GCOE Discussion Paper Series

Global COE Program
Human Behavior and Socioeconomic Dynamics

Horizontal mergers, firm heterogeneity, and R&D investments

Noriaki Matsushima, Yasuhiro Sato, and Kazuhiro Yamamoto

September 2009

Horizontal mergers, firm heterogeneity, and R&D investments[*]

Noriaki Matsushima[†]   Yasuhiro Sato[‡]   Kazuhiro Yamamoto[§]

September 8, 2009

Abstract

We investigate the incentive and the welfare implications of a merger when heterogeneous oligopolists compete both in process R&D and on the product market. We examine how a merger affects the output, investment, and profits of firms, whether firms have merger incentives, and, if so, whether such mergers are desirable from the viewpoint of social welfare. We also derive equilibrium configurations and explore their welfare properties.

1 Introduction

When firms face decisions on mergers, they expect to acquire greater market shares and/or to achieve greater efficiencies. These efficiency gains include operating synergies, diversification, financial synergies, and asset divestiture (Burgelman [4], Jensen and Ruback [13], Larsson and Finkelstein [15], and Tremblay and Tremblay [23]). The prospect of economies of scale is also an important motive for a horizontal acquisition (Capron [5]).[1] Economies of scale arise as a result of a horizontal merger if the merged firm achieves unit cost savings when it increases the scale of a given activity. They can also be achieved in functional areas including R&D, distribution, sales, and administrative activities through the spread of fixed costs.

Among the elements related to economies of scale, technological innovation has often been mentioned as one of the main reasons for M&As (de Man and Duysters [7] and Zhao [24]). One reason for this is that M&As may raise the overall R&D budgets of the companies involved (de Man and Duysters [7]). This allows them to reap economies of scale and enables them to tackle larger R&D projects than each individual firm could have done independently. For instance, since the 1990s, Alcoa has sequentially taken over Alumax and Reynolds, which were US aluminum companies, while Alcan, Pechiney, and Alusuisse carried out a three-way merger in 2003. Those mergers might enable the companies to enlarge their capacities and/or innovation activities. On the other hand, M&As may be executed in order to lessen innovation expenditure if the innovative activities of merged firms overlap strongly with each other. For instance, when oil companies merge, they expect to yield savings in costs, including transportation and plant construction costs, because they can use their installed oil plants cooperatively and can eliminate the overlaps of their oil plants functions (Matsushima [17]).

Efficiency gains from M&As are also achieved by merging with a firm with higher production efficiency if a technology spillover occurs within the merged firm. This can be a strong merger incentive when firms are heterogeneous in terms of their production efficiency.

---

[1] Capron [5] pointed out the significant risk of damaging acquisition performance when the divested assets and redeployed resources are those of the target.

This implies that, for a low-productivity firm, merging with a high-productivity firm can be considered as an alternative to R&D. As a result, the efficiency levels of firms are determined endogenously, and the achieved levels depend crucially on combinations of merged firms. Therefore, it would be worth analyzing M&As in the presence of both R&D activities related to firm efficiencies and heterogeneity of firms.

Several existing studies have examined the effects of R&D activities on M&As (Davidson and Ferrett [6] and Stenbacka [22]). In addition, some previous research concerning M&As took into account the heterogeneity of firms (Perry and Porter [19], Farrell and Shapiro [8], Barros [3], and Fumagalli and Vasconcelos [9]). None of these studies, however, incorporates both effects (R&D and heterogeneity) into their analytical models.

We investigate horizontal mergers in a Cournot model with firm heterogeneity and R&D investment. We first explore in detail the possible impacts of a merger on the output, investment, and profits of firms.[2] We then examine whether firms have merger incentives and, if so, whether such mergers are desirable from the viewpoint of social welfare. Our results imply that although the importance of R&D and firm heterogeneity leads to merger incentives, such mergers are desirable only when the share of efficient firms is high. In our setting, assuming sufficiently large cost differences between efficient and inefficient firms, if R&D is not costly, a merger between homogeneous firms tends to occur in equilibrium; otherwise, a merger between heterogeneous firms tends to occur. Moreover, if R&D is not costly, it is likely that we will observe a discrepancy between the equilibrium and optimal configurations.

We briefly review three closely related articles. Davidson and Ferrett [6] developed a Cournot model involving a cost-reducing R&D investment. In their model, firms with the same *ex ante* marginal cost produce differentiated goods, and a merged firm produces two types of goods. Investment is good specific, but investment has spillover effects. They showed that investment increases the profitability of mergers and also explored how the degree of the spillover affects this profitability. Stenbacka [22] considered the case in which

---

[2] Note that we cannot discuss the effect of the spillover of the investment, as in Davidson and Ferrett [6], because we assume that firms produce a homogeneous good.

the results of a cost-reducing investment are uncertain and the realization of the results is private information belonging to the innovating firm. Stenbacka then examined whether merger anticipations lead to *ex ante* incentives for innovating firms to reveal their cost-reducing information.[3] Barros [3] investigated a simple model highlighting the basic economic intuition about the relationship between initial market concentration and the size asymmetry of merger participants. He showed that, in more concentrated markets, mergers involve less asymmetric firms, and merger participants rank high in terms of the size distribution of firms. However, he did not take into account R&D activities.

The remainder of the paper is organized as follows. Section 2 presents the basic model and some results. Section 3 provides the results in the heterogeneous firm case. Section 4 concludes the paper.

2 Basic setup

We first provide a basic model that involves investments and merger decisions, and explain the basic characteristics of the effects of investment on mergers. Consider an industry consisting of N (≥ 3) firms initially. The model has three stages: first, it is determined whether or not predetermined pairs of firms merge; second, after the merger decisions, each (merged) firm chooses its investment level, which subsequently determines its marginal cost; and in the third stage, each firm engages in Cournot competition, given the decisions on the merger and the realized marginal costs.

More precisely, in the second stage, each (merged) firm determines how much to invest in cost-reducing R&D activities. Let x_i denote the investment level chosen by firm i. A unit increase in investment decreases the firm's marginal cost by the same margin. Thus, the total production cost incurred by the firm is given by $(z_i - x_i)q_i$, where q_i denotes the output level chosen by firm i. In this specification, z_i signifies the *ex ante* marginal cost (before the investment), whereas $z_i - x_i$ denotes the *ex post* marginal cost (after the investment). The cost of the investment is assumed to be γx_i^2. γ represents the degree of cost required for

---

[3] Atallah [2] also analyzed merger profitability in the case of cost-reducing R&D. In his numerical example, however, R&D investment does not provide incentives for mergers in most of cases.

4

R&D investment.

We first consider an equilibrium without mergers. In the third stage, upon observing x_i, the firms engage in standard Cournot (quantity) competition. The inverse demand function is specified as follows:

$$p = 1 - Q, \tag{1}$$

where Q is the total output. Each firm simultaneously chooses q_i so as to maximize its own profit. Here, we assume that z_i is the same across firms ($z_i = z$, $0 < z < 1$). In the next section, we allow z_i to differ across firms in a particular way. The profit of firm i is given by:

$$\pi_i = [P - (z - x_i)] q_i - \gamma x_i^2.$$

The Cournot competition leads to the following output levels:

$$q_i = \frac{1 - z + (N+1)x_i - \sum_{j=1}^N x_j}{n+1}, \tag{2}$$

$$Q = \frac{N(1-z) + \sum_{j=1}^N x_j}{N+1}.$$

Substituting this and (1) into the profit function, we can derive the optimal investment level chosen by firm i in the second stage:

$$x_i = \frac{N(1-z)}{\gamma(N+1)^2 - N}. \tag{3}$$

This yields $dx_i/dN < 0$ and readily proves that a merger (i.e., a reduction in the number N of firms) increases the investment level. A merger reduces the competition among firms and increases the market share of each firm, which leads to stronger incentives for cost-reducing investment. Equations (3) and (2) indicate the profit of firm i without the merger:

$$\pi_i = \frac{\gamma(1-z)^2 \left[\gamma(N+1)^2 - N^2\right]}{\left[\gamma(N+1)^2 - N\right]^2}. \tag{4}$$

From this, we can see that a firm supplies goods if and only if:[4]

$$\gamma > \frac{N^2}{(N+1)^2}.$$

---

[4] This is also the second-order condition for the investment decision.

