

# Asset-price collapse and market disruption

– A model of financial crises –

(Incomplete and preliminary)

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## Abstract

We construct a search theoretic model à la Lagos and Wright (2005), that has multiple steady-state equilibria, one of which may be interpreted as a state of financial crisis. The key ingredient is the collateral-secured loan in the decentralized matching market, in which the borrowers must put up their own land as collateral. They borrow debt for intertemporal smoothing of consumption stream and also for factor payment in production. In the crisis state, the land price is low and the debt for factor payment, i.e., liquidity, dries up. Facing liquidity shortage, all sellers choose not to participate in the matching market and the market is shut down due to the search externality. This market disruption lowers the aggregate productivity, while the low productivity justifies the low asset price in turn.

We may be able to derive a policy implication that collective debt reduction by the government intervention may solve the coordination failure and bring the economy out of the crisis equilibrium.

Keywords: Collateral-secured loan, search frictions, liquidity, chain of productions.

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# 1 Introduction

In the recent financial crises, asset-price collapses caused tightening of credit in the economy where collateral-secured loans were widespread. Examples are Japan's lost decade in the 1990s and the Asian currency crisis in 1997–1998, and the global financial crisis in 2008. Real estate, such as commercial properties and/or housing assets, are used as collateral for loans all over the world, and once the prices of collateral assets collapse credit and liquidity are severely tightened.

Macroeconomic theorists so far proposed two different kinds of models of collateral lending. In one strand of models collateral lending smooths consumption and investment intertemporally (Kiyotaki and Moore 1997; Bernanke, Gertler, and Gilchrist 1999; Iacoviello 2005). In the other models collateral lending is modeled as an intratemporal debt that is used as an instrument for factor payment for production (Carlstrom and Fuerst 1997, 1998; Jermann and Quadrini 2006; Mendoza 2006; Kobayashi, Nakajima and Inaba 2007; Kobayashi and Nutahara 2007). We call the instrument for factor payment *liquidity* in this paper. Collateral lending works either as an intertemporal smoother of consumption and investment or as an intratemporal liquidity in respective strand of existing models.

In this paper, we propose a model in which collateral lending plays both roles of an intertemporal smoother and of intratemporal liquidity. We show that the interaction between the two roles of collateral lending causes multiple steady-state equilibria to exist. There are two steady states, one of which is a bad equilibrium where the asset price collapses and a decentralized matching market is disrupted due to shortage of liquidity. In the bad equilibrium, the land price is low and the intratemporal debt, i.e., liquidity, dries up because of the shortage of collateral. Consequently, the decentralized matching market is disrupted: Since patient agents choose whether to participate in the matching market, when the land price is low all patient agents choose not to participate and the market is shutdown. The participation choice by patient agents has an external effect on impatient agents through search probability, which makes the bad equilibrium a stable equilibrium. The market disruption lowers the aggregate productivity, while

the low productivity justifies the low asset price in turn. We may be able to derive an implication for “macroprudential policy” (Borio 2003, Bernanke 2008) or financial crisis management: Collective debt reduction by the impatient agents due to the government intervention may solve the coordination failure and bring the economy out of the crisis equilibrium.

**Related literature:** This paper shows that an asset-price collapse can cause a potentially persistent, maybe decade-long, recession due to liquidity shortage. The mechanism proposed in this paper may be a possible explanation for persistent productivity declines observed in various episodes of the “great depressions.” See Kehoe and Prescott (2002, 2007) and references therein for neoclassical account for the great depressions in the 20th century: In the most of the episodes, productivity declines were the primal factor that caused the depressions, while the root causes of productivity declines are left unexplained in these research (see also Ohanian 2001). The mechanism of market disruption in our model, that is, the disruption of transactions in the matching market, is close to the disruption of chain of productions (or the division of labor among firms) in Blanchard and Kremer (1997) and Kobayashi (2004, 2006). One contribution of this paper is that the model in this paper is able to analyze the relationship between the asset price and the market disruption explicitly through adoption of the search theoretic framework of Lagos and Wright (2005). Although the Lagos-Wright framework is intended to contribute to monetary theory (see Rocheteau and Wright 2005 and Lagos and Rocheteau 2005 for application of this framework in the monetary theory literature), we use it to construct essentially a “real” model in the spirit of Carlstrom and Fuerst (1997, 1998).

In the next section, we present the model and show our basic results including policy implication for financial crisis management. In Section 3, we consider a generalized model where multiple matching markets open sequentially in the day time. Section 4 concludes. In Appendix, we briefly describe a monetary version of our model.

## 2 Model

The model is a variant of the search theoretic model developed by Lagos and Wright (2005), in which we introduce collateral lending as a payment instrument just like Ferraris and Watanabe (2008). Unlike Ferraris and Watanabe's model, in which lenders give cash to the borrowers and the total amount of cash is exogenously given by the government, banks in our model can costlessly create the payment instruments, which may be interpreted as bank notes or promissory notes.

### 2.1 Setup

The model is a closed economy à la Lagos and Wright (2005), in which there are continua of patient agents and impatient agents, who live forever. The measures of patient and impatient agents are  $M$  and 1 respectively, while  $M$  is sufficiently large:

$$M \gg 1. \tag{1}$$

There is also a unit mass of banks that can create payment instruments (i.e., bank notes) costlessly and lend them to the agents as collateral-secured loans. Time is discrete and continues from 0 to infinity:  $t = 0, 1, \dots, \infty$ . The numeraire is the consumption good. In each date  $t$ , market opens twice: The day market and the night market. Agents can consume the consumption good only in the night market. The consumption goods are not storable, that is, the consumption goods produced in the day market or the night market of date  $t$  must be consumed in the date- $t$  night market, otherwise they perish before date  $t + 1$  begins. The day market is a decentralized search market in which patient agents and impatient agents trade intermediate goods when they meet. A trade in the day market is *quid pro quo* and no trade credit is available. The night market is a centralized Walrasian market in which patient and impatient agents and banks trade consumption goods, land, bonds issued by impatient agents, and bank notes.

In the day market, when a patient agent and an impatient agent meet each other, the patient agent produces the intermediate good and sells it to the impatient agent at a competitive market price. The impatient agent pays for the intermediate good by giving

the bank notes that she borrowed from a bank in the previous night market. Patient agents need to pay a fixed cost for participating in the day market and they choose whether they participate in the day market or not at the beginning of each period  $t$ . In the night market, impatient agents issue intertemporal bonds and patient agents buy the bonds in exchange for the consumption good. Impatient agents also borrow bank notes from banks and can hold productive asset, i.e., land. For simplicity we assume that patient agents cannot hold land. Impatient agents have incomplete commitment technology. Therefore, they cannot precommit to redemption of their bonds nor repayment of their bank loans. However, the impatient agents can use land as collateral for their bonds and bank loans. Banks have complete commitment technology: They can commit themselves to that in the night market they pay one unit of consumption good in exchange for one unit of bank note that they issued in the previous night market. Therefore, bank notes are circulated as a payment instrument in the day market.

## 2.2 Optimization problem for patient agents

The state variable for a patient agent who enters the day market is the amount of bonds that she holds from the previous period. She holds bonds issued by impatient agents as her asset. When she enters the night market the bonds and the revenue she earned in the day market are the state variables. We denote the value function for a patient agent when she enters the day market by  $V^p(d_{t-1})$  and that for the night market by  $W^p(n_t, d_{t-1})$ , where  $d_{t-1}$  is bond holding and  $n_t$  is the revenue (i.e., bank notes) she gets in the day market. Both  $d_{t-1}$  and  $n_t$  are measured in the unit of date- $t$  consumption good, that is, one unit of bond or bank note issued in date- $(t-1)$  night market is a claim to one unit of date- $t$  consumption good.

At the beginning of the day market, a patient agent chooses whether she participate in the day market by paying fixed cost  $\kappa$  for market participation. The Bellman equation

for the patient agent is

$$V^p(d_{t-1}) = \max \left\{ -\kappa + \alpha(\mu_t) \max_{q_t} [-\gamma(q_t) + W^p(p_t q_t, d_{t-1})] + \{1 - \alpha(\mu_t)\} W^p(0, d_{t-1}), \quad W^p(0, d_{t-1}) \right\} \quad (2)$$

$$= \max \left\{ -\kappa + \alpha(\mu_t) \max_{q_t} [-\gamma(q_t) + W^p(p_t q_t, d_{t-1}) - W^p(0, d_{t-1})], \quad 0 \right\} + W^p(0, d_{t-1}), \quad (3)$$

where  $q_t$  is the quantity of the intermediate good produced;  $\gamma(q_t)$  is the cost for production, which satisfies  $\gamma'(q) > 0$ ,  $\gamma''(q) > 0$ , and  $\gamma(0) = 0$ ;  $p_t$  is the competitive price of the intermediate good;  $\alpha(\mu_t)$  is the matching probability for a patient agent with which she meets an impatient agent in the day market, where  $\mu_t$  ( $0 \leq \mu_t \leq M$ ) is the measure of the patient agents who participate in the day market. The expected gain for a patient agent from participating in the day market,  $\pi$ , is

$$\pi \equiv -\kappa + \alpha(\mu_t) \max_{q_t} \{-\gamma(q_t) + W^p(p_t q_t, d_{t-1}) - W^p(0, d_{t-1})\}. \quad (4)$$

