Labor Immobility in Japan: Its causes and consequences*

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Abstract

This paper builds and calibrate a model of competitive search that can reproduce a set of stylized facts concerning major impacts of the decade long stagnation and subsequent changes in the labor market in Japan. We highlight the role played by varying degrees of relation specific investments in forming employment relations. Depending upon the degree of specificity, a job slot can be immediately destructed, or, only the filled jobs survive, or, jobs survive as far as workers with specific training remain in the market. We obtain a rich variety of aggregate labor market responses to a negative productivity shock. By embedding such a system of employment in an economy plagued with limited capital mobility, we can generate upward drift in Beveridge curve, prolonged periods of labor adjustments, decline in the share of jobs with costly investment in training, and strong cohort effects. We also offer some preliminary results from numerical solutions of the model.

1 Introduction

In 1991, just before the onset of the decade long stagnation of the economy, 82% of the 44 million employees in Japan had regular[1] and full time jobs. The rest of employees, either temporary or part time or both, comprised the remaining 18%, or 7 and a half millions, out of which roughly 80% of them (5.5 million) were female. As of late 2007, around 34 million had regular and full time jobs, which now account only for 2/3 of the total employees, and the rest comprises the remaining third, or more than 17 millions.[2] The male workers in this category also rose steadily, comprising now roughly one third, or more than 5 millions. Another way to highlight the change is to compare net job losses and gains on regular, full time jobs and the rest. The size of regular and full time employees peaked in 1994 at 38 millions. Since then, we lost 3.3 million such jobs while temporary and/or part time jobs increased by 7.5 millions. In a way, the only major thing Japanese labor market accomplished in the last 15 years or so is to slowly but steadily slash permanent jobs and replace them by (more than one to one) temporary and part time jobs. This is shown in Figure 1.

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[1] 'Regular' (*Joyo*) workers refers to those under the employment contracts without pre-specified length of the employment.

[2] 11 millions of these temporary or part time workers - more than 60% of these, 25% of all the employees in Japan - earn 2 million yen (roughly 20,000US$) per year or less.

The change hit hardest younger cohorts: in 1998, among the age 15-24 cohort, 83 % of them had regular full time jobs. By 2006, this share dropped to 54%. Compared to these younger cohorts, the impact is far more modest and limited among older age groups. In 1998, 81% of workers in age 45-54 still had regular full time jobs. The ratio declined some since then and stood at 75% in 2006. [3]

As we can see from Figures 2 and 3, the stagnation of the macro economy during the 1990s first appeared in substantial declines in both accessions and separations. The general decline in labor mobility marked the start of the deep recession (around 1993), as had been the case in all the post war recessions in Japan. In the latter half of 1990s, we see the beginning of the second part, rapid increase in temporary or part time jobs. This trend accelerated as the economy emerged out of the peril of deflation spiral. The upward trend of temporary / part time workers still continues as of today.

In line with the rapid increase in temporary and part time jobs, the economy witnessed long swing and sizable outward shift of the Beveridge curve. See Figure 4. As of 2007, the registered vacancy rate stands at .037, somewhat higher than .034, the figure for 1990, the peak year before the long stagnation. Yet the unemployment rate is more than doubled compared to 2.1% in 1990. The sharp increase in unemployment from the last few years of the decade until 2002 mirrors the depressed labor market and historical low rate of accessions. Even at the peak, the separation rate is well below the level during the bubble years. In retrospect, the sharp increase was an outcome not only of the increase in separations, but also of the historical low rate of accessions. The declining mobility is masked, however, to some extent by the steady increase in the share of part time and temporary jobs wherein the average duration of single spell of jobs is much shorter. In short, it is the apparent sluggishness and declining share of the core labor force in Japan that shapes the landscape of the labor market in Japan. According to an estimate in Miyagawa et al. (2004), labor immobility alone contributed to .6% decline in labor productivity during the latter half of 1990s. It was not the labor but the capital mobility which have far more attention as the slowdown of the TFP growth and its cause has been one of the research focus on the lost decade. And, of course, we all know and still vividly remember the horror story of Zombie firms in Caballero et al. (2008).

All in all, Japanese economy in the 21st century seems to be stuck in a peculiar and precarious pseudo equilibrium characterized by low capital and labor mobility and shrinking primary sectors offering higher pays and job security.

This paper offers an interpretation of these key facts based on a model of competitive search. Our major innovation in the model analysis is to incorporate the impact of a permanent negative technology shock in an economy populated with workers with different degrees of trainability. If the cost of re-training is relatively small, worker is highly mobile. They quit the job as soon as it is hit by the negative productivity shock to search for a new job with higher productivity. Workers with somewhat higher cost decide to retain the current job after the shock, whereas once they lose the job, they move out of the depressed sector and try to search a job in a different sector. A group of workers with even higher cost of re-training may well give up mobility and cling to the job and the skill they have: they remain in the depressed sector even after they become unemployed. We show that the group of workers with highest training cost actually never take up jobs that require training. They spend their entire work life moving from one simple job to the other. Thus the model equilibrium shows the worker allocation across types of jobs according to their trainability and demonstrate that the degree of specificity and cost of training can

[3]The impact is far more sizable on the income side.

explain the heterogeneity and over all compositions of worker mobility. The model also demonstrates that a macroscopic shock which permanently changes the magnitude and frequency of negative productivity shock can reproduce qualitatively the major stylized facts we introduced in the beginning. Both long run unemployment and vacancy rates increase, the share of the non core employees also increases. Moreover, using a calibrated example economy, we show that the time needed to absorb macroeconomic shocks are highly heterogenous across types of jobs and workers. In particular, the example will demonstrate that some facets of the labor market take extremely long time to absorb the impact of the shock because they essentially moves at the speed of retirement and entry to the labor market. In particular, this is consistent with slow but steady increase in the share of temporary and part time workers.

In order to develop a model that can account for the major changes we witnessed in the last 15 years, our first task is to review and consolidate major findings taken from a fairly sizable body of empirical works on the Japanese economy during the lost decade (and its aftermath), especially those on the labor market (section 2). We will set out a simple and highly stylized 'model' economy of the Japanese economies ca. 1990, and ca. 2006, and suggest how these two can be connected through injection of 'shocks' to the system. The 'model' economies and 'shocks' are given concise representations in the model developed in section 3. We will focus our presentations of the model on its core characteristics and its implications on the three key issues. The formal presentation and the analysis of the model are relegated to Appendix. Section4[4] put the model to work to explain the impact of the great recession and its lasting impacts. We conduct a simple calibration exercise to pin down the parameter values of the model. Our primary purpose is not to offer full fledged structural estimate of the model. Instead, our focus is on the mutual consistency in the behavior of key variables. We require them to be consistent with the historical record. *[Section 4 is still incomplete.]* Section 5 concludes.

2 The Lost Decade and its Aftermath

This section reviews what we have learned about the lost decade and its aftermath in the labor market by drawing upon the past empirical studies on the lost decade, particularly those on the labor market issues, and, by so doing, we propose three major findings, against which we provide our own highly stylized model in the subsequent sections.

2.1 Two major macroeconomic shocks

2.1.1 Macro shocks

There is no doubt that severe macroeconomic shocks in the early 1990s started the long and arduous downward adjustments in the labor market. A somewhat less well known point is that there indeed were at least two, not one, shocks during the decade: the first one hit the economy in 1992-3, and the next in 1997-8. From the vintage point of 1994 or 1995, the aftermath of the first macroeconomic downturn seemed closer to the end by late 1996, at least as viewed in fig.2 and 3 wherein the hiring rate recovered to the level closer to the separation rate. It is also clear that the course of the (weak) recovery from 1994 seemed rather normal and similar to the recoveries from the previous recessions. Turnover rates declined as the economy entered the recession, as had been the case in the past. Unemployment rate climbed

[4]This section is unfinished and the results reported are tentative and used for illustrative purpose only.

up and by 1995, it rose by more than 1.5%, from the peak year of 1990-1991. Still this increase was within the comparable to those in the past recession. For example, in the recession after the first oil shock, unemployment rate increased roughly by 1% to around 3.5%. The impact of the second shock on the labor market differed in several dimensions. Unlike the past recessions, turnover rate gradually increased. From 1997 onward, towards as late as 2003, the gap between hiring and separation remained at the historical high level, plummeting unemployment rate upward. The latter half of the lost decade witnessed layoffs of the core employees at major firms, for the first time since the beginning of the high growth era. Along with waves of layoffs at major listed firms and bankruptcy and mergers in the banking sector, some firms dramatically altered the pay structure from seniority based to those individual performance base salary system. The impact of the second shock on the labor market was far larger and longer lasting. The sharp increase in unemployment during the period is marked by the increase of separation and the decline in accessions. It seems fair to say that the 1997 macro shocks was the heaviest blow[5] to the labor market, at least for two reasons. First, because the shock occurred while labor market recovery was at best weak and certainly not complete from the first shock. Second, because the shock entailed near panic in the financial market and precipitous decline and one time near collapse of the lending market.

2.1.2 Reallocation and productivity shocks

The lost decade was also the period of an unprecedented change in information technology. IT revolution directly reshaped the landscape of the competitiveness in the international markets of manufacturing, telecommunications, and software industry. It also had important impacts as an input for virtually all segments of the economy. Given the rapid pace of internet technology diffusion, it is undeniable that IT revolution did have important impacts on workplace in Japan. Internet mails (to some extent) substituted for lengthy meetings in smoke filled rooms, grocery stores quickly adopted sophisticated POS system to record daily transactions. Many case studies do find impacts of IT technology adoption on work organization, micro productivity, time use, and other dimensions of daily work.

1990s was then also the period of increased turbulence which might have called for rapid shifts in resources. Rapid technology changes and emergence of newly industrialized Asia induced de-coupling of production processes and it became common for major Japanese manufacturers to shift major production facilities over seas. By early this century, China became the largest trade partner for Japan.

Contrary to earlier anticipation of great waves of new IT firms and growth in hiring, recent studies by Anton Braun et al. (2006) and Miyagawa et al. (2006) both find that the short run response to (positive) TFP shock tends to reduce labor inputs[6]. None of these studies cover the period prior to 1990s. Thus it is difficult to tell if the finding is due to the specific period of low labor immobility, or something more structural and applicable to other periods. It is at least safe to say that the overall labor market response to the IT revolution was weak and possibly perverse. If the waves of globalization have left any visible impact, it was primarily through the process of 'restructuring'. With about the delay of a decade to the American counterpart, Japanese media now screams that Japanese jobs are destroyed by cheap

[5]In terms of fixed investment, 1992-1993 decline was much larger than 1997-1998 decline. Private fixed investment grew at annual rates of greater than 10% during the latter half of the 1980s. In contrast, it declined by 8.9% in 1992, by 10.3% in 1993, and by 5.2% in 1994. Compared to these steep declines, the decline in investment were far modest in the late 1990s, ranging % to %.

[6]Miyagawa et al. (2006) argue: " The result indicates that real inflexibility in the labor market contributes to the negative response of the labor input to the technological shock in the short run."

labor in China.[7]

2.2 Factor specificity and immobility

Employment 'rigidity' can be defined in more than one ways. Rigidity can be defined in terms of the speed of adjustment. Given the economic function of labor or other factor of mobility to shift the productive resource away from lower, towards higher, productivity sectors, employment rigidity refers to the sluggishness of labor mobility in response to various measures of productivity/profitability/wage differentials. In earlier empirical studies, the focus was distinctively on macroeconomic ones employing mostly macro time series data. They found adjustment of employment or total labor inputs towards hypothetical steady state or 'equilibrium' level is slow, at least in comparison with the comparable estimate for the United States.

With the increased use of micro data starting in early 1990s, several types of rather different studies tend to suggest that the economy during the lost decade suffered from the sluggishness of labor and other factors of immobility. By linking job creations (destructions) by new establishments (closures) to the financial performance of the firm data, Fukao and Kwon (2006) finds that exiting firms have on average *higher* TFP than those entering. Genda et al. (2008) on the other hand, finds continued increase in job destructions during the lost decade, matched with mirroring declines in job creations. They argue that depressed labor mobility, especially those with previous work experience runs counter to what we would have expected if the lost decade was also the decade of restructuring. Genda et al. (2008) thus concludes that cyclical macroeconomic shocks shaped the pattern of job creations and destructions. This view is to some extent shared by a study by Fukao et al. (2008) who finds that as late as 2006, exiting firms have higher TFP than those surviving and it is attributed to the causal relation of recovery in TFP to restructuring. Namely, modest recovery in firm level TFP growth in the current decade is due at least partially to restructuring. Although these studies disagree as to what extent the latter half of the lost decade was the time of restructuring, they all suggest that the labor mobility induced by IT revolution was weak at best, and the response of the stock variables, such as employment and unemployment are highly sluggish.

Another strand of research focused upon 'cohort effects' on employment, earnings and other key variables, and they[8] unanimously find strong and persistent effect of cohort effects: i.e., macroeconomic situation in the year they enter the labor market have long lasting and quantitatively large impacts. This is at least consistent with the view that labor mobility is severely limited after their first job. The strong cohort effect can be explained in terms of relatively heavy investment in relation specific human capital in the beginning of the work career. If you miss this crucial timing either because of the failure to land on a career job after school, or because of the quits in the mid-career, this will have lasting impact on the future prospect. Those finishing school in recession years have smaller chance to land on tenure track jobs, or face severe competition for promotion within a firm. The aggregate outcome is the visible and lasting impact of the labor market fluctuations in the year they finish school on the rest of their lives. The impact of cohort effect is found especially large for non-college graduates. Assuming that it is more costlier to train those without, than those with college educations, this finding is consistent with the view that the importance of early investment (or the lack thereof) in training is responsible for limited mobility of workers in mid-career. Available evidence suggests that net gains

[7] See two papers by Fukao and Kwon (2006) and Fukao et al. (2008)

[8] Recent studies include Kondo (2007) and Ohtake (2005).

from quits in mid-career was lower, not higher than before during the lost decade[9]. Related to these studies is the rapid increase in temporary and part time workers as we noted in Introduction. Due at least partially to the deregulation of employment matching business for profit, and partial lifting of prohibition of dispatched workers, temporary workers dispatched from those new firms increased rapidly. The survey of these workers indicate that they are less educated and they choose these jobs primarily because they could not find regular full time jobs. Studies also indicate that the probability of landing on regular, full time jobs rapidly declines with age and continued experience of these marginal jobs.[10].

We make two general observations from these studies. First, factor specificity may have played important roles in sluggish response (especially downward). Second, given the limited mobility of mid career workers, we observe persistent and quantitatively important impact from the experience during the first few years after the entry to the labor market.

2.3 Structural Breaks and Constants

As the Japanese economy finally emerged out of the decade long stagnation around 2002, the labor market was still in the midst of absorbing the shocks. It was only in 2003 to 2004 that the unemployment rate started to decline, and hiring of new school graduates picked up. Then, a simple question which is not easy to answer is: do we see the Japanese labor market after 15 turbulent years as the one essentially unchanged from the prototype employment system of the post war era, or has anything changed fundamentally? Let us start with what apparently has not changed.

2.3.1 Constants

Kato (2001) finds no evidence that the ten-year job retention rates of Japanese employees fell from the period prior to the bubble burst to the post-bubble stagnation period. The use of this alternative data source turned out to produce similar results, i.e., no evidence for the weakening lifetime employment (Chuma (1998)). On the other hand, using the most recent micro data, Kato and Kambayashi (this volume) indeed does find some weakening of the employment security for the core workers in the latter part of the lost decade . Investments in firm level training seems to have declined, but we lack any hard evidence. The system and its crucial function of the recruitment of new school graduates have not changed. It still is by far the most important means for hiring new workers. Significant gaps remain between the regular and full time versus the rest of employees in compensation, stability, and promotions inside the firm. There is no evidence indicating a major change in the overall sluggishness of employment adjustment. Together with the use of temporary workers, work hour adjustment is still predominant in adjustments in total labor inputs.

