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Dynamic Analysis of Location Choice by Multinational Firms

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Abstract

This paper constructs a North-South endogenous growth model to investigate how the organizational forms of final goods firms evolve. Initially, the final goods firms in the North obtain intermediate goods from Northern firms and produce in the North. When trade costs are sufficiently low, as the economy develops, the final goods firms produce the final goods in the North and obtain the intermediate goods from Southern firms. As the economy develops further, they produce the final goods in the South and obtain intermediate goods from Southern firms.

Keywords: Location Choice, North-South, R&D, Economic Growth **JEL classification**: F43, O31, R30

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1 Introduction

Recently, trade costs and communication costs have decreased rapidly and the volume of trade has increased. Thus, firms in developed countries can easily obtain intermediate goods from foreign countries. Indeed, many researchers argue that the importing of intermediate goods has increased. ¹ In addition, foreign direct investment (FDI) conducted by multinational firms in developed countries has been rising. UNCTAD (2008) reported that FDI inflows increased from \$ 1.4 trillion in 2000 to \$1.8 trillion in 2007. From these data, firms in developed countries use many strategies to produce final goods which relate to where they produce the final goods and from where they obtain their intermediate goods. In this paper, I investigate the relationship between the strategies of multinational firms and the trade costs and I analyze how multinational firms change their strategies to produce the final goods as economy develops.

I show examples how multinational firms change their strategies, Japanese cotton spinning companies before World War II and the scanner of Canon Inc-. After the treaty of Shimonoseki was signed in 1895, China opened some cities to Japan and Japanese firms were permitted to conduct FDI in China. However, at that time, because the wage rate in Japan was cheaper than that in China, Japanese cotton spinning companies produced the cotton yarn in Japan and they exported the cotton yarn to Chinese market. After World War I, the wage rate in Japan rose sharply. Then, Japanese cotton spinning companies conducted FDI in China. 14 Japanese cotton spinning companies conducted FDI and they managed 35 factories in 1924. On the other hand, when Canon Inc.. produced scanners in 1997, they obtained intermediate goods to domestic firms. As Taiwanese companies entered the scanner market and the price competition became severe, Canon was concerned about the high wage rate in Japan. As a result, Canon obtained intermediate goods to Taiwanese firms whose wage rates were lower. Furthermore, Canon has produced multifunction printers which have a scanner in Suzhou in China and obtained almost of all the intermediate goods from near the factory recently. From these examples, the multinational firms have changed their strategies how to produce the final goods and where to obtain the intermediate goods. In the past, multinational firms obtain their intermediate goods from the country which they locate to produce the final goods. However, in these days, multinational firms choose not only place of production but also choose where to obtain their intermediate goods. How have the multinational firms changed their strategies

¹See, for example, Campa and Goldberg (1997), Feenstra and Hanson (1996), Hummels, Ishii, and Yi (2001), Yeats (2001), and Hanson, Mataloni, and Slaughter (2001, 2005).

and what are the different strategies that the multinational firms take.

Some researchers have used static models to investigate these issues. Antràs and Helpman (2004) analyzed how final goods producers with different productivity levels choose whether to outsource the production of intermediate goods. They showed that the strategies of final goods producers crucially depend on their productivity. Grossman and Helpman (2005) investigated the choice of location from which final goods firms obtain their intermediate goods. They showed that productivity improvement in the South shifted outsourcing from North to South. However, although these studies offer useful insights into firms' final goods strategies, the researchers analyzed only static equilibria.²

In the literature on product cycles, many studies have constructed North-South growth models in which Northern firms can produce both in the North and the South. Helpman (1993), Lai (1998), Glass and Saggi (2002), Glass and Wu (2007), and Tanaka, Iwaisako, and Futagami (2009) assumed that undertaking foreign direct investment (FDI) is costless. They showed that there exists an equilibrium in which some firms produce in the North and some firms produce in the South. In a separate strand of the literature, Tanaka, Iwaisako, and Futagami (2007) assumed that final goods producers incur the costs in successfully negotiating licenses with Southern firms. When the final goods producers have successfully negotiated licensing, they can produce in the South. Although these studies considered the location of production, they did not focus on the switching point at which final goods producers change their production locations from developed countries to developing countries.