Given the aggregate measure of participating patient agents,  $\mu_t$ , a patient agent decides to participate if  $\pi > 0$ ; not to participate if  $\pi < 0$ ; and is indifferent if  $\pi = 0$ . The Bellman equation for the night market is

$$W^p(n_t, d_{t-1}) = \max_{c_t, h_t, d_t} c_t - h_t + \beta V^p(d_t), \quad (5)$$

$$\text{subject to} \quad c_t \leq d_{t-1} - \frac{d_t}{1 + r_t} + n_t + h_t, \quad (6)$$

where  $c_t$  is the consumption,  $h_t$  is the labor supply,  $r_t$  is the real interest rate, and  $\beta$  ( $0 < \beta < 1$ ) is the intertemporal discount factor. To simplify the analysis, we make the assumption that patient agents directly gain  $c_t$  units of utility from consuming  $c_t$ , which simplify the form of collateral constraint in the problem of impatient agents. As is standard in the Lagos-Wright framework, we assume that the labor input in the night market gives linear disutility and is transformed into the consumption good linearly. As shown below, this convention simplifies the analysis greatly by making all agent choose

the same value of  $d_t$  and degenerating the heterogeneity among agents with respect to revenues in the period- $t$  day market. Substituting  $h_t$  in the budget constraint into (5), we have

$$W^p(n_t, d_{t-1}) = \max_{d_t} d_{t-1} - \frac{d_t}{1+r_t} + n_t + \beta V^p(d_t). \quad (7)$$

The first-order condition (FOC) for (7) is

$$\beta V_d^p(d_t) = \frac{1}{1+r_t}. \quad (8)$$

The envelope conditions for (7) are

$$W_n^p(n_t, d_{t-1}) = 1, \quad (9)$$

$$W_d^p(n_t, d_{t-1}) = 1. \quad (10)$$

These conditions imply that  $W^p(n_t, d_{t-1}) = n_t + d_{t-1} + W_0^p$ , where  $W_0^p \equiv W^p(0, 0)$ .

Therefore, the Bellman equation for the day market can be rewritten as

$$V^p(d_{t-1}) = \max\{-\kappa + \alpha(\mu_t) \max_{q_t}[-\gamma(q_t) + p_t q_t], 0\} + d_{t-1} + W_0^p. \quad (11)$$

Since the price of the intermediate good is determined competitively,<sup>1</sup> FOC for  $q_t$  is

$$p_t = \gamma'(q_t). \quad (12)$$

The envelope condition, i.e.,  $V_d^p(d_{t-1}) = 1$ , and (8) imply that

$$\beta(1+r_t) = 1. \quad (13)$$

The measure of the patient agents who participate in the day market,  $\mu_t$ , is determined endogenously by participation decision of individual agents. The gain from participation  $\pi(\mu_t)$  can be rewritten as

$$\pi(\mu_t) = -\kappa + \alpha(\mu_t)\{-\gamma(q_t) + p_t q_t\}. \quad (14)$$

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<sup>1</sup>Although we assume for simplicity price-taking behavior of all agents in the matching market, we can easily modify our model such that the trading scheme in the day market is a bilateral bargaining or competitive search, following Rocheteau and Wright (2005).

There exist multiple equilibria corresponding to different values of  $\mu_t$ :  $\mu_t = 0$  can be an equilibrium since  $\pi(0) = -\kappa < 0$  and an impatient agent chooses not to participate when  $\pi < 0$ ; and the solution to  $\pi(\mu_t) = 0$  can be an equilibrium, too. Therefore,  $\mu_t$  is determined by the following equation in equilibrium:

$$\mu_t[\alpha(\mu_t)\{p_t q_t - \gamma(q_t)\} - \kappa] = 0. \quad (15)$$

### 2.3 Optimization problem for impatient agents

The state variables for an impatient agent who enters the decentralized day market are the amount of bonds she issued in the previous night market ( $b_{t-1}$ ); the amount of bank loans she borrowed in the previous night market ( $l_{t-1}$ ); and land she purchased in the previous night market ( $k_{t-1}$ ). The impatient agent borrows  $l_{t-1}$  from a bank in the form of bank notes, that is, she is given  $l_{t-1}$  units of bank notes when she borrows from the bank in the previous night market. Note that  $b_{t-1}$  and  $l_{t-1}$  are measured in the unit of date- $t$  consumption good. The state variables when she enters the current night market are  $b_{t-1}$ ,  $l_{t-1}$ ,  $k_{t-1}$ , the remaining bank notes ( $n'_t$ ), and the output she produced in the day market ( $y_t$ ). In the date- $t$  night market, the impatient agent must repay  $R_{t-1}l_{t-1}$  units of the date- $t$  consumption good to the banks, where  $R_{t-1}$  is the gross rate of return on the bank loan. The Bellman equation for the day market is

$$V(l_{t-1}, b_{t-1}, k_{t-1}) = \max_{y_t, q_t} \alpha(\mu_t)\mu_t W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) + \{1 - \alpha(\mu_t)\mu_t\}W(0, l_{t-1}, l_{t-1}, b_{t-1}, k_{t-1}), \quad (16)$$

$$\text{subject to } n'_t = l_{t-1} - p_t q_t \quad (17)$$

$$n'_t \geq 0, \quad (18)$$

$$y_t = Ak_{t-1}^\theta q_t^{1-\theta}, \quad (19)$$

where  $V(\cdot)$  is the value function for an impatient agent who enters the day market,  $W(\cdot)$  is that for those who enters the night market, and  $\alpha(\mu_t)\mu_t$  is the matching probability for an impatient agent to meet a patient agent in the day market.<sup>2</sup>

<sup>2</sup>We assume a standard constant-returns-to-scale matching function,  $\Lambda(\mu^p, \mu^i)$ , for the day market, where  $\mu^p$  and  $\mu^i$  are the measures of patient and impatient agents who participate in the day market. The



The Bellman equation for the night market is

$$W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) = \max_{c'_t, h'_t, l_t, b_t, k_t} U(c'_t) - h'_t + \beta' V(l_t, b_t, k_t), \quad (20)$$

$$\begin{aligned} \text{subject to } c'_t + a_t k_t + b_{t-1} + R_{t-1} l_{t-1} \\ \leq y_t + h'_t + \omega_t k_{t-1} + n'_t + \frac{b_t}{1+r_t} + a_t k_{t-1}, \end{aligned} \quad (21)$$

$$b_t + R_t l_t \leq a_{t+1} k_t, \quad (22)$$

$$l_t \geq 0, \quad (23)$$

where at the beginning of the night market one unit of land generates the dividend,  $\omega_t$ , which is exogenously given by nature;  $a_t$  is the date- $t$  land price;  $c'_t$  is the consumption;  $U(c)$  is the utility from the consumption, where  $U'(c) > 0$ ,  $U''(c) < 0$ , and  $U'(0) = +\infty$ ;  $h'_t$  is the labor supply; and  $\beta'$  is the intertemporal discount factor that satisfies

$$0 < \beta' < \beta < 1. \quad (24)$$

We also assume the Lagos-Wright convention that  $h'_t$  gives linear disutility and is transformed into the consumption good linearly. Because the impatient agent cannot precommit to repay  $b_t$  to the creditor (i.e., a patient agent) nor to repay  $R_t l_t$  to the bank in the date- $(t+1)$  night market, she must put up her land as collateral for  $b_t + R_t l_t$  when she borrow  $b_t$  and  $l_t$  in the date- $t$  night market. When the impatient agent repudiate her debt, the creditors seize the collateral and sell it off in the date- $(t+1)$  night market. Since there is no further penalty for repudiation, the debtor surely diverts and defaults on the excess amount of debt which exceeds the value of collateral. Therefore, there is no reason for the creditors to lend more than the value of collateral.<sup>3</sup> The Bellman equation

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matching probability for a patient agent is  $\Lambda(\mu^p, \mu^i)/\mu^p = \Lambda(1, \mu^i/\mu^p)$ , while that for an impatient agent is  $\Lambda(\mu^p, \mu^i)/\mu^i = \Lambda(1, \mu^i/\mu^p)\mu^p/\mu^i$ . Since  $\mu^p = \mu$  and  $\mu^i = 1$  in our model, the matching probability for a patient agent is  $\alpha(\mu)$  and that for an impatient agent is  $\alpha(\mu)\mu$ , where  $\alpha(\mu) \equiv \Lambda(1, 1/\mu)$ .

<sup>3</sup>Although we do not consider stochastic shocks to the asset prices in this paper, our model can be easily generalized to stochastic model. In such a case, the collateral constraint, (22), is rewritten as  $b_t + R_t l_t \leq E_t[a_{t+1}]k_t$ , where  $E_t[\cdot]$  is the mathematical expectations based on the information available at date  $t$ . This is the risk neutrality of the patient agent that leads her to evaluate the value of collateral

(20) can be rewritten as

$$\begin{aligned}
& W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) \\
&= \max_{c'_t, l_t, b_t, k_t} U(c'_t) - c'_t + y_t - a_t k_t + (\omega_t + a_t)k_{t-1} - R_{t-1}l_{t-1} + n'_t + \frac{b_t}{1+r_t} - b_{t-1} \\
&\quad + \beta'V(l_t, b_t, k_t),
\end{aligned}$$

subject to (22) and (23).