2.3.2 Changes

So, at the end of the day (decade!), it seems fairly clear that we see more of continuations, rather than any significant breaks in structural and organizational characteristics surrounding the Japanese labor markets. Having said this, some of the changes are still noteworthy and even important for the subsequent analysis and the model based characterizations. First of all, even if the core of employment system remained largely intact, it is undeniable that the size shrank, and continue to shrink, as can

[9]See for example, Part II, Chapter 2, Section 3 in :Ministry of Welfare and Labor (formerly Ministry of Labor),2004, *White Paper on Labor* (*Rodo Hakusyo*)

[10]See Genda and Kurosawa (2001)

be seen most easily by the steady increase in non-core employment.

The lost decade was the period also of depressed metabolism of the economy overall, and labor market is no exception. Of special importance is dwindling rate of start-ups and hence job creations by entry of new firms[11]. There has been some weak recovery in the last few years, but, even that is way too small, even after taking account of the weak over all recovery of the economy.

If there is anything genuinely new in the labor market, they are found on both ends of the quality spectrum of the Japanese labor. At one end, we find the emergence of enclaves of specific jobs where the external labor market is the dominant venue through which qualified people are matched to jobs. Such markets are now found in some of highly skilled professionals: examples include IT related jobs, such as SE, web designer, CG specialists, etc, etc. As we have noted already, we see the rapid growth of temporary and part time workers at the other end.

For different reasons, both ends of labor are substantially more mobile across firms than those in the middle. This perhaps to some extent reflects the changes in workplace that occurred during the last 20 years or so. Some of junior level clerical workers are completely disembodied from the main body of the firm[12]: The majority of them are now outsourced and partially substituted away by PC and the use of other IT technology. Having observed early failures of internalizing some professional positions[13], many firms perhaps decided to rely on either contractual outsourcing or specialized subsidiaries to handle IT related tasks. We also find some evidence showing some professional positions in the finance sector are filled by the use of head hunters and rely almost exclusively on mid career quits.

To sum up, we do see some important changes. As far as the factors related to the limited labor mobility are concerned, however, no compelling evidence exists that the Japanese economy changed.

3 The Model

This section presents the model of competitive search. We relegate formal presentation and the analysis to the Appendix. In this section, instead, we focus on the characterization of the equilibrium and demonstrate that the model captures the key observations assembled in section 2.

3.1 Focus

In Section2, We highlighted three key characteristics of the Japanese labor market.

1. Intensive investment in relation specific (human and physical) capital. This is apparently responsible for the relative immunity of the core employees.

2. Limited worker mobility. Worker mobility is largely limited to at most the first ten years since their entry to the labor market. The first jobs are particularly important. This is partially a consequence of fact 1 above. As a result, many empirical studies find statistically significant and sizable cohort effects on employment probability, job security, and earnings.

[11] See Genda et al. (2008) for relevant data.

[12] It is symbolic of the change that the Japanese word 'office lady' ,– often abbreviated as OL, referring to women at office working primarily as secretaries or lower level clerks –, almost completely disappeared from the daily conversation.

[13] Recall many troubles in integrating the computer systems after mergers of major city banks. They revealed that the "legacy" of the system is extremely costly to undo.

3. Competitive fringe of the labor market. Complementary to the market for the core employees, there exists highly flexible and competitive market for temporary / part time workers.

Against these constants of the labor markets, the lost decade have brought about three important changes.

1. Shrinkage of the core employment.
2. Outward drift of the Beveridge curve.
3. Stagnant job creations and productivity growth

In what follows, we offer a model that captures these key characteristics and changes in the Japanese labor market.

3.2 A model of competitive search

Against these background and stylized features of the Japanese labor market, we employ a variant of competitive search as our basis of the analysis[14]. Here, we describe the major features of the model without technical details.

We consider unit sized mass of population. We assume that at the rate d per unit of period, a worker retires from the labor market. The labor force is held constant by the equal amount of net inflow into the market. Each worker is endowed with distinct trait which determines the cost of training. We denote by z, the cost of training incurred when each worker is trained for the first time at a job that requires training. The variable z is a random draw from the cumulative distribution, $F(\cdot)$. We assume away any change in individual worker's z until the retirement. Job searchers and vacancies are matched through search initiated by workers. Only the unemployed workers search. Specifically, each worker contacts a vacancy at a fixed probability per unit of period. Workers can pinpoint the search in such a way he can choose the exact vacancy he will apply to. The essence of the directed search is that each worker has full knowledge of offers posted by vacancies. The firm must post employment offers in order to recruit the worker. We assume that the jobs post all the payoff relevant details of the employment contract. Moreover, they are all endowed with unlimited ability to commit to the posted contract.

Crucially, we require each job slot to choose a specific point in the continuum of "*location*". Once installed, all the jobs are totally immobile. We assume that this continuum to expand continuously over time. The idea is that a specific job is based upon installed technology and jobs are totally immobile. The technology is public goods so there is no limit to the use of any specific *location*. Arrivals of new technology is represented by the ever expanding continuum. Each *location* faces the probability of negative productivity shock. The shock permanently lowers the job productivity. Hence the '*state*' of the *location* is either 'good', or ' depressed',

[14] We believe, for our objective, the model of competitive search is a better choice, than a random matching model. For one thing, in a random matching model, mismatches occur as an inevitable outcome of matching technology. The nature of random matching is also highly problematic if it is applied to the market for the market of school leavers in Japan. The implication, that a new college graduate ends up in a job that he never wanted but he could not choose where to apply, seems difficult to justify.

Also for the analytical tractability, a random matching is a poor choice in our case. Such a model will be practically unmanageable with up to six types of employment and unemployment (state variables). Under random matching models, vacancy shadow prices will be weighted average of values associated with respective match combinations, which render the dynamics practically intractable.

depending upon if the shock already occurred. With these assumptions, we model the shock to be permanent and common to all the job slots in the same *location.* Aside from 'good' or 'depressed' state, all the locations are homogenous so that at equilibrium the new start ups of job slots are even spread in 'good' locations[15].

Each job slot can choose the location and type of job slot, and make the wage and employment contingent on the *state* of *location.* That is, the firm can offer a contract that pays the worker different wage according to the *state*, or make the employment contingent upon the *state.* Firms can commit the type of workers which they would employ. By this assumption, we can divide the market into the continuum of sub-markets indexed by worker type z. In each sub-market, there are homogeneous workers and job slots. The worker can direct their search target to the job which gives the highest expected return. If there is more than one jobs with the same highest return from the search, workers use mix strategy to spread their application probabilities over these jobs. The end outcome of competition among job slots is that only those jobs offering the market determined (highest) expected returns can attract (any number of) job searchers. In other words, at such equilibrium, individual firms and workers treat as given the expected returns from search and each job slot believes (correctly at equilibrium) that it can attract as many *expected* number of applicants as it wants as far as their posted contract provides each applicant with the market rate of expected return from application.

Consider a typical sub-market. Denote by $\psi\left(\frac{U}{V}\right)$ the probability that a vacant job slot receives at least one application, assuming there are V equally attractive vacancies and there are U equally qualified job searchers. We assume this probability is strictly increasing and concave in its argument. Then the total number of matches per unit of time is given by

$$m(V,U) = V\psi\left(\frac{U}{V}\right)$$

which is linear homogenous in V and U. The probability for a searcher to be matched to a vacancy is given by

$$\phi\left(\frac{U}{V}\right) \equiv \frac{m(V,U)}{U} = \frac{V}{U}\psi\left(\frac{U}{V}\right)$$

3.2.1 Two types of jobs

We assume an economy wherein production technology is embodied in each job slot as it is created. With instant and free access to any technology for a newly created job, zero profit condition for job slots always hold.

Denote by ρ the probability of shock per each period. If a job slot is hit by a shock, its productivity is reduced to $\theta(< 1)$ and it will remain at that level forever[16]. Henceforth we call the collection of jobs with permanently lower productivity as d-sector, in contrast to g-sector where jobs are yet to experience such shocks. Namely, d-sector collects all the jobs located in the depressed *locations.* Given the symmetry of locations which are all held implicit in the model, each *location* in g-sector has

[15] As you might have guessed there is no new start ups of job slots in the depressed locations.

[16] The idea is that installed jobs runs the risk of being left behind as newer technology arrives to the industry they reside. Rather than modeling explicitly the productivity growth of new technology, we model such technology shock affecting negatively the relative position of the existing jobs. This shortcut simplifies the model greatly, but, interested readers should consult Mortensen and Pissarides (1998) and Hornstein et al. (2007) for explicit modeling of technology advancement in search theoretic models.

the same size and they are indistinguishable. The same holds true for locations in d-sector.

To highlight the key heterogeneity among the types of employment, we consider two types of jobs. Each job slot can be occupied by a single worker. We represent the competitive fringe of the labor market by type *s jobs* which requires no training and any worker can be employed to produce the same q^s units of (homogenous) output per period of time. Each type s job slot can be instantaneously and costlessly created but it costs p_s per period to maintain with or without a worker in place. Destructions are automatic and instantaneous if they stop paying the cost. On the other hand, type c jobs require investment in training. For simplicity, we assume that it takes once and for all investment by the amount z to train a worker who has never received any kind of training, or, ϵz if he has been trained at a different firm[17] in the same *location*, and $(\epsilon + m)z$ if he received training in the past at a firm belonging to a different *location*[18]. Naturally, we assume

$$0 \leq \epsilon \leq \epsilon + m \leq 1$$

We also denote by $p_c(> p_s)$ per period rental cost of type c job slot.

Finally, we assume that exogenous separations occur in either type of jobs and denote by δ_i, the probability per period that such separations occur within each period.

Notice that the specification above incorporate varying degree of transferability of skills across jobs and industries. Since workers are always free to apply for any vacancy (provided that they meet the specified qualifications as detailed below), the mobility of workers are made endogenous. In particular, workers without any previous training is perfectly mobile because he has not sunk any investment in training. Note that this mobility reflects not only the nature of training costs as specified above, but it also reflects the nature of competitive search: the mobility is ensured by the assumption that a worker can pinpoint the job offer he applies[19].

3.2.2 Constrained social efficiency of competitive search equilibrium

The market equilibrium mediated through competitive search is known to be constrained socially efficient: the market allocation maximize the net social surplus, given the matching and production technology. Intuitively, the efficiency of the equilibrium can be understood by a simple analogy of the competitive search to a perfect competition. In the latter, all the participants take as given the equilibrium price and maximizes her expected utility. The marginal condition ensures the optimality as the equilibrium price equates (locally non-increasing) marginal benefit for the buyer to the (locally non-decreasing) marginal cost of output. In the competitive search equilibrium, job searchers and vacant job slots take the expected returns from job search

[17]We ignore the measure zero probability of being matched to the same job slot where he received the previous training.

[18]Notice that the formulation accommodates various special cases. For example, if $m = 0$, the training contain some firm specific element but it is general across industries. $\epsilon = 0$ on the other hand corresponds to a case wherein training produces human capital shared by all the firms in an industry. So a completely firm specific human capital implies$\epsilon = 1$ so that none of the previous trainings are helpful outside the firm they are trained.

[19]This is an extreme but useful property of the competitive search. Given the time and money school leavers and the firms invest in the market for new school graduates, conventional model of random matching seems to ill fit the nature of the friction in the market. After all, random matching assume that workers cannot choose the type of jobs they apply. In this sense, immobility is due to matching technology. Needless to say, 'churning' do exists even in the markets for the new school graduates, but they are not central to the issue of labor immobility.

as determined in the market and maximizes respective surplus. The constrained efficiency is ensured by the equality between marginal (and non-increasing) gains in the probability of match (and production) to the opportunity cost (i.e., applying to the other jobs). The joint outcome of these decisions generate allocation identical to that of social planner because expected returns from search acts as the price at which each job slot can attract as many *expected number of* applicants as possible. Search friction transforms competitively priced input (the expected number of applicants) into the probability of match and output (and hence profit). This transformation is determined completely by matching technology and it is strictly increasing and concave. The end outcome of this maximizations on the both sides of the market is the first order optimality condition which ensures the efficiency.

In our current model, we added inter-temporal element with irreversible investment. This does not alter the fundamental property of the efficiency of the equilibrium in that we allow the job slot to commit to posted offer which can specify in full all the contingent elements of pay and employment. [20]

3.2.3 Market equilibrium

At the market equilibrium, each worker (and the corresponding search market) is indexed by the cost of training and work experience. Consider a typical group of workers whose training cost level is $\overline{z}$. If a worker has never been trained before, his choice is to apply to a type s job or a type c job without requirement for any prior training. The market equilibrium value of such offers can be denoted as $U_g(n, z)$ wherein n as the first argument in U_g signifies he has never been trained at any firm before. Since the type s job does not require any training, its equilibrium offer is independent of $\overline{z}$, so we may denote this by U_s. Hence his best choice is simply

$$U(n, z) = \max\left[U_g(n, z), U_s\right]$$

As higher training cost reduces the joint surplus at type c jobs, $U_g(n, z)$ is decreasing in z. Thus there exists a threshold value of z such that untrained workers choose type c job if and only if her training cost is equal to or lower than z^s. Given the stationarity of the economy as well as individual worker's life, a worker's optimal choice of job at equilibrium is also stationary and it depends only upon her training cost and work experience. Specifically, it is evidently optimal, for a worker exogenously separated from type s job, to resume searching for a type s job *again* because type s job does not offer training opportunity nor is it required. Since a trained worker costs strictly less to retrain, a trained worker never apply to a type s job. Hence the decision to search for a type s (c) job is *final* and they will continue to do so after exogenous separations. To be blunt, type s is a dead end job.

For a worker separated from type c job, his job search will be different from those without prior training because his training cost at a new type c job is either ϵz or $(\epsilon + m)z$, depending upon whether he remains in the *location* where he received the training in the past. Again, their choice is completely characterized by respective value functions, $U_i(g, z), U_i(d, z)$ $(i = g, d)$ wherein subscript i signifies the sector in

[20] Note that this is certainly not meant to deny the possibility that some types of competitive search with training can generate inefficient allocation. For example, if a job slot cannot commit to a contingent contract and chooses the action which is ex post optimal after the shock, such an equilibrium may entail inefficiency. See subsection 4 in Appendix. In short, the market equilibrium with competitive equilibrium in our paper is an exact analogue to Arrow Debreu model of general competitive equilibrium.

Another note on the wage schedule. Since employment continues more than one periods, competitive search can be supported by more than one types of wage offers provided that the expected present value of the offer is the same. See appendix for the details.

which a worker searches for a job. If the *location* he is trained in the past belongs to g-sector, there is no need to move to a different *location* where a larger re-training is needed. On the other hand, if the *location* he received training is now in d-sector and productivity of jobs are lower, it might make sense to move out of the *location* and search for a job in g-sector. In Appendix, we prove the following

$$\frac{\partial U_g(d,z)}{\partial z} < \frac{\partial U_d(d,z)}{\partial z} < 0$$

$$\exists z^u \text{ such that}$$

$$U_g(d,z) \gtreqless U_d(d,z)$$

$$as$$

$$z \lesseqgtr z^u$$

Namely, the impact of training index z is smaller if she stays in the same sector than if he moves out to a g-sector, i.e., $U_d(d,z)$ is flatter than $U_g(d,z)$. This makes sense because if he moves out, re-training cost is $(\epsilon + m)z$, whereas if he stays, it costs only ϵz. Thus the dependence of his value function on training cost is proportionately smaller if he stays. If the training cost is sufficiently small, it is evidently optimal to incur re-training cost and move to a *location* in *g-sector* where jobs are more productive. Thus there exists threshold z^u such that those with training cost lower than the threshold will move out of *d-sector.* The remaining question is what if the worker is employed when the job is hit by the productivity shock. The optimal choice is again characterized by yet another threshold, z^e. If the training cost is smaller than the threshold, it is optimal to terminate the employment and move to *g-sector* and search for a job. Since his value of being employed is higher than being unemployed and searching for a job in the d-sector, we have

$$z^e < z^u$$

Consequently, three thresholds completely characterize the competitive equilibrium in each market indexed by z.