5

When this condition does not hold, competition in investment is so intense that no firm can make positive profits.

We now examine how the existence of the investment decision affects the profitability and desirability of a merger. Here, in order to make the analysis as simple as possible, we consider a pairwise merger; i.e., a merger between two firms only. A pairwise merger only reduces the number of firms from n to $n-1$, implying that the profit π_M of a merged firm becomes:

$$\pi_M = \frac{\gamma \left(1-z\right)^2 \left[\gamma N^2 - \left(N-1\right)^2\right]}{\left(\gamma N^2 - N + 1\right)^2}.$$

In this section, we use simple gains from the merger as a criterion of a merger incentive.[5] Therefore, when we consider a pairwise merger, we compare π_M with the joint profit of firms involved in the merger described in (4). If the former is larger than the latter, we consider that this merger is profitable and that these two firms have an incentive to merge. More formally, a pairwise merger is profitable if:

$$\pi_M - 2\pi_i > 0. \tag{5}$$

We can show that whether this inequality holds depends on the (premerger) number n of firms and the investment cost parameter γ:

$$\pi_M - 2\pi_i = \gamma \left(1-z\right)^2 \left\{ \frac{2\left[N^2 - \gamma\left(N+1\right)^2\right]}{\left[N - \gamma\left(N+1\right)^2\right]^2} - \frac{\left(1-N\right)^2 - \gamma N^2}{\left(1 - N + \gamma N^2\right)^2} \right\}. \tag{6}$$

Because $0 < z < 1$, whether (5) holds true depends on the sign of the term in curly brackets, which depends only on γ and N. The following figure describes the region where a pairwise merger is profitable in the $\gamma - N$ plane.

[Figure 1 here]

---

[5] This criterion is consistent with the equilibrium concept of the core (see Horn and Persson [11]). In fact, when firms are homogeneous, an allocation involving a merger that satisfies this criterion is in the core under the restriction of a pairwise merger. In the next section, where we introduce firm heterogeneity, we use the core as an equilibrium concept and provide an analysis of the equilibrium.

The shaded areas of Figure 1-(i) represent the combinations of the investment cost parameter γ and the number of firms N under which a pairwise merger is profitable. Note first that in our model, a merger changes (a) the degree of competition and (b) the level of investment. As shown by Salant *et al.* [21], in standard Cournot competition, only the effect on (a) is relevant, and a merger is not profitable unless a merged firm occupies more than 80% of the market share. In contrast, once we introduce the cost-reducing investment, a pairwise merger is more likely to be profitable because of the effect on (b). Equation (3) implies that a pairwise merger increases the investment of each firm in cost-reducing R&D. A larger investment leads to a lower marginal cost and raises the profit of each firm for a given market share. Because of this effect, a pairwise merger that is not profitable under standard Cournot competition may become profitable. For example, when $\gamma = 1$, a pairwise merger is profitable for $N \leq 4$, and the merged firm occupies one-third ($\doteqdot 33\%$) at most of the market share.

Second, for a given γ, a pairwise merger is more likely to be profitable for a smaller N. This is because the fewer the number of firms, the more a pairwise merger increases the market share, which is likely to increase the additional profits of a merger.[6]

Finally, when investment is less costly, a pairwise merger is profitable for a larger N. In fact, a pairwise merger becomes profitable for any number N (≥ 3) of firms if the cost parameter γ becomes sufficiently small (The proof is given in Appendix A).

Summarizing the above arguments, we have the following proposition.

Proposition 1 *The cost-reducing R&D investment makes it more likely that a pairwise merger will be profitable. A pairwise merger is profitable for any number of firms if the investment cost is sufficiently small.*

We next examine the desirability of a pairwise merger. We use the social surplus W as

---

[6] A change in the market share due to a pairwise merger is given by:

$$\frac{1}{n-1} - \frac{1}{n} = \frac{1}{n(n-1)},$$

which is larger for a smaller number of firms.

the criterion of welfare:

$$W = \text{consumer surplus } (CS) + \text{sum of firms' profits} = \frac{Q^2}{2} + \sum_{i=1}^{N} \pi_i. \qquad (7)$$

Denoting the premerger and postmerger surpluses as W and W_M, respectively, a merger is desirable if and only if $W_M - W > 0$. The shaded areas of Figure 1-(ii) describe the combinations of γ and N under which a merger is desirable from the viewpoint of social welfare. For a given γ, a pairwise merger is less likely to be desirable for a smaller N, as the merger effect on the degree of competition is larger. Moreover, when investment is less costly, a pairwise merger is desirable for any number N (≥ 3) of firms if the cost parameter γ is sufficiently small (The proof is given in Appendix B):

Proposition 2 *The cost-reducing R&D investment makes it more likely that a pairwise merger will be desirable. A pairwise merger is desirable for any number of firms if the investment cost is sufficiently small.*

Figure 1-(iii) shows the areas in which profitability and desirability do not go together. A pairwise merger is profitable but undesirable when investment is costly and the number of firms is small ($\gamma > 1.5$ and N is close to 3). In this case, the effect of reducing competition among firms is large enough for a merger to be profitable, but the merger effect of enhancing investment is small, leading to decreases in the consumer surplus. The latter effect dominates the former, and the social surplus decreases. A pairwise merger is unprofitable but desirable when the number of firms is large and the cost of investment is not small (the upper-right area of Figure 1-(iii)). With a large number of firms, the effect of a merger on the degree of competition is small, and it scarcely reduces the consumer surplus. Hence, because of increases in the investment, the consumer surplus increases. In contrast, a small impact on the degree of competition leads to an unprofitable merger. The former effect dominates the latter, and a merger increases the social surplus.

3   Heterogeneous firms

Now, we introduce the heterogeneity of firms: we allow z to differ across firms in the most simple way. Firms are either efficient and equipped with a low *ex ante* marginal cost z_l or inefficient and equipped with a high *ex ante* marginal cost z_h, where $z_l < z_h$.

We assume that if a pair of firms merge, the value of z_i becomes the minimum value between them. In this setting, a merger between heterogeneous firms is similar to the elimination of the less efficient of the merging firms. Therefore, the incentive for a merger can possibly come from two sources: one is the efficiency gain that arises in the case of merger between heterogeneous firms, and the other is the gain from investment that is explained in the previous section.

Suppose that $z_l = c$ and $z_h = c + \delta$, where c and δ are positive constants. δ represents the difference between efficient and inefficient firms. We can apply the same calculation method to derive the equilibrium outcomes in the two cases. The *ex ante* marginal cost of a firm can be represented by $c + c_i$, where c_i $(i = 1, 2, \ldots, N)$ takes value of either 0 or δ.