The FOCs are

$$U'(c'_t) = 1, \tag{25}$$

$$\frac{1}{1+r_t} - \xi_t + \beta'V_b(l_t, b_t, k_t) = 0, \tag{26}$$

$$\beta'V_k(l_t, b_t, k_t) = a_t - \xi_t a_{t+1}, \tag{27}$$

$$\beta'V_l(l_t, b_t, k_t) + \eta_t - R_t \xi_t = 0, \tag{28}$$

where  $\xi_t$  and  $\eta_t$  are the Lagrange multipliers for (22) and (23), respectively. The envelope conditions are

$$\begin{aligned}
W_y(t-1) &= 1, & W_l(t-1) &= -R_{t-1}, & W_{n'}(t-1) &= 1, \\
W_b(t-1) &= -1, & W_k(t-1) &= \omega_t + a_t,
\end{aligned}$$

where  $W_z(t-1)$  is the derivative of  $W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1})$  with respect to  $z$  ( $= y_t, n'_t, l_{t-1}, b_{t-1}$ , or  $k_{t-1}$ ). These conditions imply that  $W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) = y_t - R_{t-1}l_{t-1} + n'_t - b_{t-1} + (\omega_t + a_t)k_{t-1} + W_0$ , where  $W_0 \equiv W(0, 0, 0, 0, 0)$ . The Bellman equation for the day market can be rewritten as

$$\begin{aligned}
V(l_{t-1}, b_{t-1}, k_{t-1}) &= \max_{q_t} \alpha(\mu_t) \mu_t \{ A k_{t-1}^\theta q_t^{1-\theta} - p_t q_t \} + (1 - R_{t-1})l_{t-1} - b_{t-1} \\
&\quad + (\omega_t + a_t)k_{t-1} + W_0,
\end{aligned} \tag{29}$$

$$\text{subject to } p_t q_t \leq l_{t-1}, \tag{30}$$

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as the mathematical expectation of the value of the asset in the next period. If the utility function of the patient agents is not linear, the collateral constraint in the stochastic model is no longer as simple as the above one.

The FOC is

$$p_t = \frac{(1 - \theta)Ak_{t-1}^\theta q_t^{-\theta}}{1 + x_t}, \quad (31)$$

where  $x_t$  is the Lagrange multiplier for (30). The envelope conditions are

$$V_l(l_{t-1}, b_{t-1}, k_{t-1}) = \alpha(\mu_t)\mu_t x_t, \quad (32)$$

$$V_b(l_{t-1}, b_{t-1}, k_{t-1}) = -1, \quad (33)$$

$$V_k(l_{t-1}, b_{t-1}, k_{t-1}) = \omega_t + \alpha(\mu_t)\mu_t \theta Ak_{t-1}^{\theta-1} q_t^{1-\theta} + a_t. \quad (34)$$

Note that the collateral constraint (30) becomes binding with probability  $\alpha(\mu_t)\mu_t$  if it binds at all, since it becomes relevant only when the impatient agent successfully matches with a patient agent. Conditions (26)–(28) and (32)–(34) imply the following system of equations that determines the dynamics of the models:

$$\beta' \alpha(\mu_{t+1})\mu_{t+1} x_{t+1} + \eta_t = R_t \xi_t, \quad (35)$$

$$1 = (1 + r_t)\{\xi_t + \beta'\}, \quad (36)$$

$$a_t = \xi_t a_{t+1} + \beta' \left[ \omega_{t+1} + \alpha(\mu_{t+1})\mu_{t+1} \theta Ak_t^{\theta-1} q_{t+1}^{1-\theta} + a_{t+1} \right]. \quad (37)$$

## 2.4 Banks

A bank lends  $l_t$  units of bank notes to an impatient agent in the date- $t$  night market. The bank has the commitment technology that she can give one unit of the consumption good in exchange for one unit of the bank note in the date- $(t + 1)$  night market. The borrower repays  $R_t l_t$  in the form of the consumption good to the bank in the date- $(t + 1)$  night market, where  $R_t$  is determined competitively. If the borrower repudiate her debt, the bank can seize collateral from the borrower but cannot impose any further penalty on the borrower. The bank's profit in the unit of the date- $(t + 1)$  consumption good is  $R_t l_t - l_t$ . In the date- $t$  night market, the bank chooses  $l_t$  to maximize  $R_t l_t - l_t$ , where there is no restriction on the supply of  $l_t$ . Therefore, in equilibrium,

$$R_t = 1. \quad (38)$$

This result parallels to that about the intraperiod debt in Carlstrom and Fuerst's (1998) model. Note that  $R_t = 1$  is guaranteed by the fact that the bank can create costlessly an unlimited amount of the bank notes. If the bank needs to give cash to the borrower and the total supply of cash is fixed exogenously,  $R_t$  may exceed 1 when the total amount of cash is small. See Ferraris and Watanabe (2008) for this case.

## 2.5 Equilibrium

The total supply of land is fixed in this economy:

$$k_t = K. \quad (39)$$

The bond market clears:

$$\forall t, \quad Md_t = b_t. \quad (40)$$

The equilibrium of this economy is defined as follows.

**Definition 1** *A competitive equilibrium consists of prices,  $\{p_t, R_t, r_t, a_t\}$ , and quantities,  $\{c_t, c'_t, h_t, h'_t, q_t, k_t, d_t, b_t, l_t, \mu_t\}$ , that satisfy that (i) given the prices, the quantities satisfy (12) and (15), which are the optimality conditions for patient agents; (ii) given the prices and  $\{\mu_t\}$ , quantities solve impatient agents' optimization problems, (20) and (29), and banks' optimization problem; and (iii) equilibrium conditions, (39) and (40), are satisfied.*

The equilibrium path of this economy is described by the prices,  $\{p_t, R_t, r_t, a_t, x_t, \xi_t, \eta_t\}$ , and the quantities,  $\{c'_t, q_t, k_t, b_t, d_t, l_t, \mu_t\}$ , where they are the solution to the following system of equations: (12), (13), (15), (22), (25), (30), (31), (35), (36), (37), (38), (39), (40), and

$$\eta_t p_t q_t = 0. \quad (41)$$

Equations (13) and (36) imply that  $1 + r_t = 1/\beta$  and  $\xi_t = \beta - \beta'$ . We can show the following proposition:

**Proposition 1** *In equilibrium, either  $(q_t, \mu_t) = (q_h, \mu_h)$  or  $(q_t, \mu_t) = (0, 0)$ , where  $q_h = q(\mu_h)$  and  $\mu_h$  is the solution to  $\Pi(\mu) = \kappa$ ; where  $\Pi(\mu) \equiv \alpha(\mu)\{\gamma'(q(\mu))q(\mu) - \gamma(q(\mu))\}$  and  $q(\mu)$  is an increasing function defined by*

$$\gamma'(q_t)q_t^\theta = \frac{\alpha(\mu_t)\mu_t(1-\theta)AK^{-\theta}}{\alpha(\mu_t)\mu_t + \frac{\beta}{\beta'} - 1}. \quad (42)$$

*Proof:* In equilibrium either  $\eta_t = 0$  or  $\eta_t > 0$ . In the case where  $\eta_t > 0$ , the nonnegativity condition (23) is binding. Since  $l_t = p_t q_t = 0$ ,  $q_t = 0$  in equilibrium. Equation (15) implies that since the gain from participating the day market is zero for a patient agent, all patient agents decide not to participate and  $\mu_t = 0$  in this case. Therefore, in equilibrium where  $\eta_t = 0$ ,  $(q_t, \mu_t) = (0, 0)$ . In the case where  $\eta_t = 0$ , it must be the case that  $\mu_t \neq 0$ , since otherwise (35) implies that  $\xi_t = 0$ , which contradicts the fact that  $\xi_t = \beta - \beta' > 0$ . Therefore,  $\mu_t > 0$  when  $\eta_t = 0$ . With  $\mu_t > 0$ , (35) implies that  $x_t = (\beta - \beta')/\{\alpha(\mu_t)\mu_t\}$ . Therefore, (12), (31), and (39) imply that  $q_t$  satisfies (42). Note that (42) determines  $q_t$  as an increasing function of  $\mu_t$ , that is,  $q_t = q(\mu_t)$ , since the left-hand side of (42) is monotonically increasing in  $q_t$  and the right-hand side is monotonically increasing in  $\mu_t$ . Since  $\mu_t > 0$  and  $q_t = q(\mu_t)$  in the case where  $\eta_t = 0$ , (15) implies that  $\mu_t$  is determined by  $\Pi(\mu_t) = \kappa$ . Since (42) implies that  $q(0) = 0$ ,  $\Pi(0) = 0$ . Since the matching probability for a patient agent,  $\alpha(\mu)$ , converges to zero as  $\mu \rightarrow \infty$ , it must be the case that  $\lim_{\mu \rightarrow \infty} \Pi(\mu) = 0$ . Since  $\alpha(\mu)$  is decreasing in  $\mu$  and  $\gamma'(q(\mu))q(\mu) - \gamma(q(\mu))$  is increasing in  $\mu$ ,  $\Pi(\mu)$  has only one peak as shown in Figure 1 if functional forms for  $\alpha(\cdot)$  and  $\gamma(\cdot)$  are given appropriately.<sup>4</sup>

