfers among them?

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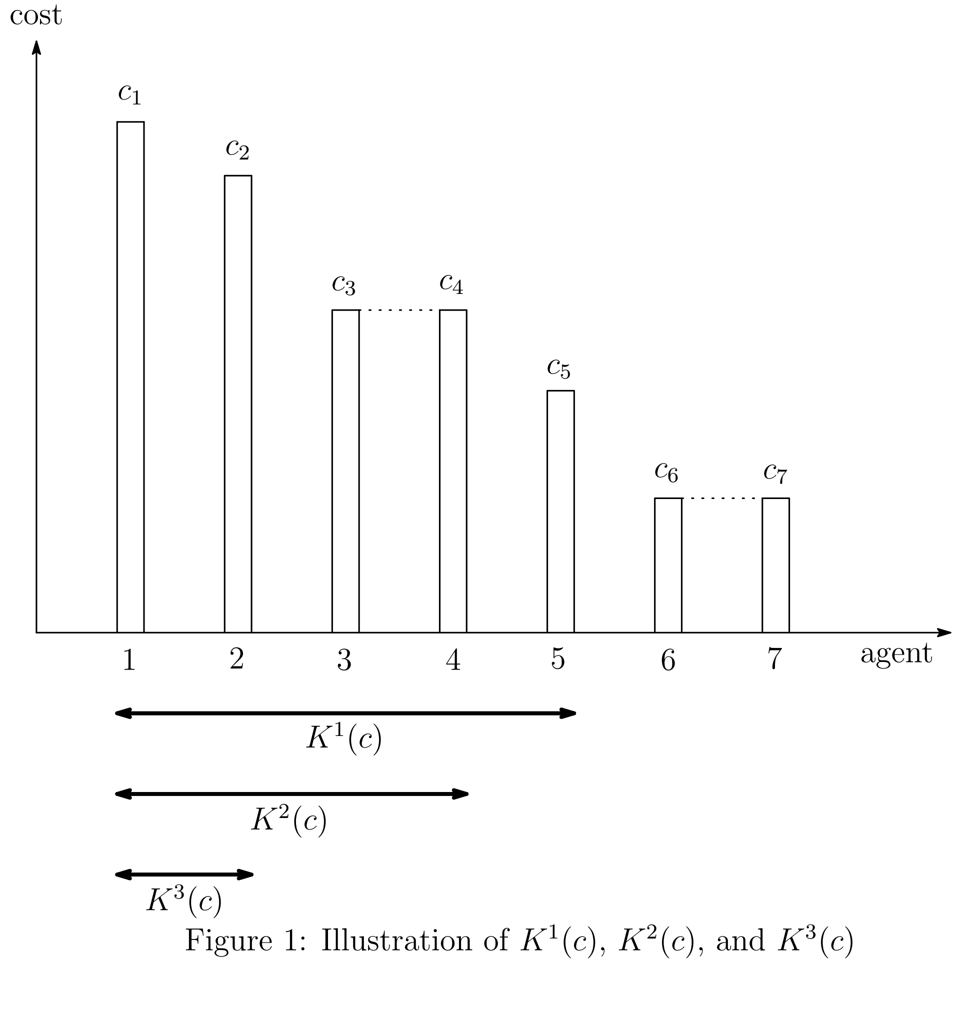

Figure 1: Illustration of $K^1(c)$, $K^2(c)$, and $K^3(c)$

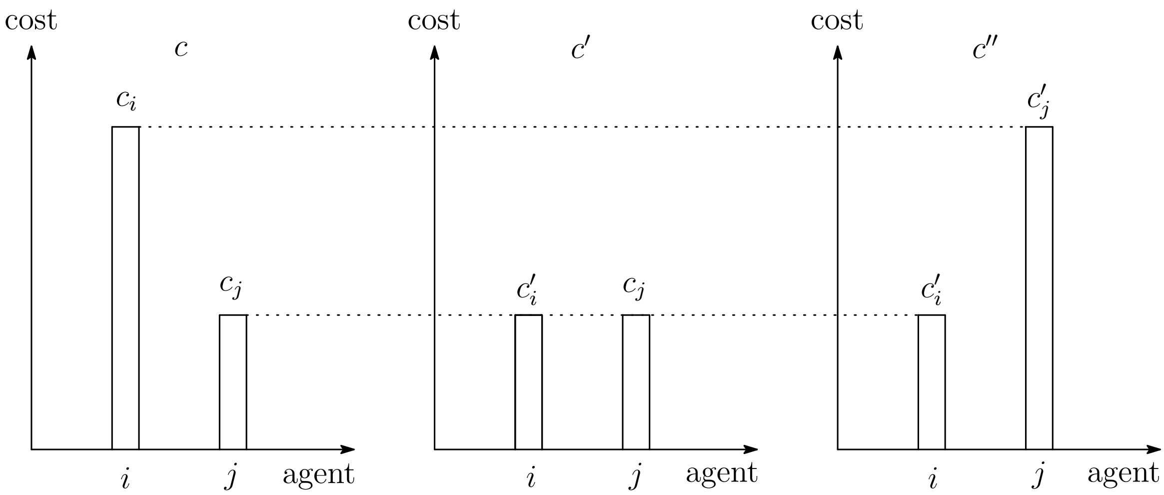

Figure 2: Illustration of the proof of theorem 1, case 2 of claim