When there are N firms, the profit functions of the firms are denoted by:

$$\pi(N, \mathbf{c}; c_i) = \left[1 - Q - (c + c_i - x_i)\right] q_i - \gamma x_i^2,$$

where $\mathbf{c} \equiv (c_1, c_2, \ldots, c_N)$. Given the values of x, the first-order conditions lead to:

$$q(N, \mathbf{c}; c_i) = \frac{1 - c - N(c_i - x_i) + \sum_{j \neq i}(c_j - x_j)}{N + 1}. \tag{8}$$

Substituting q into the profit functions, we derive the optimal investment levels x:

$$x(N, \mathbf{c}; c_i) = \frac{N \left\{ \left[\gamma(N + 1) - N\right] (1 - c) - N \left[\gamma(N + 1) - 1\right] c_i + \gamma(N + 1) \sum_{j \neq i} c_j \right\}}{\left[\gamma(N + 1)^2 - N\right] \left[\gamma(N + 1) - N\right]}. \tag{9}$$

¿From the optimal investment levels, we have the equilibrium quantities supplied by the firms:

$$q(N, \mathbf{c}; c_i) = \frac{\gamma(N + 1) \left\{ \left[\gamma(N + 1) - N\right] (1 - c) - N \left[\gamma(N + 1) - 1\right] c_i + \gamma(N + 1) \sum_{j \neq i} c_j \right\}}{\left[\gamma(N + 1)^2 - N\right] \left[\gamma(N + 1) - N\right]}.$$

The profits and the consumer surplus are given by:

$$\pi(N, \mathbf{c}; c_i) = \frac{\gamma \left[\gamma(N+1)^2 - N^2 \right]}{\left[\gamma(N+1)^2 - N \right]^2 \left[\gamma(N+1) - N \right]^2}$$

$$\times \left(\left[\gamma(N+1) - N \right] (1-c) - N \left[\gamma(N+1) - 1 \right] c_i + \gamma(N+1) \sum_{j \neq i} c_j \right)^2,$$

$$CS = \frac{\gamma^2(N+1)^2 \left[N(1-c) - \sum_i c_i \right]^2}{2 \left[\gamma(N+1)^2 - N \right]^2}.$$

n efficient and m inefficient firms exist   We consider the case in which n efficient and m inefficient firms exist (the number of firms is $N = n + m$). Using (8) and (9), we know that an efficient firm invests and produces more than an inefficient firm:[7]

$$x(n, m, \mathbf{c}; 0) - x(n, m, \mathbf{c}; \delta) = \frac{\delta N}{\gamma(N+1) - N} > 0, \tag{10}$$

$$q(n, m, \mathbf{c}; 0) - q(n, m, \mathbf{c}; \delta) = \frac{\gamma \delta(N+1)}{\gamma(N+1) - N} > 0.$$

Moreover, the profit functions and the consumer surplus are given by:

$$\pi(n, m, \mathbf{c}; 0) = \frac{\gamma \left[\gamma(N+1)^2 - N^2 \right] \left\{ \left[\gamma(N+1) - N \right] (1-c) + \gamma \delta(N+1)(N-n) \right\}^2}{\left[\gamma(N+1)^2 - N \right]^2 \left[\gamma(N+1) - N \right]^2},$$

$$\pi(n, m, \mathbf{c}; \delta) = \frac{\gamma \left[\gamma(N+1)^2 - N^2 \right] \left\{ \left[\gamma(N+1) - N \right] (1-c) - \delta \left[\gamma(N+m-1)(N+1) - N \right] \right\}^2}{\left[\gamma(N+1)^2 - N \right]^2 \left[\gamma(N+1) - N \right]^2},$$

$$CS = \frac{\gamma^2(N+1)^2 \left[N(1-c) - \delta(N-n) \right]^2}{2 \left[\gamma(N+1)^2 - N \right]^2}.$$

In this case, δ must satisfy the following inequality because each firm's quantity supplied must be positive.

$$0 < \delta < \frac{(1-c)((N+1)\gamma - N)}{(n+1)(N+1)\gamma - N} \equiv \bar{\delta}. \tag{11}$$

This determines the relevant interval of γ:

$$\gamma > \frac{N}{N+1}.$$

---

[7] The following inequalities hold true because we assume that $\gamma > 1$.

10

3.1 Merger effects

In this subsection, we analyze the effects of a merger on each firm's levels of investment, output, and profit. In our current framework, a pairwise merger takes one of the following three forms.

Type (I) A merger of efficient firms.

Type (II) A merger of heterogeneous firms.

Type (III) A merger of inefficient firms.

Because we assumed that the marginal cost of a merged firm is equal to the lesser marginal cost of the firms that compose the merger, a Type (I) merger is represented by a reduction in the number n of efficient firms, and a merger of Types (II) and (III) is described by a decline in the number $m = N - n$ of inefficient firms.

We first see how mergers of different types of firms affect firms. From Section 2, we know that a merger increases the investment level if firms are symmetric. In the following three cases, the relation between the investment level and a merger is satisfied:

$$\frac{\partial x(n, m, \mathbf{c}; 0)}{\partial n} < 0, \quad \frac{\partial x(n, m, \mathbf{c}; \delta)}{\partial n} < 0, \quad \frac{\partial x(n, m, \mathbf{c}; \delta)}{\partial m} < 0.$$

However, this relation does not always hold:

$$\frac{\partial x(n, m, \mathbf{c}; 0)}{\partial m} > 0 \text{ iff } \bar{\delta} > \delta > \frac{(1-c)(N^2-1)((N+1)\gamma - N)^2}{(N+1)^3((n+2)N-n) - N^2(N+1)^2(n+2)\gamma + N^2(N^2+n)},$$
(12)

$$\frac{\partial x(n, m, \mathbf{c}; 0)}{\partial m} < 0 \text{ otherwise.}$$

When the cost difference between efficient and inefficient firms is large (δ is large), a merger including an inefficient firm decreases the investment levels of the efficient firms.

We now mention the mechanism behind the inequalities (Ishida *et al.* [12]). When a firm enlarges its effort to reduce its marginal cost, two effects exist: (1) direct gain and (2) strategic gain. The direct gain stems from the quantity supplied by the firm. In other words, the significance of the cost saving is correlated to the quantity supplied. The strategic gain

stems from decreases in the quantities supplied by the firm's rivals (because of an increase in their market price). This is because the improvement of a firm's efficiency reduces the rivals' quantities supplied because of strategic substitutability. The two gains depend on the number of competitors. In general, the direct gain decreases with the number of competitors (see Section 2). However, the negative effect of the increase in competitors is small when those competitors are inefficient. The strategic gain may increase with the number of competitors because a unit decrease in a firm's marginal cost can affect more firms when the number of competitors is large. Because the quantity supplied by each efficient firm is large, the strategic gain (the price increase) has a significant impact on the incentive to invest. Therefore, if an additional competitor is less efficient, an increase in inefficient competitors enhances the R&D incentives of the efficient firms.

We now check how different the impact of a merger on the strategic variables is for different types of mergers:[8]

$$-\left(\frac{\partial x(n,m,\mathbf{c};0)}{\partial n} - \frac{\partial x(n,m,\mathbf{c};0)}{\partial m}\right) > 0, \tag{13}$$
$$-\left(\frac{\partial x(n,m,\mathbf{c};\delta)}{\partial n} - \frac{\partial x(n,m,\mathbf{c};\delta)}{\partial m}\right) > 0.$$

A Type (I) merger has a larger impact on firms' investments than a merger of Type (II) or (III). An efficient firm occupies a larger market share than an inefficient firm, which implies that a Type (I) merger, which reduces the number of efficient firms, weakens the competition among firms and increases the incentive for investment more than do mergers of other types. A similar analysis is possible regarding the output of each firm:

$$-\left(\frac{\partial q(n,m,\mathbf{c};0)}{\partial n} - \frac{\partial q(n,m,\mathbf{c};0)}{\partial m}\right) > 0, \tag{14}$$
$$-\left(\frac{\partial q(n,m,\mathbf{c};\delta)}{\partial n} - \frac{\partial q(n,m,\mathbf{c};\delta)}{\partial m}\right) > 0.$$

A Type (I) merger increases the output of each firm by more than does a merger of other types. This result arises from two effects: first, a Type (I) merger weakens the competition among firms more do other types of merger; second, it increases the cost-reducing investment

---

[8] All of the explicit derivations are given in Appendix C.

by more than do other types of mergers. As shown in Appendix C, a larger value of γ makes these tendencies weaker, implying that the lower cost of investment strengthens the impacts of a Type (I) merger relative to those of a merger of Type (II) or (III).