Figure 1

We assume that  $\kappa$  is sufficiently small and functional forms for  $\alpha(\cdot)$  and  $\gamma(\cdot)$  are given appropriately such that there exists unique  $\bar{\mu}$  such that  $\Pi(\mu)$  is increasing in  $\mu$  for  $0 \leq \mu \leq \bar{\mu}$  and decreasing for  $\mu \geq \bar{\mu}$ . And we assume that  $\kappa$  is sufficiently small such that  $\Pi(\mu) = \kappa$  has exactly two solutions,  $\mu_l$  and  $\mu_h$ , where  $0 < \mu_l < \mu_h < M$ . Note that we assumed that the total measure of patient agents,  $M$ , is sufficiently large.  $\Pi(\mu)$

<sup>4</sup>For example we can set that  $\alpha(\mu) = \min\{1, \mu^{-\delta}\}$  and  $\gamma(q) = q^\epsilon$ , where  $0 < \delta < 1 < \epsilon$ .

is increasing in the neighborhood of  $\mu_l$  and decreasing in the neighborhood of  $\mu_h$ . The equilibrium with  $\mu = \mu_l$  is unstable because a slight deviation from  $\mu_l$  is amplified by participation decisions of individual agents. The equilibrium with  $\mu_h$  is stable, since a deviation from  $\mu_h$  is corrected by participation decisions by individual agents. So we can focus on the solution  $\mu_t = \mu_h$ . We have shown that for appropriate parameters and functional forms,  $(q_t, \mu_t) = (q_h, \mu_h)$  in equilibrium where  $\eta_t = 0$ . *Q.E.D.*

Since there is no technological innovation nor capital accumulation in this model, there are no time-varying state variables relevant to the aggregate dynamics of the economy. So any sequence of  $\{q_t, \mu_t\}_{t=0}^{\infty}$  that satisfies  $\forall t, (q_t, \mu_t) \in \{(q_h, \mu_h), (0, 0)\}$ , can be an equilibrium path: Given any such sequence of  $\{q_t, \mu_t\}_{t=0}^{\infty}$ , the sequence of  $\{a_t, b_t\}_{t=0}^{\infty}$  is determined by (37) and

$$b_t = a_{t+1}K - p_h q_h \frac{\mu_{t+1}}{\mu_h}, \quad (43)$$

where  $p_h = \gamma'(q_h)$ . Therefore, an equilibrium path of this model is a sunspot equilibrium in the sense that any path that satisfies Proposition 1 can be the equilibrium outcome solely depending on the agents' expectations.

## 2.6 Steady States

In what follows in this paper we focus on the steady-state equilibria. Steady-state values of the asset price,  $a$ , and bonds,  $b$ , are determined by

$$a = \frac{\beta'}{1-\beta} \{\omega + \alpha(\mu)\mu\theta AK^{\theta-1}q^{1-\theta}\}, \quad (44)$$

$$b + pq \leq aK, \quad (45)$$

$$(46)$$

The welfare of patient and impatient agents are measured by the values of their value functions at the beginning of the daytime in the steady state:

$$V^p = d = \frac{b}{M}, \quad (47)$$

$$V = \frac{\alpha(\mu)\mu}{1-\beta'} \{AK^{\theta}q^{1-\theta} - pq\} + \frac{\omega K}{1-\beta'} - \frac{1-\beta}{1-\beta'} b, \quad (48)$$

where  $V^p$  and  $V$  are the welfare of patient and impatient agents, respectively. If we define the total welfare of the economy in the steady state,  $E$ , as the sum of the patient and impatient agents' welfare, it can be written as

$$E \equiv MV^p + V = \frac{\alpha(\mu)\mu}{1-\beta'} \{AK^\theta q^{1-\theta} - pq\} + \frac{\omega K}{1-\beta'} + \frac{\beta-\beta'}{1-\beta'} b. \quad (49)$$

There exist two types of steady state in our model: One with  $(q, \mu) = (q_h, \mu_h)$  and the other with  $(q, \mu) = (0, 0)$ .

### 2.6.1 Normal state

We call steady state with  $(q, \mu) = (q_h, \mu_h)$  “normal state.” The asset price and the amount of bond are determined by

$$a_n = \frac{\beta' \{\omega + \alpha(\mu_h)\mu_h \theta AK^{\theta-1} q_h^{1-\theta}\}}{1-\beta}, \quad (50)$$

$$b_n = a_n K - p_h q_h, \quad (51)$$

where  $q_h = q(\mu_h)$  and  $p_h = \gamma'(q_h)$ . In the equilibrium where  $\mu$  is positive, collateral is used to issue intertemporal bonds for consumption smoothing and also it is used to borrow bank notes for factor payment in the decentralized day market. The day market in which agents form chains of productions is operative in the equilibrium where  $\mu > 0$ , while it is shutdown in the equilibrium we describe in the following section.

### 2.6.2 Crisis state

We call the steady state with  $(q, \mu) = (0, 0)$  the “crisis state,” because the decentralized day market is shutdown in this steady state. The asset price and bonds are determined by

$$a_c = \frac{\beta' \omega}{1-\beta}, \quad (52)$$

$$b_c = a_c K. \quad (53)$$

Obviously from (50) and (52), the asset price is lower in the crisis state than in the normal state:  $a_n > a_c$ . The debt-asset ratio,  $b/(aK)$ , is higher in the crisis state than in

the normal state:

$$\frac{b_n}{a_n K} = \frac{1}{a_n K} (a_n K - p_h q_h) < 1 = \frac{b_c}{a_c K}. \quad (54)$$

It is shown as follows that the total welfare in the normal state,  $E_n$ , is higher than that in the crisis state,  $E_c$ . Define  $\Delta \equiv (1 - \beta')(E_n - E_c)$ . Then (49) implies

$$\Delta = \alpha(\mu_h)\mu_h \{AK^\theta q_h^{1-\theta} - \gamma'(q_h)q_h\} + (\beta - \beta') \left\{ \frac{\beta'}{1 - \beta} \alpha(\mu_h)\mu_h \theta AK^\theta q_h^{1-\theta} - \gamma'(q_h)q_h \right\}. \quad (55)$$

Since (42) implies  $\gamma'(q)q = (1 - \theta)\beta'\alpha(\mu)\mu AK^\theta q^{1-\theta} / \{\beta'\alpha(\mu)\mu + \beta - \beta'\}$  and  $\{\alpha(\mu)\mu + \beta - \beta'\} / \{\beta'\alpha(\mu)\mu + \beta - \beta'\} < 1/\beta'$ , this equation implies

$$\Delta > \left\{ 1 + \frac{(\beta - \beta')\beta'}{1 - \beta} \right\} \theta \alpha(\mu_h)\mu_h AK^\theta q_h^{1-\theta} > 0. \quad (56)$$

Therefore, the total welfare,  $E$ , is higher in the normal state than in the crisis state.

## 2.7 Coordination failure in the crisis state and policy implication

In date- $t$  night market, impatient agents choose  $l_t$ , given  $\mu_{t+1}$ ; and at the beginning of date- $(t + 1)$ , the measure of participating patient agents,  $\mu_{t+1}$ , is determined by participation choices of individual patient agents, which are made taking  $\mu_{t+1}$  and  $l_t$  as given.

In the crisis state, patient agents collectively set  $\mu_{t+1} = 0$  and impatient agents choose  $l_t = 0$  for all  $t$ . Given that  $l_t = 0$ , a patient agent has no incentive to participate in date- $(t + 1)$  day market: If she participate, she can meet with an impatient agent with probability 1 because  $\mu_{t+1} = 0$ . (Note that all impatient agents participate in the day market.) But the impatient agent has no payment instrument, and therefore the patient agent can get nothing in the day market, while she must pay  $\kappa$  when she enters the day market. Thus the expected gain from participation is negative for a patient agent.

Given that  $\mu_{t+1} = 0$ , an impatient agent has no incentive to borrow from a bank in date- $t$  night market and bring bank notes into date- $(t + 1)$  day market: Since she can meet with a patient agent in the day market with zero probability, the bank notes are



useless. Meanwhile, she wants to maximize the amount of bonds that she issues,  $b_t$ , since the market rate,  $r_t = \beta^{-1} - 1$  is cheaper than her subjective rate of time discount, i.e.,  $(\beta')^{-1} - 1$ . Therefore, all impatient agents set  $b_t$  at its maximum possible value,  $a_{t+1}K$ , and they set  $l_t = 0$ .