Morita (2010) constructed an endogenous growth model of the variety-expansion type in which final goods firms choose the intermediate firms from which they obtain their intermediate goods. In an earlier paper, I explored transition paths and the switching point at which final goods producers obtained goods from foreign suppliers rather than from domestic suppliers. However, Morita (2010) focused on the one strategy of multinational firms where to obtain their intermediate goods and could not explain that the multinational firms perform FDI as the economy develops. Compared to Morita (2010), I generalize that model to allow Northern firms not only to obtain intermediate goods to

²Some researchers have used dynamic models to investigate these issues. Naghavi and Ottaviano (2009) used a North-South endogenous growth model with offshoring to investigate the extent to which final goods producers outsource the production of intermediate goods to the South. They analyzed two types of equilibrium: one in which all producers outsources the production of intermediate goods, and another in which none of them outsource. In addition, Gao (2007) developed an endogenous growth model and analyzed the relationship between trade costs and the location in which intermediate goods are produced. He showed that, as trade costs fall, the number of intermediate goods produced in the South increases. However, these studies analyze only the steady state.

either the North or South, but also to base their production of final goods in either the North or South. This enables me to explain the different behaviors of Japanese cotton spinning companies before World War II and Canon Inc-.

In this model, I assume that it is costless to trade intermediate goods within a country but not between countries. In addition, I assume that when final goods firms produce their final goods in the South, they incur a management cost because they have to monitor the Southern workers. The final goods firms choose the location that maximizes profit. I show why firms change the location of final goods production as the economy develops. Suppose that, initially, a final goods firm produces final goods in the North and obtains intermediate goods from Northern firms. When transportation costs are sufficiently low, as the economy develops, the final goods firm obtains the intermediate goods from Southern firms. Then, as the economy develops further, the final goods firm obtains the intermediate goods from the South and also produces the final goods in the South. However, when transportation costs are sufficiently high, the final goods firms does not choose the strategy that they locate in the North and obtain the intermediate goods from South like Japanese cotton spinning companies before World War II. Then, in this case, as the economy develops, the final goods firm obtains the intermediate goods from firms and also produces the final goods from Southern firms and also produces the final goods in the South.

The remainder of the paper is structured as follows. In Section 2, I introduce the model. In Section 3, I derive the equilibrium path of the model and prove that it is unique. Section 4 concludes this paper.

2 The Model

I develop a dynamic general equilibrium model where final goods producers obtain the intermediate goods from intermediate producers either in their home country or in foreign countries and they produce the final goods in their home country or in foreign countries. Our model has a similar structure to the model of Grossman and Helpman (1991, Ch. 3).

The world economy consists of two countries, the North and the South denoted by $l \in \{N, S\}$. The population size in the world is one. Each individual lives forever and is endowed with L^l units of labor services, which are supplied inelastically at each point of time. There exist three types of goods: a homogeneous good, intermediate goods, and final goods. The homogeneous good is produced only in the South, ³ and the final goods

 $^{^{3}}$ In this paper, a homogeneous good is produced in both countries with one unit of labor per unit. This paper focuses on the case that the wage rate of North is larger than the wage rate of South and

can be produced in the both countries. The intermediate goods can be produced in either country. Individuals consume the homogeneous good and the final goods. R&D activities can be conducted only in the North.⁴

The final goods firms purchase intermediate goods either from the Northern firms or from the Southern firms, and produce the final goods in the North or in the South in order to maximize their profits. Therefore, there are four possible strategies. First, the final goods firms obtain the intermediate goods from the Northern firms and produce the final goods in the North. Second, they obtain the intermediate goods from the Southern firms and produce the final goods in the North. Third, they obtain the intermediate goods from the Northern firms and produce the final goods in the South. Finally, they obtain the intermediate goods from the Southern firms and produce the final goods in the South. In the first case, there are three sectors in the North: the R&D sector, the final goods sector, and the intermediate sector. In the South, there is only one sector: the homogeneous good sector. In the second case, there are two sectors in the North: the R&D sector and the final goods sector. In the South, there are two sectors: the homogeneous good sector and the intermediate sector. In the third case, there are two sectors in the North: the R&D sector and the intermediate sector. In the South, there are also two sectors: the homogeneous good sector and the final goods sector. Finally, in the fourth case, there is only one sector in the North: the R&D sector. In the South, there are now three sectors: the homogeneous good sector, the intermediate sector, and the final goods sector.

2.1 Individuals

Individuals in both countries have identical preferences:

$$\int_{t}^{\infty} e^{-\rho(\nu-t)} U(\nu) d\nu, \quad 0 < \rho < 1, \tag{1}$$

where ρ is the constant subjective discount rate. $U(\nu)$ is the instantaneous utility per person at time ν and is specified as follows:

$$U(\nu) = y(\nu) + \frac{1}{\mu}X(\nu)^{\mu}, \quad 0 < \mu < 1,$$
(2)

the market of the homogeneous good is perfect competitive. Therefore, a homogeneous good is never produced in the North in equilibrium.