Next, we examine how a particular type of merger differently affects different firms:

$$-\frac{\partial}{\partial n}\left(x(n,m,\mathbf{c};0) - x(n,m,\mathbf{c};\delta)\right) < 0,$$
$$-\frac{\partial}{\partial m}\left(x(n,m,\mathbf{c};0) - x(n,m,\mathbf{c};\delta)\right) < 0,$$
$$-\frac{\partial}{\partial n}\left(q(n,m,\mathbf{c};0) - q(n,m,\mathbf{c};\delta)\right) < 0,$$
$$-\frac{\partial}{\partial m}\left(q(n,m,\mathbf{c};0) - q(n,m,\mathbf{c};\delta)\right) < 0.$$

Any type of merger affects inefficient firms more than it affects efficient firms. This result comes from the assumption of the quadratic cost function. As seen in (10), the level of investment is larger for an efficient firm than for an inefficient firm. When the cost function is quadratic, even a small change in the investment level leads to large changes in the investment cost for an efficient firm, which makes an efficient firm less sensitive to a particular change. Appendix C shows that such differences between efficient and inefficient firms in terms of their reactions to a particular merger become larger as the investment becomes less costly (as the value of γ becomes smaller).

Summarizing the above arguments, we obtain the following proposition.

Proposition 3 *(a) A merger between efficient firms (a Type (I) merger) has larger impacts on the levels of investment and output than does a merger involving an inefficient firm (a merger of Type (II) or (III)). (b) A reduction in the number of firms of a particular type has larger impacts on an inefficient firm than on an efficient firm. (c) These tendencies become more prominent as the cost of investment declines.*

Although we have uncovered the effects of a merger on investment and output, it is very difficult to know whether a merger is profitable and desirable for arbitrary values of δ (firm difference) and γ (investment cost) with n efficient and m inefficient firms. In order to obtain clear-cut results, we need to specify either a pair of δ and γ or a pair of n and m.

The following propositions shows the profitability and desirability of a merger at the limits of δ and γ, which are comparable to the results shown in Propositions 1 and 2 (the proof is given in Appendix D).

Proposition 4 *Suppose that firm differences are sufficiently large and that the investment cost is sufficiently small. A Type (I) or Type (II) merger is almost always profitable. A Type (III) merger is always profitable. A merger of any type is desirable if the ratio of efficient firms is sufficiently high.*

As shown in Appendix D, a Type (I) or Type (II) merger is profitable when the ratio of efficient firms is higher than some threshold values (T_{pI} and T_{pII}), which are given as follows. A Type (I) merger:

$$\frac{n}{N} > T_{pI} \equiv \frac{\sqrt{N[N^2(N+2)-1]/2}}{N^2(N-1)+1}.$$

A Type (II) merger:

$$\frac{n}{N} > T_{pII} \equiv \frac{\sqrt{N[N^2(N+2)-1]}}{N^2(N-1)+1}.$$

We can readily check that T_{pI} and T_{pII} decline as the number N of total firms increases. When $N = 3$, $T_{pI} \doteq 0.14$ and $T_{pII} \doteq 0.2$. Hence, in this case, any type of merger is profitable if at least one firm is efficient. These threshold values decline to $T_{pI} \doteq 0.04$ and $T_{pII} \doteq 0.05$ ($T_{pI} \doteq 0.0085$ and $T_{pII} \doteq 0.012$) when $N = 5$ ($N = 10$). Therefore, we can safely say that, even if firms are heterogeneous, a pairwise merger is very likely to be profitable if the difference between firms is large and the investment cost is small even if firms are heterogeneous.

In contrast, we obtain very different results in terms of the desirability of a merger when comparing the case with heterogeneous firms and the case with homogeneous firms. From Appendix D, the threshold value for desirability for any merger (T_d) is given as:

$$\frac{n}{N} > T_d \equiv \frac{2[N^2(N-1)+1]^2}{N(2N^5 - 4N^3 + N - 1)}.$$

We can see that T_d is increasing in N. When $N = 3$, $T_d \doteq 0.63$, implying that a merger is desirable if there is more than one efficient firm although it is undesirable if there is only one efficient firm. T_d increases to 0.70 and 0.82 when N becomes 5 and 10, respectively. As shown in Proposition 3-(b), a particular type of merger has larger impacts on the output of an inefficient firm than on that of an efficient firm. This implies that the lower the share of efficient firms is, the more a merger reduces the total output. This effect dominates the investment-enhancing effect of a merger when the share of efficient firms is lower than a certain threshold level, leading to welfare loss.

In the next subsection, we specify the number of firms as three and examine fully which type of merger is likely to occur under particular values of δ and γ. In the case of three firms, we can go beyond the arguments on profitability to arguments on the equilibrium.

3.2 Equilibrium and its efficiency—the case of three firms

Here, we demonstrate how the effects uncovered in the previous section interact to yield equilibrium configurations by using three firm cases. We follow Horn and Persson [11] and use the *core* as an equilibrium concept. In so doing, we consider only a pairwise merger and assume that each merger consists of two firms. This assumption has been used frequently in previous studies and is appropriate in analyzing merger decisions, especially in the three-firm model, because monopoly is prohibited in most countries.[9] The ownership structure of the industry is assumed to be formed through a cooperative game of coalition formation. With three firms, it is interesting to consider the following two cases: (i) one efficient and two inefficient firms exist, and (ii) two efficient and one inefficient firms exist. In these cases, we have four possible market structures.

Type (0) No merger.

Type (I) A merger of efficient firms.

Type (II) A merger of heterogeneous firms.

---

[9] See Davidson and Ferrett [6], Lommerud *et al.* [16], and Qiu and Zhou [20] for recent examples.

Type (III) A merger of inefficient firms.

In case (i), there are one efficient firm (labeled A) and two inefficient firms (labeled E and F). Therefore, we have the following three possible market structures and four ownership structures.

Type (0) No merger: $M_0^{(i)} = \{A, E, F\}$.

Type (II) A merger of heterogeneous firms: $M_2^{(i)} = \{AE, F\}$, $M_2^{(i)\prime} = \{AF, E\}$.

Type (III) A merger of inefficient firms: $M_3^{(i)} = \{A, EF\}$.

In case (ii), there are two efficient firms (labeled A and B) and one inefficient firms (labeled E), and the following three possible market structures and four ownership structures are relevant.

Type (0) No merger: $M_0^{(ii)} = \{A, B, E\}$.

Type (I) A merger of efficient firms: $M_1^{(ii)} = \{AB, E\}$.

Type (II) A merger of heterogeneous firms: $M_2^{(ii)} = \{AE, B\}$, $M_2^{(ii)\prime} = \{A, BE\}$.

The solution procedure is based on Horn and Persson [11]. They treat the merger process as a cooperative game of coalition formation, where the players are free to communicate and to write binding contracts. When owners of firms agree on a merger, they can decide on any division of the firm's profits. However, payments between coalitions are not allowed. The approach then involves a comparison of any two possible ownership structures M_i and M_j. M_i is said to "dominate" M_j if the combined profits of the "decisive" group of owners are larger in M_i than in M_j. The decisive group of owners consists of the owners that are expected to be able to influence whether M_i will be formed instead of M_j and vice versa. In our model, although owners belonging to identical coalitions in M_i and M_j cannot affect whether M_j will be formed instead of M_i, all remaining owners can influence this choice and thus they are decisive.[10] For instance, when we compare Type (0) (no merger, $M_0^{(i)}$) and

---

[10] See Horn and Persson [11] for a detailed discussion on these points.

Type (III) (a merger of inefficient firms, $M_3^{(i)}$) in case (i), firm A does not merge in both ownership structures, and firms E and F merge in $M_3^{(i)}$ and do not merge in $M_0^{(i)}$. In this case, E and F are the decisive owners, whereas A is not, implying that $M_3^{(i)}$ dominates $M_0^{(i)}$ if the profit of the merged firm in $M_3^{(i)}$ is larger than the sum of profits of firms E and F in $M_0^{(i)}$. The ownership structure is in the core if the structure is undominated by any other structures. We use the core as the equilibrium concept in the following analysis.