Note that the coordination failure that causes the crisis equilibrium to exist occurs due to the existence of both the intertemporal consumption loan,  $b_t$ , and the loan for intratemporal payment,  $l_t$ . Both debts are necessary to make multiple equilibria. For example, if we did not introduce consumption loan,  $b_t$ , in our model, the crisis state would be eliminated and the normal state would become the only steady-state equilibrium because  $l_t$  is always positive under the collateral constraint,  $l_t \leq a_{t+1}k_t$ . The novel feature of our model, that is, collateral-secured loan is used as both intertemporal consumption loan and intratemporal liquidity, enables the multiple steady-state equilibria to exist.

**Policy implication:** As we saw above, there is no incentive for impatient agents to reduce their bonds individually in the crisis state. Therefore, there may arise effectiveness of the government intervention to coordinate the expectations and eliminate the crisis equilibrium. For example, if the government intervenes and imposes the restriction that all impatient agents must set their intertemporal liabilities,  $b_t$ , such that  $b_t \leq a_{t+1}k_t - p_h q_h$ , then the crisis state is eliminated and the economy can immediately jump to the normal state. The government-coordinated debt restriction may be interpreted as a simplified model of the government policies during the episodes of financial crises, such as the 700 billion-dollar TARP (Troubled Assets Relief Program) scheme initiated by the Treasury in 2008, in which the US government will buy and dispose of the nonperforming assets and will ultimately bailout the debt-ridden households and firms. In the past episodes of financial crises all over the world, the governments of the crisis affected countries undertook various crisis-management policies to urge financial institutions to dispose of nonperforming assets and to reduce the debt burdens in the private sector. In light of our model, we can consider that the essence of these crisis-management policies may be to restrict  $b_t$  in order to restore the supply of liquidity,  $l_t$ .

Another policy implication would be that if the binding collateral constraint for borrowers is crucial in a financial crisis, policies targeting at lenders may not be effective to attain economic recovery. For example, rehabilitation of banking sector by capital injections and liquidity provision to banks by the central bank may not be effective to resolve the market disruptions due to a financial crisis unless the debt reduction of borrowers are properly addressed.

### 3 Generalization – Chains of productions

There may be several directions for generalization of this model. One of the most intriguing generalizations would be to incorporate fiat money in this model in a meaningful way, while it turns out to be impossible in our patient-and-impatient-agents framework. Because of the difference of time discount factors, impatient agents never hold nominal money in a steady-state equilibrium. It is also easily shown that even when an interest-bearing money, i.e., bank deposits, is introduced, the impatient agents do not hold deposit money in a steady state (See Appendix).

In this section, we consider a generalization that makes our stylized model a little closer to the reality of chains of productions and business cycles by introducing multiple matching markets which open distinct subperiods in the day time. We can assume that impatient agents go through the matching markets sequentially during the day time, in each of which they produce the consumption good using land and the intermediate good produced by patient agents. We assume a simple structure of chains of productions that an impatient agent who could not produce in a matching market cannot enter the subsequent matching markets and must go to the night market directly; and only an impatient agent who successfully produces in a matching market can enter the next matching market. In equilibrium, several matching markets may be operative, while the other matching markets are shutdown. Although we have only two steady states in the basic model, there arise naturally more than two steady-state equilibria in the generalized model, each of which is distinguished by the number of open matching markets or, equivalently, by the level of land price. In a steady state with a higher land price,

more matching markets are operative. The steady states in the generalized model may be interpreted as representing various stages in the business cycles, such as a boom, a shallow recession, a deep recession, etc. Thus the generalized model may be potentially useful to analyze both ordinary business cycles and extraordinary financial crises in a unified framework.

The generalized model shows a new theoretical possibility of expectations-driven business cycles: Changes in asset prices may cause changes in the current and future productivity through changes in the expectations on the number of operative matching markets, while the productivity changes justify the fluctuations in the asset prices. In short, the generalized model show that the expectations on the operative matching markets may drive the business cycles, while the expectations are justified by the induced productivity changes.<sup>5</sup> The feature of our model is exactly the same as sunspot equilibrium models in that the number of operative matching markets, the asset price, and the aggregate productivity are endogenously selected from candidates of steady states, and there is no selection mechanism that is based on the economic fundamentals.

### 3.1 Setup

The model is the same as the basic model described in the previous section except that there are  $N$  subperiods in the day time, where  $N \geq 2$ , and a distinct matching market opens in each subperiod. We call the matching market in  $i$ -th subperiod D- $i$  market, where  $i = 1, 2, \dots, N$ .

### 3.2 Optimization of impatient agents

An impatient agent enters D-1 market in the first subperiod of the day time of date  $t$ , bringing  $l_{t-1}$  units of bank notes which are borrowed from a bank,  $b_{t-1}$  units of debt

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<sup>5</sup>This type of the expectations-driven business cycles may be interesting because in the existing models of the expectations-driven business cycles pioneered by Beaudry and Portier (2004), the expectations on future productivity is exogenously given, while in our model the expectations change endogenously. See for examples of the existing models of the expectations-driven business cycles Christiano, Ilut, Motto and Rostagno (2007), Jaimovich and Rebelo (2008), and Kobayashi, Nakajima, and Inaba (2007).

obligations, and  $k_{t-1}$  units of land from date- $(t-1)$  night market. The impatient agent searches for a patient agent and if she successfully meets a patient agent she buys the intermediate good from the patient agent and produces the consumption good; and only if she successfully produces in D-1 market, she enters D-2 market in the second subperiod, bringing the output of D-1 market and the remaining bank notes. If she could not meet a patient agent in D-1 market, she cannot participate in any subsequent markets in the day time and must go directly to the night market. This process is repeated  $N$  times going from D-1 to D- $N$  markets. In D- $i$  market, an impatient agent meets a patient agent with probability  $\alpha(\mu_t^{(i)})\mu_t^{(i)}$ , where  $\mu_t^{(i)}$  is the *relative* measure of patient agents who participate in D- $i$  market, that is, the measure of participating patient agents divided by the measure of impatient agents who participate in D- $i$  market.<sup>6</sup> We denote variables in D- $i$  market by putting superscript  $(i)$  on those variables.

The Bellman equation for D- $i$  market ( $1 \leq i \leq N$ ) is as follows. The state variables in the value function are  $y_t^{(i-1)}$ , the consumption good that the agent carries at the beginning of D- $i$  market;  $n_t^{(i-1)}$ , the remaining bank notes that the agent carries at the beginning of D- $i$  market;  $l_{t-1}$ , the bank loan that the agent borrowed in date- $(t-1)$  night market;  $b_{t-1}$ , the bond; and  $k_{t-1}$ , the land.

$$V^{(i)}(y_t^{(i-1)}, n_t^{(i-1)}, l_{t-1}, b_{t-1}, k_{t-1}) = \max_{y_t^{(i)}, q_t^{(i)}, n_t^{(i)}} \alpha(\mu_t^{(i)})\mu_t^{(i)} V^{(i+1)}(y_t^{(i)}, n_t^{(i)}, l_{t-1}, b_{t-1}, k_{t-1}) \\ + \{1 - \alpha(\mu_t^{(i)})\mu_t^{(i)}\} W(y_t^{(i-1)}, n_t^{(i-1)}, l_{t-1}, b_{t-1}, k_{t-1}), \quad (57)$$

$$\text{subject to } n_t^{(i)} = n_t^{(i-1)} - p_t^{(i)} q_t^{(i)}, \quad (58)$$

$$n_t^{(i)} \geq 0, \quad (59)$$

$$y_t^{(i)} = A^{(i)} \{k_{t-1}\}^\theta \{q_t^{(i)}\}^{1-\theta} + y_t^{(i-1)}, \quad (60)$$

where  $V^{(i)}(\cdot)$  is the value function for an impatient agent entering D- $i$  market;  $V^{(N+1)}(\cdot) \equiv W(\cdot)$ ;  $y_t^{(0)} = 0$ ; and  $n_t^{(0)} = l_{t-1}$ . Note that the production technology given in (60)

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<sup>6</sup>Using the relative measures,  $\mu_t^{(i)}$ , the measure of impatient agents who participate in D- $i$  market is written as  $\prod_{j=1}^{i-1} \alpha(\mu_t^{(j)})\mu_t^{(j)}$  and the measure of patient agents who participate in D- $i$  market is  $[\prod_{j=1}^{i-1} \alpha(\mu_t^{(j)})\mu_t^{(j)}]\mu_t^{(i)}$ .

guarantees that the production in D- $i$  market is independent from the output in D- $j$  market for  $i \neq j$ . Note also that the assumption of chains of productions that only those who successfully produce in D- $i$  market can enter D- $(i + 1)$  market greatly simplifies the analysis by avoiding the curse of dimensionality. This is because this assumption guarantees that all impatient agents who participate in a matching market have identical trading history and have identical values of state variables.