Figure 5 summarizes the discussion above, with slightly different formulation which is used in the formal presentation of the equilibrium in Appendix. The figure takes the cost of training z on the horizontal axis, and on the vertical axis we take the value of the contracts at the equilibrium. Given the future probability of negative productivity shock, the contract offer contains contingency in the event of the shock. First, notice that the value of type s contract is a horizontal line since the job does not require any training and the output is the same irrespective of z or previous training. The first threshold, z^s is the point where the value of type c job offer to untrained worker cuts the horizontal line. Thus workers with training cost smaller than threshold search for a type c job. The decision on mobility after the shock is an outcome of competition among offers with different contingency. Note that this decision concerns only those with previous training. Untrained workers can move costlessly across different sub-markets so they never stay in *locations* hit by the negative productivity shock.

Then, suppose a worker has previous training and have been searching for a job in the *location* when the industry is hit by the negative productivity shock. Whether or not he should search in the same *location* or move out depends upon the two value functions, each representing the stay or move. At market equilibrium which is socially optimal, this decision is equivalent to whether or not an offer with conditional employment contract that stipulates separation after the shock has higher value than the offer available for the trained worker in d-sector. Thus the second threshold, z^u, is given by the intersection of the two value functions. Finally, consider if a worker employed should retain the job when the productivity shock occurs. In terms of competition of offers, this is tantamount to the comparison between the offer with unconditional employment and the one that stipulates separation at the moment of the shock. The intersection of these two schedule determines the last threshold, z^e.

3.2.4 Worker mobility under three types of equilibrium

For workers with training cost above z^s, they only apply for type s jobs. While employed at a type s job, he may be separated for exogenous reasons (that occur with probability δ_s), retires (with probability d) or the job is hit by the negative productivity shock. Depending upon the configuration of parameters, either the job is destructed and employment is terminated, or, the employment continues until separations occur for exogenous reasons[21].

Workers with training cost lower than z^s search for a type c job. Their employments are terminated either by exogenous separations or by retirement of workers. Upon the productivity shocks, we obtain three types of configurations. Those workers with training cost between z^u and z^s will remain in the same industry until their retirement, even when the industry is hit by the negative productivity shock. Thus workers are necessarily less mobile than type s workers. Among workers in the next tier, i.e., those with training cost between z^u and z^e, their mobility depend upon the timing of the productivity shock. If he is employed when the industry is hit by the productivity shock, he retains the job until he is separated by exogenous reasons. Once unemployed, however, he moves out of the *location* now in d-sector and seeks a job in g-sector. Therefore this tier of workers are more mobile than the first tier. Finally, workers with the training cost lower than z^e is the most mobile among workers who choose type c jobs. He moves out of the industry whenever it is hit by the productivity shock, even if he is employed at the moment of shock.

In sum, the model predicts the worker mobility is U-shaped across index of training cost: those with highest training cost actually never receive training and mobile, while workers with training cost below z^e will always move after the shock.

3.2.5 Zero profit condition and optimal queue length

In appendix, we show that market equilibrium is characterized by the first order condition for the optimal queue length, i.e., the average number of applicants to a job. For a population of workers with given training cost z, there exists potentially three types of offers for type c job. One type of offers is for the untrained workers. The second is for those trained in a different job in the same *location.* Finally, an offer to a worker who had been trained in a different *location.* Note that the last group of workers must have been in a *location* hit by the productivity shock. Otherwise, given the symmetry of the *location* continuum, there is no reason to move.

In sum, there are potentially four distinct sub-markets each with distinct offers, depending upon whether or not and at where a worker is trained, as shown in Table 1. Let us start with job slots in d sector. As we explained above, no worker moves to a job in d sector. If he is trained, there is no gain from moving into a *location* with lower productivity. By the same reason, untrained worker has no reason to search for a job in *d-sector.* Consequently, the market is potentially active only for those who have been trained in the same *location* already. As we have shown above,

[21] Because this type of jobs do not require training, the optimal choice is the same for all the type s jobs. Hence only one of the two possible cases is observed at equilibrium. It should be noted also that the surplus from employment is due entirely to the search friction and we should expect that job destructions and separations are more likely outcome in comparison with type c jobs.

	Location type	
Worker type	g-sector	d-sector
not trained	(n,g)	not active
trained in the same location	(g,g)	(d,d) if $z \geq z^u$
trained elsewhere	(d,g) if $z \leq z^u$	not active

Table 1: Table of sub-markets

this segment is inactive if $z \leq z^u$. For a job in g-sector, two types of sub-markets, (n,g), and (g,g) are always active, whereas (d,g) is inactive if $z \geq z^u$. The zero profit condition for each active sub-market uniquely determines the optimal queue length. By complementary slackness condition, there is no vacancy and the market is inactive if the zero profit condition is not binding.

3.3 The Full Market Equilibrium and Impacts of Macroscopic Technology Shocks

3.3.1 The Full Market Equilibrium

The full market equilibrium is determined by two sets of conditions. First set contains equations which jointly determine optimal queue lengths and corresponding values of employed and unemployed workers. The model equilibrium is such that these sets of conditions do not contain state variables, i.e., employment and unemployment with or without training. The equilibrium values of queue lengths for respective offers are used to obtain the values of offers, or, equivalently,the shadow prices for unemployed workers for given training cost and the location of the last training. They together thus determine "prices" that jointly support the efficient allocation of workers to quene and hence to jobs.

The 'price' block of the model is determined completely within themselves, independently from the state variable dynamics. Since the dynamics of 'price' block is totally unstable, the solution, or the market equilibrium is such that the equilibrium prices are always at respective steady state values. Consequently they jump immediately to the new steady state values whenever the model is disturbed by a change in a parameter. Employment and unemployment evolves over time towards the steady state with the speed which partially depends upon 'prices', the details of which are shown in Appendix.

Worker mobility at the market equilibrium is depicted in the three diagrams shown in Figure 6 through Figure 8. Figure 6 shows the possible moves for the least mobile group, i.e., those with training cost between z^u and z^s. The arrows directing from g sector to d sector is caused by the shock and they never endogenously leave the job they are matched for the first time. Even if they are separated from the first job, they remain in the same *location.* Figure 7 shows the moves for the middle tier of the workers in type c jobs. Note the arrow from u_d to e_g indicating the move from the depressed *location* to search for a job in healthy *location.* Finally, the most mobile group is shown in Figure 8. They never stay in d sector.

We now employ the model equilibrium to trace out the development of the labor market in Japan from the onset of the lost decade until early years of the new century.

3.3.2 Two types of shocks: increased turbulence and deeper impact

Needless to say, conventional types of macro shocks must have hit the economy during the lost decade. After all, no one doubts that the economy experienced severe decline in the aggregate demand. To the extent that those shocks were cyclical, however, tracing out the impact of such shocks is not our main focus, thus we ignore them. It seems fair to say that at least two type of technological shocks also occurred during the period and both of them are likely to be permanent. The advent of information technology (IT) most likely shifted comparative advantages of industries and organization forms. Another important factor is rapid shifts in comparative advantages in the international markets. Lower trade barriers, emergence of East Asian countries, etc all made the market more volatile. We model this increased turbulence as an upward shift in ρ. At the same time, rapid shifts in technology frontier made it more costly to retain and continue to use the current technology. We model this as a deeper impact of negative productivity shock to the existing jobs, i.e., lower θ.

Although both types of changes reduce the overall efficiency of type c jobs, they differ in their impacts on the labor market dynamics. To begin with, note that the endogenous labor flows in our model is driven by the mobility from d-sector to g-sector. An increase in ρ *reduces* the net productivity gains from this mobility because if the probability of permanent productivity shock increases, the expected duration of a job in g-sector is reduced. Thus moving from d sector to g sector generates smaller gains as the new *location* is more likely then before the macro shock to become d-sector. Thus such a shift lowers the mobility after the productivity shock. Overall impact of the shock on labor mobility is therefore negative.

On the other hand, an decrease in θ does the opposite: the impact of productivity shock is larger so that the gains from the mobility is also larger. As a consequence, workers are more likely to move out of d-sector to search for a job in g-sector. Thus the labor mobility should increase as a result of the shock. Therefore, a combination of changes in these two parameters tend to reinforce each other in terms of lowering overall productivity of type c jobs (vis a vis type s sector), whereas the impacts on short run labour mobility tend to canceled out each other. As a result, both types of shocks shifts the overall labor allocation away from type c jobs and type s positions will expand.

3.3.3 Short run and long run impacts

Suppose these types of shocks are totally unexpected and changes are also permanent. The immediate impacts of these types of shocks differ depending upon whether or not the impact is favorable to the d-sector employment or not.

An increase in ρ lowers the difference in values of g-sector and d-sector jobs as g-sector jobs are more likely to move to d-sector. In this case, however, it still remains that an increase in employment at d-sector can occur only through the increase inflows out of $g$sector to d, and, also through increased matching at d-sector unemployment pool. In short, employment level at d-sector cannot jump. Consequently, shocks which favors the declining sector will move the economy only gradually over time.

On the other hand, shocks which favor the g-sector jobs can bring about immediate job destructions at d-sector. Thus the change will produce immediate job destructions and resulting move to unemployment pool for the trained (at d sector) workers to g sector. We would expect to have a sudden increase in separation and accession rate. In other words, the model predicts that the decline in turnover rates should be gradual and long lived, whereas an increase tend to be more sporadic and rapid.

In the long run, both of these shocks reduce the overall surplus of type c employment, reduce the equilibrium quene length, and increase the steady state unemployment. The end result is outward shift of Beveridge curve and lower vacancy/unemployment rate.

3.3.4 Long and persistent impacts

Apparent sluggishness of the Japanese labor market in absorbing macro economic shocks has been the research agenda for a long time. The development in the lost decade and its aftermath only made this characteristics more puzzling, given the severity and the length of the stagnation. The model developed in this paper represents the overall mobility as the aggregated outcome of individual worker mobilities who differ in their trainability and their experience. We have shown above that a group of workers in type c jobs with relatively high training costs are highly immobile. They are totally immobile across "*locations*". They cling to the skills they acquired in the previous job even after the *location* lost its productivity edge permanently. For those jobs to disappear, the workers occupying these jobs need to retire. This makes the process of adjustment extremely slow[22]. Unlocking employment relations with relation specific investment is costly for themselves as well as to the economy as a whole. On the other hand, it is misleading to project the aggregate outcome based solely on this group. By definition, this group is under represented in the labor mobility data (because they do not move, at least not endogenously). The over all labor mobility, hence the time needed to absorb macroeconomic shocks, are the aggregated outcomes of heterogenous workers' individual mobility decision, thus it is sensitive to the share of workers in terms of trainability and work experience.

Needless to say, the impacts on individual workers mobility and incomes differ. The other side of relative immobility of trained workers is that the joint surpluses protect their jobs, while those jobless, especially those without training, take the blunt of the shock.

The available evidence suggests the lost decade was also the time when the aggregate mobility diminished. Population composition was changing rapidly and new entry to the labor market declined sharply. The distortions in the capital market which manifested so vividly during the last years of the century only contributed to decelerate the labor mobility. This impact was felt on both creations and destructions of jobs. Decline in new start ups must have reduced the available new openings, whereas the survival of "zombie" firms contributed to the longevity of employment in depressed sectors above and beyond warranted under unfettered competition.

3.3.5 Cohort Effects

Two macro technology shocks, represented by changes in θ or ρ, shift the relative positions of value functions and hence positions of three thresholds. For example, suppose that z^s threshold moves left after the shock, say from z^s_0, to $z^s_1 (< z^s_0)$. Workers with training cost between z^s_0 and z^s_1 will have dramatically different mobility and income, depending upon the timing of the first employment. If a worker is lucky to get a job before the shock, he must have been employed at type c job and he must have received training, thus making him largely immune from the shock because his value function reflects the investment which is already sunk. On the other hand, those who fail to be employed (strictly speaking experience employment) at type c

[22] According to Census of Agriculture, 66% of farmers (those considered primary in the sense farming is their main economic activity) are aged 60 or older. The share was 36% in 1985. Aged as they are, there are still more than 6 million who at least partially work as farmers, and more than 3 million whose main economic activity is farming.

job before the shock now find themselves no longer needed in type c jobs. Only jobs available now for them is type s which offer no training. This impact is permanent and cannot be undone, thus leaving the visible scar to the work history of those cohorts.

Intuitions also suggest that this change occurs over an extended period of time. A worker who was lucky enough to jump into the boat of secure, life time job in type c will live a life not very different from the one before the shock, and this situation lasts as long as he remains active. The economy soaks in full impact only when those last lucky ones eventually retires from the labor market[23].

3.4 Summary and Limitations

This section presented without technical details the essence of the model of competitive search. Our model captures some of the key findings on developments of the Japanese labor market since 1990s. Sluggishness of the overall response to the shock is modeled as the aggregate mobility of heterogenous workers with different training costs and work experiences. We highlighted the fact that for some of workers it is never optimal to move to a different job even if the current job is hit by permanent productivity loss. Our model equilibrium is consistent with the presence of strong cohort effects. We also have demonstrated that the combinations of shocks may cancel out each other on their impacts on short run mobility, even though they both increase the long run unemployment rate in the long run.

Before closing this section, we should note the limitations of our model as employed for the narrative of the Japanese labor market in the last 15 years. To begin with, our model cannot capture some of the key macroeconomic issues, which must have occupied the minds of policy makers during the lost decade, such as the fear of deflation spiral, or for that matter, any macroeconomic dynamics associated with price rigidity, market failures, etc. Needless to say, these concern shaped many of the policies which we analyze below. Admittedly, it is unfair to consider only the type of efficiency issues in the framework of equilibrium analysis because by construction any intervention that alters the equilibrium allocation reduces efficiency.

Far more important and serious limitation of the model is somewhat more subtle. In spite the complexity, the model equilibrium is fairly straight forward in implications and its complexity of the model analysis remain within manageable range. This relative clarity and sharpness are bought by the competitive search and zero profit condition on vacancies. If we acknowledge the time delay and irreversibility of capital investment, this condition is highly restrictive and evidently not applicable[24]. Historical records of job creations or vacancies show clearly the underlying sluggishness of the start ups in response to new opportunities. Banks forbearance, a variety of government policies to subsidize interest rate payments for depressed industries only aggravated the problem. For these reasons, the model analysis necessarily fail to keep track of cyclical fluctuations in accessions and separations as we observed during the lost decade. In the next section, we offer some calibrations results with due precautions as required by these limitations.

[23] The crucial point is that the training investment is sunk cost. Therefore, even if a second technology shock completely washes out the change in either θ or ρ at the first shock, those training investment still have permanent impacts on the economy as long as they remain in the labor market.

[24] This trade off between the model transparency and poor prediction power (at least in terms of cyclical functions) is not uncommon among calibrated models of the labor market based upon costly search. See, for example, Fujita and Ramey (2007) for their simulations results and the better fit obtained by introducing sunk cost for job creations.

parameter	value	description
r	0.005	discount rate
d	0.0063	retirement probability
μ_z	20.0	mean of training cost
σ_z	14.5	standard deviation of training cost
ϵ	0.15	generality of experience
m	0.33	mobility cost
ρ	0.013	the frequency of productivity shock
θ	0.88	the scale of negative shock
δ^c	0.006	separation rate of type j job
δ^s	0.0188	separation rate of type s job
q^c	2.0	productivity of complicated task
q^s	1.0	productivity of simple task
p^c	0.575	rental cost of type c capital
p^s	0.165	rental cost of type s capital
A	0.46	efficiency of matching function
η	0.3	parameter of matching function

Table 2: Benchmark Values

4 Model Calibration (incomplete, extremely tentative)

4.1 Steady State Equilibrium with benchmark specifications.

We use the model outline in section 3 to trace out the development of the labor market for the period 1991-2006. The model has 16 free parameters as shown in Table 2. Our strategy is simple. We pin down these parameter values by employing various sources of the past empirical studies We show our tentative benchmark values of 16 parameters as shown in Table 2 . Below we have a brief description on how we set the value for each parameter.

