Employment and Hours of Work*

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Abstract

This paper develops a dynamic model of the labor market in which the degree of substitution between employment and hours of work is determined as part of a search equilibrium. Each firm chooses the demand for working hours and the number of vacancies, and the wage profile is determined by Nash bargaining. The wage profile is generally nonlinear in hours of work, and it defines the trade-off between employment and hours of work. Hours of work are longer than optimal because only part of the cost is reflected in the wage profile. The Hosios condition is not enough for efficiency. When there are two industries, workers employed by firms with higher recruitment costs work longer and earn more. That is, "good jobs" require longer hours of work. Interestingly, technology differentials cannot account for working hours differentials.

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1 Introduction

There are extensive and intensive margins for adjusting labor input: the number of workers and hours of work per employee. Understanding how firms utilize these two margins is crucial for understanding the long-run trend in hours of work (Prescott, 2004; Pissarides, 2007), the movements of employment and hours over the business cycle (Burnside et al., 1993), the likely effect of regulation regarding hours of work (Hoel, 1986; Booth and Schiantarelli, 1987; Hunt, 1998; Marimon and Zilibotti, 2000), and cross-sectional differences in hours of work (Hamermesh, 1993), to name a few.

The last few decades have witnessed declines in hours of work in major developed economies (Rogerson, 2006; Pissarides, 2007). Despite a decline in average working hours, a disparity in working hours across workers has gradually widened. This disparity is particularly distinguished for workers in their 30s. The Japanese Labour Force Survey (2006) documented that the share of male employees aged between 35 and 39 years who work more than 60 hours per week rose from 18.9% in 1993 to 23.5% in 2003.[1] Similarly, the share of those who work less than 35 hours per week also rose from 6.4% to 7.1%. An important question is, what is the major cause of the dispersion in hours of work?

In this paper, we construct a dynamic equilibrium model of labor demand under *search frictions* and focus on how firms utilize employment and hours of work. The idea is that hours of work can be chosen instantly whereas employment adjustment is frictional. A novel feature of this paper is that the source of labor adjustment costs is search frictions, and that the cost is influenced by labor market tightness. Thus, the degree of substitution between employment and hours of work is determined as part of the search equilibrium. This sharply contrasts with the traditional models of working hours (Hoel, 1986; Booth and Schiantarelli, 1987; Calmfors and Hoel, 1988; Cahuc and Zylberberg, 2004) and the labor adjustment models (Sargent, 1978; Hamermesh, 1993), in which the degree of substitution between working hours and employment is structurally given by the

[1] For the US economy, Kuhn and Lozano (2005) investigated this issue and documented that "highly educated, high-wage, salaried men" had the greatest increase in long working hours. See also Hamermesh (1993).

production function or the adjustment cost function.

To understand the role of search frictions, consider the standard neoclassical model (Lucas and Rapping, 1969; Prescott, 2004), in which workers face a labor–leisure choice and choose hours of work optimally, taking the market wage rate as given. In the neoclassical framework, *firms can employ any quantity of labor at the market wage rate.* Thus, from the firms' viewpoint, it does not matter whether workers work longer or shorter hours because the competitive market ensures that the quantity of labor demanded equals the quantity of labor supplied. On the other hand, with search frictions and bilateral trading, if employees work shorter, then the firm must pay extra search costs to maintain its labor input. Thus, firms do care about working hours.

The basic model is an extension of Smith (1999), which is particularly useful for our purpose because the number of workers at each firm is determined endogenously, although modeling the firm size in a search equilibrium is generally a formidable task.[2] We incorporate the choice of working hours into Smith's (1999) framework. Each firm chooses the number of vacancies and hours of work per employee to maximize the value of the firm.

A novel feature of the model is that wage bargaining determines the wage profile, which relates earnings and hours of work. In particular, the wage profile is shown to be a nonlinear function of hours of work, and this reflects the concave production technology and the workers' convex utility cost of longer hours of work. Interestingly, the wage profile is consistent with the standard upward-sloping labor supply curve, and it defines the trade-off the firm faces when choosing employment and hours of work.

Smith's (1999) main result is that firms overemploy. The mechanism is intuitive. The social planner does not take into account the wage rate when the optimal employment is chosen because wages are simply transfers among the members of the society. However, firms do care about the wage rate. When the production technology is concave, an increase in employment results in a reduction in the wage rate. Firms have incentives to exploit this opportunity, resulting in overemployment. This mechanism also works in our framework.

[2] Bertola and Caballero (1994), Bertola and Garibaldi (2001), Cahuc and Wasmer (2001), and Cahuc et al. (2008) also developed search models with large firms.

However, it is not a trivial matter to assess whether hours of work are too long or too short. This is because there are two opposite effects at work. One effect comes directly from Smith's (1999) overemployment result because higher employment tends to induce shorter hours of work. The other effect comes from the choice of hours. When the social planner chooses hours of work, he or she is concerned with the disutility form longer hours of work because it constitutes the social cost of longer hours of work. However, when firms choose hours of work, they are concerned only with the wage rates. This implies that firms care about only part of disutility that is reflected in the wage profile. As a result, firms choose longer hours of work than the social planner does. We show that overall, hours of work are too long if firms choose working hours. Interestingly, hours of work are too short when workers choose working hours. This is a result of overemployment; it reduces the marginal product and hence the marginal hourly wage rate, reducing workers' incentives to work longer.

It has become increasingly important to ask whether regulating hours of work increases employment (Hoel, 1986; Booth and Schiantarelli, 1987; Hunt, 1998; Marimon and Zilibotti, 2000). According to our model, under regulation of working hours, a reduction of hours of work generally increases employment *at each firm*. Whether it expands the aggregate employment depends crucially on the initial condition. Suppose that firms are to choose hours of work, in which case hours of work are too long under laissez-faire. Starting at laissez-faire, regulating hours of work increases the aggregate employment and reduces unemployment.

We also study two other forms of regulation. One is about wage regulation. We consider a scenario in which the wage profile is perfectly regulated, and find that a wage profile that mimics the worker's disutility function implements the efficient levels of employment and hours of work. The key is to match the marginal hourly wage rate to the marginal disutility from longer hours of work. The other regulation we consider is entry regulation. This issue is becoming very important in the recent literature (Bertrand and Kramarz, 2002; Blanchard and Giavazzi, 2003; Fang and Rogerson, 2007). We consider a scenario in which the number of firms is regulated, and find that a decrease in the number of firms as a result of tighter regulation leads to shorter hours of work and a greater number of employees at each firm. However, it results in higher unemployment because there are fewer firms. In other words, expansions in employment at individual firms cannot compensate for the decrease in the number of firms.

Hamermesh (1993) documented that there are sizable differences in working hours across industries. For example, in 1990 in the US, the average weekly hours of work were 44 hours for the mining industry, 40.8 hours for manufacturing, 32.6 hours for services, and 28.8 hours for retail trade. We extend the model to ask why some individuals work longer than others. This finding is interesting because it suggests that there are large differences in working hours even among industries, each of which consists of a variety of jobs. Accordingly, we focus on job characteristics rather than worker characteristics as determinants of dispersion in hours of work.

We first address *within-industry* differentials in hours of work, such as the full-time-part-time differentials. To account for the differentials, we adopt a model of a single industry in which the labor market is pooled for two types of jobs. The key to working hours differentials is the differences in recruitment costs. Hamermesh (1993), for example, documented that firms spend much higher costs to recruit workers with skills or education than to fill jobs that do not require much skill or training. Our result implies that jobs with high recruitment costs are characterized as "good jobs" (Acemoglu, 2001), and workers with such jobs work longer than those with "bad jobs".

Finally, we address *interindustry* differentials in hours of work. There are two industries and the labor markets are segmented. Workers apply to one of the industries, so arbitrage dictates between the two markets. We show that the industry with a higher recruitment cost pays more and requires longer hours of work. The jobs in this industry are considered as "good jobs", and tightness of the labor market in this industry is lower because more workers apply to this industry. The key finding is that recruitment costs account for interindustry dispersion in hours of work. Interestingly, we find that differences in technology cannot account for working hours differentials.

2 The Model

2.1 Environment

This section constructs a dynamic model of the demand for hours and employment with search frictions. The model developed in this section is an extension of Smith (1999). Consider an economy consisting of workers and firms. The measure of workers is normalized to one. There is a large number of firms, with the exact number to be determined by free entry. Time is discrete and all agents discount the future at the common discount rate r.

The number of matches M is determined by the standard constant returns to scale matching technology: $M = m(U, V)$, where U is the total number of job seekers and V is the number of aggregate job vacancies. A vacancy is matched with a worker during a period with probability q, where

$$q \equiv \frac{M}{V} = m\left(\frac{U}{V}, 1\right). \tag{1}$$

It is easy to verify that an increase in labor market tightness $V/U \equiv \theta$ decreases the matching probability q. Similarly, the probability that a worker is matched with a vacancy is given by $M/U = m(1, V/U) = \theta q(\theta)$. Let λ be the exogenous rate of job destruction. Then, in any steady state, the flow into employment $\theta q(\theta)U$ must equal the flow into unemployment $\lambda(1-U)$. Thus:

$$m\left(U, V\right) = \lambda(1-U), \tag{2}$$

which defines the Beveridge curve, as shown in Figure 1.