Since the problem for an impatient agent in the night market is identical to the basic model, the Bellman equation for the night market is

$$W(y_t, n_t, l_{t-1}, b_{t-1}, k_{t-1}) = \max_{c'_t, h'_t, n_{t+1}^{(0)}, l_t, b_t, k_t} U(c'_t) - h'_t + \beta' V^{(1)}(0, n_{t+1}^{(0)}, l_t, b_t, k_t), \quad (61)$$

$$\begin{aligned} \text{subject to} \quad c'_t + a_t k_t + b_{t-1} \\ \leq y_t + h'_t + \omega_t k_{t-1} + n_t - R_{t-1} l_{t-1} + \frac{b_t}{1 + r_t} + a_t k_{t-1}, \end{aligned} \quad (62)$$

$$b_t + R_t l_t \leq a_{t+1} k_t, \quad (63)$$

$$l_t \geq 0, \quad (64)$$

$$n_{t+1}^{(0)} \leq l_t. \quad (65)$$

The same arguments as those in the previous section imply that the value function for the night market can be written as

$$W(y_t, n_t, l_{t-1}, b_{t-1}, k_{t-1}) = y_t + n_t - R_{t-1} l_{t-1} - b_{t-1} + (\omega_t + a_t) k_{t-1} + W_0. \quad (66)$$

Using this expression, the reduced form of the value function  $V^{(1)}(\cdot)$  is derived, given  $\{p_t^{(i)}, \mu_t^{(i)}\}_{i=1}^N$ : First,  $V^{(N)}(\cdot)$  is solved using (66); and  $V^{(i)}(\cdot)$  is solved backwardly from  $i = N - 1$  to  $i = 1$ . Thus we have

$$\begin{aligned} V^{(1)}(0, n_t^{(0)}, l_{t-1}, b_{t-1}, k_{t-1}) = \sum_{i=1}^N \max_{q_t^{(i)}, n_t^{(i)}} \left[ \prod_{j=1}^i \alpha(\mu_t^{(j)}) \mu_t^{(j)} \right] \left[ A^{(i)} k_{t-1}^\theta \{q_t^{(i)}\}^{1-\theta} - p_t^{(i)} q_t^{(i)} \right] \\ - R_{t-1} l_{t-1} - b_{t-1} + (\omega_t + a_t) k_{t-1} + W_0, \end{aligned} \quad (67)$$

$$\text{subject to} \quad p_t^{(i)} q_t^{(i)} + n_t^{(i)} \leq n_t^{(i-1)}, \quad \text{for } i = 1, 2, \dots, N, \quad (68)$$

$$n_t^{(i)} \geq 0, \quad \text{for } i = 1, 2, \dots, N. \quad (69)$$

Given  $\{p_t^{(i)}, \mu_t^{(i)}\}_{i=1}^N$ , the FOCs are

$$p_t^{(i)} = \frac{(1 - \theta)A^{(i)}k_{t-1}^\theta \{q_t^{(i)}\}^{-\theta}}{1 + x_t^{(i)}}, \quad (70)$$

$$x_t^{(i-1)} = \beta' \alpha(\mu_t^{(i)}) \mu_t^{(i)} x_t^{(i)} + \eta_t^{(i-1)}, \quad \text{for } i = 1, 2, \dots, N, \quad (71)$$

where  $x_t^{(i)}$  and  $\eta_t^{(i)}$  are the Lagrange multipliers for (68) and (69), respectively, and the FOCs for (61) and the envelope conditions for (67) imply that  $x_t^{(0)} = \beta - \beta'$  and  $\eta_t^{(0)} = 0$ .

### 3.3 Optimization of patient agent

We assume that at the beginning of each date a patient agent choose whether she participate in a decentralized matching market in the day time. A patient agent can participate in at most one market among D-1,  $\dots$ , D- $N$  markets. Thus the optimization problem for a patient agent is identical to the basic model. Therefore,  $\mu_t^{(i)}$ , the relative measure of patient agents who participate in D- $i$  market, is determined by

$$\left[ \alpha(\mu_t^{(i)}) \{p_t^{(i)} q_t^{(i)} - \gamma(q_t^{(i)})\} - \kappa \right] \mu_t^{(i)} = 0. \quad (72)$$

If  $\mu_t^{(i)} > 0$ ,  $q_t^{(i)}$  is determined by the FOC:

$$\gamma'(q_t^{(i)}) = p_t^{(i)}. \quad (73)$$

### 3.4 Equilibrium

We can define a competitive equilibrium of the  $N$ -market model as follows.

**Definition 2** *A competitive equilibrium consists of prices,  $\{\{p_t^{(i)}\}_{i=1}^N, R_t, r_t, a_t\}$ , and quantities,  $\{c_t, c'_t, h_t, h'_t, k_t, d_t, b_t, l_t, \{q_t^{(i)}, n_t^{(i)}, \mu_t^{(i)}\}_{i=1}^N\}$ , that satisfy that (i) given the prices, the quantities satisfy (72) and (73), which are the optimality conditions for patient agents; (ii) given the prices and  $\{\mu_t^{(i)}\}_{i=1}^N$ , the quantities solve impatient agents' optimization problems, (67) and (61), and banks' optimization problem; and (iii) equilibrium conditions, (39) and (40), are satisfied.*

The equilibrium values of variables in D- $i$  market are determined by solving (70), (71), (72) and (73) forwardly, given  $x_t^{(0)} = \beta - \beta'$  and  $\eta_t^{(0)} = 0$ , where  $\eta_t^{(i)} = 0$  if D- $i$  market

is operative. There exist multiple steady-state equilibria. D- $i$  market is either open or shutdown for  $i = 1, 2, \dots, N$  in a steady-state equilibrium. Since the assumption of chains of productions guarantees that if  $\mu_t^{(i)} = 0$  then  $\mu_t^{(j)} = 0$  for all  $j \geq i + 1$ , there exist at most  $N + 1$  steady-state equilibria: Each equilibrium is distinguished by the number of the matching markets which is not shutdown. We can define  $I$ -equilibrium ( $I = 0, 1, \dots, N$ ) as a steady-state equilibrium where the first  $I$  matching markets are operative, that is,  $\mu^{(i)} > 0$  for  $1 \leq i \leq I$  and  $\mu^{(i)} = 0$  for  $I + 1 \leq i \leq N$ . In  $I$ -equilibrium, the equilibrium variables in D- $i$  market are determined by the following system of equations, given  $x^{(0)} = \beta - \beta'$ :

$$p^{(i)} = \frac{(1 - \theta)A^{(i)}k_{t-1}^\theta \{q^{(i)}\}^{-\theta}}{1 + x^{(i)}}, \quad (74)$$

$$x^{(i-1)} = \beta' \alpha(\mu^{(i)}) \mu^{(i)} x^{(i)}, \quad (75)$$

$$\alpha(\mu^{(i)}) \{p^{(i)} q^{(i)} - \gamma(q^{(i)})\} - \kappa = 0, \quad (76)$$

$$\gamma'(q^{(i)}) = p^{(i)}, \quad (77)$$

for  $i = 1, 2, \dots, I$ ; and  $q^{(i)} = \mu^{(i)} = 0$  for  $i = I + 1, \dots, N$ .

The values of  $\{p^{(i)}, q^{(i)}, \mu^{(i)}, x^{(i)}\}_{i=1}^N$  are determined by solving the system of equations (74)–(77) forwardly from  $i = 1$  to  $i = N$ , given  $x^{(0)} = \beta - \beta'$ . Note that if D- $i$  market is open in both  $I$ -equilibrium and  $I'$ -equilibrium, the values of variables in D- $i$  market are identical in  $I$ - and  $I'$ -equilibria for  $I \neq I'$ .<sup>7</sup>

The asset price in  $I$ -equilibrium is determined by

$$a(I) = \frac{\beta' \{\omega + \sum_{i=1}^I [\prod_{j=1}^i \alpha(\mu^{(j)}) \mu^{(j)}] \theta A^{(i)} K^{\theta-1} \{q^{(i)}\}^{1-\theta}\}}{1 - \beta}, \quad (78)$$

or, equivalently,  $a(I) = a(I - 1) + \frac{\beta'}{1 - \beta} [\prod_{j=1}^I \alpha(\mu^{(j)}) \mu^{(j)}] \theta A^{(I)} K^{\theta-1} \{q^{(I)}\}^{1-\theta}$ . Obviously,  $a(I)$  is increasing in  $I$ , that is, the asset price is higher in the equilibrium where more matching markets are operative. The amount of bonds in  $I$ -equilibrium is determined

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<sup>7</sup>Note also that for a certain range of parameter values there may exist  $J (\leq N)$  such that the above system of equations (74)–(77) has no positive solution  $\{p^{(i)}, q^{(i)}, \mu^{(i)}, x^{(i)}\}$  for  $i = J$ . In this case, D- $J$ , D- $(J + 1)$ ,  $\dots$ , D- $N$  markets are always shutdown in any equilibrium. In this case, there exist only  $J$  steady-state equilibria: 0-, 1-,  $\dots$ ,  $(J - 1)$ -equilibria.

by

$$b(I) = a(I)K - \sum_{i=1}^I p^{(i)}q^{(i)}. \quad (79)$$

The welfare of an agent can be defined as the value of her value function at the beginning of the daytime in the steady state. Thus the welfare of a patient agent in  $I$ -equilibrium,  $V^p(I)$ , and that of an impatient agent,  $V^{(1)}(I)$ , are

$$V^p(I) = d(I) = \frac{b(I)}{M}, \quad (80)$$

$$V^{(1)}(I) = \frac{1}{1-\beta'} \left[ \omega K - (1-\beta)b(I) + \sum_{i=1}^I \left\{ \prod_{j=1}^i \alpha(\mu^{(j)})\mu^{(j)} \right\} \left( A^{(i)}K^\theta \{q^{(i)}\}^{1-\theta} - p^{(i)}q^{(i)} \right) \right]. \quad (81)$$