- The one period of the model corresponds to a quarter of a year.
- The annual discount rate is set at 2%, roughly the long run growth rate of the Japanese economy. Thus we set $r = 0.005$
- Assuming the average work career to last 40 years, we set the quarterly retirement probability at $d = 0.0063$
- q^s, per period output for a type s job in the state g, is used as the benchmark so we set q^s=1
- q^c , the output for a type c in the state g ise set at 2.
- Matching function. We assume that it is a simple Cobb Doughlas type.

  $$M = AV^{\eta}U^{1-\eta}$$

  with the coefficient for vacancy, $\eta = 0.3$. This specification approximate the coefficient estimates in Kano and Ohta (2005) and Kambayashi and Ueno (2006). We also set $A = 0.45$ so that at the bench mark equilibrium, the average monthly job finding rate is around 15%
- According to Fukao and Kwon (2006), the 7 year (between 1994-2001) survival rate of firms in the upper 50% bracket is estimated as 68%. Thus we set $\rho = 0.013$ so that $\exp[-28\rho] \doteqdot .68$.

- We use the results from Ito and Lechevalier (2008) for the across firm distribution of labor productivity of full time jobs. Specifically, we set $\theta = 0.88$ so that the log productivity differential between upper 25% and lower 25% corresponds to their results.

- On exogenous separation rate for type c jobs. We use the average (1991-2006) separation rate for regular full time jobs, 11.3%. We subtract 51.7% of them as those separations with tenure less than 2 years. This adjustment is made on the ground that our model assume away separations due to unknown match specific productivity, which are likely to be concentrated among employees with shallow tenure. In net, we set the exogenous separation rate in such a way that the total separation rate (including endogenous ones due to negative shocks) be equal to 5.9%=.517×11.3%. The best choice turns out to be $\delta_c = 0.006$.

- The average (1991-2006) ratio of separation rate for temporary/part time job to that of the regular, full time job is 2.6. Thus we set δ_s such that the equilibrium separation rate (including endogenous separations) is equal to 15.2% per year, i.e.,2.6× 5.9%. $\delta_s = 0.025$ satisfies this condition.

- We have almost no information directly relevant to this parameter for Japan, but Koike (2005) asserts the firm specific components of human capital is at most 10-20%. Thus we set $\epsilon = 0.15$. We know even less on magnitude of m. We chose $m = .33$ as the value (jointly with other parameter specifications) gives us .52, the target rate of job changes with industry changes to the total accessions (see the explanation for this target ratio below).

- Distribution of the (first job) training cost. We assume $F(\cdot)$ to be a normal distribution with mean μ_z and standard deviation, σ_z, truncated at $x = 0$ and re-normalized density such that the probability sums up to unity. The value of σ_z is set at 14.5 so that at the mean, $\mu_z(= 20)$, the re-normalized density also has the peak.

- $\mu_z = 20$. This corresponds to 2.5 years' worth of output for a type c job in the state g. Ariga et al. (2006) reports average On the Job Training is 82.5 hours per year among the surveyed sample firms in Manufacturing, which corresponds to 4-5% of annual work hours, which in turn corresponds to 2 years of 40 year work life. Ariga et al. (2006) also have been conducting by annual surveys at two auto assembly plants and they also find the average monthly training hours around 8-10 hours per month among assembly line employees. This suggests annual training hours around 100. This translates into 5% of total work hours.

- We set p^c =0.575, and p^s =0.165, so that the equilibrium aggregate unemployment-vacancy rate is unity.

- We set the target share of type s employment at the bench mark equilibrium to be at .085, the figure for male workers at 1991.

- We set the target ratio of job accessions involving *location* changes to the total accessions at .52, which is the estimated ratio of job changes involving changes in industry affiliation of jobs in Japan.

	benchmark	θ shock	ρ shock	two shocks
Unemployment rate	0.036	0.042	0.036	0.044
Vacancy/Unemployment	1.00	1.05	1.03	1.10
Thresholds				
$\bar{z}$	40.3	37.0	39.5	35.4
$\tilde{z}$	29.6		24.7	32.9
$\hat{z}$	9.9	16.6	7.1	12.8
Job Finding Rate	0.45	0.46	0.46	0.46
part-time job	0.56	0.56	0.56	0.56
full-time job	0.43	0.43	0.43	0.43
Annual worker turnover rate (full-time %)	11.8	14.0	11.6	13.9
Annual worker turnover rate (part-time %)	30.5	30.5	34.5	34.5
The share of accessions with location change	0.52	0.64	0.51	0.68
Employment share (%)				
at part-time job	8.6	12.9	9.4	15.3
at full-time job in d	41.3	27.0	52.0	36.9
at full-time job in g	50.1	60.1	38.7	47.8
Gross output	1.75	1.71	1.72	1.65

Table 3: Steady States

4.1.1 Steady States of the model under base parameterization and macro technology shocks

We configured parameters in order for the steady state equilibrium of the model to mimic closely the normalized actual data on unemployment rate, accessions and separations rates for the regular-full time and part time/temporary workers. The results are shown in the first column of Table 3. In the second column, we show the steady state values when we lower the value of θ from .88 to .84. Similarly, the third column is for the case wherein ρ is decreased from .018 to .013. Finally, the last column show the steady state when two changes are combined.

The steady state values of the variables listed in Table meets the targets we outlined above: the share of type s employment is 8.6%, corresponding to the target figure, 8.5%, shown above; the mobility across *locations* is 52% of the total separation; and, the vacancy size matches total unemployment rate. Job finding rate is also close to the target. The unemployment rate at the bench mark is 3.6%.

About 50% of employment is at type c jobs which are yet to be hit by the technology shocks, whereas 40% have type c jobs which are already in depressed state. Although we set exogenous separation rate of type s at 7.5% per year, 5% higher than the corresponding rate for type c jobs, the annual turnover rate for type s is 30.5 (thus separation rate is 15.2) compared to 11.8 (5.9%). The result indicates that more than a half of the net difference in separation rate, 10.7%, is accounted for by the difference in endogenous separation. This of course reflects the equilibrium effect of training investment. The share of aggregate training cost is 8.6% of gross output, which is probably over estimate and this may account for some of the extremely sluggish response in some of the state variables, as we will show below.

The long run effects by two types of macro technology shocks replicate qualitative characteristics we verbally explained in section 3.3. In terms of quantitative effects, we lack any empircal counterparts to evaluate. We only warn you an obvious but important feature of the numerical values of equilibria. Namely, the impact on compositions of employment are highly sensitive to the specification of the distri-

	θ shock	
	50% benchmark	75% benchmark
Unemployment rate	1.5	1.5
Employment share (%)		
at part-time job	26	52
at full-time job in g	0.5	1
	ρ shock	
	50% benchmark	75% benchmark
Unemployment rate	5.5	24
Employment share (%)		
at part-time job	33	63
at full-time job in g	6.5	14

Table 4: Transition paths

bution function for training cost. With this due warning, we offer some tentative assessments.

Compared to the modest impact on gross output by either type of shocks (2-3%), changes in employment shares are larger: roughly speaking, 1% GDP change due to a decline in θ corresponds to : 3% increase in type s job, 4.3% increase in the share of type c jobs in g sector, and 6% decrease in the share of type c jobs in the depressed sector In the case of an increase in ρ, 1% decline in GDP is matched by .5 % increase in the share of type s job, 6.7% decrease in the share of type c in g, and 6.3% share increase of type c jobs in$d$. If we consider these ratios to be too large to be credible, an economy with lower training cost will show even more larger response ratios of these shares relative to changes in GDP. The implication is that the alarming increase in type s jobs [or in terms of data, steady increase in temporary or part time jobs] may not be so serious in terms of the accompanying loss in output.

The impact on the steady state unemployment is modest compared to these changes: .6% increase in θ shock, almost no change by ρ, and about .8% when two shocks are combined. GDP declines by 5.8% with the combined shock so that the ratio of GDP change to the change in unemployment rate is around 10. Setting aside the exact number, the low sensitivity of unemployment to GDP change in Japan is a well known feature.

4.2 Dynamics

We traced the impact of two types of shock over time. We show two results. Figure 9- 12 show the dynamic paths of major variables after shocks. In Table 4 we tabulate the time needed for each of major variables to absorb the long run impacts. This is measured in term of time needed to move half way (50%) of the difference between the two steady states, and the time needed for 75% benchmark. We notice almost immediately that the dynamics differ sharply between the two shocks. The transition to the new steady state is relatively fast in the case of θ shock, which is exactly what we anticipated in section 3.3. In the case of the employment composition between g sector and d sector, the transition from the benchmark steady state to the one with the shock is 75% complete within 1 year. The only exception is the slow adjustment in the share of type s relative to the combined share of type c. It takes 25 years to reach 50% benchmark, and 52 years to reach 75%[25]. This is also exactly what we

[25] You may know already why, but we note that the large numbers shown for the transition time for the share of type s employment is due to the simplifying asumption: i.e., workers are assumed

	benchmark	capital subsidy	firing cost	training
Unemployment rate	0.044	-0.001	-0.001	-0.001
Vacancy/Unemployment	1.10	-0.01	± 0	-0.04
Thresholds				
z_s	35.4	+0.6	± 0	+1.3
z_u	32.9	-1.0	± 0	± 0
z_e	12.8	-0.6	-0.8	± 0
Job Finding Rate	0.46	± 0	± 0	± 0
Annual worker turnover rate (full-time %)	13.9	-0.3	-0.3	-0.1
The share of accessions with location change	0.68	-0.1	± 0	-0.1
Employment share (%)				
at part-time job	15.3	-0.9	± 0	-2.1
at full-time job in d	36.9	+1.7	+1.1	+1.5
at full-time job in g	47.8	-0.8	-1.1	+0.5
Gross output	1.65	+0.01	± 0	+0.02

Table 5: Policy effects

would have expected: the share moves roughly in line with the speed of retirement and entry.

The shock in ρ generates far long adjustment process. Unemployment rate reaches 50% benchmark only after 5.5 years, and more than two decades to reach 75% level. As expected, the adjustment in the share of type s takes longer than in the θ shock. The difference is relatively smaller compared to other variables: 33 years to reach 50%, and 63 years to reach 75%. Again we note that the relatively smaller difference is attributable to the fact that the pace of entry and exit from the labor market determines the base speed of adjustment. With the risk of over simplification, the sluggishness of the s sector response is akin to the impact of a change in education attainment on the average education of labor force. By the same token, a policy (say, a subsidy for training) intended to reversal the trend must also takes an excruciatingly long time to generate visible impact. Perhaps, even the Liberal Democratic Party cannot wait that long!

4.3 Injecting various government policies

Table 5 show the long run impacts of three types of policies. In the extreme left column, we show the bench mark steady state that corresponds to the steady state shown in the last column of Table 3. Namely, each policy is introduced to the steady state of the economy after the two shocks.

In the second column, we show the steady state associated with a program that subsidizes the rental cost for type c jobs in d-sector. In the third column, we show the impact of legal restriction on firing which we represent by wasteful costs incurred upon all types of separations in type c jobs. In the last column, we show the impact of a subsidy to training. The subsidy is given to all the unemployed workers when he receives training for the first time. Re-training are excluded from the program.

We set the magnitude of each program in such a way that the expenditure (which we presume to be financed by lump sum tax) is equal to .2% of GDP, which translates to roughly 1 trillion yen per year in the current Japanese economy. Note that the

ageless and face a constant probability of retirement. But, remember, the counterpart, the rate of new entry is not very far from the reality.

impacts are computed under the assumption that the program is carried out with the same magnitude forever.

Each program has more or less the same impact on ling run unemployment rate: about .1% decline.

The program that subsidizes rental cost of job slot in d sector corresponds closely to various government programs to assist the structurally depressed industries. Subsidies on interest payments are the most common. Our experiment reduces rental cost of type c jobs by 1.5%. As a result, the share of d-sector job increases by 1.7%, which is partially offset by .7% decline in g-sector job. A good news is that the overall profitability of type c jobs increases. Hence the share of type s jobs decline by .9%. Somewhat surprising result is the slight decline in vacancy/unemployment ratio after subsidizing the cost of vacancy. This result obtains because of the composition effect: the vacancy/unemployment ratio is higher in type s jobs, than in type c at the benchmark. The decline in the share of type s jobs thus reduces the share of type s vacancies. Not surprisingly, the labor mobility is somewhat (.3%) reduced by such a policy as the policy help survival of jobs in *d sector.*

In the second experiment, we considered the impact of penalizing separations. Unlike other two, we gauge the total size of firing cost imposed to .2% of GDP. This firing cost is considered social wastes. The results shown in the second column are mostly near zero except for the two variables. The biggest impact is found in the threshold z^e, i.e., those above the threshold optimally choose to be separated from the current employment once the negative productivity hits the *location.* Thus the imposition of firing cost reduces the mobility of the most mobile workers. The only good news is that the suppressed separations reduce the unemployment. In turn, the share of jobs in g sector declines by 1.1%, which is matched exactly by the rise in the share of d sector jobs.

The last experiment is the impact of subsidy for training. The .2% of GDP outlay in this experiment corresponds to a 3.3% proportional reduction of the first time training cost, z. Thus a simple way to visualize the experiment is to replace z by $.97z$ for all those who receive training for the first time. As a result, the share of type c employment increases by 2.1%, out of which 1.5% is due to the increase in type c jobs in d-sector. The reason is that the subsidy matters more to the workers closer to the threshold between type c and type s. Those are the workers with highest training cost among those in type c jobs. Consequently, the impact eventually translates into the concentrated increase in the depressed sector jobs. This point can be confirmed by comparing the impact on thresholds.

5 Conclusion (necessarily incomplete as Section 4)

We have shown that the delayed response of the labor market to a shock can be understood as the equilibrium outcome in an economy with search friction and (at least partially) relation specific investment in training. That would be hardly a news, though[26]. Our modest contribution is to show that such an economy also have other features which we find in the Japanese labor market. We have shown that a relatively modest change in a parameter governing the nature of productivity shock is enough to cause long but steady increase in the share of jobs that do not require training. This impact causes history dependence because of the sunk cost nature of training. The same cohort of workers with the same innate ability can have a totally different job history. The effect will last as long as this induced heterogeneity is eventually washed away by the retirement of the cohort from the labor market. We also have

[26]See for an early contribution by Fukao and Otaki (1993).

shown that a model size of negative macro technology shock is enough to explain a sizable (say 1%) increase in the long run unemployment rate, without much visible impact on the unemployment-vacancy ratio. I.e., Beveridge curve shifts out. When we allow two technology parameters to change, the combined outcome shows all of these changes, yet the steady state values of employment turnovers differ little from the baseline. The big jump occurs, however in the share of employment in type s jobs. Even after the deepening of the productivity shock, the new steady state has roughly the same share of workers clinging to the jobs with lower productivity as the effect of higher probability of permanent shock almost cancels out the first effect. A large share of those will search and work in those jobs until retirement. The end result is that the shrinking primary sector jobs in the g sector.

If these characterizations look too bleak, I only suggest to come to visit one of Japanese villages with rice fields, or one of 2,921 mostly deserted fisherman's ports[27], or anyone tending the cashier at a convenience store. Those folks will tell you their own versions of the stories why they do what they have been doing for the entire working life.

A Appendix: Formal Description of Market Equilibrium

A.1 Set up

As we explained in the main text, we assume the constant population of workers. All the participants in the labor market are risk-neutral and use the common discount rate r.