One efficient and two inefficient firms exist   There are two types of pairwise mergers: Type (II), a merger between heterogeneous firms; and Type (III), a merger between inefficient firms. Figure 2 illustrates the combinations of the cost parameter (γ) and the degree of firm difference $(\delta/(1-c))$ that lead to equilibrium.

[Figure 2 here]

Note first that, because both types of mergers imply a decrease in the number of inefficient firms, neither of the ownership structures in the two types of mergers dominates the other. Hence, a particular ownership is in the core if it is undominated by the ownership structure in the no-merger case. A Type (II) merger takes place in equilibrium for a broad area, whereas we observe a Type (III) merger when the investment cost is not large, which comes from Proposition 3 (c). When firms are less different and R&D investments are very costly, the no-merger case is in the core because of the "merger paradox" (Salant et al. [21]). When firms are very different and R&D investments are not costly (see the upper left area in Figure 2), a merger between inefficient and efficient firms diminishes the merged firm's incentive to invest (see (12)), and then this merger is not profitable. On the other hand, it is likely that two inefficient firms will merge because the merger mitigates the efficient firm's incentive to invest (see (12)).

We now discuss how those pairwise mergers change social welfare. The two types of pairwise mergers lead to the same duopoly market with one efficient and one inefficient firm. From the results in the previous subsections, we have the conditions under which those pairwise mergers enhance/harm social welfare.

[Figure 3 here]

We now mention the mechanism behind the effects of mergers depicted in Figure 3. When the cost difference is moderate, a merger is desirable because the elimination of an inefficient firm improves the production efficiency in this market (Lahiri and Ono [14]). When the cost difference is large enough and R&D investments are not costly, a merger is not desirable. The existence of inefficient firms induces the efficient firm to engage in more R&D (see (12)), which then accelerates the scale economies of R&D investments engaged in by the efficient firm.

Two efficient firms and one inefficient firm exist There are two types of pairwise mergers: Type (I), a merger between efficient Firms; and Type (II), a merger between heterogeneous firms. The following two figures describe the equilibrium and its welfare properties.

[Figure 4 here]

[Figure 5 here]

In this case, again, a Type (II) merger takes place in equilibrium over a broad area of the Figure. In contrast, a Type (I) merger occurs only when the degree of firm difference is sufficiently large and the investment cost is sufficiently small. A Type (I) merger leads to a duopoly of efficient and inefficient firms, whereas a Type (II) merger results in a duopoly of two efficient firms. Proposition 3 (a) implies that the former has larger impacts on the price than the latter. This effect is amplified as γ gets smaller (Proposition 3 (c)). Moreover, a large firm difference indicates that the merged firm in a Type (I) merger is more likely to have a higher market share, implying that this type of merger is more likely to be profitable. Meanwhile, from Proposition 3 (b), an outside firm (an inefficient firm) in a Type (I) merger gains more than the merged firm and this ownership structure is in equilibrium.

We now mention the mechanism behind the effects of mergers depicted in Figure 5. The mechanism behind a Type (II) merger is similar to the previous case in which only one efficient firm exists. When the cost difference is small and R&D investments are not costly,

18

a Type (I) merger is desirable even though it does not take place. The desirability of the merger stems from the scale economies of R&D investments, but it does not take place because of the "merger paradox" (Davidson and Ferrett [6] and Salant *et al.* [21]). When the cost difference is large and R&D investments are not costly, a Type (I) merger is not desirable even though it takes place. The desirability of the merger stems from the increase in the degree of market concentration, but again the merger does not take place for the reasons discussed in the previous paragraph. Because the inefficient firm is highly inefficient, a Type (I) merger gives a strong market power to the merged firm.

3.3 Discussions on firm heterogeneity

In this subsection, we consider the case in which three firms are all different: firms A and C, respectively, are efficient and inefficient, and their *ex ante* marginal costs are c and $c+\delta$, respectively. Firm B is the second most efficient firm, and its *ex ante* marginal cost is $c+\alpha\delta$, where $0 \leq \alpha \leq 1$.

[Figure 6 here]

Figure 6 shows that the desirability of a merger is nonmonotonic with respect to α when R&D investments are not costly ($\gamma = 1$). That is, given the efficiencies of the most and the least efficient firms, the second most efficient firm with an intermediate value of α does not enhance social welfare. This is because it crowds out the incentive of the most efficient firm to invest. If it is less efficient (the value of α is higher), it enhances the incentive of the most efficient firm to invest. Therefore, if R&D investments are not costly (γ is small), inefficient firms can be more beneficial than ones with intermediate efficiency levels. When R&D investments are costly ($\gamma = 4$), the standard intuition mentioned by Lahiri and Ono [14] holds. As the efficiency of a firm becomes worse, the existence of the firm tends to be harmful from the viewpoint of social welfare.

4 Conclusion

We investigated the incentive and the welfare implications of a merger when heterogeneous oligopolists compete both in process R&D and on the product market. We showed that although firms have merger incentives in the presence of R&D, such mergers improve social welfare only when the share of efficient firms is sufficiently high. Under a sufficient degree of firm heterogeneity, a low R&D cost leads to a merger between homogeneous firms in equilibrium, whereas a high R&D cost implies a merger between heterogeneous firms. Moreover, if R&D is not costly, we observe a discrepancy between the incentives for, and the desirability of, mergers.

Appendix A: *Profitability of a pairwise merger for a sufficiently small γ.*

¿From (6), $\pi_M - 2\pi_i$ is positive if and only if:

$$\Delta = 2\left[N^2 - (N+1)^2\,\gamma\right]\left(1 - N + N^2\gamma\right)^2 + \left[N^2\gamma - (1-N)^2\right]\left[N - (N+1)^2\,\gamma\right]^2 > 0.$$

For a given number N (≥ 3) of firms, firms supply goods as long as $\gamma > N^2/(N+1)^2$. Therefore, the lower bound of the relevant interval of γ is $N^2/(N+1)^2$. Evaluating Δ at this lower bound, we have:

$$\lim_{\gamma \to N^2/(N+1)^2+0} \Delta = \frac{(N-1)^2 N^2 (2N^2 - 1)}{(N+1)^2} > 0 \qquad \text{for } N \geq 3.$$

Appendix B: *Desirability of a pairwise merger for a sufficiently small γ.*

From (7), we have:

$$W_M - W = \frac{(N-1)\left[4N - 2 + N^2(\gamma - 2) + N^3\gamma\right]}{(1 - N + N^2\gamma)^2} + \frac{2N^3 - N(N+2)(N+1)^2\gamma}{\left[N - (N+1)^2\gamma\right]^2}.$$

This is positive if and only if:

$$\Gamma = (N-1)\left[4N - 2 + N^2(\gamma - 2) + N^3\gamma\right]\left[N - (N+1)^2\gamma\right]^2$$
$$+ \left[2N^3 - N(N+2)(N+1)^2\gamma\right]\left(1 - N + N^2\gamma\right)^2$$
$$> 0.$$

Evaluating Γ at this lower bound, we have:

$$\lim_{\gamma \to N^2/(N+1)^2+0} \Gamma = \frac{2N^2}{(N+1)^4} + \frac{N^3\left\{N\,(2N-3)\left[3 + 2N^2\left(N^2 - 2\right)\right] - 2\right\}}{(N+1)^4} > 0 \qquad \text{for } N \geq 3.$$

Appendix C: *Derivations of the effects of mergers.*

From (8) and (9), we obtain:

$$-\left(\frac{\partial x(n,m,\mathbf{c};0)}{\partial n}-\frac{\partial x(n,m,\mathbf{c};0)}{\partial(N-n)}\right)=-\left(\frac{\partial x(n,m,\mathbf{c};\delta)}{\partial n}-\frac{\partial x(n,m,\mathbf{c};\delta)}{\partial(N-n)}\right)$$
$$=\frac{\gamma\delta N(N+1)}{[\gamma(N+1)-N]\,[\gamma(N+1)^2-N]}>0,$$
$$-\left(\frac{\partial q(n,m,\mathbf{c};0)}{\partial n}-\frac{\partial q(n,m,\mathbf{c};0)}{\partial(N-n)}\right)=-\left(\frac{\partial q(n,m,\mathbf{c};\delta)}{\partial n}-\frac{\partial q(n,m,\mathbf{c};\delta)}{\partial(N-n)}\right)$$
$$=\frac{\gamma^2\delta(N+1)^2}{[\gamma(N+1)-N]\,[\gamma(N+1)^2-N]}>0.$$