⁴Lai (1998), Glass and Saggi (2002), Grossman and Helpman (1991, ch. 11), Grossman and Helpman (2005), and Tanaka (2006) constructed a North-South model in which innovation sector is located only in the North and imitation sector is located only in the South. They assume that South does not have the know-how needed to invent new goods. This paper omits the imitation sector because I focus on the strategies of the final goods firms.

where μ is a parameter, $y(\nu)$ stands for consumption of the homogeneous good at time ν , and $X(\nu)$ is a composite good at time ν that is made up of differentiated final goods. For simplicity, I drop the time index ν from all the variables. A composite good X is given by:

$$X = \left[\int_0^n x(i)^\alpha di\right]^{\frac{1}{\alpha}}, \qquad 0 < \alpha < 1,$$
(3)

where x(i) represents consumption of different final goods i and n denotes the number of final goods and n is endogenous variable. $\frac{1}{1-\alpha}$ is the elasticity of substitution between any two varieties of final goods in a given sector. If α is close to one, the goods are nearly perfect substitutes and the sector is highly competitive. If α is close to zero, the goods are distinct products and the sector becomes monopolistic. I assume that $\alpha > \mu$ to ensure the concavity of preferences over the x(i).

The utility maximization problem of the individual can be solved in two steps. The first step is to solve the following static optimization problem:

$$\max_{x,x(i)} \quad y + \frac{1}{\mu} X^{\mu},$$

subject to $y + \int_0^n P(i)x(i)di = E,$

where the homogeneous good is chosen to be the numeraire, P(i) stands for the price of the product *i*, and *E* is the total expenditure. From the first-order condition, I can obtain the following inverse demand function:

$$P(i) = X^{\mu - \alpha} x(i)^{\alpha - 1}, \ i \in [0, n] \,. \tag{4}$$

The second step is to solve the intertemporal optimization problem. From (4), the indirect utility function is given by:

$$U = E - \left(1 - \frac{1}{\mu}\right) \left[\int_0^n P(i)^{\frac{\alpha}{\alpha - 1}} di\right]^{\frac{\mu(\alpha - 1)}{\alpha(\mu - 1)}}.$$
(5)

As is clear from the indirect utility function (5), the marginal utility of expenditures is constant. The intertemporal budget constraint is given by

$$\int_{0}^{\infty} e^{-\bar{r}(s)} E^{l}(s) ds = A^{l}(0) + \int_{0}^{\infty} e^{-\bar{r}(s)} w_{l}(s) ds,$$
(6)

where $\bar{r}(s)$ is the average interest rate between times 0 and s and $\bar{r}(s) = \int_0^s r(t) dt/t$. $A^l(0)$ is the present value of asset owned by a worker in country l, and w_l is the wage rate in country l. In this model, I focus on the equilibrium that individuals consume at each period. Therefore, the market interest rate at time t, r(t), must be equal to the subjective discount rate:

$$r(t) = \rho \qquad \text{for all } t . \tag{7}$$

2.2 Production

2.2.1 The homogeneous good sector

Production of the homogeneous good uses labor only. The homogeneous good is produced only in the South. The production of one unit of the homogeneous good requires one unit of Southern labor. I assume that the homogeneous good market is perfectly competitive. Thus, the price of the homogeneous good becomes equal to the wage rate in the South. As the homogeneous good is chosen to be the numeraire, the wage rate in the South becomes unity, that is, $w_S = 1$. I assume that the homogeneous good can be traded costlessly.

2.2.2 The intermediate goods sector

The intermediate goods can be produced in the both countries. Production of one unit of each intermediate good requires one unit of labor. I assume that perfect competition prevails in the intermediate goods markets in both countries. Thus, the marginal cost of intermediate goods produced in the North is the wage rate in the North, w_N . In this paper, I focus on the case that $w_N > 1$. The marginal cost of intermediate goods produced in the South is the wage rate in the South, $w_S = 1$. I assume that no transportation cost is incurred in transferring the intermediate goods when the final goods firms and the intermediate good firms exist in the same country, but the final goods firms have to pay an iceberg cost τ ($\tau > 1$) to obtain intermediate goods firms is as follows:

$$p_{N,N}^m = w_N,\tag{8}$$

$$p_{N,S}^m = \tau, \tag{9}$$

$$p_{S,N}^m = \tau w_N,\tag{10}$$

$$p_{S,S}^m = 1,$$
 (11)

where $p_{j,k}^m$ is the price of intermediate goods when final goods firms are in country j, $j \in \{N, S\}$ and they obtain the intermediate goods from the suppliers in country k, $k \in \{N, S\}$. Hereafter, if there are two subscripts, the first subscript denotes where final

goods firms are located, and the second subscript denotes the country from which the final goods firms obtain the intermediate goods.