We consider two types of jobs, type c and type s.The productivity of both jobs depend on the state of the *location.* There is a continuum of the *location* and the state of each *location* evolves according to the following simple stochastic process. There are two possible states, 'g' (good) and 'd' (depressed). The collection of the *locations* in each state is called *sector.* Thus g-[d-] sector collects all the *g- [d-] locations.* Each *location* in state g turns to the state d with probability ρ, representing the permanent productivity shock. In order to prevent all the existing *locations* becoming *d,* we assume the continuum of the *location* expands over time so that there always exists strictly positive mass of the locations in state g. Let q^c [q^s] denote the output per unit of time of a type c[s] job in g-sector filled by a properly trained worker. As we explained in the main text, each worker is endowed with innate aptitudes for type c jobs and we represent by z the worker continuum in terms of the training cost needed. The value z represents the cost of the first training. We assume that such a cost is payable in once and each training is complete once the cost is incurred. We denote by F the distribution of z.

The basic building blocks of the model are the firms' job offers and workers' application strategies. The job offer must stipulate employment and compensation in all payoff relevant contingencies: i.e., required type of the worker, the type of job slot, the *location*, and wage schedule which can be contingent on the state of *location.* Such a job offer maximizes the profit given the application strategies of each type of unemployed workers. Workers are distinguished by training cost z and the *last* training she has taken. As we will see shortly, no worker receives training for the first time at a type c job which is *already* in the state d. Still, an unemployed worker could have received training at a job which is *currently* in g-sector, or, in d-sector, or, never been trained before. Although a worker can search for a job in any *location,* if the job where he received the training is currently in g sector, he has no reason to search in a different *location* because all the *locations* in the g sectors are identical because of the imposed symmetry of the locations. A worker may move out, however, from the *location* where he received training if the *location* has moved to the d-sector and job productivity is lower. Therefore a sub market in d - sector is populated only by those who received the training in the same *location,* whereas sub markets for jobs in g sector can be populated by workers with training in g as well as d.

Let $e = g, d$, denote the current state of the *location* where he received training and denote by $e = n$ for the untrained. Let $T \equiv \{g, d, n\} \times Z$ denote the set of types of unemployed worker. Although a worker can be trained more than once, what matters is the most recent training[28]. Thus the notation of the type of training refers to the most recent one. Note that the training type of

[27]The labor force statistics in 2005 reports 230 thousands listed as fishermen in Japan. Thus there are only 78 fisherman per seaport. On the other hand, during the peak hours, each train (12 cars) in Yamanote line accommodates more than 3,000 men and women at a time.

[28]As no one choose to go back to the *location* now in the state *d,* all the past trainings before the last must have been in the *locations* which are now all in d sector.

the worker can change over time for two reasons. First the state of the *location* where he received training can change. Second, once he is employed, he receives re-training.

The job offer can be identified by the pair of wage schedule $w = (w_g, w_d)$ and contract types, wherein w_g [w_d]denotes the wage offer contingent upon the *location* being in the state g [d]. For each cohort of workers with given training cost index z, the following types of contract types are possible. Let us start with contract types in type c jobs. First, the offer can be contingent upon the state of the *location.* By contingent contract, we mean that the employment is terminated once the location is hit by the shock. For simplicity, we also assume that in this type of contract the job slot itself is self-destroyed when hit by the productivity shock[29]. Since we only consider a permanent shock, the state d is permanent. A job in g-sector can also offer an unconditional employment contract, except for the exogenous separations. Namely, an unconditional contract guarantees employment after the shock (but a worker can of course walk away any time they want). Since the state d is permanent, all the employment offer is unconditional guarantee. Hence we need to consider contingent employment offer only for jobs currently in g-sector. By the same token, non-trivial *contingent wage schedule* applies only to one type, i.e., the one that promises unconditional employment.

For untrained workers, no offers from jobs in d sector can compete with those in g sector because, for the untrained workers, the only difference between the two is lower productivity in d. Thus all the untrained workers will choose *locations in g sector.* We impose symmetry on the selection among the totally homogenous locations in the g sector so that each *location* is populated with the same density of the untrained workers.

We now consider contract types for type s jobs. It is easy to see that there can be only two types of contracts, conditional and unconditional employment in g-sector because unfilled job slots are immediately discarded as soon as the location is hit by the shock. This is the case because type s jobs do not need any training. Thus there is no advantage of retaining the vacancies in d-sector. Consequently there is no active job slot that will post an offer. Again we assume each location in g sector has the same size of the workers searching for a job[30].

Consequently we need to consider the following five types of contracts: unconditional type c in the g-sector($\bar{g}$), conditional type c in the g-sector ($\hat{g}$), (unconditional) type c in the d-sector (d), unconditional type s ($\bar{s}$), and conditional type s ($\hat{s}$). While unconditional employment contract is never terminated by the employer, conditional employment contract is terminated when the sector is hit by a permanent productivity shock without any compensations.

Since the firm can make an offer contingent on the type of worker, we can define the type contingent employment contract by the pair $\{\mathcal{C}, \mathcal{W}\}$, where $\mathcal{C} : T \to C$ and $\mathcal{W} : T \to \mathbb{R}^2_+$ are mapping from worker type to offer types and wage schedule, respectively. $C \equiv \{\bar{g}, \hat{g}, d, \bar{s}, \hat{s}\}$ is the set of type of contract. Thus each sub-market is fully specified by the job offer. In each sub market, all the job offers are of the same type, and all the workers are of the same type. Denote by x the ratio of job searchers to the vacancy, which we call queue length. As explained in the main text, in each period, with probability A, a worker can send an application to a job that she chooses. Then the probability that a vacancy receives at least one application is given by $\psi(x)$, which is increasing and strictly concave in x.

Now that the description of each sub market is complete, an allocation of the economy can be fully specified by a tuple $\{\mathcal{C}, \mathcal{W}, x\}$ where $x : T \to \mathbb{R}_+$ is the queue length in each sub market. Given the allocation $\{\mathcal{C}, \mathcal{W}, x\}$, we can compute the expected present value of profit stream for a job slot and the corresponding value of the expected income stream for a worker. Let $V_c(w, x)$ be such value for a vacancy that post the offer (c, w), where $c \in C$ is type of offer. Then we have

$$rV_c(w, x) = -p(c) + \psi(x)[J_c(w) - V_c(w, x)]. \tag{A1}$$

where $p(c) = p_c$ if $c = \bar{g}, \hat{g}, d$ and $p(c) = p_s$ otherwise. The value of filled job depends upon the state contingent wage schedule $w = (w_g, w_d)$ and the type of contract c and given by

$$\begin{aligned}
rJ_d(w) &= \theta q^c - w_d - p_c - (\delta_c + d)J_d(w), \\
rJ_{\bar{g}}(w) &= q^c - w_g - p_c - (\delta_c + d)J_{\bar{g}}(w) + \rho[J_d(w) - J_{\bar{g}}(w)], \\
rJ_{\hat{g}}(w) &= q^c - w_g - p_c - (\delta_c + d + \rho)J_{\hat{g}}(w), \\
rJ_{\bar{s}}(w) &= q^s - w_g - p_s - (\delta_s + d)J_{\bar{s}}(w) + \rho[J^s_d(w) - J_{\bar{s}}(w)], \\
rJ_{\hat{s}}(w) &= q^s - w_g - p_s - (\delta_s + d + \rho)J_{\hat{s}}(w),
\end{aligned} \tag{A2}$$

[29] This simplifying assumption is used only to avoid further crowding of notations. As we impose the zero profit condition, all the existing vacancy at equilibrium will have zero value anyway.

[30] Notice that because of the free access to the underlying technology, single location can accommodate any size of job vacancies and workers. Although uneven distribution across *locations* are immaterial as far as they remain in g sector, the impact of the technology shock on the aggregate labor market obviously will depend upon the size of the location hit by the shock. We avoid this unnecessary complications by imposing symmetry.

where $J^s_d(w)$ is given by

$$rJ^s_d(w) = \theta q^s - w_d - p_s - (\delta_s + d)J^s_d(w)$$

In words, the first is the value function for an active contract in state d. The first three terms sum up to the net flow profit, and the last term corresponds to the capital loss upon worker retirement or exogenous separations. The second and the third are the value function of type c jobs for contracts in g state. The last two equations are for type s jobs in the state g. Note that a permanent shock to the sector generates capital loss in the value of filled job only if the job is currently in the g-sector. The unconditional employment contract continues even after the sector is hit by permanent shock but it incurs the capital loss due to the change of state. On the other hand, the conditional employment contract terminates and the job slot is destroyed if the sector is hit by permanent shock. Thus the value of job turns to be zero.

Next, we consider the value of unemployed worker given the allocation and the equilibrium value of each type of unemployed worker. The rational unemployed workers choose their application strategies taking their future change of training into account. Here, we incorporate the equilibrium value into the value function instead of considering the future decision directly. Let $U_c(t, w, x, U^*)$ be the value of type t unemployed worker given the wage schedule, contract type, queue length, and equilibrium value of unemployed worker, which we denote by U^*. In this formulation, we treat as given the value of different types of unemployed workers, even though in the future he may become one of those types. We thus focus on the optimal choice of the current application strategy. We have

$$\begin{aligned} rU_c(t, w, x, U^*) =& \phi(x)[E_c(z, w, U^*) - U_c(t, w, x, U^*) - \kappa(c, t)] \\ & - dU_c(t, w, x, U^*) + I(e)\rho[U^*(d, z) - U_c(t, w, x, U^*)], \end{aligned} \tag{A3}$$

where $I(\cdot)$ is indicator function that takes value one if $e = g$ and zero otherwise. The last term in the square bracket is the capital loss associated with productivity shock which is applicable only for a worker trained in g-sector.

The training cost $\kappa(c, t)$ is given by[31]

$$\kappa(c, t) = \begin{cases} 0 & \text{if} \quad c = \bar{s}, \hat{s} \\ z & \text{if} \quad e = n \text{ and } c \neq \bar{s}, \hat{s} \\ \epsilon z & \text{if} \quad (e, c) = (d, d), (g, \bar{g}), (g, \hat{g}) \\ (\epsilon + m)z & \text{otherwise} \end{cases} \tag{A4}$$

In words, the first case applies to type s jobs irrespective of contract types as no training is required for this type of job. If a worker is never trained and he applies for a type c job, full training cost must be incurred, irrespective of the contract type. In the third line, we show the following: a worker have to pay ϵz if he decides to search and apply for a type c job in the same location where he received the last training. Finally the same worker has to incur $(\epsilon + m)z$ if he decides to move out of the current location and being matched to a type c job in the g-sector.

The value of employed worker depends on the training cost index z and wage. They are represented by the following value functions for employment.

$$\begin{aligned} rE_d(z, w, U^*) =& w_d - \delta_c[E_d(z, w, U^*) - U^*(d, z)] - dE_d(z, w, U^*), \\ rE_{\bar{g}}(z, w, U^*) =& w_g - \delta_c[E_{\bar{g}}(z, w, U^*) - U^*(g, z)] - dE_{\bar{g}}(z, w, U^*) \\ & + \rho\left[E_d(z, w, U^*) - E_{\bar{g}}(z, w, U^*)\right], \\ rE_{\hat{g}}(z, w, U^*) =& w_g - \delta_c[E_{\hat{g}}(z, w, U^*) - U^*(g, z)] - dE_{\hat{g}}(z, w, U^*) \\ & + \rho\left[U^*(d, z) - E_{\hat{g}}(z, w, U^*)\right], \\ rE_{\bar{s}}(z, w, U^*) =& w_g - \delta_s[E_{\bar{s}}(z, w, U^*) - U^*(n, z)] - dE_{\bar{s}}(z, w, U^*) \\ & + \rho\left[E^s_d(z, w, U^*) - E_{\bar{s}}(z, w, U^*)\right], \\ rE_{\hat{s}}(z, w, U^*) =& w_g - \delta_s[E_{\hat{s}}(z, w, U^*) - U^*(n, z)] - dE_{\hat{s}}(z, w, U^*) \\ & + \rho\left[U^*(n, z) - E_{\hat{s}}(z, w, U^*)\right], \end{aligned} \tag{A5}$$

where

$$rE^s_d(z, w, U^*) = w_d - \delta_s[E^s_d(z, w, U^*) - U^*(n, z)] - dE^s_d(z, w, U^*)$$

This completes the specifications of all the value functions for potentially active contract and job types.

[31] Who actually pays the training cost is immaterial. If you so wish, we could add another dimension in the contract type, depending upon who pays the training cost. Since training cost is not contingent upon the state (although its consequence does depend upon the state in the future), adding this dimension is redundant. To put it differently, if a firm offers a contract in which they pay the training cost, the equilibrium value of the offer will be the same as the current one in that wage schedule will be adjusted accordingly. Needless to say, who pays the training cost *does* matter at least potentially, if we allow incompleteness of the contract. See section 5 of this appendix below.

A.2 Market equilibrium

We now complete the specification of the model by imposing the zero profit condition for all the active vacancies. Note by the complementary slackness, expected net value of inactive vacancies must be strictly negative.

We are ready to define the market equilibrium.

Definition 1. *The market equilibrium is defined by the set* $\{\mathcal{C}^*, \mathcal{W}^*, x^*, U^*, V^*\}$ *that satisfies the following conditions.*

1. *For any type* $t \in T$*, firms post vacancies so as to maximize their values under the constraint that the offer must guarantee the equilibrium value* $U^*(t)$*, i.e., given that queue lengths are determined so as to be consistent with the equilibrium value of unemployment* $U^*(t)$*. Therefore, we have*

$$(\mathcal{C}^*(t), \mathcal{W}^*(t), x^*(t)) \in \arg\max_{c,w,x} \; V_c(w, x^*) \equiv V^*(t)$$

subject to

$$U^*(t) \geq U_c(t, w, x, U^*)$$

and $x \geq 0$ *with complementary slackness, where* U^* *solves*

$$\forall t \in T \quad U^*(t) = U_{\mathcal{C}^*(t)}(t, \mathcal{W}^*(t), x^*(t), U^*)$$

2. *By the free entry condition, the maximized value of active vacancies,* $V^*(t)$*, must be equal to zero.*

In equilibrium, a job slot computes the value of deviation based on the belief that the queue length that corresponds to alternative job offer should be adjusted so as to guarantee to the unemployed workers the market determined present value of the expected income stream. In order to obtain the market equilibrium, we can solve the problem,above, or, equivalently, we can solve the dual. That is, we maximize the value of the unemployed worker conditional upon profit maximization of all the active job offers and the zero profit condition.

$$\begin{aligned} U^*(t) &= \max_{c,w,x} \; U_c(t, w, x, U^*) \\ &\quad s.t. \quad V_c(w, x) = 0 \end{aligned} \tag{A6}$$

Before we move on to analyze the market equilibrium defined above, we offer verbal explanations why the market equilibrium defined above coincides with that of a social planner. Given the full array of contractual arrangements, the market equilibrium defined above evidently solves the resource allocation problem for a social planner endowed with the same search technology, technology evolutions underlying productivity shocks to job slots. In order to see through the logic behind, let us assume for the time being that both job slots and workers are identical among themselves and assume away also the productivity shocks, etc. None of these additional factors matters for this explanation. Under this simplified setting, an individual offer is simple a wage rate at matched worker receives.The competitive search equilibrium is such that the invididual agents take the value of the unemployed worker, N^* given. From the viewpoint of each job slot, its own wage offer must satisfy the constraint

$$\phi(x)M(w) = N^*$$

Namely the product of the probability that an application results in an offer ,$\phi(x)$, and the value of the offer, $M(w)$,should be equal to the market determined N^*. Since

$$\phi(x) \equiv \frac{\psi(x)}{x}$$

we have

$$N^* = \frac{\psi(x)M(w)}{x},$$

namely, this is the trade off between more attractive (hence higher wage) offer and the probability that a job is filled. Since the offer competition gurantees that N^* is the shadow price of the unemployed worker, the profit maximization condition ensures that the optimal choice of quene length, x, coincides that of a social planner. [32].

[32]See Moen and Rosen (2004) for a more formal proof in a similar model. The formal proofs (omitted) for our case involves straight forward but lengthy derivations of optimal policy for a social planner that solves the corresponding Hamiltonian defined upon the net social output. The solutions of course coincide with those given here for market equilibrium.