2.2 Firms

The production technology is given by $f(L)$, where $L \equiv hl$ is total labor input, h denotes hours of work per employee and l is the number of employees at each firm. We assume that $f'(L) > 0 > f''(L)$ and $f(0) = 0$. In particular, we assume that production technology is specified by $f(L) = AL^{\alpha}$, where $A > 0$ and $\alpha \in (0, 1)$.

The instantaneous payoff to a firm with l employees is given by $f(hl) - W(h, l)l - kv - c$, where v is the number of vacancies, $k > 0$ is the cost of each vacancy, which may include a recruitment cost and an equipment cost (Acemoglu, 2001), and $c \geq 0$ is a fixed cost. Each worker earns $W(h,l)$, where $W(h,l)$ is the *wage function* or wage profile to be determined as part of equilibrium. The wage profile $W(h,l)$ is negotiated in each period, so the profile is treated as a function of the state variable l. It is important to note that $w(h,l) \equiv W(h,l)/h$ is the *average* hourly wage rate, which is generally different from the *marginal* hourly wage rate $W_h(h,l)$ if the wage profile is a nonlinear function of h. For the firm's decision making, what matters is the marginal hourly wage rate, so we focus on $W(h,l)$ rather than $w(h,l)$.

Let $J(l)$ be the value of an operating firm with l employees. The Bellman equation is given by

$$J(l) = \max_{h,v} \left\{ f(hl) - W(h,l)l - kv - c + \delta J(l_{+1}) \right\}, \tag{3}$$

where $l_{+1} = (1-\lambda)l + q(\theta)v$ is the next period's employment level, $\delta \equiv 1/(1+r)$ is the discount factor, and λ is the exogenous rate of job destruction. The first-order conditions with respect to h and v are:

$$f'(hl) - W_h(h,l) = 0, \tag{4}$$

$$-k + q(\theta)\delta J'(l_{+1}) = 0, \tag{5}$$

respectively. The envelope condition yields

$$J'(l) = f'(hl)\,h - W(h,l) - W_l(h,l)l + (1-\lambda)\,\delta J'(l_{+1}). \tag{6}$$

In any steady state, the flow into unemployment equals the flow into employment:

$$\lambda l = q(\theta)v. \tag{7}$$

Then, (5) and (6) imply

$$\frac{(r+\lambda)k}{q(\theta)} = f'(hl)h - W(h,l) - W_l(h,l)l. \tag{8}$$

2.3 Workers and Wages

The value of being employed by a firm of size l, denoted by $J^E(l)$, satisfies

$$J^E(l) = W(h,l) - e(h) + \lambda\delta J^U + (1-\lambda)\delta J^E(l_{+1}), \tag{9}$$

where $\delta \equiv 1/(1+r)$ is the discount factor, λ is the exogenous separation rate, $e(h)$ is disutility from work, and J^U is the values of being unemployed. We assume that $e'(\cdot) > 0$, $e''(\cdot) > 0$ and $\lim_{h\to\infty} e(h) = \infty$.

We assume that workers and a firm share the total surplus. In addition, we assume that workers are not unionized. Consider a bargaining process between a firm and workers of measure Δ. The threat point for the firm is $J(l-\Delta)$ because failing to agree on a contract implies losing the workers. The total match surplus is therefore $J(l) - J(l-\Delta) + \Delta(J^E(l) - J^U)$. If the firm's share of the surplus is given by $1-\beta \in [0,1]$, then we have $\beta[J(l) - J(l-\Delta)] = (1-\beta)\Delta(J^E(l) - J^U)$. In the limit as $\Delta \to 0$,

$$\beta J'(l) = (1-\beta)\left[J^E(l) - J^U\right]. \tag{10}$$

This is the key equation for rent sharing.[3] Note that this amounts to maximizing the generalized Nash product $[J'(l)]^{1-\beta}[J^E(l) - J^U]^\beta$ with respect to $W(h,l)$, taking J^U as given.

We need to solve the bargaining outcome (10) for the wage rate. First, use (5) and (6) to obtain

$$J'(l) = \frac{1+r}{r+\lambda}\left[f'(hl)\,h - W(h,l) - W_l(h,l)l\right]. \tag{11}$$

Similarly, from (9), we obtain

$$J^E(l) - J^U = \frac{1+r}{r+\lambda}\left[W(h,l) - e(h) - \frac{r}{1+r}J^U\right], \tag{12}$$

Substituting (11) and (12) into (10), we obtain

$$W(h,l) = \beta\left[f'(hl)\,h - W_l(h,l)l\right] + (1-\beta)\,e(h) + \frac{(1-\beta)\,r}{1+r}J^U, \tag{13}$$

which is a differential equation about the unknown wage function for each h and θ.

Proposition 1 *The wage function is given by*

$$W(h,l) = l^{-1/\beta}\int_0^l i^{\frac{1}{\beta}-1}\left[f'(hi)\,h\right]di + (1-\beta)\,e(h) + \frac{(1-\beta)\,r}{1+r}J^U.$$

[3] Smith (1999) also considered wage determination in a model similar to ours, and derived the wage rate using the intuition from the sequential bargaining theory (Osborne and Rubinstein, 1990). In contrast, our method exploits the Nash sharing rule together with the envelope condition in the dynamic programming problem.

Proof. In Appendix. ■

It is easy to verify that $\partial W(h,l)/\partial J^U > 0$. Thus, as in Pissarides (2000), the wage rate increases with the value of unemployment. Since the value of unemployment is increasing in labor market tightness, this implies that the wage rate increases with the tightness.

For $f(L) = AL^{\alpha}$, the wage function reduces to

$$W(h,l) = \frac{\alpha A h^{\alpha} l^{\alpha-1}}{\alpha + \frac{1-\beta}{\beta}} + (1-\beta)\, e\,(h) + \frac{(1-\beta)\, r}{1+r} J^U, \tag{14}$$

from which it is easy to verify that

$$W_h(h,l) = \frac{\alpha^2 A\, [hl]^{\alpha-1}}{\alpha + \frac{1-\beta}{\beta}} + (1-\beta)\, e'\,(h) > 0, \tag{15}$$

$$W_l(h,l) = \frac{(\alpha-1)\, \alpha A h^{\alpha} l^{\alpha-2}}{\alpha + \frac{1-\beta}{\beta}} < 0. \tag{16}$$

These two results are worth emphasizing. First, the wage profile is not a linear function of h. For α sufficiently close to unity, $W_{hh} > 0$ holds. In other words, as long as the production technology is sufficiently less concave, the marginal hourly wage rate is increasing in hours of work, justifying the observed positive association between hours of work and the hourly wage rage. Murphy and Topel (1997), for example, found a positive cross-sectional relationship between working hours and the hourly wage rate, and interpreted this relationship as the standard labor supply curve based on a worker's labor–leisure choice. Our result suggests that the positive association between hours of work and the wage rate represents the wage profile as a result of wage bargaining, and this is derived without a worker's labor–leisure choice.

Finally, (16) states that the wage rate is decreasing in the number of employees. As pointed by Smith (1999), this effect is due to the concavity of the production technology. We will see that because of this effect, firms have incentives to overemploy to reduce the wage rates. It is easy to see that this term is zero for the linear production technology ($\alpha = 1$).[4]

[4] We do not consider the case with $\alpha = 1$ because the return function in the Bellman equation will be linear in l and hence unbounded. See Stokey and Lucas (1989).