2.2.3 The final goods sector

Production of each final goods requires labor and a variety-specific intermediate good. When the final goods firms exist in the North, they use Northern labor and Northern intermediate goods. I assume that their unit cost function is as follows:

$$c_N(w_N, p_{N,k}^m) = w_N^{1-\beta}(p_{N,k}^m)^{\beta}, k \in \{N, S\},$$
(12)

where $p_{N,k}^m$ is the price of the intermediate goods produced in country k when final goods firms are in the North, β is constant, and $0 < \beta < 1$. On the other hand, when the final goods firms produce the final goods in the South, I assume that they have to hire not only Southern workers but also Northern workers in order to manage and train Southern workers. The final goods firms have to incur the cost because Northern workers have to cross international borders and live in the South, which has a different culture and language from the North, in order not only to produce but also to monitor the Southern labor. Specifically, I assume that management costs take the iceberg form to simplify this analysis. Hence, the cost of Northern labor is ϵw_N , $\epsilon > 1$. Then, I assume that the unit cost function of the final goods firms is as follows:

$$c_{S}(\epsilon w_{N}, p_{S,k}^{m}) = A(\epsilon w_{N})^{\gamma(1-\beta)} w_{S}^{(1-\gamma)(1-\beta)}(p_{S,k}^{m})^{\beta},$$

= $A(\epsilon w_{N})^{\gamma(1-\beta)}(p_{S,k}^{m})^{\beta}, k \in \{N, S\},$ (13)

where $p_{S,k}^m$ is the price of the intermediate goods produced in country k when the final goods firms are in the South, γ is a constant parameter, and $0 < \gamma < 1$. A is also a constant parameter.

By using (4), I can obtain the revenue of each final goods producer as follows:

$$R_{j,k}(i) = P(i)x(i),$$

= $X_{j,k}^{\mu-\alpha} x_{j,k}(i)^{\alpha}, \quad j,k \in \{N,S\}, \ i \in [0,n].$ (14)

where $R_{j,k}$ is the revenue of the final goods producer, $X_{j,k}$ is the composite good and $x_{j,k}(i)$ is the consumption of different final goods *i*. I consider the profit maximization problem of the final goods producers in both countries. In the Appendix A.1, I derive the profit function when the final goods firms exist in the North and in the South. Using (8),

(9), (10), and (11), I can write the profit functions as follows:

$$\pi_{N,N} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu}{1-\mu}} (1-\alpha), \tag{15}$$

$$\pi_{N,S} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu(1-\beta)}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}} (1-\alpha),$$
(16)

$$\pi_{S,N} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} (\tau w_N)^{\frac{-\beta\mu}{1-\mu}} (1-\alpha), \tag{17}$$

$$\pi_{S,S} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} (1-\alpha).$$
(18)

Comparing (17) to (18), I can show that the profit of the final goods firms in the South when they obtain the intermediate goods from the Southern firms is larger than that when they obtain the intermediate goods from the Northern firms, that is, $\pi_{S,S} > \pi_{S,N}$, because $\tau w_N > 1$. Therefore, when they are in the South, the final goods firms never purchase the intermediate goods from the Northern firms. Then, I obtain the following lemma.

Lemma 1. When the final goods firms are in the South, they obtain the intermediate goods only from the Southern firms.

I label the economy *Regime NN* when the final goods firms are in the North and obtain the production of intermediate goods from the Northern firms; *Regime NS* when the final goods firms are in the North and obtain the intermediate goods from the Southern firms; and *Regime SS* when the final goods firms are in the South and obtain the intermediate goods from the Southern firms.

2.3 The R&D sector

The R&D activities of the present model follow Grossman and Helpman (1991, Ch. 3). The final goods producers in the North enter into the R&D race and finance the cost of R&D by issuing equity in the stock market. The equity is bought by individuals who live in either country. The stock value of the final goods producers at time t is equal to the discounted sum of its profit stream from time t. Suppose that the final goods firms is first in one regime and they next move to another regime at time s and further to another regime at time s'. Then, the stock value of the final goods producers at time t is given by:

$$v_{j,k} = \int_{t}^{s} e^{-\rho(\tau-t)} \pi_{j,k} d\tau + e^{-\rho(s-t)} \int_{s}^{s'} e^{-\rho(\tau-s)} \pi_{j',k'} d\tau + e^{-\rho(s'-t)} \int_{s'}^{\infty} e^{-\rho(\tau-s')} \pi_{j'',k''} d\tau, \ j,j',j'',k,k',k'' \in \{N,S\},$$
(19)

where $v_{j,k}$ is the stock value. Differentiating (19) with respect to time t yields the following no-arbitrage conditions:

$$\rho v_{N,N} = \pi_{N,N} + \dot{v}_{N,N},\tag{20}$$

$$\rho v_{N,S} = \pi_{N,S} + \dot{v}_{N,S}, \tag{21}$$

$$\rho v_{S,S} = \pi_{S,S} + \dot{v}_{S,S}.$$
 (22)