A larger value of γ makes these tendencies weak because:

$$\frac{\partial}{\partial\gamma}\left(\frac{\gamma\delta N(N+1)}{[\gamma(N+1)-N]\,[\gamma(N+1)^2-N]}\right)=\frac{\delta N(N+1)\left[N^2-\gamma^2(N+1)^3\right]}{[\gamma(N+1)-N]^2\,[\gamma(N+1)^2-N]^2}<0,$$
$$\frac{\partial}{\partial\gamma}\left(\frac{\gamma^2\delta(N+1)^2}{[\gamma(N+1)-N]\,[\gamma(N+1)^2-N]}\right)=\frac{\gamma\delta N(N+1)^2\left[2N-\gamma(N+1)(N+2)\right]}{[\gamma(N+1)-N]^2\,[\gamma(N+1)^2-N]^2}<0.$$

Moreover, differences between efficient and inefficient firms in their reactions to a particular type of merger are given by:

$$-\frac{\partial}{\partial n}\left(x(n,m,\mathbf{c};0)-x(n,m,\mathbf{c};\delta)\right)=-\frac{\partial}{\partial(N-n)}\left(x(n,m,\mathbf{c};0)-x(n,m,\mathbf{c};\delta)\right)$$
$$=-\frac{\partial}{\partial n}\left(q(n,m,\mathbf{c};0)-q(n,m,\mathbf{c};\delta)\right)$$
$$=-\frac{\partial}{\partial(N-n)}\left(q(n,m,\mathbf{c};0)-q(n,m,\mathbf{c};\delta)\right)$$
$$=-\frac{\gamma\delta}{[\gamma(N+1)-N]^2}<0.$$

These differences are smaller for a larger value of γ because:

$$\frac{\partial}{\partial\gamma}\left(-\frac{\gamma\delta}{[\gamma(N+1)-N]^2}\right)=\frac{\delta\left[\gamma(1+N)+N\right]}{[\gamma(N+1)-N]^3}>0.$$

Appendix D: *Profitability and desirability in the limits of δ and γ.*

A pairwise merger is profitable if (5) holds true. For each type of merger, this condition becomes as follows.

A Type (I) pairwise merger (i.e., a merger of efficient firms) is profitable if $\pi(n-1,m,\mathbf{c};0)-2\pi(n,m,\mathbf{c};0)>0$.

A Type (II) pairwise merger (i.e., a merger of heterogeneous firms) is profitable if $\pi(n, m - 1, \mathbf{c}; 0) - \pi(n, m, \mathbf{c}; 0) - \pi(n, m, \mathbf{c}; \delta) > 0$.

A Type (III) pairwise merger (i.e., a merger of inefficient firms) is profitable if $\pi(n, m - 1, \mathbf{c}; 0) - 2\pi(n, m, \mathbf{c}; \delta) > 0$.

The upper bound of the relevant interval of δ is $\bar{\delta}$, given in (11), and the lower bound of the relevant interval of γ is $N/(N + 1)$. Evaluating gains from a merger at these bounds, we obtain:

$$\lim_{\gamma \to N/(N+1)+0} \lim_{\delta \to \bar{\delta}+0} \pi(n - 1, m, \mathbf{c}; 0) - 2\pi(n, m, \mathbf{c}; 0)$$

$$= \frac{(1 - z)^2 \{n^2 N[N^2(N + 2) - 1] - 2[N^2(N - 1) + 1]^2\}}{n^2(N + 1)[N^2(N - 1) + 1]^2},$$

$$\lim_{\gamma \to N/(N+1)+0} \lim_{\delta \to \bar{\delta}+0} \pi(n, m - 1, \mathbf{c}; 0) - \pi(n, m, \mathbf{c}; 0) - \pi(n, m, \mathbf{c}; \delta)$$

$$= \frac{(1 - z)^2 \{n^2 N[N^2(N + 2) - 1] - [N^2(N - 1) + 1]^2\}}{n^2(N + 1)[N^2(N - 1) + 1]^2},$$

$$\lim_{\gamma \to N/(N+1)+0} \lim_{\delta \to \bar{\delta}+0} \pi(n, m - 1, \mathbf{c}; 0) - 2\pi(n, m, \mathbf{c}; \delta)$$

$$= \frac{(1 - z)^2 N(N^2 + N - 1)}{[N^2(N - 1) + 1]^2} > 0.$$

We thus see that when δ is sufficiently large and γ is sufficiently small, a pairwise merger of Type (I) is profitable if:

$$\frac{n}{N} > \frac{\sqrt{N[N^2(N + 2) - 1]/2}}{N^2(N - 1) + 1}.$$

Similarly, a pairwise merger of Type (II) is profitable if:

$$\frac{n}{N} > \frac{\sqrt{N[N^2(N + 2) - 1]}}{N^2(N - 1) + 1}.$$

The desirability of a merger comes from the following results:

$$\lim_{\gamma \to N/(N+1)+0} \lim_{\delta \to \bar{\delta}+0} \left[-\frac{\partial W}{\partial n} \right] = \lim_{\gamma \to N/(N+1)+0} \lim_{\delta \to \bar{\delta}+0} \left[-\frac{\partial W}{\partial m} \right]$$

$$= \frac{(1 - z)^2 \{n(2N^5 - 4N^3 + N - 1) - 2[N^2(N - 1) + 1]^2\}}{2n(N + 1)[N^2(N - 1) + 1]^2} > 0.$$

This implies that a merger is desirable if:

$$\frac{n}{N} > \frac{2[N^2(N - 1) + 1]^2}{N(2N^5 - 4N^3 + N - 1)}.$$

23

References

[1] Andrade, G., M. Mitchell, and E. Stafford, 2001, New evidence and perspectives on mergers, *Journal of Economic Perspectives* 15, 103–120.

[2] Atallah, G., 2005, R&D and merger profitability, *Seoul Journal of Economics* 18, 325–354.

[3] Barros, P. P., 1998, Endogenous mergers and size asymmetry of merger participants, *Economics Letters* 60, 113–119.

[4] Burgelman, R. A., 1983, A process model of internal corporate venturing in the diversified major firm, *Administrative Science Quarterly* 28, 223–244.

[5] Capron, L., 1999, The long-term performance of horizontal acquisitions, *Strategic Management Journal* 20, 987–1018.

[6] Davidson C., and B. Ferret, 2007, Mergers in multidimensional competition, *Economica* 74, 695–712.

[7] de Man, A.-P., and G. Duysters, 2005, Collaboration and innovation: a review of the effects of mergers, acquisitions, and alliances on innovation, *Technovation* 25, 1377–1387.

[8] Farrell, J., and C. Shapiro, 1990, Horizontal mergers: An equilibrium analysis, *American Economic Review* 80, 107–126.

[9] Fumagalli, E., and H. Vasconcelos, 2009, Sequential cross-border mergers, *International Journal of Industrial Organization* 27, 175–187.

[10] Hijzen, A., H. Görg, and M. Manchin, 2008, Cross-border mergers and acquisitions and the role of trade costs, *European Economic Review* 52, 849–866.

[11] Horn, H., and L. Persson, 2001, The equilibrium ownership of an international oligopoly, *Journal of International Economics* 53, 307–333.

[12] Ishida, J., T. Matsumura, and N. Matsushima, 2008, When market competition benefits firms, OSIPP Discussion Papers 08-05, Osaka University.