3 Equilibrium

3.1 Characterization

Following Smith (1999), we assume free entry of firms and that a firm entering the market opens a large number of vacancies to achieve the steady state level of employment l in the next period, so that there are no transitional dynamics or (transitory) size distribution of firms. Because the rate of filling a vacancy is $q(\theta)$, in order to achieve l in the next period, the firm must create exactly $l/q(\theta)$ vacancies today. Thus, the value of entry is given by

$$J(0) = -\frac{kl}{q(\theta)} - c + \delta J(l). \tag{17}$$

Equation (17) is interpreted as follows. The new firm creates $l/q(\theta)$ vacancies in order to employ l workers in the next period, and pays $kl/q(\theta)$ and c. Because the firm will employ l workers in the next period, the continuation value is exactly $J(l)$. The number of firms is determined by the free entry condition $J(0) = 0$. Thus, from (3) and (17), we obtain

$$f(hl) - W(h,l)l = \frac{r+\lambda}{q(\theta)} kl + (1+r)c. \tag{18}$$

Equation (7) can be rewritten as $\lambda l = [m(U,V)/V] \times v$, which implies that, assuming symmetric equilibrium, the number of firms is

$$N \equiv \frac{V}{v} = \frac{m(U,V)}{\lambda l} = \frac{1-U}{l}. \tag{19}$$

Consider the unemployed. The probability that an individual finds a job is $M/U = \theta q(\theta)$. Then, the value of being unemployed is written recursively as

$$J^U = b + \theta q(\theta)\delta J^E(l_{+1}) + (1-\theta q(\theta))\delta J^U, \tag{20}$$

where b is the (exogenous) unemployment benefit. Substitute (10) and (5) into (20) to obtain the equilibrium value of J^U as

$$\frac{r}{1+r} J^U = b + \theta q(\theta)\delta \left[J^E(l) - J^U\right] = b + \theta q(\theta)\delta \frac{\beta}{1-\beta} J'(l) = b + \frac{\beta}{1-\beta}\theta k. \tag{21}$$

Definition 2 *A steady-state equilibrium under free entry is a set of variables h, l, θ, U, and V that satisfy $\theta = V/U$, (2), (4), (8), and (18), equipped with the wage profile (14).*

Under free entry, U and V are determined by the Beveridge curve after θ is determined, so we focus on the determination of h, l, and θ. The equilibrium is then summarized by the following:

$$f'(hl) = W_h(h,l), \tag{22}$$

$$f(hl) = f'(hl)hl - W_l(h,l)l^2 + (1+r)c, \tag{23}$$

$$\frac{(r+\lambda)k}{q(\theta)} = W_h(h,l)h - W(h,l) - W_l(h,l)l, \tag{24}$$

where the wage function is given by (14) and J^U is given by (21). Equation (22) is from the firm's choice of hours of work (4), (23) is derived by substituting (8) into the free entry condition (18), and (24) is derived by substituting (4) into the choice of employment (8). With $f(L) = AL^{\alpha}$, these equations reduce to:

$$\frac{\alpha}{\alpha\beta + 1 - \beta} AL^{\alpha-1} = e'(h), \tag{25}$$

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta + 1 - \beta} AL^{\alpha} = (1+r)c, \tag{26}$$

$$\frac{(r+\lambda)k}{q(\theta)} + \beta\theta k = (1-\beta)\left[e'(h)h - e(h) - b\right], \tag{27}$$

where $L \equiv hl$ is the total labor input. It is easy to see that the equilibrium is uniquely determined. First, the equilibrium labor input L is determined by the free entry condition (26). Given L, (25) determines h. Finally, (27) determines θ. The determination of equilibrium is depicted in Figure 2. The upward-sloping curve is from (27), which represents the optimal choice of employment: as labor market tightness increases, the firm substitutes away from employment. However, the choice of hours is independent of tightness because the the firm's demand for hours of work is determined by comparing the marginal product and the marginal wage rate $W_h(h,l)$, both of which are independent of tightness. This is reflected by the horizontal line in this figure.

Proposition 3 *Suppose $e(h) = \varepsilon h^2$. (a) An increase in A increases h and θ. (b) An increase in k has no effect on h and decreases θ. (c) An increase in ε decreases h and θ.*

Proof. (a) Equation (26) implies that an increase in A reduces L. Since (25) and (26) imply $e'(h)L = (1+r)c\alpha/(1-\beta)(1-\alpha)$, it is evident that L and h are negatively related. Thus, an increase in A increases h. (b) Note that k appears only in (27). An increase in k will shift the employment curve upward in Figure 2. (c) From (26), changes in ε have no effect on L. Given this, (25) and (27) are rewritten as $H = 2\varepsilon h$ and $(r+\lambda)k/q(\theta) + \beta\theta k = (1-\beta)[\varepsilon h^2 - b]$, where H is a constant. Eliminate h from these two expressions to obtain $(r+\lambda)k/q(\theta)+\beta\theta k = (1-\beta)[H^2/4\varepsilon - b]$, from which it is easy to verify that $d\theta/d\varepsilon < 0$. ■

The results are intuitive. An increase in productivity induces potential firms to enter the market, which tightens the labor market and raises the expected cost of a vacancy. Firms will respond to this by substituting employment for hours of work. An increase in the cost of vacancy directly raises the expected cost of a vacancy. This has two effects, direct and indirect. The direct effect is that firms respond this increase in the vacancy cost by substituting employment for hours of work. The other, indirect, effect is to discourage entry, which reduces tightness and firms substitute hours for employment. The overall effect of an increase in recruitment cost on hours of work is neutral. Finally, an increase in the marginal disutility from work will reduce hours of work even though hours of work is the firm's choice variable. The key is that the wage profile is determined by bargaining. An increase in the marginal disutility of work is reflected in the wage profile, so firms face a greater marginal wage rate for choosing longer hours of work.

Here we present a numerical example. The parameters are $A = 0.9$, $\alpha = 0.8$, $\beta = 0.4$, $r = 0.01$, $\lambda = 0.2$, $c = 0.1$, $k = 0.1$, $b = 0.1$, $e(h) = h^2$, and $q = \theta^{-0.5}$. The equilibrium is given by $L = 0.83$, $h = 0.41$, $l = 2.04$, and $\theta = 0.58$. Figure 3 shows the equilibrium wage profile. Since we have chosen the parameters so that the disutility function is sufficiently convex and the production function is not too concave, the wage profile is convex as shown in the figure.

3.1.1 Neutrality Result

Does it matter who faces the cost of longer hours of work? Here, we briefly discuss an alternative environment in which firms face the cost of longer hours of work. Let $J(l)$ be the value of an operating firm with l employees. The Bellman equation is given by $J(l) = \max_{h,v}\{f(hl) - e(h)l -$

$W(h,l)l - kv - c + \delta J(l_{+1})\}$, where $l_{+1} = (1-\lambda)l + qv$. The first-order conditions with respect to h and v are $f'(hl) - e'(h) - W_h(h,l)h = 0$ and $-k + q\delta J'(l_{+1}) = 0$, respectively. The envelope condition yields $J'(l) = f'(hl)\,h - e(h) - W(h,l) - W_l(h,l)l + (1-\lambda)\delta J'(l_{+1})$. The wage equation becomes $W(h,l) = \beta[f'(hl)h - e(h) - W_l(h,l)l] + (1-\beta)rJ^U/(1+r)$. The corresponding wage function is

$$W(h,l) = \frac{\alpha\beta A h^{\alpha} l^{\alpha-1}}{\alpha\beta + 1 - \beta} - \beta e(h) + (1-\beta)\frac{r}{1+r}J^U.$$

The equilibrium conditions are: $f'(hl) = e'(h) + W_h(h,l)$, $f(hl) = f'(hl)hl - W_l(h,l)l^2 + (1+r)c$, and $(r+\lambda)k/q = e'(h)h - e(h) + W_h(h,l)h - W(h,l) - W_l(h,l)l$. It is easy to verify that these conditions reduce to (25)–(27).

Proposition 4 *It does not matter who faces the cost of longer hours of work.*

3.2 Welfare

In this section, we consider the efficiency of the equilibrium. Following Smith (1999), we consider the social planner's problem for $r = 0$ and focus on the steady-state welfare: $N[f(hl) - e(h)l - c] + bU - kV$, where N is the number of firms in steady state. Thus,

$$\max_{h,l,U,V} \frac{1-U}{l}[f(hl) - e(h)l - c] + bU - kV \text{ subject to } m(U,V) = \lambda(1-U).$$

It is easy to establish that the efficient allocation must satisfy:

$$\begin{aligned} f'(L) &= e'(h), & (28)\\ f(L) &= f'(L)L + c, & (29)\\ \frac{f(L)-c}{l} - e(h) - b &= \frac{m_U + \lambda}{m_V}k, & (30)\\ m(U,V) &= \lambda(1-U). & (31) \end{aligned}$$

The efficiency conditions (28)–(31) are comparable with the decentralized conditions. First, we compare (29) with (23). Even with $r = 0$, these two conditions are not identical because of the term $W_l(h,l) < 0$.

By $W_l(h,l) < 0$, we can conclude that the equilibrium L is larger than the efficient one, replicating Smith's (1999) result.[5] The term $W_l(h,l) < 0$ is the source of externality in Smith (1999), in the sense that a firm has an incentive to overemploy to reduce the wage rates of all employees by exploiting the relationship between the marginal product of labor and the wage rate. The source of this externality is easily understood by the objective function for the social planner. For the social planner, the wages are transfers among the members of the society. Thus, for efficiency, wages should not influence agents' decisions. However, for individual agents, wages matter in a crucial manner. The effect of overemployment on h is negative. However, the opposite effect is at work.

The same mechanism works for the efficiency of hours of work. The social planner is only concerned with the marginal disutility from longer hours of work. However, because of the concave production technology, an increase in hours of work has a negative impact on the wage rate. This will encourage firms to choose longer hours of work. In addition, as the wage function (14) clarifies, only a fraction of $e'(h)$ is reflected in the wage profile, which also encourages firms to expand hours of work.