The final goods producers hire labor to develop blueprints. I assume that knowledge spillovers in the R&D activities exist; that is, the more innovations that have been created previously, the lower is the cost of innovating. I assume that L^A units of labor for R&D activities for a time interval dt produce $dn = (L^A n/a)dt$ new products. The cost of the R&D activities is $w_N L^A dt$ because the R&D sector is located only in the North. Producing the blueprints creates a value for the final goods producers of $v_{j,k}dn$ because each blueprint has a market value of $v_{j,k}$. I assume that there is free entry into the R&D race. Therefore, the following free-entry condition must hold:

$$v_{j,k} \le \frac{aw_N}{n}$$
, with equality whenever $\dot{n} \equiv \frac{dn}{dt} > 0, \ j,k \in \{N,S\}$. (23)

2.4 The labor market

Labor market equilibrium requires that labor supply equals to labor demand. Under *Regime NN*, the demand for labor in the North comes from the intermediate sector, the final goods sector, and the R&D sector. In the South, the demand for labor comes only from the homogeneous good sector. Therefore, the labor market equilibrium conditions are as follows:

$$L^{N} = L^{A} + L^{M}_{N,N} + L^{F}_{N,N},$$
$$L^{S} = y_{N,N},$$

where $y_{j,k}$ is the labor demand for production of the homogeneous good in the South, $L_{j,k}^{M}$ is the labor demand for production of the intermediate goods in the North, and $L_{j,k}^{F}$ is the labor demand for the production of the final goods when the final goods firms producing in *j* country obtain the intermediate goods from *k* country. Under *Regime NS*, the demand for labor in the North comes from the R&D sector and the final goods sector. In the South, the demand for labor comes from the homogeneous good sector and the intermediate goods sector. Therefore, the labor market equilibrium conditions are as follows:

$$L^{N} = L^{A} + L^{F}_{N,S},$$
$$L^{S} = y_{N,S} + L^{M}_{N,S}$$

Finally, under *Regime SS*, the demand for labor in the North comes from the R&D sector and the final goods sector. In the South, the demand for labor comes from the homogeneous good sector, the intermediate goods sector and the final goods sector. Therefore, the labor market equilibrium conditions are as follows:

$$L^{N} = L^{A} + L^{F}_{S,S},$$

$$L^{S} = y_{S,S} + L^{M}_{S,S} + L^{F,S}_{S,S}$$

where $L_{S,S}^F$ is the labor demand for the production of the final goods in the South.

In the Appendix A.2, I derive the labor demand in each sector and under each regime. Under *Regime NN*, I can rewrite the labor market clearing conditions in both countries as follows:

North
$$L^{N} = \frac{a\dot{n}}{n} + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} w_{N}^{\frac{-\mu}{1-\mu}},$$
 (24)

South
$$L^{S} = E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_{N}^{\frac{-\mu}{1-\mu}}.$$
 (25)

Using (9), (A.11), (A.12), and (A.13), I can rewrite the labor market clearing conditions of *Regime NS* as follows:

North
$$L^{N} = \frac{a\dot{n}}{n} + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (1-\beta) \alpha^{\frac{1}{1-\mu}} w_{N}^{\frac{\beta\mu-1}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}},$$
 (26)

South
$$L^{S} = E + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (\alpha\beta - \tau) \alpha^{\frac{\mu}{1-\mu}} w_{N}^{\frac{-\mu(1-\beta)}{1-\mu}} \tau^{\frac{\mu(1-\beta)-1}{1-\mu}}.$$
 (27)

Finally, using (11), (A.14), (A.15), (A.16), and (A.17), I can rewrite the labor market clearing conditions of *Regime SS* as follows:

North
$$L^{N} = \frac{a\dot{n}}{n} + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha \gamma (1-\beta) \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_{N})^{\frac{-\mu\gamma(1-\beta)}{1-\mu}-1},$$
(28)

South
$$L^{S} = E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{\mu}{1-\mu}} (\epsilon w_{N})^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} \left[1 - \alpha\beta - \alpha(1-\gamma)(1-\beta)\right].$$
(29)

3 The Equilibrium Path

In this section, I examine the dynamics of the economy. First, I consider the dynamic behaviors of *Regime NN*, *Regime NS*, and *Regime SS* separately. Finally, I integrate these dynamic behaviors.