We can define the total welfare of the economy in  $I$ -equilibrium by  $E(I) \equiv MV^p(I) + V^{(1)}(I)$ , which is written as

$$E(I) = \frac{1}{1-\beta'} \left[ \omega K + (\beta - \beta')b(I) + \sum_{i=1}^I \left\{ \prod_{j=1}^i \alpha(\mu^{(j)})\mu^{(j)} \right\} \left( A^{(i)}K^\theta \{q^{(i)}\}^{1-\theta} - p^{(i)}q^{(i)} \right) \right]. \quad (82)$$

It is shown as follows that  $E(I)$  is increasing in  $I$ , that is, the welfare of the economy increases as the number of operative matching markets increases. Define  $\Delta(I) \equiv (1 - \beta')\{E(I) - E(I-1)\}$ .

$$\Delta(I) = \left[ \left\{ 1 + \frac{(\beta - \beta')\beta'\theta}{1 - \beta} \right\} \Gamma(I) - \{\Gamma(I) + \beta - \beta'\} \frac{1 - \theta}{1 + x(I)} \right] A^{(I)}K^\theta \{q^{(I)}\}^{1-\theta}, \quad (83)$$

where  $\Gamma(I) = \prod_{i=1}^I \alpha(\mu^{(i)})\mu^{(i)}$ . Since  $x(I) = (\beta - \beta')/[(\beta')^I \Gamma(I)]$  and  $\{\Gamma(I) + \beta - \beta'\}(\beta')^I / \{(\beta')^I \Gamma(I) + \beta - \beta'\} < 1$ , it is shown that

$$\Delta(I) > \left\{ 1 + \frac{(\beta - \beta')\beta'}{1 - \beta} \right\} \theta \Gamma(I) A^{(I)}K^\theta \{q^{(I)}\}^{1-\theta} > 0, \quad (84)$$

which implies that  $E(I) > E(I-1)$  for all  $I \geq 1$ .

**Policy implication:** Multiple steady-state equilibria arise in this generalized model due to the same coordination failure that we discussed in the previous section. This



model is essentially a sunspot equilibrium model and the equilibrium selection depends on the macroeconomic expectations. Government policy or regulation that restricts the amount of debt issued by impatient agents may improve the coordination failure and may change the equilibrium. For example, if the government imposes the restriction that  $b_t \leq a_{t+1}k_t - \sum_{i=1}^J p^{(i)}q^{(i)}$ , then the candidates of the steady-state equilibrium that can be realized become restricted to  $J$ -,  $(J + 1)$ -,  $\dots$ ,  $N$ -equilibria. In other words, the steady states from 0-equilibrium to  $(J - 1)$ -equilibrium are eliminated from the candidates of realizable steady-state equilibrium. Therefore, this model implies that monetary policy and/or financial regulations that restrict the aggregate level of debt may raise the aggregate productivity and the asset prices through increasing liquidity that enhances economic transactions in decentralized matching markets. This policy implication may be consistent with the historical episodes of financial crises. Japan and Sweden, for example, experienced collapses of land prices and emergences of nonperforming loans problems almost simultaneously in the early 1990s. Sweden disposed of bad loans aggressively in 1992–1994, and then attained a V-shaped economic recovery in the middle of the 1990s, while Japan postponed the disposal of nonperforming loans and its debt-ridden economy experienced decade-long slow growth.

## 4 Conclusion

In this paper, we propose a model in which collateral-secured loan is used to smooth consumption intertemporally and also used as a payment instrument, i.e., liquidity, intratemporally. We show that the interaction between the two roles of collateral lending causes multiple steady-state equilibria to exist. The role of liquidity to mitigate the search friction in the decentralized market plays the key role to generate the multiple equilibria. In our basic model, there are two stable steady states, one of which is a bad equilibrium where the asset price collapses and the decentralized matching market is disrupted due to shortage of liquidity. In the bad equilibrium, the land price is low and the intratemporal debt, i.e., liquidity, dries up because of the shortage of collateral. Consequently, the decentralized matching market is shutdown, since sellers choose not

to participate in the market facing buyers' liquidity shortage. This market disruption lowers the aggregate productivity, while the low productivity justifies the low asset prices in turn. The asset price is lower and the debt/asset ratio,  $b_t/(a_{t+1}K)$ , is higher in the bad equilibrium. Therefore, a policy implication for financial crisis management is that if the government imposes the restriction that all impatient agents must set their debt burden,  $b_t$ , such that  $b_t \leq a_{t+1}k_t - p_h q_h$ , then the crisis state is eliminated and the economy may jump to the normal state. This result seems to support the effectiveness of debt reduction policy as financial crisis management. Thus our analysis in this paper may shed some light on assessment of policies for financial crisis management that are explicitly concerned with the asset prices and the aggregate amount of debt.

## References

- Beaudry, P. and F. Portier (2004). "An exploration into Pigou's theory of cycles." *Journal of Monetary Economics* 51(6): 1183–1216.
- Bernanke, B. S. (2008) "Reducing systemic risk." Speech at the Federal Reserve Bank of Kansas City's Annual Economic Symposium, Jackson Hole, Wyoming, August 22, 2008.
- Bernanke, B. S., M. Gertler, and S. Gilchrist (1999). "The financial accelerator in a quantitative business cycle framework." In J. B. Taylor and M. Woodford eds., *Handbook of Macroeconomics* 1C. 1341–1393.
- Blanchard, O., and M. Kremer (1997). "Disorganization." *Quarterly Journal of Economics* 112(4): 1091–1126.
- Borio, C. (2003). "Towards a macroprudential framework for financial supervision and regulation?" BIS Working Papers No.128. Bank for International Settlements.

Carlstrom, C. T., and T. S. Fuerst (1997). “Agency costs, net worth, and business fluctuations: A computable general equilibrium analysis.” *American Economic Review* 87(5): 893–910.

Carlstrom, C. T., and T. S. Fuerst (1998). “Agency costs and business cycles.” *Economic Theory* 12: 583–597.

Christiano, L. J., C. Ilut, R. Motto, and M. Rostagno (2007). “Monetary policy and stock market boom-bust cycles.” European Central Bank Working Paper Series 955.

Ferraris, L., and M. Watanabe (2008). “Collateral secured loans in a monetary economy.” *Journal of Economic Theory* (forthcoming).

Iacoviello, M. “House prices, borrowing constraints, and monetary policy in the business cycle.” *American Economic Review* 95 (3): 739–764.

Jaimovich, N. and S. Rebelo (2008). “Can news about the future drive the business cycle?” *American Economic Review* (forthcoming).

Jermann, U., and V. Quadrini (2006). “Financial innovations and macroeconomic volatility.” Mimeo. University of Southern California.

Kehoe, T. J., and E. C. Prescott (2002). “Great depressions of the 20th century.” *Review of Economic Dynamics* 5 (1): 1–18.

Kehoe, T. J., and E. C. Prescott eds.(2007). *Great depressions of the twentieth century* Federal Reserve Bank of Minneapolis.

Kiyotaki, N., and J. Moore (1997). “Credit cycles.” *Journal of Political Economy* 105

(2): 211–48.

Kobayashi, K. (2004). “A theory of debt disorganization.” *Journal of Restructuring Finance* 1 (1): 1–11.

Kobayashi, K. (2006). “Payment uncertainty, the division of labor, and productivity declines in great depressions.” *Review of Economic Dynamics* 9 (4): 715–741.

Kobayashi, K., T. Nakajima, and M. Inaba (2007). “Collateral constraint and news-driven cycles.” RIETI Discussion Paper Series 07-E-013.

Kobayashi, K. and K. Nutahara (2007). “Collateralized capital and news-driven cycles.” *Economics Bulletin* 5 (17): 1–9.

Lagos, R., and G. Rocheteau (2005). “Inflation, output and welfare.” *International Economics Review* 46: 495–522.

Lagos, R., and R. Wright (2005). “A unified framework for monetary theory and policy analysis.” *Journal of Political Economy* 113 (3): 463–484.

Mendoza, E. G. (2006). “Endogenous sudden stops in a business cycle model with collateral constraints: A Fisherian deflation of Tobin’s  $q$ .” NBER Working Paper 12564.

Ohanian, L. E. (2001). “Why did productivity fall so much during the Great Depression?” *American Economic Review* 91 (2): 34–38.

Rocheteau, G., and R. Wright (2005). “Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium.” *Econometrica* 73 (1): 175–202.

## A Appendix

In this Appendix, we confirm that our results in the previous section hold even if we introduce money in our model. As is the case for Carlstrom and Fuerst's (1997) model, the setting of our model that there are two different types of agents, patient and impatient, implies that impatient agents do not hold cash for payment in a steady-state equilibrium. In this Appendix, we consider a slightly stronger form of money than cash. We confirm that money does not matter for our results in a modified version of our model in which we introduce deposit money that can earn interest.