Proposition 5 *h is above the optimal level h^*.*

Proof. Let h^* and L^* denote their efficient levels. (26) and (29) imply $L/L^* = [1-\alpha/(1-\beta)]^{1/\alpha}$. Since $e'(h^*)/e'(h) = (1-\beta)L/L^*$, we have $e'(h^*)/e'(h) = (1-\beta)[1-\alpha/(1-\beta)]^{1/\alpha} < 1$, which implies $h^* > h$. ■

Proposition 6 *The equilibrium allocation is efficient if and only if $\eta(\theta) = \beta$ and $\beta = 0$ hold, where $\eta(\theta) \equiv -q'(\theta)\theta/q(\theta)$.*

Proof. When $\beta = 0$, (25) and (26) reduce to $\alpha A L^{\alpha-1} = e'(h)$ and $(1-\alpha)AL^{\alpha} = (1+r)c$. These conditions coincide with (28) and (29) at $r = 0$. Note that $[1-\eta(\theta)]q(\theta) = m_V$ and $[1-\xi(\theta)]\theta q(\theta) = m_U$ hold, where $\eta(\theta) \equiv -q'(\theta)\theta/q(\theta)$ is the elasticity of $q(\theta)$ with respect to θ,

[5] Remember that in Smith (1999), there is no employment–hours split, so too large L implies overemployment.

and $\xi(\theta) = 1 - \eta(\theta)$ is the elasticity of $\theta q(\theta)$ with respect to θ. It is then easy to rewrite (30) as

$$\frac{f(L) - c}{l} - e(h) - b = \frac{[1 - \xi(\theta)]\theta q(\theta) + \lambda}{[1 - \eta(\theta)]q(\theta)} k. \tag{32}$$

Now consider (18). At $r = 0$, this is rewritten as

$$\frac{f(hl) - c}{(1 - \beta) l} - \frac{\alpha\beta A h^{\alpha} l^{\alpha - 1}}{(1 - \beta)[\alpha\beta + 1 - \beta]} - e(h) - b = \frac{\beta\theta q(\theta) + \lambda}{(1 - \beta) q(\theta)} k.$$

It is evident that this expression is identical to (32) if and only if $\eta(\theta) = \beta$ holds and $\beta = 0$. ■

Interestingly, the Hosios (1990) condition ($\eta(\theta) = \beta$) is not enough for efficiency. The reason is the presence of Smith's (1999) externality, which exists as long as the production function is concave. To eliminate it, one needs to impose $\beta = 0$, under which the firm takes all the bargaining surplus. In such a case, firms no longer have incentives to overemploy to cut the wage rates because externality is internalized.

3.2.1 Does it matter who chooses hours of work?

Does it matter who chooses hours of work? To address this question, suppose that each employed chooses hours of work to maximize the value of employment. Then, $J^E(l) = \max_h \{W(h, l) - e(h) + \lambda\delta J^U + (1 - \lambda)\delta J^E(l_{+1})\}$. The first-order condition is $W_h(h, l) = e'(h)$, or

$$\frac{\alpha^2 A [hl]^{\alpha - 1}}{\alpha\beta + 1 - \beta} = e'(h). \tag{33}$$

Other equilibrium conditions are

$$\begin{aligned} f(hl) &= f'(hl) hl - \frac{(\alpha - 1) \alpha A [hl]^{\alpha}}{\alpha + \frac{1-\beta}{\beta}} + (1 + r) c, \\ \frac{r + \lambda}{q(\theta)} k + \beta\theta k &= (1 - \beta) \left[e'(h) h - e(h) - b\right] + f'(hl) h - e'(h) h. \end{aligned}$$

It is evident that for L, it does not matter who chooses hours of work. By comparing (33) with (25) and (28), we can conclude that hours of work are shorter than optimal.

Now turn to the scenario in which hours of work are determined in the bargaining stage. In particular, consider the case in which the wage rate and hours of work are determined so as to maximize the generalized Nash product. The Nash bargaining problem is given by $\max_{W,h} [J'(l)]^{1-\beta} [J^E(l) -$

$J^U]^\beta$ subject to (6) and (9), taking $J'(l_{+1})$, J^U, and $J^E(l_{+1})$ as given. The first-order conditions with respect to W and h are (10) and

$$(1-\beta)\left[J^E - J^U\right]\frac{\partial\left[J'(l)\right]}{\partial h} + \beta J'(l)\frac{\partial\left[J^E(l) - J^U\right]}{\partial h} = 0. \tag{34}$$

Reduce these expressions to

$$\frac{\partial\left[J'(l)\right]}{\partial h} + \frac{\partial\left[J^E(l) - J^U\right]}{\partial h} = 0, \tag{35}$$

where $\partial[J'(l)]/\partial h = f'(hl)+f''(hl)hl-W_h(h,l)-W_{lh}(h,l)l$ and $\partial[J^E(l)-J^U]/\partial h = W_h(h,l)-e'(h)$. The wage function is the same as before and is given by (14). Thus, (35) becomes $f'(hl)+f''(hl)hl-W_{lh}(h,l)l-e'(h)=0$, or

$$f'(hl) + f''(hl)\,hl = \frac{(\alpha-1)\,\alpha^2 A\,[hl]^{\alpha-1}}{\alpha + \frac{1-\beta}{\beta}} + e'(h)\,.$$

With $f(L) = AL^\alpha$, we obtain $[\alpha\beta + 1 - \beta]^{-1}\alpha^2 AL^{\alpha-1} = e'(h)$, which is identical to (33). From this and other equilibrium conditions, it is easy to establish that employment and hours of work in this economy coincide with those when hours of work are chosen by each worker.

Let h^* denote the efficient level of hours of work, h^w denote hours of work chosen by workers, h^b denote hours of work chosen in bargaining, and h^f denote hours of work chosen by firms. Then we can summarize the results as follows.

Proposition 7 *$h^w = h^b < h^* < h^f$.*

Thus, it does matter who is to make the choice of hours of work. Its policy implication is important. If hours of work are determined by workers, then, for efficiency, policies must be designed to induce longer hours of work. On other hand, if firms choose hours of work, then polices that reduce hours of work may improve efficiency.

3.3 Implications for Regulation

3.3.1 The Employment Effect of Restricting Working Hours

There is a policy debate about whether regulating hours of work can increase the aggregate employment (Booth and Schiantarelli, 1987, Marimon and Zilibotti, 2000). According to Proposition 7, regulating hours of work may be welfare-enhancing if firms choose hours of work under laissez-faire. Suppose that hours of work can be perfectly controlled by regulation. Does restricting working hours stimulate employment? The purpose of this section is to address this issue.

The Bellman equation is given by $J(l) = \max_v\{f(hl) - W(l)l - kv - c + \delta J(l_{+1})\}$, where $l_{+1} = (1-\lambda)l + q(\theta)v$. Here, hours of work h is a policy parameter (Marimon and Zilibotti, 2000). The first-order conditions with respect to v is $-k + q(\theta)\delta J'(l_{+1}) = 0$, and the envelope condition is given by (6). From these equations, $(r+\lambda)k/q = f'(hl)h - W(l) - W'(l)l$. With free entry, we have $f(hl) = W(l)l + (r+\lambda)kl/q + (1+r)c = f'(hl)hl - W'(l)l^2 + (1+r)c$. The wage function is given by (14) and J^U is given by (21). After some algebra, it is easy to show that the equilibrium is a pair of L and θ that satisfy

$$\begin{aligned}\frac{(1-\beta)(1-\alpha)AL^{\alpha}}{\alpha\beta+1-\beta} &= (1+r)c,\\ \frac{r+\lambda}{q(\theta)}k + \beta\theta k &= \frac{(1-\beta)\alpha AL^{\alpha-1}}{\alpha\beta+1-\beta}h - (1-\beta)\left[e(h)+b\right],\end{aligned}$$

where $L = hl$.

Proposition 8 *Suppose working hours h are perfectly regulated. A reduction in hours of work increases the number of employees at each firm. It increases the aggregate vacancies and decreases unemployment if and only if $\alpha AL^{\alpha-1} > [\alpha\beta+1-\beta]e'(h)$.*

This implies that, starting at the laissez-faire, regulating hours of work will have no impact on the aggregate number of vacancies and unemployment because $\alpha AL^{\alpha-1} = [\alpha\beta+1-\beta]e'(h)$ holds at laissez-faire. As h reduces further, the left-hand side of the condition decreases and the condition is satisfied.

3.3.2 Regulated Wages

Suppose that the wage rates are regulated. In particular, we assume that the wage profile $W(h)$ is written by the regulator and is exogenous. To simplify the analysis, we assume that the wage function is continuous and differentiable with respect to h. In addition, $W'(h) > 0$.