3.1 Dynamic behavior under *Regime SS*

In Regime SS, the equilibrium conditions are (7), the no-arbitrage condition, (22), the free-entry condition, (23), and the labor market clearing condition, (28). From the labor market clearing condition, I can derive the differential equation for the number of firms, n, as follows:

$$\dot{n} = \frac{n}{a} \left[L^N - \alpha \gamma (1 - \beta) n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}-1} \right].$$
(30)

In the Appendix A.3, I derive the differential equation for the wage rate in the North, w_N as follows:

$$\dot{w_N} = \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1}{a} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} \left(\epsilon w_N\right)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} \left(1 - \alpha + \frac{\alpha\gamma(1-\beta)}{\epsilon}\right).$$
(31)

These two equations, (30) and (31), constitute the dynamic system of *Regime SS*. Then, the equation for $\dot{n} = 0$ locus is represented by:

$$w_N = \epsilon^{-1} \left(\frac{\alpha \gamma (1 - \beta)}{L^N} \right)^{\frac{1-\mu}{\Phi}} \left(\frac{A}{\alpha} \right)^{\frac{-\mu}{\Phi}} n^{\frac{\mu(1-\alpha)}{\alpha\Phi}}, \tag{32}$$

where $\Phi = 1 - \mu + \mu \gamma (1 - \beta)$. The $\dot{n} = 0$ locus is concave and $\lim_{n \to \infty} \frac{\partial w_N}{\partial n} = 0$. On the other hand, the equation for $\dot{w}_N = 0$ locus is given by:

$$w_N = \epsilon^{-1} \left(\frac{\epsilon(1-\alpha) + \alpha\gamma(1-\beta)}{a\rho + L^N} \right)^{\frac{1-\mu}{\Phi}} \left(\frac{A}{\alpha} \right)^{\frac{-\mu}{\Phi}} n^{\frac{\mu(1-\alpha)}{\alpha\Phi}}.$$
 (33)

The $\dot{w_N} = 0$ locus has a similar shape to $\dot{n} = 0$ locus. Thus, the origin is the only intersection point of the two curves, $\dot{n} = 0$ and $\dot{w_N} = 0$. Figure 1 depicts the phase diagram for this system on the (n, w_N) plane. When $\frac{\alpha a \rho}{1-\alpha} < L^N$ holds, $\dot{w_N} = 0$ locus is above $\dot{n} = 0$ locus for any $w_N > 0$. Hereafter, I focus on $\frac{\alpha a \rho}{1-\alpha} < L^N$.⁵ On the saddle path, the number of firms and the wage rate in the North increases perpetually. In this model, I assume that the knowledge spillovers in the R&D activities exist. Therefore, an increase in the number of the firms raises the demand of labor in the North. Then, the wage rate in the North increases steadily.

I show the saddle path stability in *Regime SS*. Let us define $z_{SS} \equiv n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (\epsilon w_N)^{\frac{-\Phi}{1-\mu}} (\frac{A}{\alpha})^{\frac{-\mu}{1-\mu}}$. Then, differentiating z_{SS} with respect to time t and using (30) and (31) yields the following differential equation for z_{SS} :

$$\frac{\dot{z}_{SS}}{z_{SS}} = \frac{\mu(1-\alpha)}{\alpha(1-\mu)} \left(\frac{\dot{n}}{n}\right) - \frac{\Phi}{1-\mu} \left(\frac{\dot{w}_N}{w_N}\right)$$
$$= \left[\frac{\mu-\alpha-\alpha\mu\gamma(1-\beta)}{\alpha(1-\mu)}\frac{L^N}{a} - \frac{\Phi}{1-\mu}\rho\right] + \frac{\Gamma_{SS}z_{SS}}{a(1-\mu)}$$

⁵Even when $\frac{\alpha a \rho}{1-\alpha} > L^N$, the following discussion can be applied.



Figure 1: Phase diagram under *Regime SS*

where

$$\Gamma_{SS} \equiv (1-\mu) \left[\epsilon (1-\alpha) + \alpha \gamma (1-\beta) \right] + \mu \gamma (1-\beta) \left[(\epsilon-1)(1-\alpha) + \alpha \gamma (1-\beta) \right] > 0.$$

From $\epsilon > 1$, $\frac{z_{SS}}{z_{SS}} = 0$ schedule is a straight line through the negative intercept with a positive slope in Figure 2. There is a unique steady state at E. The steady state value is as follows:

$$z_{SS}^* = \frac{\left[\alpha(\mu + \Phi) - \mu\right]L^N + a\alpha\rho\Phi}{\alpha\Gamma_{SS}}.$$
(34)

Because the steady state E is unstable and z_{SS} is a jump variable, z_{SS} jumps to z_{SS}^* at the initial time. In the steady state, the relationship between the number of final goods, n, and the wage rate in the North, w_N becomes as follows:

$$w_N = \epsilon^{-1} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{\Phi}} (z_{SS}^*)^{\frac{-1+\mu}{\Phi}} n^{\frac{\mu(1-\alpha)}{\alpha\Phi}}.$$
(35)

This equation represents the stable saddle path. Therefore, the dynamic equilibrium follows the stable saddle path shown by the solid locus with arrow (I) in Figure 1.