We now solve the problem (A6) in two steps. First, we calculate the value of unemployment worker given the type of employment contract, and then compare these contracts. To be specific, we derive the following value at first

$$U_c(t, U^*) \equiv \max_{w,x} U_c(t, w, x, U^*)$$
$$s.t. \quad V_c(w, x) = 0$$

then solve the functional equations

$$U^*(t) = \max_c U_c(t, U^*).$$

In order to derive market equilibrium, we must look for the optimal employment contract for each type of worker. The following points that we already made are helpful in deriving the desired functional. First, recall that the untrained or those trained in the g sector should apply to the job in the g-sector since they have no advantage to work in d-sector. In addition, the worker who search for type s jobs should also apply to the job in the g-sector. In both cases, they have no proper training or no trainings necessary so that they have no point choosing the d sector jobs which, after all, have lower productivity. The crucial remaining problem is whether or not to move for those who received training in the *location* which now belongs to d-sector. When the unemployed worker applies to the job in the g-sector, should she choose the unconditional employment contract or not?

Next, note also that the trained workers never apply to type s job in the steady state equilibrium. This is evident from the stationarity of the optimal policy: the fact that he received trained in the past implies that it was optimal to apply for a type c job. Then, it should be optimal to do so now, as well. Therefore the third remaining question is which type of untrained worker should apply to type s job.

The following proposition gives the answer to the second. For the last question, we have to wait until section A.4.

Proposition 1. *The comparison between the value of unconditional contract and conditional contract can be implemented by the checking the following inequality.*

$$U_{\bar{g}}(t, U^*) \gtreqless U_{\hat{g}}(t, U^*) \Leftrightarrow (r + d)U^*(d, z) \lesseqgtr \theta q^c - p_c$$

The intuition behind the Proposition 1 is as follows. From (A2) and (A5), the joint surplus that is gained by a match in d-sector is given by

$$E_d(z, w, U^*) + J_d(z, w, U^*) - U^*(d, z) = \frac{\theta q^c - p_j - (r + d)U^*(d, z)}{R_j}$$

Therefore, Proposition 1 says that the unconditional employment contract is preferred to the conditional contract if and only if the joint surplus from declining job is positive. The proposition also implies that the comparison between unconditional contract and conditional contract hinges only upon innate trait z, it does not change according to career path of worker. The unemployed worker trained in the [current] g-sector always should apply to the type c job in the g-sector. Hence the equilibrium value must satisfy

$$U^*(g, z) = \begin{cases} U_{\bar{g}}((g, z), U^*) \text{ if } (r + d)U^*(d, z) \leq \theta q^c - p_j \\ U_{\hat{g}}((g, z), U^*) \text{ if } (r + d)U^*(d, z) > \theta q^c - p_j \end{cases}$$

Therefore, the optimal application of type (g, z) unemployed worker depends on the expected income of type (d, z) unemployed worker. We solve the optimal application problem of type (d, z) worker, which is the most important decision in our model, in the following section.

A.3 The optimal policy for trained workers

In order to derive the optimal policy of type (d, z) [those trained in the *location* which is currently in d-sector] unemployed worker, we consider the case wherein the worker always chooses the same type of contract irrespective of his past choices. The option values that correspond to these strategies can be defined recursively

$$\tilde{U}_c(t) \equiv U_c(t, \tilde{U}_c) = \max_{w,x} \; U_c(t, w, x, \tilde{U}_c)$$
$$s.t. \quad V_c(w, x) = 0$$

Here we can find the equilibrium value of type (d, z) unemployed worker by comparing these values. If the type (d, z) worker should apply to the job in the d-sector, then his re-training (costing ϵz) would not change his training type. Therefore stationarity implies that he should apply to the same type of job in the future as well. On the other hand, by Proposition 1, if it is optimal to move out and search for a new job in g sector, he should choose the same policy in the future even though re-training could change the type of worker. Therefore, in equilibrium, type (d, z) unemployed worker should apply to the same type of contract even after he receives re-training. That is, we have

$$U^*(d,z) = \max_{c \in \{d, \bar{g}, \hat{g}\}} \tilde{U}_c(d,z)$$

We can show that the optimal contract for type (d, z) worker is monotone in z.

Lemma 1. *Let $x_c(t)$ be the queue length that maximizes the present value of the expected income stream of type t unemployed given the type of contract c and the subsequent value $U(t)$. Then the value can be written as the function of queue length,*

$$U_c(t,U) = \frac{\Delta(x_c(t))p(c) + I(e)\rho U(d,z)}{r + d + I(e)\rho}$$

where $\Delta(x) = \psi'(x)/(\psi(x) - \psi'(x)x)$. Moreover, if $U(t) = U_c(t, U)$, then we have $x_c(g, z) \leq x_c(d, z)$ for any c and z. That is, type (g, z) worker has larger probability to receive an offer than type (d, z) worker if he seek the same type of job.

Proposition 2. *Suppose that type (d, z^u) unemployed worker is indifferent between staying in the d-sector and re-entering the g-sector, i.e., $\tilde{U}_d(d, z^u) = \tilde{U}_g(d, z^u)$, where $\tilde{U}_g(d, z) = \max\{\tilde{U}_{\bar{g}}(d, z), \tilde{U}_{\hat{g}}(d, z)\}$. Then, we have*

$$U^*(d,z) = \begin{cases} \tilde{U}_d(d,z) \text{ if } & z \geq z^u \\ \tilde{U}_g(d,z) \text{ if } & z < z^u \end{cases}$$

In addition $U^*(d, z)$ is decreasing in z.

Proposition 2 can be stated in words as follows. The difference in z yields the difference in value of unemployed worker which is proportional to the additional training cost in future. Thus the impact of z is smaller if you stay at d-sector because re-training cost is proportionally larger if you move out to g-sector, i.e., the negative slope of U is steeper if you move out, than at d-sector. Therefore, only the high ability worker $(z > z^u)$ should move out to a new *location* in the g-sector after the productivity shock, whereas the low ability worker $(z < z^u)$ should stay in the same *location* after the shock.

We can also show that the choice between unconditional contract and conditional contract is monotone in z because $U^*(d, z)$ is decreasing in z. Let define z^e by $U^*(d, z^e) = (\theta q^c - p_c)/(r + d)$. Then those with lowest training cost $(z < z^e)$ should move every time after the shock , i.e., they choose type $\hat{g}$ job if $z^e < z^u$. (see Figure 5)

A.4 The optimal policy for untrained workers

Given the optimal choice of experienced workers, as summarized in the two thresholds, z^u and z^e, we can finally solve the optimal strategies of untrained workers. The untrained workers must incur the same amount of training cost z regardless of the state of sector as long as they apply to type c job, thus they should apply to the job in the g-sector if they apply to type c job. The choice between unconditional contract and conditional contract is determined by z^e as in the case of type (g, z) worker. That is, we have

$$U_g(n,z) \equiv \max\{U_{\bar{g}}(n,z), U_{\hat{g}}(n,z)\} = \begin{cases} U_{\bar{g}}((n,z),U^*) \text{ if } z \geq z^e \\ U_{\hat{g}}((n,z),U^*) \text{ if } z < z^e \end{cases}$$

The last remaining problem is who should apply for a type s job. Since this type of job does not require training, the choice should be unanimous for those who choose type s. Let $U_s = \max\{U_{\bar{s}}, U_{\hat{s}}\}$ denote the value of unemployed worker who searches for type s job. If $U_s > U_g(n, z)$, type z worker should apply to type s job when he enters the labor market. Let z^s be the threshold that satisfies $U_s = U(u, z^s)$. Since we can show that $U_g(n, z)$ is decreasing in z, the worker should apply to type s job if $z > z^s$.

Suppose that $z^e < z^u < z^s$. Then, we can summarize the optimal strategy of each ability of unemployed worker as follows. First, the most talented workers $(z < z^e)$ always apply to type c job in the g-sector and leave the job if the sector is hit by permanent shock. Second, $z \in [z^e, z^u)$ workers also apply to the type c job in the g-sector but they stay in the d-sector as long as they are employed. Third, type $z \in [z^u, z^s)$ workers apply to the type c job in the g-sector when they enter the labor market and stay in the same sector even though they become unemployed. Finally, the least adaptable $z \geq z^s$ worker always apply to the type s job in the g-sector.

Now we have made the full circle and the market equilibrium is completely determined except for the evolution of the state variables, which are shown in section A.6. Notice that we have the complete system of equations which jointly determine the equilibrium values of unemployment and employment. See the last section of this appendix for the details of derivations for the optimal queue lengths. These values are independent from the dynamics through which state variables converge to the steady state.

A.5 Ex Post Optimality and Incomplete Contract

In the definition of market equilibrium, we assumed that job slots offer two types of contract: contingent and non-contingent employment contracts. The former stipulates that the employment is terminated at the moment of the productivity shock. On the other hand, in unconditional contract, wage is made contingent upon the state. If we deprive of full commitment ability, and assume instead agents are restricted to offer non-contingent wage and employment. In that case, time inconsistency problem may arise: namely, when they post the vacancy, their optimal choice of offer entails unconditional employment at a wage rate which is also unconditional. Ex post, when the job slot filled by a worker is hit by the productivity shock, the job slot may well find it optimal to renege on the promise as the expected return under the depressed state may well be negative.

Now let us return to the contingent wage schedule contract. Whether wage schedule is state contingent or not matters only for the case in which a job slot posts an unconditional employment contract. In the proof of Lemma 1, we have shown how the queue length for each type of worker is determined by the optimal (first order) condition, whereas the optimal wage schedule is derived by substituting the queue length into the zero profit conditions.

Since single zero profit condition can pin down only the discounted sum of the state contingent wage, there are (infinitely) many wage schedules that satisfy this condition. That is, the optimal wage schedule of the unconditional employment contract is not unique. The conditions, by which the retention of employment in d-sector is made *ex post* optimal for both sides, are:

$$\frac{\theta q^c - w_d - p_c}{R_c} \geq 0, \tag{A7}$$

$$\frac{w_d + \delta_c U^*(d,z)}{R_c} \geq U^*(d,z). \tag{A8}$$

which are equivalent (respectively to)

$$\begin{aligned} J_d(w) &\geq 0, \\ E_d(z, w, U^*) &\geq U^*(d,z). \end{aligned}$$

where $R_c = r + d + \delta_c$. Therefore, if w_d satisfies

$$(r+d)U^*(d,z) \leq w_d \leq \theta q_c - p_j,$$

then the unconditional contract is *ex post* optimal. We can show that this condition can be satisfied when the joint surplus from the retention of job in d-sector is non-positive. This always holds true if type $\bar{g}$ contract is *ex ante* optimal.

Proposition 3. *In equilibrium, the firm can make the type $\bar{g}$ contract ex post optimal by the appropriate state contingent wage schedule if the type $\bar{g}$ contract is ex ante optimal.*

A.6 Steady State

Given the equilibrium allocation $\{\mathcal{C}^*, \mathcal{W}^*, x^*\}$, we consider the flow and distribution of workers at steady state. Let $e_s(z)$ and $u(e,z)$ denote the proportion of employment in the state s sector and the proportion of unemployed with experience e among trait z workers, respectively. Let define function $\iota_e(z)$ and $\iota_u(z)$ as

$$\iota_e(z) = \begin{cases} 1 & \text{if } z \geq z^e \\ 0 & \text{if } z < z^e \end{cases}, \quad \iota_u(z) = \begin{cases} 1 & \text{if } z \geq z^u \\ 0 & \text{if } z < z^u \end{cases}$$

Then the flows of workers that apply to type c jobs ($z \leq z^s$) are given by[33] (see Figures 6-8)

$$\begin{aligned} \dot{u}(n,z) &= d - (\phi_n + d)u(n,z) \\ \dot{u}(g,z) &= \delta_j e_g(z) - (\phi_g + \rho + d)u(g,z) \\ \dot{u}(d,z) &= \rho(u(g,z) + (1-\iota_e(z))e_g(z)) + \delta_j e_d(z) - (\phi_d + d)u(d,z) \\ \dot{e}_d(z) &= \iota_e(z)\rho e_g(z) + \iota_u(z)\phi_d u(d,z) - (\delta_j + d)e_d \\ \dot{e}_g(z) &= \sum_{e \in \{n,g\}} \phi_e u(e,z) + (1-\iota_u(z))\phi_d u(d,z) - (\rho + \delta_j + d)e_g(z) \end{aligned} \tag{A9}$$

[33]Simple but extremely tedious computations will show that the state variable subsystem is locally stable. It involves confirming for each sub-case the linearized transition matrix to have non-positive eigen values only. We have not encountered any (non-local) instability in numerical computations we used for the analysis in Section 4 of the main text.

where $\phi_e = \phi(x^*(e,z))$. From (A9), at steady state, we have

$$\begin{aligned}
u(n,z) &= \frac{d}{d+\phi_n} \\
u(g,z) &= \frac{d\delta\phi_n\omega_2(z)}{(d+\phi_n)\Omega(z)} \\
u(d,z) &= \frac{d\phi_u\rho\omega_1(z)}{(d+\phi_n)\Omega(z)} \\
e_d(z) &= \frac{d\phi_u\rho[\iota_e(z)(d+\rho+\phi_g)\omega_2(z)+\iota_u(z)\phi_d\omega_1(z)]}{(d+\delta_j)(d+\phi_n)\Omega(z)} \\
e_g(z) &= \frac{d(d+\rho+\phi_g)\phi_n\omega_2(z)}{(d+\phi_n)\Omega(z)}
\end{aligned}$$

where

$$\begin{aligned}
\Omega(z) &= (d+\rho)(d+\rho+\delta_j+\phi_g)\omega_2(z) - (1-\iota_u(z))\rho\phi_d\omega_1(z) \\
\omega_1(z) &= (d+\delta_j)[(1-\iota_e(z))(d+\rho+\phi_g)+\delta] + \delta_j\iota_e(z)(d+\rho+\phi_g) \\
\omega_2(z) &= d(d+\delta_j+\phi_d) + (1-\iota_u(z))\delta\phi_d
\end{aligned}$$

On the other hand, low ability $z > z^s$ worker always apply type s job. The steady state distribution for these worker, which is independent of trait of worker, is simply given by

$$\begin{aligned}
u(n,z) &= \begin{cases} \frac{d+\delta_s}{d+\delta_s+\phi_n} & \text{if } U_{\bar{s}} \geq U_{\hat{s}} \\ \frac{d+\delta_s+\rho}{d+\delta_s+\rho+\phi_n} & \text{if } U_{\bar{s}} < U_{\hat{s}} \end{cases} \\
e_s &= \begin{cases} \frac{\phi_n}{d+\delta_s+\phi_n} & \text{if } U_{\bar{s}} \geq U_{\hat{s}} \\ \frac{\phi_n}{d+\delta_s+\rho+\phi_n} & \text{if } U_{\bar{s}} < U_{\hat{s}} \end{cases}
\end{aligned}$$

where e_s is the share of employment at type s job, which is independent of z, for $z > z^s$.