The Bellman equation is given by $J(l) = \max_{h,v}\{f(hl) - W(h)l - kv - c + \delta J(l_{+1})\}$, where $l_{+1} = (1-\lambda)l + q(\theta)v$. First-order conditions are $f'(hl) - W'(h) = 0$ and $-k + q(\theta)\delta J'(l_{+1}) = 0$, respectively. The envelope condition is $J'(l) = f'(hl)h - W(h) + (1-\lambda)\delta J'(l_{+1})$. The key is that absence of wage bargaining, we have $W_l(h) = 0$, which rules out the bargaining externality because firms have no incentive to overemploy to reduce the wage rates. With free entry, the equilibrium is given by $f'(hl) = W'(h)$, $f(hl) = f'(hl)hl + (1+r)c$, and $(r+\lambda)k/q(\theta) = W'(h)h - W(h)$.

Proposition 9 *The following wage function implements the efficient levels of employment and hours of work: $W(h) = e(h) + \bar{W}$, where $\bar{W}$ is constant.*

Proof. It is evident that total employment is efficient at $r = 0$. The choice of hours of work will be efficient if h satisfies $f'(hl) = e'(h)$. Thus, the regulator must make sure that $W'(h) = e'(h)$. Take integral on both sides to obtain $W(h) = e(h) + \bar{W}$, where $\bar{W}$ is an arbitrary constant. ■

This proposition suggests that, for efficiency, the regulator must set the marginal increase in the wage rate to match the marginal disutility from longer hours of work. Regulators are usually concerned with the minimum level of earnings, $\bar{W}$. However, it does not matter for efficiency of employment and hours of work. From $(r+\lambda)k/q(\theta) = W'(h)h - W(h)$, we can conclude that an increase in $\bar{W}$ decreases θ, leading to a higher unemployment rate. In other words, the policy maker faces a trade-off between $\bar{W}$ and unemployment.

3.3.3 Entry Regulation

Some critics argue that entry regulation can serve as a useful labor market policy because lower product market competition may increase employment. Using a model of monopolistic competition and rent sharing, Blanchard and Giavazzi (2003) showed that tougher entry regulation leads to lower employment. Bertrand and Kramarz (2002) argued that stronger entry regulation slowed down employment growth in France. Does entry regulation increase employment or reduce it? To address this important issue, we modify the model so that the number of firms is regulated.

Suppose that the number of firms N is constant, and is sufficiently large. We treat N as a continuous variable for the ease of exposition. Equation (19) in this environment must be

$$\sum_{i=1}^{N} l_i = 1 - U(\theta), \tag{36}$$

where $U'(\theta) < 0$ from the Beveridge curve. Thus,

$$\begin{aligned} \frac{\alpha}{\alpha\beta + 1 - \beta} A l_i^{\alpha-1} &= h_i^{1-\alpha} e'(h_i), & (37) \\ \frac{(r+\lambda)k}{q(\theta)} + \beta\theta k &= (1-\beta)\left[e'(h_i)h_i - e(h_i) - b\right] & (38) \end{aligned}$$

hold for each firm $i = 1, ..., N$, and the free entry condition is replaced with (36). We look for a Nash equilibrium in which individual l_i is determined as the solution to (36)–(38), taking as given all other firms' employment levels. It is easy to verify that (37) and (38) define $l_i = \Phi(\theta)$ for all i in the symmetric equilibrium, where $\Phi'(\theta) < 0$. Thus, (36) reduces to $N\Phi(\theta) = 1 - U(\theta)$, which determines the equilibrium tightness.

Proposition 10 *A decrease in N decreases h and θ, and increases l and U.*

Proof. The symmetric equilibrium must satisfy $N\Phi(\theta) = 1 - U(\theta)$, where the left-hand side is decreasing and the right-hand side is increasing. Thus, a decrease in N decreases θ. Expression (38) implies that a decrease in θ is associated with a decrease in h for all firms. Since $l = \Phi(\theta)$ is decreasing, a decrease in θ must be associated with an increase in l for all firms. U increases because it is decreasing in θ. ∎

Since hours of work are longer than optimal when the condition in Proposition 5 is satisfied, entry regulation is useful in reducing hours of work. However, although it expands employment at each individual firm, it increases aggregate unemployment because the number of firms is limited.

An alternative way to model entry regulation is to introduce a fixed cost of entry while maintaining the assumption free entry. In this case, the free entry condition is replaced with $J(0) = F$, where $F > 0$ is the cost of entry, which is the policy parameter. The equilibrium is characterized by (25)–(27) and (26) is replaced with

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta + 1 - \beta} A L^{\alpha} = (1+r)c + rF.$$

It is then easy to establish that an increase in F increases L and reduces h and θ. In other words, if entry regulation is implemented as a fixed entry fee, either pecuniary or non-pecuniary, then a stronger regulation reduces hours of work and unemployment. Thus suggests that the exact form of entry regulation matters: a direct control of the number of firms reduces employment while entry cost expands employment.

3.4 Endogenous Entry

In the basic model, we have assumed that $J(0) = 0$. This implies that there is entry of firms as long as $J(0) > 0$. However, from a potential entrant's point of view, the opportunity cost of starting up a company must be the value of being a worker (Fonseca et al, 2001). Thus, we assume that an entrepreneur starts up a company if and only if $J(0) \geq J^U$. We maintain the assumption of a unit measure of workers. Thus, if $J(0) < J^U$, then firms exit without increasing total measure of workers.[6]

Under endogenous entry, we replace $J(0) = 0$ with $J(0) = J^U$. Thus, $J^U = -kl/q - c + \delta J(l)$, so (23) is replaced with $f(hl) = f'(hl)hl - W_l(h,l)l^2 + (1+r)c + rJ^U$, where J^U is given by (21). The equilibrium under endogenous entry is characterized by (25), (27), and

$$\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} AL^{\alpha} = (1+r)\left[c + b + \frac{\beta}{1-\beta}\theta k\right], \tag{39}$$

from which we define $L = L(\theta)$ with $L' > 0$. Thus, the equilibrium is characterized by a pair of h and θ that satisfy

$$\begin{aligned} \frac{\alpha}{\alpha\beta+1-\beta} A[L(\theta)]^{\alpha-1} &= e'(h), & (40) \\ \frac{(r+\lambda)k}{q(\theta)} + \beta\theta k &= (1-\beta)\left[e'(h)h - e(h) - b\right]. & (41) \end{aligned}$$

Equilibrium determination is depicted in Figure 4. Equation (41) gives the upward-sloping curve as in Figure 2. Equation (40) gives the downward-sloping curve. The intuition is roughly as follows.

[6] Note that there is a continuum of workers while the number of entrepreneurs equals the number of firms, which is finite. Thus, it requires a mass of entrepreneurs to change the total measure of workers, so we can safely assume that the measure of workers is invariant to entrepreneurs' decisions.

An increase in labor market tightness induces firms to substitute employment for longer hours of work, which defines the upward-sloping curve. On other hand, an increase in tightness makes entrepreneurship less attractive, which reduces the number of firms. Incumbent firms' reactions are to increase the total labor input, which reduces the marginal product of labor. Since firms want to match the marginal product with the marginal disutility of longer hours, the demand for hours decreases.

Comparative statics results are summarized below.

Proposition 11 *(a) An increase in A increases h and θ. (b) An increase in k has an ambiguous effect on h and decreases θ. (c) An increase in λ increases h and decreases θ.*

4 Dispersion in Hours of Work

4.1 Preliminaries

According to the Groningen Growth & Development Centre (GGDC) 60-Industry Database,[7] in 2003 in the US, the annual hours of work per employee were 2,306 hours for the mining industry, 1,615 hours for retail trade, and 1,350 hours for hotel and catering. In 2003 in the European Union, the annual hours work were 1,738 hours for the mining industry, 1,515 hours for retail trade, and 1,503 hours for hotel and catering. In 2002 in Japan, the annual hours of work per employee were 2,055 hours in the mining industry, 1,661 hours for retail trade, and 1,661 hours for hotel and catering.

Armed with the model developed in the preceding sections, we investigate possible determinants of dispersion in working hours. There could be a variety of reasons why people work differently, and we do not intend to give a comprehensive list of those reasons. Instead, we focus on job characteristics (rather than worker characteristics such as differences in preferences) as determinants of dispersion in hours of work. In particular, we present a simple model with two types of firms and explore the possibility of differentials in working hours. The key question in this section is:

[7] 60-Industry Database is available at http://www.ggdc.net/index-dseries.html. For a comprehensive account of this database, see Rogerson (2006), whose focus is on (the evolution of) cross-country differentials in hours of work.

which structural parameters are responsible for generating working hours differentials, and which parameters are not?

In this section and the next, we consider two models of hours dispersion. First, we develop a model of within-industry hours dispersion. This model assumes that the labor market is pooled for all jobs, so a job seeker can receive a job offer from a firm of either industry. Next, we consider a model of interindustry hours dispersion. This model assumes segmented labor markets, so a job seeker in an industry cannot receive a job offer from a firm of the other industry.