3.2 Dynamic behavior under *Regime NN*

In Regime NN, the equilibrium conditions are (7), the no-arbitrage condition, (20), the free-entry condition, (23), and the labor market clearing condition, (24). From the labor market clearing condition, I can derive the differential equation for the number of final goods, n, as follows:

$$\dot{n} = \frac{n}{a} \left[L^N - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} w_N^{\frac{-1}{1-\mu}} \right].$$
(36)



Figure 2: Phase diagram about z_{SS}

Using (7), (20), (23), and (36), I can obtain the differential equation for the wage rate in the North, w_N , in the same way as (31). Then, the differential equation for the wage rate in the North is

$$\dot{w_N} = \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1}{a} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu}{1-\mu}}.$$
(37)

These two equations, (36) and (37), constitute the dynamic system of *Regime NN*. The equation for $\dot{n} = 0$ locus is given by the following:

$$w_N = \left(\frac{1}{L^N}\right)^{1-\mu} \alpha n^{\frac{\mu(1-\alpha)}{\alpha}}.$$
(38)

The $\dot{n} = 0$ locus is concave and $\lim_{n\to\infty} \frac{\partial w_N}{\partial n} = 0$. On the other hand, the equation for $\dot{w}_N = 0$ locus is given by:

$$w_N = \left(a\rho + L^N\right)^{-(1-\mu)} \alpha^{\mu} n^{\frac{\mu(1-\alpha)}{\alpha}}.$$
(39)

The $\dot{w_N} = 0$ locus has a similar shape to $\dot{n} = 0$ locus. Thus, the origin is the only intersection point of the two curves $\dot{n} = 0$ and $\dot{w_N} = 0$. Figure 3 depicts the phase diagram for this system on the (n, w_N) plane when the inequality $\frac{\alpha a \rho}{1-\alpha} < L^N$ holds.

Similarly to *Regime SS*, I can obtain the stable saddle path (See Appendix A.4). In the steady state, the relationship between the number of final goods, n, and the wage rate in the North, w_N becomes as follows:

$$w_N = \left(\frac{A}{\alpha}\right)^{-\mu} \left(\frac{(\alpha - \mu + \alpha\mu)L^N + a\alpha\rho}{\alpha\left\{1 - \mu(1 - \alpha)\right\}}\right)^{-1 + \mu} n^{\frac{\mu(1 - \alpha)}{\alpha}}.$$

This equation represents the stable saddle path. Therefore, the dynamics in *Regime NN* follows the stable saddle path shown by the arrow (II) in Figure 3.

3.3 Dynamic behavior under *Regime NS*

In *RegimeNS*, the equilibrium conditions are (7), the no-arbitrage condition, (21), the free-entry condition, (23), and the labor market clearing condition, (26). In the same way



Figure 3: Phase diagram under *Regime NN*

as in the former section, I can derive $\dot{n} = 0$ locus and $\dot{w}_N = 0$ locus. From the labor market clearing condition, I can derive the differential equation for the number of firms, n, as follows:

$$\dot{n} = \frac{n}{a} \left[L^N - \alpha^{\frac{1}{1-\mu}} (1-\beta) n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} w_N^{\frac{-(1-\beta\mu)}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}} \right].$$
(40)

In the same way as (31), I can obtain the differential equation for the wage rate in the North, w_N , as follows:

$$\dot{w_N} = \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1 - \alpha\beta}{a} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu(1-\beta)}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}}.$$
(41)

These two equations, (40) and (41), constitute the dynamic system of *Regime NS*. Then, the equation for $\dot{n} = 0$ locus is represented by:

$$w_N = \left(\frac{1-\beta}{L^N}\right)^{\frac{1-\mu}{1-\beta\mu}} \alpha^{\frac{1}{1-\beta\mu}} \tau^{\frac{-\beta\mu}{1-\beta\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\beta\mu)}}.$$
(42)

The $\dot{n} = 0$ locus is concave and $\lim_{n\to\infty} \frac{\partial w_N}{\partial n} = 0$. On the other hand, the equation for $\dot{w}_N = 0$ locus is given by:

$$w_N = \left(\frac{1-\alpha\beta}{a\rho+L^N}\right)^{\frac{1-\mu}{1-\beta\mu}} \alpha^{\frac{\mu}{1-\beta\mu}} \tau^{\frac{-\beta\mu}{1-\beta\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\beta\mu)}}.$$
(43)

The $\dot{w_N} = 0$ locus has a similar shape to $\dot{n} = 0$ locus. Thus, the origin is the only intersection point of the two curves, $\dot{n} = 0$ and $\dot{w_N} = 0$. Figure 4 depicts the phase diagram for this system on the (n, w_N) plane. When $\frac{\alpha a \rho}{1-\alpha} < L^N$ holds, $\dot{w_N} = 0$ locus is above $\dot{n} = 0$ locus for any $w_N > 0$.