We assume that agents can hold intertemporal bank deposits that earn interest with the rate  $r_t^d$ , instead of bonds. Impatient agents can use the principal and interest of their deposits as a payment instrument in the decentralized market in addition to the bank notes that they borrow from banks.

### A.1 Optimization of patient agents

Since patient agents hold bank deposit,  $d_t$ , instead of bonds,  $b_t$ , we get the Bellman equations for our monetary model by substituting  $d_t$  and  $r_t^d$  for  $b_t$  and  $r_t$  in the previous section. Note that  $d_t$  is measured in the unit of date- $(t + 1)$  consumption good.

$$V^p(d_{t-1}) = \max \{ -\kappa + \alpha(\mu_t)[- \gamma(q_t) + W^p(p_t q_t, d_{t-1}) - W^p(0, d_{t-1})], \quad 0 \} + W^p(0, d_{t-1}), \quad (85)$$

and

$$W^p(l_t, d_{t-1}) = \max_{d_t} d_{t-1} - \frac{d_t}{1 + r_t^d} + l_t + \beta V^p(d_t). \quad (86)$$

Similar arguments as those in the previous section imply

$$\beta(1 + r_t^d) = 1. \quad (87)$$

### A.2 Optimization of impatient agents

Since impatient agents can hold bank deposits,  $d_t'$ , as their assets, we need to include  $d_t'$  as the state variables. In this modified model, impatient agents borrow intertemporal

loan,  $b_t$ , from banks instead of borrowing from patient agents directly. The interest rate for  $b_t$  is  $r_t$ , where  $r_t \geq r_t^d$ . The Bellman equation for the day market is

$$V(l_{t-1}, b_{t-1}, d'_{t-1}, k_{t-1}) = \alpha(\mu_t)\mu_t \max_{y_t, q_t, l_t, d''_t} W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) \\ + (1 - \alpha(\mu_t)\mu_t)W(0, l_{t-1}, l_{t-1} + d'_{t-1}, b_{t-1}, k_{t-1}), \quad (88)$$

$$\text{subject to } n'_t = d'_{t-1} + a_t k_{t-1} - p_t q_t, \quad (89)$$

$$n'_t \geq 0, \quad (90)$$

$$y_t = A k_{t-1}^\theta q_t^{1-\theta}, \quad (91)$$

where  $n'_t$  is the total of bank notes and deposits that the impatient agent brings into the night market. The Bellman equation for the night market is

$$W(y_t, n'_t, l_{t-1}, b_{t-1}, k_{t-1}) = \max_{c'_t, l_t, b_t, d'_t, k_t} U(c'_t) - c'_t + y_t - a_t \{k_t - k_{t-1}\} + \omega_t k_{t-1} + n'_t - \frac{d'_t}{1 + r_t^d} - R_t l_t \\ + \frac{b_t}{1 + r_t} - b_{t-1} + \beta' V(l_t, b_t, d'_t, k_t), \quad (92)$$

subject to  $b_t + R_t l_t \leq a_{t+1} k_t + d'_t$ . FOCs and envelope conditions for  $b_t$  and  $d'_t$  imply

$$1 = (1 + r_t)\xi_t + \beta'(1 + r_t)(1 + \alpha(\mu_{t+1})\mu_{t+1}x_{t+1}), \quad (93)$$

$$1 \geq \beta'(1 + r_t^d)\{1 + \alpha(\mu_{t+1})\mu_{t+1}[R_{t+1} + R_{t+1}x_{t+1} - 1]\}, \quad (94)$$

where  $d'_t = 0$  if (94) holds with strong inequality. Therefore, equations (93) and (94) imply that  $d'_t = 0$  if  $R_{t+1} = 1$  and  $r_t^d < r_t$ , which, we will show below, is the case in the equilibrium.

### A.3 Optimization of banks

Banks accept deposits  $d_t$  from patient agents and  $d'_t$  from impatient agents, and invest them into intertemporal lending  $b_t$  and the real cash reserve  $m_t$  under a technological constraint that total deposits<sup>8</sup> cannot exceed  $\phi m_t$  where  $\phi$  ( $\phi > 1$ ) is a technological

<sup>8</sup>Here we make a slightly problematic assumption that the intraperiod bank notes,  $l_t$ , that the bank issues at the day market are not constrained by cash holding of the bank. Otherwise the constraint (97) would be  $d_t + d'_t + l_t \leq \phi m_t$  and the intraperiod rate of interest would exceed one, that is,  $R_t > 1$ . In this case, the analysis would be complicated and our results hold only for a certain range of parameter values. For simplicity, we assume in this paper that the constraint for the bank is (97).

parameter for production of payment services. The Optimization problem of bank in period  $t$  is written as follows:

$$\max_{l_t, b_t, d_t, d'_t} b_t + \frac{m_t}{1 + \pi_t} - (d_t + d'_t) + (R_t - 1)l_t, \quad (95)$$

$$\text{subject to } \frac{b_t}{1 + r_t} + m_t + l_t \leq \frac{d_t + d'_t}{1 + r_t^d} + l_t, \quad (96)$$

$$\frac{d_t + d'_t}{1 + r_t^d} \leq \phi m_t, \quad (97)$$

where  $\pi_t$  is the inflation rate. Note that  $l_t$  denotes the bank notes lent to impatient agents. The bank's problem can be rewritten as follows: Given  $r_t$ ,  $r_t^d$  and  $R_t$ ,

$$\max_{l_t, m_t} [(r_t - r_t^d)\phi + (1 + \pi_t)^{-1} - (1 + r_t)]m_t + (R_t - 1)l_t. \quad (98)$$

Since  $l_t < \infty$  and  $m_t < \infty$  in the equilibrium,

$$R_t = 1, \quad (99)$$

$$1 + r_t = \frac{\beta^{-1}\phi - (1 + \pi_t)^{-1}}{\phi - 1}. \quad (100)$$

As long as  $1 + \pi_t \geq \beta$ ,  $r_t$  exceeds  $r_t^d = \beta^{-1} - 1$ , and therefore,  $d'_t = 0$  from (93) and (94). It is easily shown that qualitatively the same results as those in Section 2 holds for the steady-state equilibria of the model in this Appendix as long as  $1 + \pi \geq \beta$ , since  $d'_t = 0$  in this case.

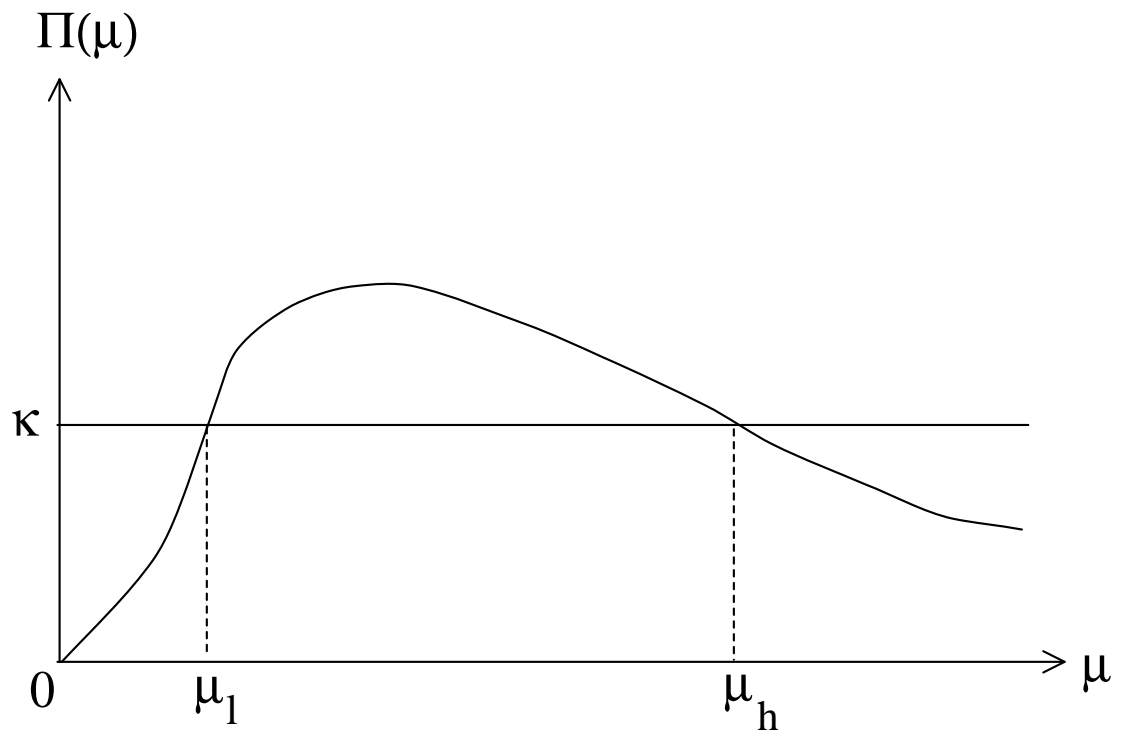


Figure 1: Determination of the measure of participating patient agents in Normal State