4.2 Within-Industry Differentials

Suppose that there is a single industry, but there are two types of firms, type 1 and type 2. As before, the matching technology is $m(U,V)$, and we define the market tightness as $\theta \equiv V/U$. The probability that a vacancy is filled is $q(\theta) \equiv m(U/V, 1)$, and the probability that a worker finds a job is $\theta q(\theta)$. We assume a single labor market. Thus, all firms and workers face the same labor market tightness θ.

The Bellman equation for a firm of type $j = 1, 2$ is given by $J_j(l_j) = \max_{h_j, v_j} \{A_j [h_j l_j]^\alpha - W^j(h_j, l_j) l_j - k_j v_j - c_j + \delta J_j(l_{+1}^j)\}$, where $l_{+1}^j = (1-\lambda) l_j + q(\theta) v_j$. Notice that both types of firms face the same market tightness. This reflects the assumption that the two firms are in the same industry so the markets are not segmented, and that workers will accept all types of jobs.[8] We assume that the bargaining weight is the same for all firms.[9] In addition, we assume that all firms face the same separation rate.[10] The first-order conditions with respect to h_j and v_j, and the envelope condition are $\alpha A_j [h_j l_j]^{\alpha-1} = W_h^j(h_j, l_j)$, $q(\theta)\delta J_j'(l_j) = k_j$, and $(r+\lambda)\delta J_j'(l_j) = \alpha A_j [h_j l_j]^{\alpha-1} h_j - W^j(h_j, l_j) - W_l^j(h_j, l_j) l_j$, respectively.

[8] To ensure this, we need to focus on equilibria where there is a nonnegative surplus for a relationship to share. This requires that the fixed cost c_i should not be too large.

[9] Here, we choose to avoid focusing on the bargaining weight because the foundation of this parameter is the relative frequency of making an offer in the context of strategic bargaining (Osborne and Rubinstein, 1990).

[10] It is evident that firms facing greater separation rates would choose longer working hours. A model with on-the-job search (Burdett and Mortensen, 1998) is required to generate a search equilibrium with endogenously dispersed separation rates.

As before, we assume that workers and the firm share the rent. The firm's share of rent is denoted by $1-\beta$, where $\beta \in [0,1]$. The Nash bargaining requires $\beta J_j'(l_j) = (1-\beta)(J_j^E - J^U)$ for $j = 1, 2$, where the value of being employed by a firm of type j is $J_j^E = P_j + \lambda\delta J^U + (1-\lambda)\delta J_j^E$, where $P_j \equiv W^j(h_j, l_j) - e(h_j)$. It is easy to establish that the wage function is given by

$$W^j(h_j, l_j) = \frac{\alpha A_j h_j^\alpha l_j^{\alpha-1}}{\alpha + \frac{1-\beta}{\beta}} + (1-\beta)\, e\,(h_j) + \frac{(1-\beta)\, r}{1+r} J^U. \tag{42}$$

Let ϕ denote the equilibrium fraction of type 1 vacancies. The probability that a job seeker finds a position is $\theta q(\theta)$. Given that a job seeker finds a position, he or she is employed by a firm of type 1 with probability ϕ. The value of being unemployed is $J^U = b + \theta q(\theta)[\phi\delta J_1^E + (1-\phi)\delta J_2^E] + (1-\theta q(\theta))\delta J^U$, or

$$\frac{r}{1+r} J^U = b + \theta q\,(\theta) \left[\phi\delta \left(J_1^E - J^U\right) + (1-\phi)\, \delta \left(J_2^E - J^U\right)\right]. \tag{43}$$

Noting $J_j^E - J^U = (1-\beta)^{-1}\beta(1+r)k_j/q(\theta)$, we have

$$\frac{r}{1+r} J^U = b + \frac{\beta\theta\,[\phi k_1 + (1-\phi)\, k_2]}{1-\beta}. \tag{44}$$

Lemma 12 *$P_1 > P_2$ holds if and only if $k_1 > k_2$.*

Proof. From the employee's value functions, it is easy to verify that $J_1^E - J_2^E = (r+\lambda)^{-1}(1+r)[P_1 - P_2]$. Similarly, the bargaining outcome and the vacancy choice imply $J_j^E - J^U = (1-\beta)^{-1}\beta(1+r)k_j/q(\theta)$ for $i = 1, 2$. From these equations, we obtain

$$P_1 - P_2 = \frac{\beta}{1-\beta}\frac{r+\lambda}{q(\theta)}(k_1 - k_2).$$

It is then easy to establish that $P_1 > P_2$ holds if and only if $k_1 > k_2$. ■

In other words, jobs with higher recruitment costs are "good jobs" (Acemoglu, 2001).

Assuming endogenous entry ($J(0) = J^U$),[11] the equilibrium conditions are

$$\begin{aligned}
\frac{\alpha}{\alpha\beta+1-\beta} A_j L_j^{\alpha-1} &= e'(h_j), && (45)\\
\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} A_j L_j^{\alpha} &= (1+r)\left[c_j + b + \frac{\beta\theta\left[\phi k_1 + (1-\phi)k_2\right]}{1-\beta}\right], && (46)\\
\frac{(r+\lambda)k_j}{q(\theta)} + \beta\theta k_j &= (1-\beta)\left[e'(h_j)h_j - e(h_j) - b\right], && (47)
\end{aligned}$$

for $j = 1, 2$. The equilibrium is characterized by 6 equations in 6 unknowns, L_1, L_2, h_1, h_2, θ, and ϕ. Equation (47) suggests that, given the equilibrium labor market tightness θ, there is a one-to-one relationship between k_j and h_j. It suggests that within-industry differentials in hours of work can arise only when $k_1 \neq k_2$. Remember that k_j captures the cost of creating a vacancy, and it works similarly to the cost of creating a job in Acemoglu (2001), in which the difference in the cost of equipment for each job creates the difference in job characteristics such as wages and productivity. To emphasize:

Proposition 13 *Within-industry differentials in working hours arise if and only if $k_1 \neq k_2$. In particular, $k_1 > k_2$ implies that $h_1 > h_2$. Technology differentials cannot account for differentials in hours of work.*

Jobs with a greater recruitment cost are "good jobs" as shown in Lemma 12. Further, Proposition 13 implies that good jobs are associated with longer hours work. Since there are two types of jobs in a single industry, a possible interpretation of the result is that it describes the differentials in hours of work between full-time and part-time jobs, or differentials between jobs that require training and those that do not require training. To see the difference in recruitment costs, we quote Hamermesh (1993, p. 208):[12]

> A survey of employers in the Rochester, New York, area in 1965–66 found an average hiring cost for all occupations of \$910, but an average for professional and managerial

[11] Unfortunately, the basic model cannot be used to study heterogeneous firms because it cannot support an equilibrium with heterogeneous firms. Thus, the model we employ in what follows is the one with endogenous entry presented in Section 3.4. The consequence of adoping the model with free entry is presented in Appendix B.

[12] All values are in 1990 US dollars.

> workers of \$4,600. A survey in Los Angeles in 1980 found hiring and training costs of \$13,790 for salaried workers, and \$5,110 for production workers. [...] In a nationwide survey of large employers in 1979 the cost of hiring a secretary was \$680, but for a college graduate was \$2,200.

Thus, there is a sizable difference in recruitment costs, and jobs that require higher skills are more costly to fill. The model suggests that such jobs pay more, and require longer hours of work.

4.3 Interindustry Differentials

To account for interindustry hours dispersion, this section studies a directed search version of the model in the spirit of Acemoglu (2001, Section III.B) and Moen (1997). The key feature is that there are two industries and their labor markets are segmented: once entered, a worker will never receive a job offer from a firm of the other industry. However, each unemployed can freely choose one of the industries at the beginning of each period. Arbitrage dictates and the values of seeking a job in the two industries must be balanced in equilibrium.

Suppose that the markets for the two types of jobs are segmented, and each worker can apply to either industry 1 or industry 2. We assume that the two industries face the same matching technology. Let U_j denote the number of unemployed workers applying to a firm of industry j. Similarly, V_j is the number of vacancies of industry j. Thus, the probability that a worker applying to industry j finds a job in that industry is $\theta_j q(\theta_j)$, where $\theta_j \equiv V_j/U_j$. Let μ be the fraction of workers in industry 1. Then, (2) is replaced with $m(U_1, V_1) = \lambda(1 - U_1)\mu$ and $m(U_2, V_2) = \lambda(1-U_2)(1-\mu)$, which define the Beveridge curves for industries 1 and 2, respectively.