A.7 Proofs

Proof of Proposition 1

From (A3), we have

$$\begin{aligned}
U_c(t,U^*) = &\max_{w,x} \frac{\phi(x)[E_c(z,w,U^*)-\kappa(c,t)]+I(e)\rho U^*(d,z)}{r+d+\phi(x)+I(e)\rho} \\
&s.t. \quad V_c(w,x) = 0
\end{aligned}$$

Note that $\kappa(\bar{g},t) = \kappa(\hat{g},t)$ for any $t \in T$. From (A4),

$$\begin{aligned}
E_{\bar{g}}(z,w,U^*) &= \frac{R_c w_g + \rho w_d + \delta(R_c U^*(g,z)+\rho U^*(d,z))}{R_c(R_c+\rho)} \\
E_{\hat{g}}(z,w,U^*) &= \frac{w_g + \delta U^*(g,z) + \rho U^*(d,z)}{R_c+\rho}
\end{aligned}$$

Use the zero profit condition to substitute for w, we get

$$U_c(t,U^*) = \max_x \frac{\phi(x)[\tilde{E}_c(z,x,U^*)-\kappa(c,t)]+I(e)\rho U^*(d,z)}{r+d+\phi(x)+I(e)\rho}$$

where

$$\begin{aligned}
\tilde{E}_{\bar{g}}(z,x,U^*) &= \frac{\tilde{q}-p_c}{R_c} + \frac{\delta(R_c U^*(g,z)+\rho U^*(d,z))}{R_c(R_c+\rho)} - \frac{p_c}{\psi(x)}, \\
\tilde{E}_{\hat{g}}(z,x,U^*) &= \frac{q^c - p_c + \delta U^*(g,z)+\rho U^*(d,z)}{R_c+\rho} - \frac{p_c}{\psi(x)}
\end{aligned}$$

where $\tilde{q} \equiv \frac{R_j+\rho\theta}{R_j+\rho} q^c$. Since we have the following, the proof is complete.

$$\forall x \quad \tilde{E}_{\bar{g}}(z,x,U^*) \lesseqgtr \tilde{E}_{\hat{g}}(z,x,U^*) \Leftrightarrow (r+d)U^*(d,z) \gtreqless \theta q^c - p_c$$

Proof of Lemma 1

The optimal queue length solves the following problem

$$U_c(t,U) = \max_x \frac{\phi(x)[\tilde{E}_c(z,x,U) - \kappa(c,t)] + I(e)\rho U(d,z)}{r+d+\phi(x)+I(e)\rho}$$

where $\tilde{E}_c(z,x,U^*)$ is the value of employment that satisfies the zero profit condition. By the first order condition, we have

$$(r+d+\psi'(x)+I(e)\rho)p_c = \gamma(x)[(r+d+I(e)\rho)(\tilde{E}_c(z,x,U)+p(c)/\psi(x)-\kappa(c,t)) - I(e)\rho U(d,z)]$$

where $\gamma(x) = \psi(x) - \psi'(x)/x$. By arranging terms, we get

$$\tilde{E}_c(z,x,U) - \kappa(c,t) = \frac{\Delta(x)x}{\psi(x)}p(c) + \frac{[\psi'(x_c(t))/\gamma(x_c(t))]p(c) + I(e)\rho U(d,z)}{r+d+I(e)\rho}$$

By substituting this into the objective function, we complete the proof of the first half of the proposition.

Second, we will show the second half of the proposition. By using the results above, we write

$$U_c((g,z),U) = \frac{\Delta(x_c(g,z))p(c) + \rho U_c((d,z),U)}{r+d+\rho},$$

$$U_c((d,z),U) = \frac{\Delta(x_c(d,z))p(c)}{r+d}.$$

Since $U_c((g,z),U) \geq U_c((d,z),U)$ and $\Delta'(x) < 0$, we have $x_c(g,z) \leq x_c(d,z)$.

Proof of Proposition 2

Let define $\bar{z}^u$ and $\hat{z}^u$ by $\tilde{U}_d(d,\bar{z}^u) = \tilde{U}_{\bar{g}}(d,\bar{z}^u)$ and $\tilde{U}_d(d,\hat{z}^u) = \tilde{U}_{\hat{g}}(d,\hat{z}^u)$, respectively. We will prove that $z \gtreqless \bar{z}^u \Leftrightarrow \tilde{U}_d(d,z) \gtreqless \tilde{U}_{\bar{g}}(d,z)$ and that $z \gtreqless \hat{z}^u \Leftrightarrow \tilde{U}_d(d,z) \gtreqless \tilde{U}_{\hat{g}}(d,z)$, which will complete the proof if we let $z^u = \max\{\bar{z}^u, \hat{z}^u\}$.

First, we show that $\partial\tilde{U}_{\bar{g}}(d,\bar{z}^u)/\partial z \leq \partial\tilde{U}_d(d,\bar{z}^u)/\partial z < 0$, which assures that $z \gtreqless \bar{z}^u \Leftrightarrow \tilde{U}_d(d,z) \gtreqless \tilde{U}_{\bar{g}}(d,z)$. Since $I(d) = 0$ and $p(c) = p_c$ for $c = d, \bar{g}, \hat{g}$, lemma 1 implies $x_d(d,\bar{z}^u) = x_{\bar{g}}(d,\bar{z}^u)$ and $x_{\bar{g}}(g,\bar{z}^u) \geq x_{\bar{g}}(d,\bar{z}^u)$. Let $x' = x_d(d,\bar{z}^u) = x_{\bar{g}}(d,\bar{z}^u)$ and $x'' = x_{\bar{g}}(g,\bar{z}^u)$.

From (A3) and (A5), $\tilde{U}_d(d,z)$ is written by

$$\tilde{U}_d(d,z) = \max_{w_d,x} \frac{w_d - R_c\epsilon z}{(r+d)(R_c+\phi(x))}\phi(x) \quad s.t. \quad V_d(w,x) = 0$$

By the envelope theorem, we have

$$\frac{\partial\tilde{U}_d(d,\bar{z}^u)}{\partial z} = -\frac{R_c\epsilon\phi(x')}{(r+d)(R_c+\phi(x'))}$$

Similarly, we have

$$\tilde{U}_{\bar{g}}(d,z) = \max_{w_g,w_d,x} \frac{R_c w_g + \rho w_d - R_c(R_c+\rho)(\epsilon+m)z + \delta_c R_c \tilde{U}_{\bar{g}}(g,z)}{(r+d)(R_c+\rho)(R_c+\phi(x)) + \phi(x)\delta_c R_c}\phi(x) \quad s.t. \quad V_{\bar{g}}(w,x) = 0$$

and

$$\tilde{U}_{\bar{g}}(g,z) = \max_{w_g,w_d,x} \frac{\phi(x)(R_c w_g + \rho w_d - R_c(R_c+\rho)\epsilon z) + \rho[R_c(R_c+\rho) + \phi(x)\delta_c]\tilde{U}_{\bar{g}}(d,z)}{R_c(r+d+\rho)(R_c+\rho+\phi(x))} \quad s.t. \quad V_{\bar{g}}(w,x) = 0$$

Again by the envelope theorem, we have

$$\frac{\partial\tilde{U}_{\bar{g}}(d,\bar{z}^u)}{\partial z} = \frac{-(R_c+\rho)R_c(\epsilon+m) + \delta_c R_c \cdot \partial\tilde{U}_{g^u}(g,z)/\partial z}{(r+d)(R_c+\rho)(R_c+\phi(x')) + \phi(x')\delta_c R_c}\phi(x') \tag{A10}$$

$$\frac{\partial\tilde{U}_{\bar{g}}(g,\bar{z}^u)}{\partial z} = \frac{-R_c(R_c+\rho)\epsilon\phi(x'') + \rho[R_c(R_c+\rho) + \phi(x'')\delta_c]\partial\tilde{U}_{g^u}(d,z)/\partial z}{R_c(r+d+\rho)(R_c+\rho+\phi(x''))} \tag{A11}$$

By substituting (A11) into (A10), we have

$$\frac{\partial\tilde{U}_{\bar{g}}(d,z)}{\partial z} = -\frac{R_c\phi(x')[(r+d+\rho)(R_c+\rho+\phi(x''))(\epsilon+m) + \delta_c\phi(x'')\epsilon]}{(r+d)[(r+d+\rho)(R_c+\phi(x'))(R_c+\rho+\phi(x'')) + \delta\phi(x')(R_c+\phi(x''))]}$$

Taking the ratio

$$\frac{\partial \tilde{U}_{\bar{g}}(d,\bar{z}^u)/\partial z}{\partial \tilde{U}_d(d,z_u^u)/\partial z} = \frac{(r+d+\rho)(R_c+\phi(x'))(R_c+\rho+\phi(x''))(\epsilon+m)/\epsilon+\delta_c\phi(x'')(R_c+\phi(x'))}{(r+d+\rho)(R_c+\phi(x'))(R_c+\rho+\phi(x''))+\delta_c\phi(x')(R_c+\phi(x''))}$$

Since $m \geq 0$ and $x'' \geq x'$, the numerator is not less than the denominator. Therefore, we have $\partial \tilde{U}_{\bar{g}}(d,\bar{z}^u)/\partial z \leq \partial \tilde{U}_d(d,\bar{z}^u)/\partial z < 0$.

Since we can also show $\partial \tilde{U}_{\hat{g}}(d,\hat{z}^u)/\partial z \leq \partial \tilde{U}_d(d,\hat{z}^u)/\partial z < 0$ by the same procedure, the first part of the proposition is proved.

The second part of the proposition is obvious because $\partial \tilde{U}_c(d,z)/\partial z$ is non-positive for $c = d, \bar{g}, \hat{g}$.

Proof of Proposition 3

If type $\bar{g}$ contract is *ex ante* optimal, we have $(r+d)U^*(d,z) \leq \theta q^c - p_c$ from Proposition 1. Therefore by setting w_d so as to satisfy (8), the firm can make the contract *ex post* optimal.

A.8 The Optimal Queue Lengths

We briefly indicate how to solve for the optimal queue lengths. As in the proof of Lemma 1, the optimal queue length satisfies

$$\begin{aligned}&(r+d+\psi'(x)+I(e)\rho)p_c\\ &= \gamma(x)[(r+d+I(e)\rho)(\tilde{E}_c(z,x,U)+p(c)/\psi(x)-\kappa(c,t))-I(e)\rho U(d,z)]\end{aligned}$$

As an example, we consider the case in which the unemployed trained workers choose to stay in the same *location*, i,e, $z \in (z^s, z^e)$. The other cases can be solved in a similar fashion.

Let x_d^* be the optimal queue length for type d worker. Since the unemployed worker chooses the type d contract, $c = d$ and $\kappa(c,t) = \epsilon z$, then x_d solves

$$\begin{aligned}&(r+d+\psi'(x_d^*))p_c\\ &= \gamma(x_d^*)(r+d)[(\tilde{E}_d(z,x,U)+p(c)/\psi(x_d^*)-\epsilon z]\\ &= \gamma(x_d^*)(r+d)\left[\frac{\theta q^c - p_c + \delta_c U^*(d,z)}{R_c} - \epsilon z\right]\end{aligned}$$

From Lemma 1, we have $U^*(t) = \frac{\Delta(x^*(t))p(c)+I(e)\rho U^*(d,z)}{r+d+I(e)\rho}$, define, thus

$$\begin{aligned}&(r+d+\psi'(x_d^*))p_c\\ &= \gamma(x_d^*)(r+d)\left[\frac{\theta q^c - p_c}{R_c} + \frac{\delta_c}{R_c}\frac{\Delta(x_d^*)p_c}{r+d} - \epsilon z\right]\end{aligned}$$

Since $\Delta(x) = \psi'(x)/\gamma(x)$, we have

$$p_c(R_c+\psi'(x_d^*)) = \gamma(x_d^*)(\theta q^c - p_c)$$

On the other hand, the optimal queue length for type g worker, x_g^*, satisfies

$$\begin{aligned}&(r+d+\psi'(x_g^*)+\rho)p_c\\ &= \gamma(x_g^*)[(r+d+\rho)(\tilde{E}_{g^u}(z,x,U)+p(c)/\psi(x_g^*)-\epsilon z)-\rho U^*(d,z)]\\ &= \gamma(x_g^*)\left[(r+d+\rho)\left(\frac{\tilde{q}-p_c}{R_c}+\frac{\delta_c(R_cU^*(g,z)+\rho U^*(d,z))}{R_c(R_c+\rho)}-\epsilon z\right)-\rho U^*(d,z)\right]\\ &= \gamma(x_g^*)\left[(r+d+\rho)\left(\frac{\tilde{q}-p_c}{R_c}-\epsilon z\right)+\frac{\delta_c R_c\Delta(x_g^*)p_c-\rho(r+d)(r+d+\rho)U^*(d,z)}{R_c(R_c+\rho)}\right]\end{aligned}$$

By rearranging terms, we have

$$R_c(R_c+\rho+\psi'(x_g^*))p_c = \gamma(x_g^*)\left[(R_c+\rho)(\tilde{q}-p_c-R_c\epsilon z)-\rho\Delta(x_d^*)p_c\right]$$

Similarly, the optimal queue length for type n worker, x_n^*, satisfies

$$\begin{aligned}&(r+d+\psi'(x_n^*))p_c\\ &= \gamma(x_n^*)(r+d)[\tilde{E}_{g^u}(z,x,U)+p(c)/\psi(x_n^*)-z]\\ &= \gamma(x_n^*)(r+d)\left[\frac{\tilde{q}-p_c}{R_c}+\frac{\delta_c(R_cU^*(g,z)+\rho U^*(d,z))}{R_c(R_c+\rho)}-z\right]\\ &= \gamma(x_n^*)(r+d)\left[\left(\frac{\tilde{q}-p_c}{R_c}-z\right)+\delta_c\frac{(r+d)R_c\Delta(x_g^*)p_c+\rho(r+d+\rho+R_c)\Delta(x_d^*)p_c}{R_c(R_c+\rho)(r+d+\rho)(r+d)}\right]\end{aligned}$$

Thus, we have

$$\begin{aligned}&(r+d+\psi'(x_n^*))p_c\\ &= \gamma(x_n^*)\left[(r+d)\left(\frac{\tilde{q}-p_c}{R_c}-z\right)+\delta_c\frac{(r+d)R_c\Delta(x_g^*)p_c+\rho(r+d+\rho+R_c)\Delta(x_d^*)p_c}{R_c(R_c+\rho)(r+d+\rho)}\right]\end{aligned}$$

(The end of the Appendix)

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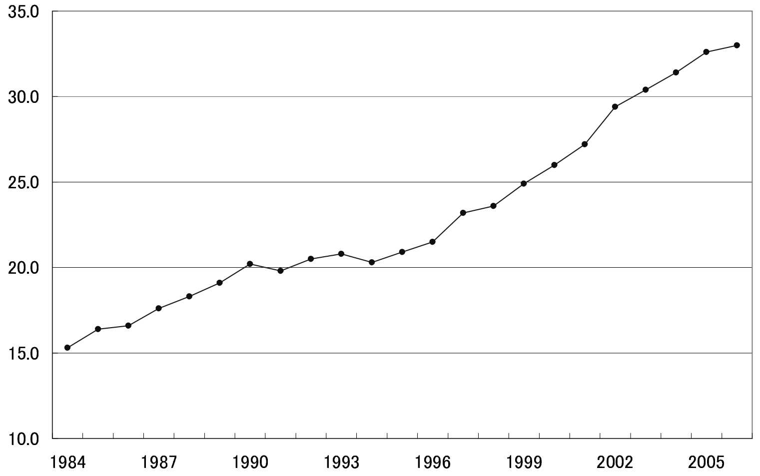