[13] Jensen, M.C., and R. S. Ruback, 1983, The market for corporate control: The scientific evidence, *Journal of Financial Economics* 11, 5–50.

[14] Lahiri, S., and Y. Ono, 1988, Helping minor firms reduces welfare, *Economic Journal* 98, 1199–1202.

[15] Larsson, R., and S. Finkelstein, 1999, Integrating strategic, organizational, and human resource perspectives on mergers and acquisitions: A case survey of synergy realization, *Organization Science* 10, 1–26.

[16] Lommerud, K.E., O.R. Straume, and L. Sørgard, 2006, National versus international mergers in unionized oligopoly, *RAND Journal of Economics* 37, 212–233.

[17] Matsushima, N., 2001, Horizontal mergers and merger waves in a location model, *Australian Economic Papers* 40, 263–286.

[18] Matsushima, N., Y. Sato, and K. Yamamoto, 2008, Trade and mergers in the presence of firm heterogeneity, *Discussion Papers in Economics and Business* 08-35, Osaka University.

[19] Perry, M., and R. Porter, 1985, Oligopoly and the incentive for horizontal merger, *American Economic Review* 75, 219–227.

[20] Qiu, L. D., and W. Zhou, 2006, International mergers: Incentives and welfare, *Journal of International Economics* 68, 38–58.

[21] Salant, S., S. Switzer, and R. Reynolds, 1983, Losses from horizontal merger: The effects of an exogenous change in industry structure on Cournot–Nash equilibrium, *Quarterly Journal of Economics* 98, 185–199.

[22] Stenbacka, L.R., 1991, Mergers and investments in cost reduction with private information, *International Journal of Industrial Organization* 9, 397–405.
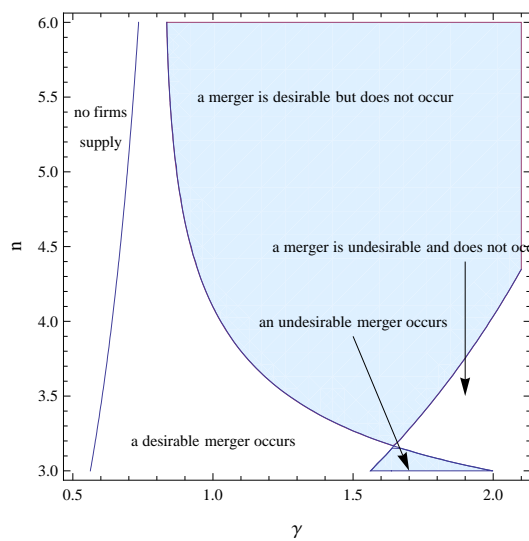
[23] Tremblay, V.J., and C.H. Tremblay, 1988, The determinants of horizontal acquisitions: Evidence from the US brewing industries, *Journal of Industrial Economics* 37, 21–45.

[24] Zhao, X., 2009, Technological innovation and acquisitions, *Management Science* 55, 1170–1183.
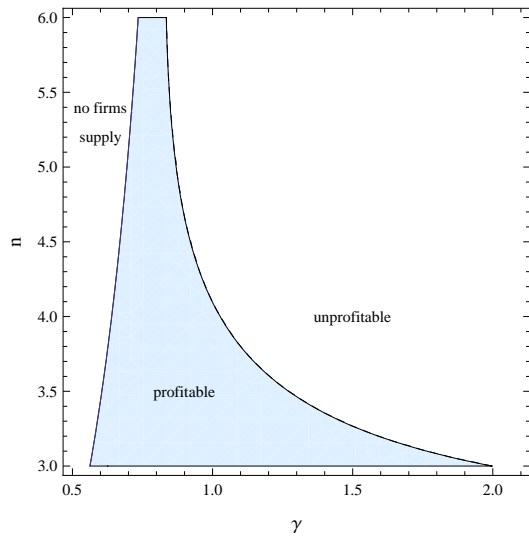
(i) profitability
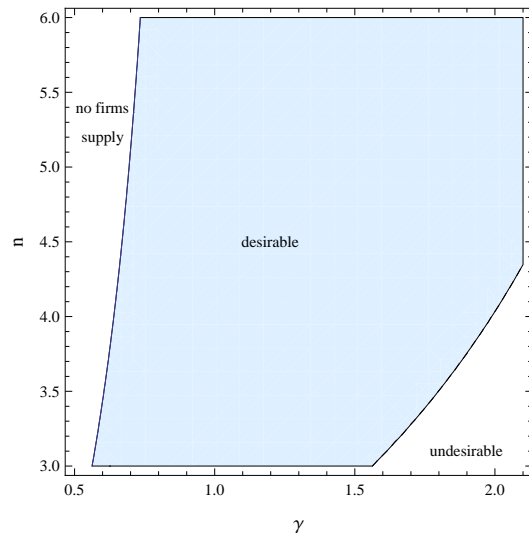
(ii) desirability

(iii) discrepancy

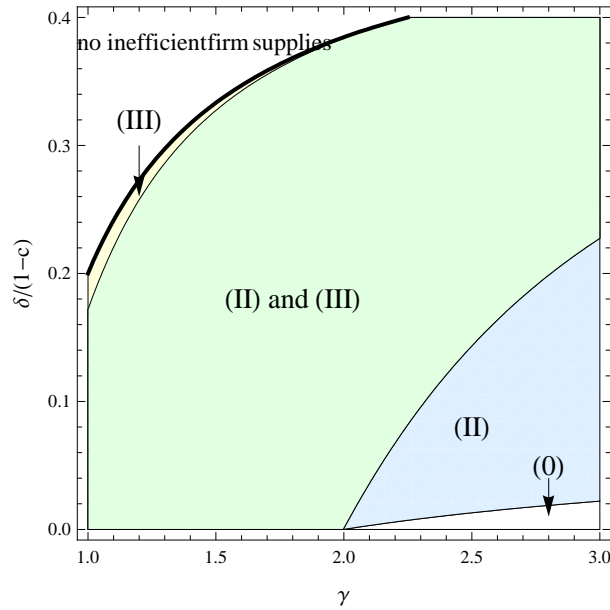Figure 1: Profitability and desirability of a pairwise merger with investment.

Figure 2: Equilibrium: one efficient and two inefficient firms.

Note: Type (0)=No merger, Type (II)=A merger of heterogeneous firms,
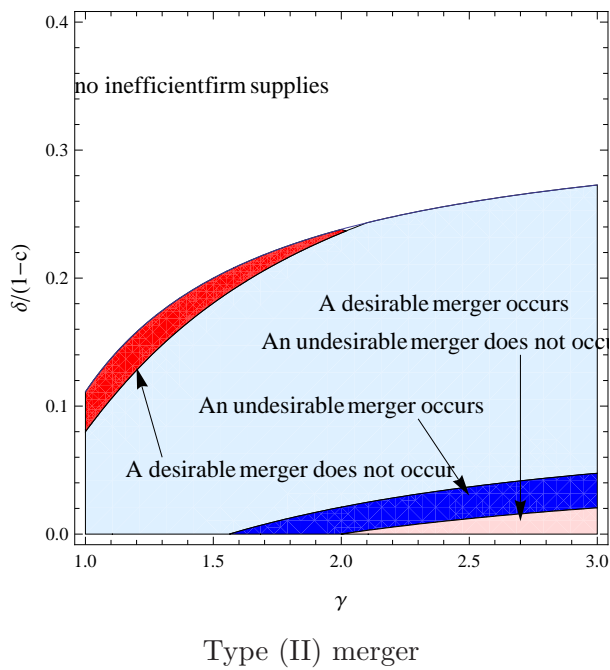   Type (III)=A merger of inefficient firms.
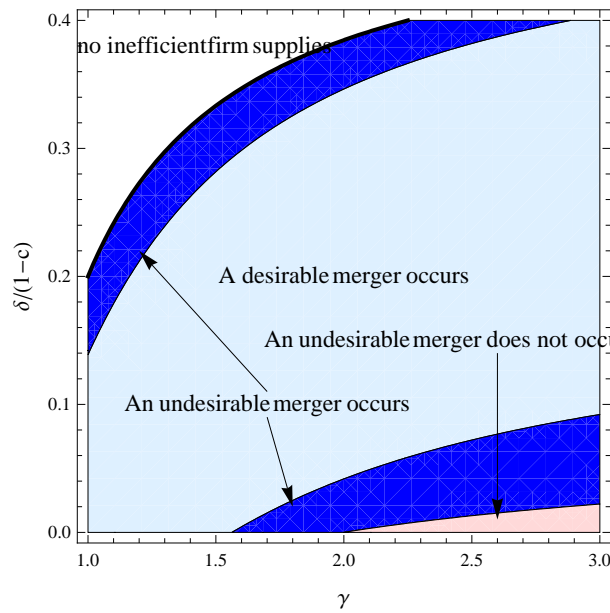
Figure 3: Social welfare and the desirability of mergers: one efficeint and two inefficient firms.