Following Acemoglu (2001), we assume that each industry produces a distinct commodity. Let $p_1(\mu)$ and $p_2(\mu)$ be the prices of the product in industries 1 and 2, respectively. We assume that $p_1(\mu)$ is decreasing and $p_2(\mu)$ is increasing.[13]

As before, we assume that the bargaining weight is the same for all industries. In addition, we assume that all firms face the same separation rate. The Bellman equation for a firm in

[13] In general, the price function is more complex. However, we simply postulate the price function in order to avoid further complication of the analysis. For a derivation of the price function, see Acemoglu (2001), for example.

industry $j = 1, 2$ is given by $J_j(l_j) = \max_{h_j, v_j}\{p_j(\mu)A_j[h_j l_j]^\alpha - W^j(h_j, l_j)l_j - k_j v_j - c_j + \delta J_j(l^j_{+1})\}$, where $l^j_{+1} = (1-\lambda)l_j + q(\theta_j)v_j$. The first-order conditions with respect to h_j and v_j, and the envelope condition are $\alpha p_j(\mu)A_j[h_j l_j]^{\alpha-1} = W^j_h(h_j, l_j)$, $q(\theta_j)\delta J'_j(l_j) = k_j$, and $(r+\lambda)\delta J'_j(l_j) = \alpha p_j(\mu)A_j[h_j l_j]^{\alpha-1}h_j - W^j(h_j, l_j) - W^j_l(h_j, l_j)l_j$, respectively.

The Nash bargaining requires $\beta J'_j(l_j) = (1-\beta)(J^E_j - J^U_j)$ for $j = 1, 2$. The value of applying to industry $j = 1, 2$ is $J^U_j = b + \theta_j q(\theta_j)\delta J^E_j + (1 - \theta_j q(\theta_j))\delta J^U_j$. Similarly, the value of being employed is $J^E_j = P_j + \lambda\delta J^U_j + (1-\lambda)\delta J^E_j$ for $j = 1, 2$, where $P_j = W_j(h_j, l_j) - e(h_j)$. From these expressions, we obtain

$$\frac{r}{1+r}J^U_j = b + \frac{\beta}{1-\beta}\theta_j k_j$$

(see also (21)). For the two sectors to coexist, $J^U_1 = J^U_2$ must hold, which requires

$$k_1\theta_1 = k_2\theta_2 \tag{48}$$

in any equilibrium.

Lemma 14 *$P_1 > P_2$ holds if and only if $k_1 > k_2$.*

Proof. From the employee's value functions, it is easy to verify that $J^E_1 - J^E_2 = (r+\lambda)^{-1}(1+r)[P_1 - P_2]$. Similarly, the bargaining outcome and the vacancy choice imply $J^E_j - J^U = (1-\beta)^{-1}\beta(1+r)k_j\theta_j/\theta_j q(\theta_j)$ for $i = 1, 2$. From these equations, we obtain

$$P_1 - P_2 = \frac{\beta}{1-\beta}(r+\lambda)k_1\theta_1\left[\frac{1}{\theta_1 q(\theta_1)} - \frac{1}{\theta_2 q(\theta_2)}\right].$$

It is then easy to establish that $P_1 > P_2$ holds if and only if $\theta_2 > \theta_1$. Finally, (48) implies $k_1 > k_2$. ■

Jobs with higher recruitment costs are "good jobs" as in Lemma 12. The industry with a greater recruitment cost faces a less tight labor market because this industry pays more and attract more workers.

With endogenous entry, the equilibrium is characterized by

$$\begin{aligned}
\frac{\alpha}{\alpha\beta+1-\beta} p_j(\mu) A_j L_j^{\alpha-1} &= e'(h_j), && (49)\\
\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} p_j(\mu) A_j L_j^{\alpha} &= (1+r)\left[c_j + b + \frac{\beta}{1-\beta}\theta_j k_j\right] && (50)\\
\frac{(r+\lambda)k_j}{q(\theta_j)} + \beta\theta_j k_j &= (1-\beta)\left[e'(h_j)h_j - e(h_j) - b\right], && (51)
\end{aligned}$$

$k_1\theta_1 = k_2\theta_2$, $m(U_1, V_1) = \lambda(1-U_1)\mu$, $m(U_2, V_2) = \lambda(1-U_2)(1-\mu)$, $\theta_1 = V_1/U_1$, $\theta_2 = V_2/U_2$. Thus, the equilibrium is the solution to the system of 11 equations in 11 unknowns, L_1, L_2, h_1, h_2, θ_1, θ_2, U_1, U_2, V_1, V_2, and μ.

Proposition 15 *Suppose that all industries share the same β, λ, and $e(h)$. Interindustry dispersion in working hours arises if and only if $k_1 \neq k_2$. In particular, $k_1 > k_2$ implies $h_1 > h_2$. Technology differentials cannot account for differentials in hours of work.*

Proof. Equation (51) implies that h_j and the left-hand side of (51) is positively related. Since (48) holds, the second term of the left-hand side of (51) is the same for industries 1 and 2. So we can focus on comparing $(r+\lambda)k_1/q(\theta_1)$ and $(r+\lambda)k_2/q(\theta_2)$. Thus,

$$\frac{(r+\lambda)k_1}{q(\theta_1)} - \frac{(r+\lambda)k_2}{q(\theta_2)} = \frac{(r+\lambda)}{q(\theta_1)}\frac{k_2\theta_2}{\theta_1} - \frac{(r+\lambda)k_2}{q(\theta_2)} = k_2\theta_2\left[\frac{r+\lambda}{\theta_1 q(\theta_1)} - \frac{r+\lambda}{\theta_2 q(\theta_2)}\right] > 0$$

because $k_1 > k_2 \Leftrightarrow \theta_2 > \theta_1 \Leftrightarrow \theta_2 q(\theta_2) > \theta_1 q(\theta_1)$. This establishes that the left-hand-side of (51) is higher for industry 1 than industry 2. ■

If the labor markets are segmented for different industries and workers choose either of the two, then firms face different labor market tightness. As pointed out by Acemoglu (2001), firms facing a greater recruitment cost (or equipment cost) pay more. Thus, more workers apply to the high-paying industry, reducing the tightness in that industry. Proposition 15 establishes that such jobs require longer hours of work. Interestingly, differences in technology A_j cannot account for working hours differentials; they account for differences in the total labor input L_j across industries. Since employment at each firm satisfies $l_j = L_j/h_j$, both technology and recruitment costs account for differences in firm size across industries.

5 Conclusion

This paper has studied the choice of employment and hours of work in a dynamic model with search frictions and wage bargaining. It revealed that the bargaining outcome defines the wage profile that the firm faces when choosing the composition of employment and hours of work. We found that differences in recruitment costs are responsible for dispersion in hours of work both across industries and within an industry. In particular, "good jobs" (Acemoglu, 2001) pay more and require longer hours of work. Interestingly, technology differentials cannot account for working hours differentials.

An advantage of our approach is that the wage profile is derived as part of an equilibrium. Since the wage profile reflects both the production technology and the worker's disutility from labor, hours of work may be determined either by the firm's value maximization or by a worker's labor–leisure choice. In addition, the wage profile is consistent with the empirical labor supply curve. We found that hours of work determined by a worker's labor–leisure choice are too short. In contrast, hours of work determined by the firm's value maximization are too long, suggesting that the regulator must induce shorter hours of work. Our model highlights the empirical question as to who is to make the choice of hours of work. We also found that the Hosios condition is not enough for efficiency.

A limitation of our approach is that the basic model with free entry cannot support an equilibrium with heterogeneous firms. We have modified the basic model by replacing the free entry assumption with endogenous entry. Generally, one needs a richer model of multiple industries to account for heterogeneity of job characteristics in equilibrium, and much of this line of research is left for future research. A possible direction is suggested by Cahuc et al. (2008).

In this paper we did not consider heterogeneity of workers. Kuhn and Lozano (2005) documented that the observed increase in hours of work in the US was concentrated among skilled men. A deeper understanding of dispersion in hours of work requires a model with heterogeneous skills. It is interesting to introduce skilled and unskilled workers into the model of this paper to investigate whether the model can account for Kuhn and Lozano's (2005) findings.

In this paper we focused solely on the steady state. The choice of employment and hours of work is important for understanding business cycles. It is often documented that during a recession, firms cut employment rather than reducing hours of work enough to maintain the level of employment. Since employment adjustment is costly, it seems rational to hoard labor during a recession if the aggregate business condition comes back shortly (Burnside et al., 1993; Bertola and Caballero, 1994). This suggests that firms face a large and persistent negative shock during a recession. An important future work is to extend the model of this paper to explain the substitution between employment and hours of work over the business cycle.