Figure 1: Share of Non-Regular Staff

Figure 2: Job Accession and Separations: Full time employees

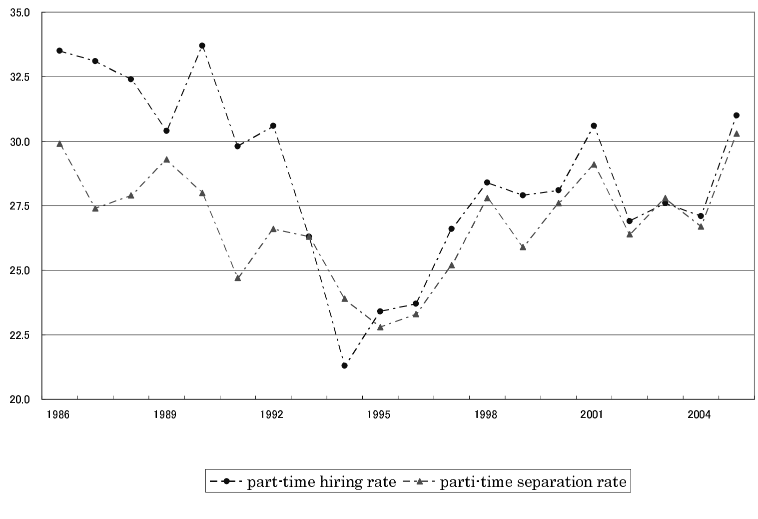

Figure 3: Job Accession and Separations: Part time employees

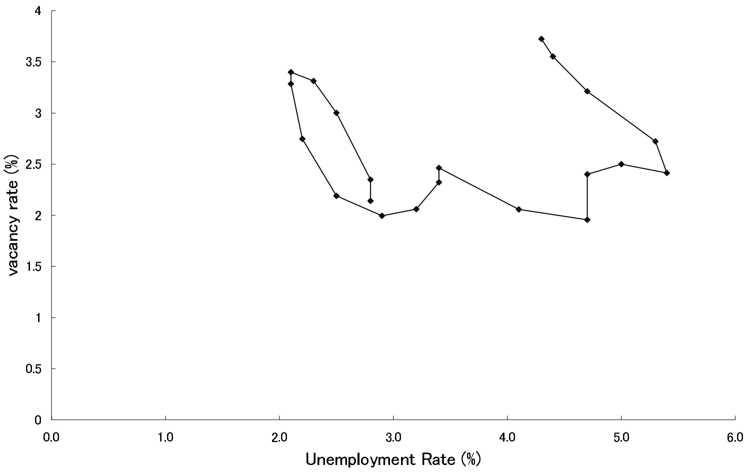

Figure 4: Beveridge Curve

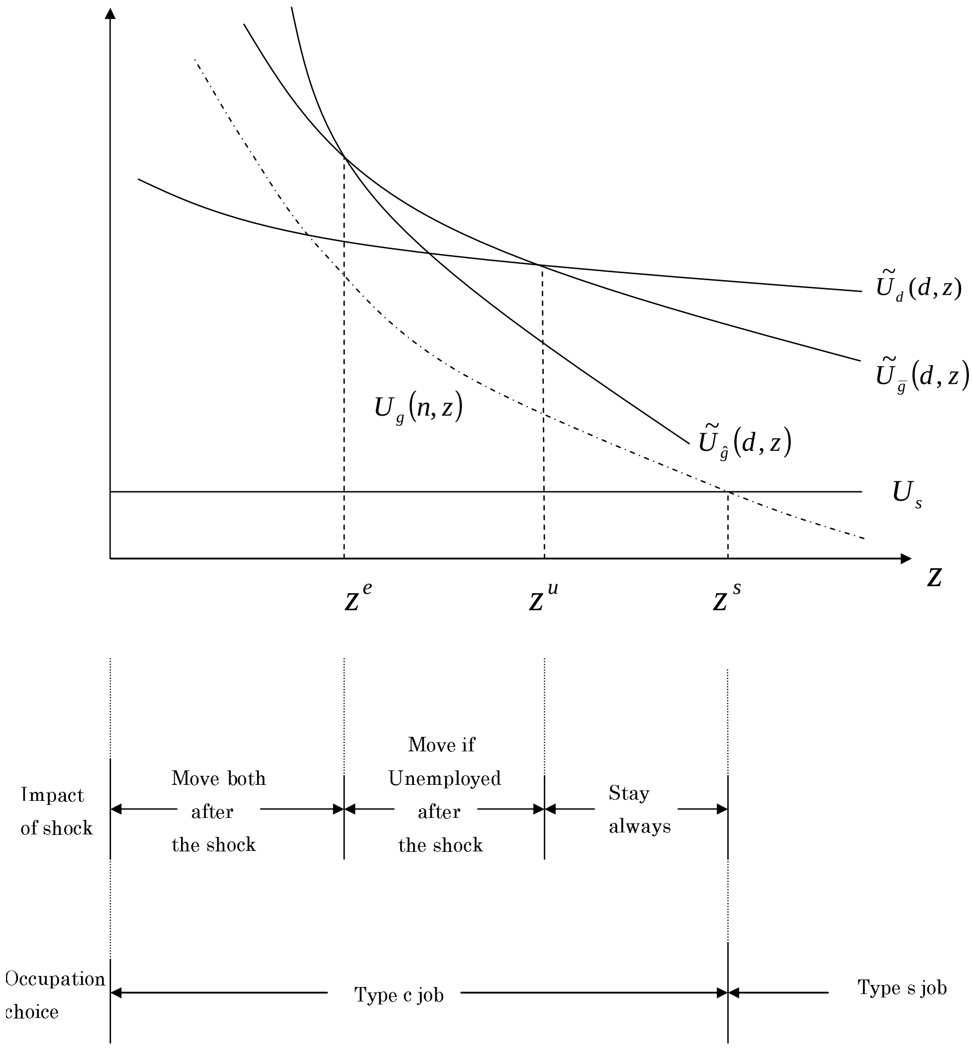

Figure 5: Optimal Application

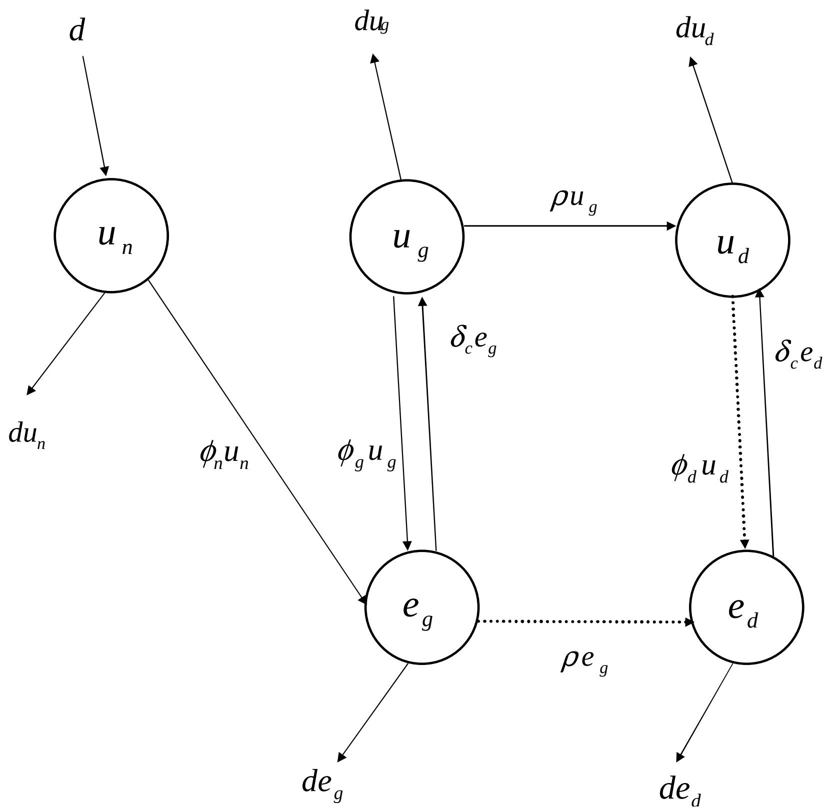

Figure 6: $z^u < z < z^s$

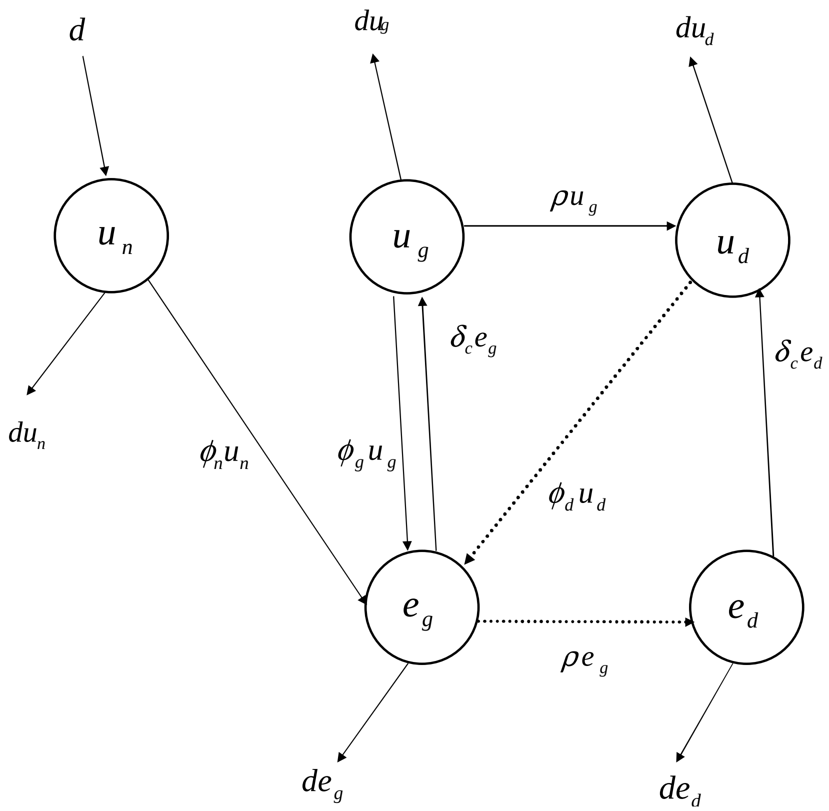

Figure 7: $z^e < z < z^u$

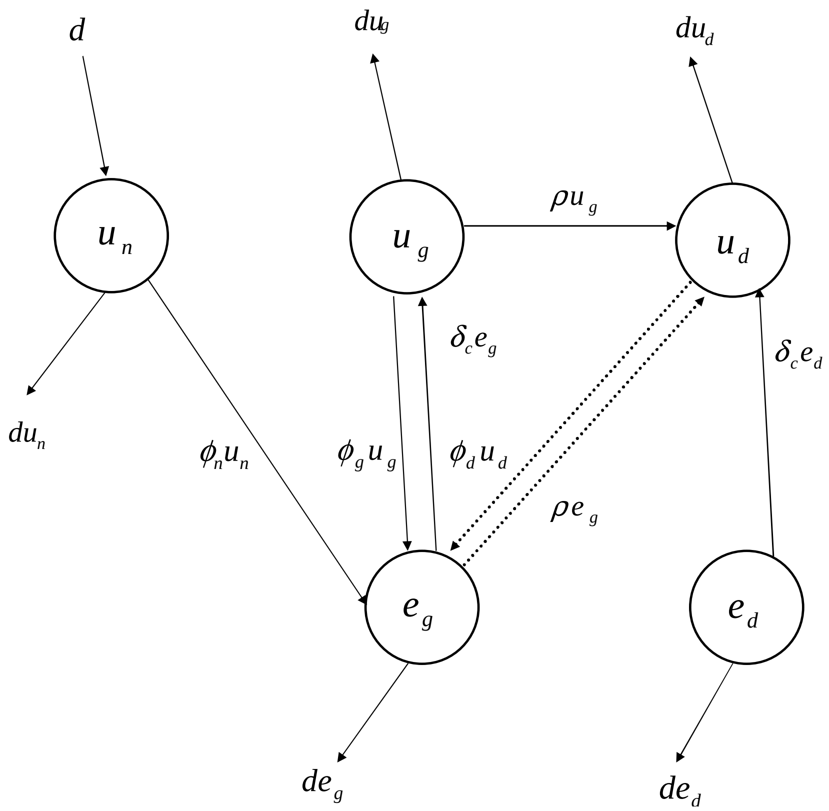
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Figure 8: $z < z^e$

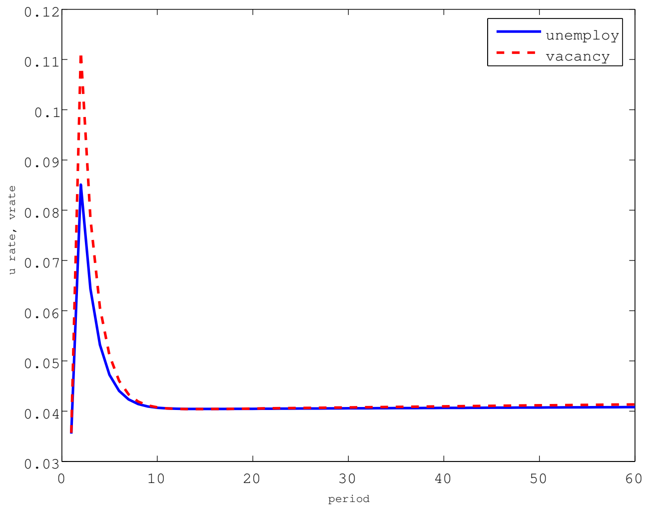

Figure 9: Simulated Dynamics of Unemployed and Vacancy: θ shock

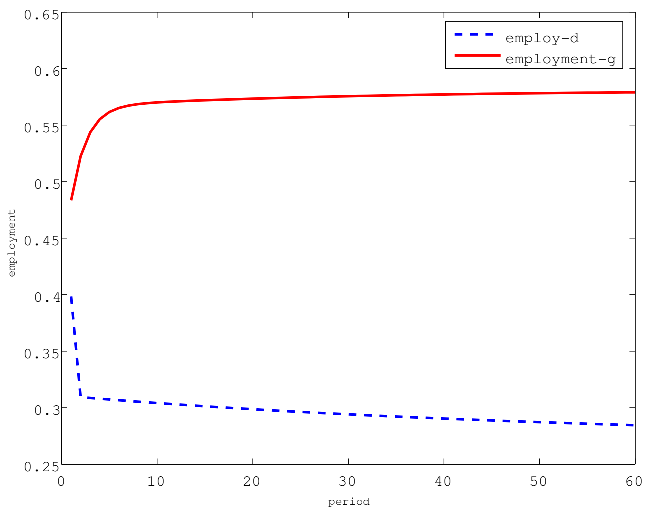

Figure 10: Simulated Dynamics of Employment Composition: θ shock

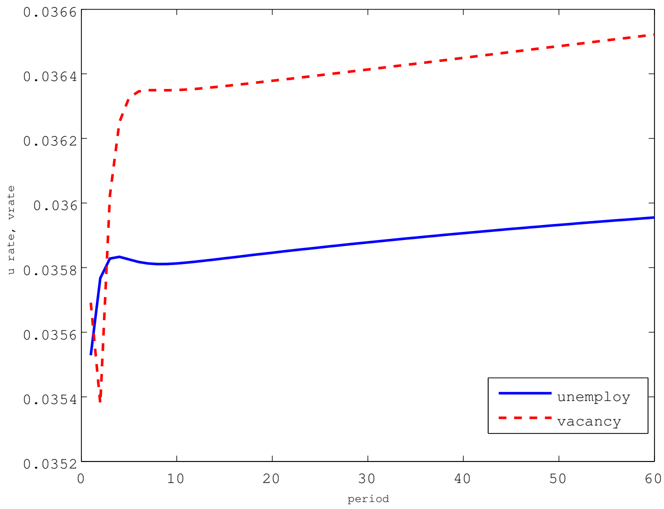

Figure 11: Simulated Dynamics of Unemployed and Vacancy: ρ shock

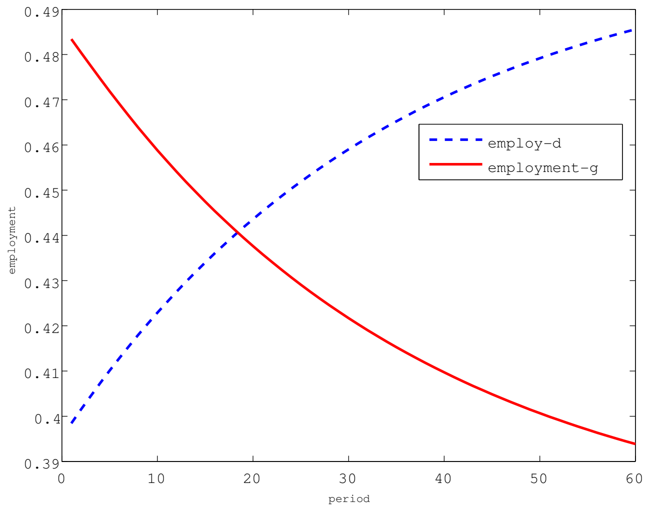

Figure 12: Simulated Dynamics of Employment Composition: ρ shock