28
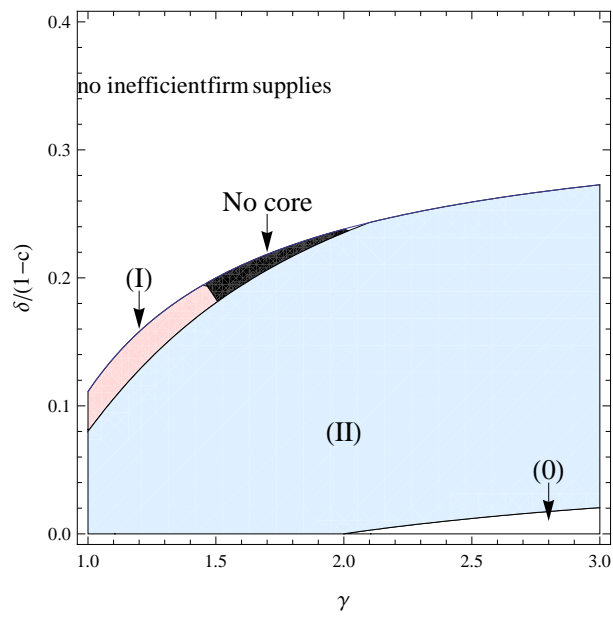
Figure 4: Equilibrium: two efficient and one inefficient firms.

Note: Type (0)=No merger, Type (I)=A merger of efficient firms,
        Type (II)=A merger of heterogeneous firms.
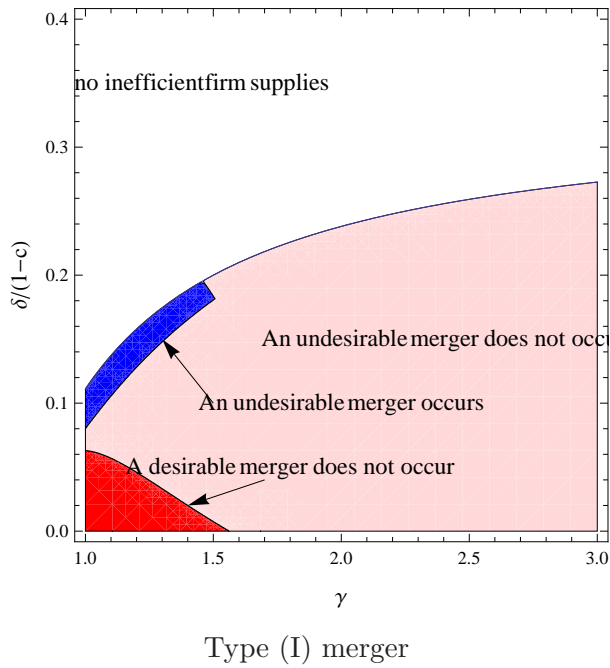
Type (I) merger

Type (II) merger

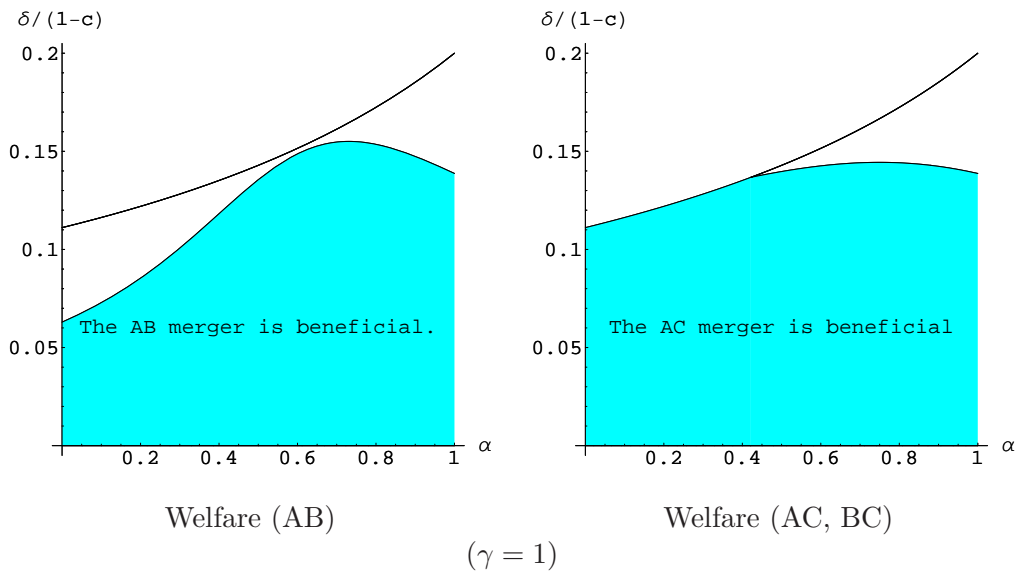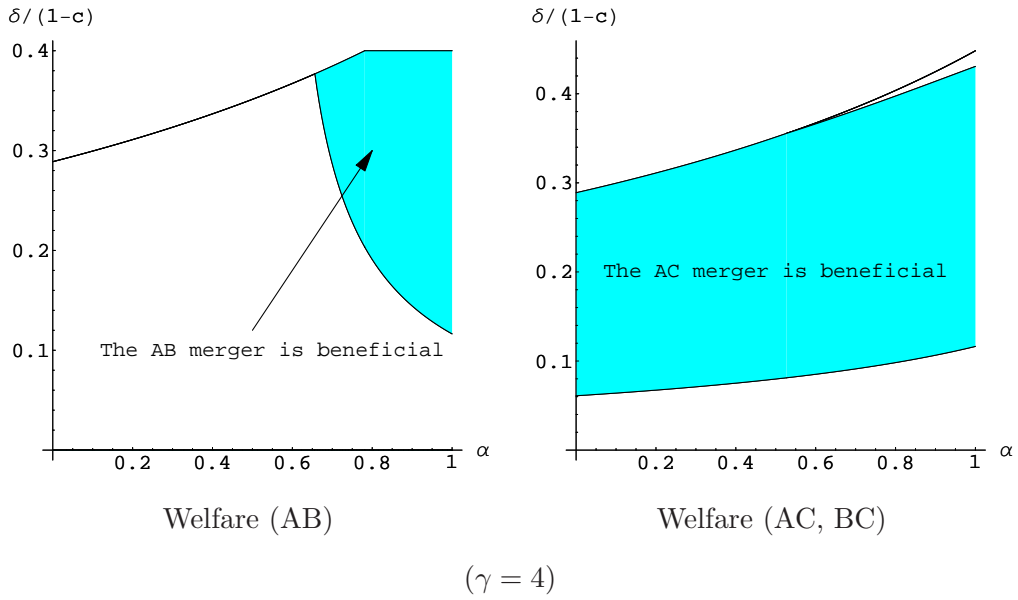Figure 5: Social welfare and the desirability of mergers: two efficeint and one inefficient firms.

Figure 6: The desirability of mergers: three different firms.