Appendix

A Proof of Proposition 1

Rewrite (13) as

$$W_l(h,l) + \frac{1}{\beta l} W(h,l) = \frac{f'(hl)\,h}{l} + \frac{1-\beta}{\beta l}\left[e(h) + \frac{r}{1+r} J^U\right].$$

Let $y(l) \equiv W(h,l)$, $a(l) \equiv B/l \equiv \beta^{-1}/l$, and

$$g(l) \equiv \frac{f'(hl)\,h}{l} + \frac{1-\beta}{\beta l}\left[e(h) + \frac{r}{1+r} J^U\right] \equiv \pi(l) + \frac{\Pi}{l}.$$

Then the equation can be written more compactly as

$$y' + a(l)y = g(l), \tag{52}$$

which is a linear ordinary differential equation with variable coefficients, and its solution is known to be easily derived. A useful reference is Bellman and Cooke (1995, pp. 40–41.), from which the general solution to (52) is given by

$$y(l) = \exp\left\{-\int_0^l a(s)ds\right\}\left[\int_0^l \exp\left\{\int_0^i a(s)ds\right\} g(i)di + C\right],$$

where C is a constant. Since $y(l)$ is the wage function, it is (economically) reasonable to assume $y(0) = 0$, which leads to $C = 0$. Thus,

$$y(l) = \exp\left\{-\int_0^l a(s)ds\right\}\int_0^l \exp\left\{\int_0^i a(s)ds\right\} g(i)di = \int_0^l \exp\left\{-\int_i^l a(s)ds\right\} g(i)di.$$

Notice that

$$\exp\left\{-\int_i^l a(s)ds\right\} = \exp\left\{-B\int_i^l \frac{1}{s}ds\right\} = \exp\left\{-B\left[\ln s\right]_i^l\right\} = \exp\left\{-B\left[\ln l - \ln i\right]\right\} = l^{-B} i^B.$$

Thus,

$$\begin{aligned} y(l) &= \int_0^l l^{-B} i^B\left[\pi(i) + \frac{\Pi}{i}\right] di = l^{-B}\int_0^l i^B\left[\pi(i)\right] di + \Pi l^{-B}\int_0^l i^{B-1} di \\ &= l^{-B}\int_0^l i^B\left[\pi(i)\right] di + \frac{\Pi}{B}. \end{aligned}$$

Thus, the wage function we are looking for is

$$W(h,l) = l^{-1/\beta} \int_0^l i^{\frac{1}{\beta}-1} \left[f'(hi)\,h\right] di + (1-\beta)\left[e(h) + \frac{r}{1+r} J^U\right],$$

which is a generalization of the wage functions derived by Bertola and Caballero (1994), Bertola and Garibaldi (2001) and Cahuc and Wasmer (2001).

B Firm Heterogeneity under Free Entry

This section briefly discusses why the free entry assumption must be replaced with endogenous entry. Under free entry ($J(0)=0$), the equilibrium conditions are

$$\begin{aligned}
\frac{\alpha}{\alpha\beta+1-\beta} A_j L_j^{\alpha-1} &= e'(h_j), && (53)\\
\frac{(1-\beta)(1-\alpha)}{\alpha\beta+1-\beta} A_j L_j^{\alpha} &= (1+r)\,c_j, && (54)\\
\frac{(r+\lambda)\,k_j}{q(\theta)} + \beta\theta k_j &= (1-\beta)\left[e'(h_j)\,h_j - e(h_j) - b\right]. && (55)
\end{aligned}$$

Note that ϕ does not appear in (54), while (46) contains ϕ. It is easy to verify that (54) determines L_j, and given this, (53) determines h_j for $j=1,2$. However, (55) contradicts the fact that all firms face the same tightness θ.

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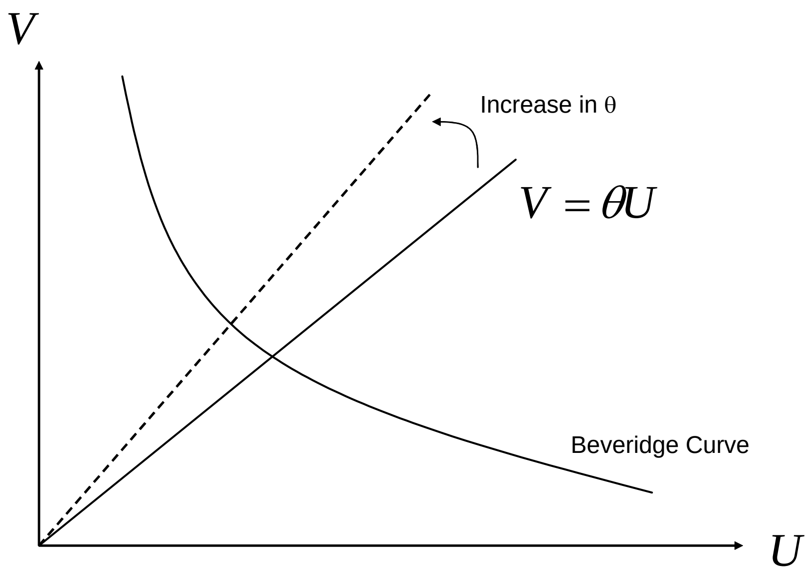

Figure 1

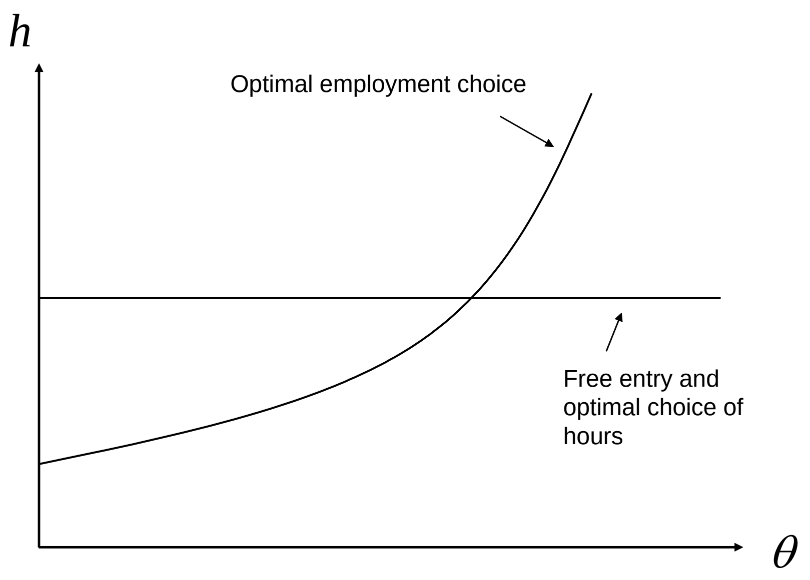

Figure 2

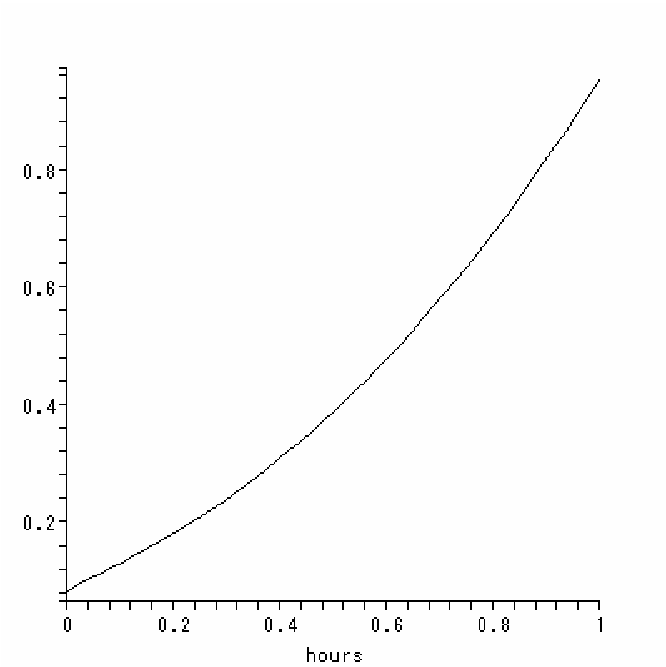

Figure 3: Wage Profile

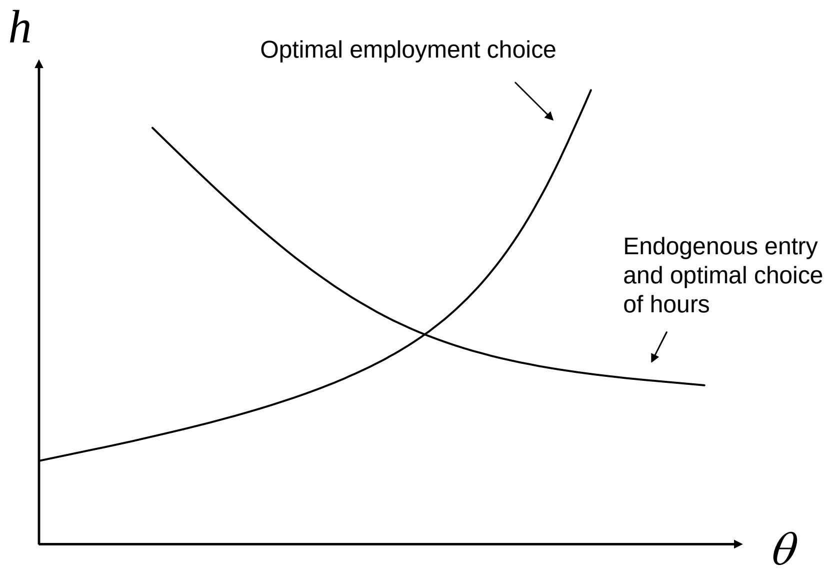

Figure 4