Figure 4: Phase diagram under *Regime NS*

Similarly to the previous regimes, I can obtain the stable saddle path (See Appendix A.4). In the steady state, the relationship between the number of final goods, n, and the wage rate in the North, w_N becomes as follows (See appendix A.4):

$$z_{NS}^* = \frac{\left[(\alpha - \mu + \alpha\mu(1 - \beta))L^N + (1 - \beta\mu)a\alpha\rho\right]}{\alpha\Gamma_{NS}}$$

This equation represents the stable saddle path. Therefore, the dynamics in *Regime NS* follows the stable saddle path shown by the arrow (III) in Figure 4.

3.4 Boundaries between the three regimes

The final goods firms have to decide where they produce the final goods and from where they purchase the intermediate goods. Then, the final goods firms compare levels of the profit in each regime and choose a regime where they can earn the largest profit. I investigate the condition under which the profit in *Regime NN* become the same as the profits in *Regime NS*; that is $\pi_{N,N} = \pi_{N,S}$. From (15) and (16), the boundary condition between *Regime NN* and *Regime NS* is given by:

$$w_N = \tau. \tag{44}$$

Next, I show the condition under which the profit in *Regime NN* becomes the same as the profits in *Regime SS*; that is $\pi_{N,N} = \pi_{S,S}$. From (15) and (18), the boundary condition between *Regime NN* and *Regime SS* becomes as follows:

$$w_N = A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}},\tag{45}$$

where $\Psi = 1 - \gamma(1 - \beta)$ and $0 < \Psi < 1$. Finally, from (16) and (18), I investigate the boundary condition between *Regime NS* and *Regime SS* as follows:

$$w_N = A^{\frac{1}{\Psi-\beta}} \tau^{\frac{-\beta}{\Psi-\beta}} \epsilon^{\frac{\gamma}{1-\gamma}}.$$
(46)

From the three boundary conditions, (44), (45), and (46), I can obtain the following proposition (see Appendix A.5 for the proof).

Proposition 1. When transportation costs are small and management costs are large, that is, $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, Regime of the economy is chosen from three regimes, Regime NN, Regime NS, and Regime SS. On the other hand, when transportation costs are large and management costs are small, that is, $\tau \ge A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, Regime of the economy is chosen from two regimes, Regime NN and Regime SS.

Thus, when the inequality, $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, holds, the economy has three regimes, that is, Regime NN, Regime NS, and Regime SS. When the wage rate in the North is low, Figure 5 shows that the economy is in *Regime NN*. As the wage rate in the North increases, the economy turns into *Regime NS*. Finally, a further increase in the wage rate in the North get the economy turn into *Regime SS*. When trade costs are sufficiently small, obtaining the intermediate goods from southern firms becomes easier for the final goods firms. When management costs and in the parameter of cost function, A are sufficiently large, the cost of FDI becomes large compared to purchasing the intermediate goods from Southern firms. Then, the final goods firms obtain the intermediate goods from the Southern firms and there exists *Regime NS*. On the other hands, when the inequality, $\tau \geq A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, holds, Figure 6 shows that the economy has only *Regime NN* and *Regime SS*. When the wage rate in the North is high, the economy is in *Regime SS*. Intuitively, when transportation costs are sufficiently high, the cost of obtaining the intermediate goods from abroad is high. Therefore, the final goods firms do not purchase the intermediate goods from abroad. When management costs is sufficiently small, the cost of producing the final goods in the South becomes lower and the final goods firms does not purchase the intermediate goods from abroad.

Lemma 2. When $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$ holds, a decrease in the transportation costs and an increase in the management costs enlarge the area of Regime NS. On the other hand, when $\tau \ge A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$ holds, an increase in the management costs enlarges the area of Regime NN and an increase in trade costs does not affect the sizes of those regimes.

Proof: See the Appendix A.6.



When $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, Regime of the economy is chosen form three regimes, that is NN Regime, NS Regime, and SS Regime from proposition 1. A decrease in trade costs means that the cost of purchasing the intermediate goods from abroad decreases and firms purchase the intermediate goods from the foreign countries easier. An increase in the management costs means that the cost of producing in the foreign countries is increasing. Then, the producing the final goods in the North to purchase the intermediate goods from abroad is relatively more profitable than producing in the South. Therefore, the area of Regime NS is enlarged. When $\tau \geq A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, Regime of the economy is chosen from two regimes, that is NN Regime, and SS Regime from proposition 1. Then, if the management costs increase, the cost of producing in the South increases and final goods firms prefer Regime NN to Regime SS. On the other hand, an increase in the transportation costs does not affect the behavior of final goods firms. Because in Regime NN and Regime SS, final goods firms do not freight the intermediate goods to cross the border, the final goods firms do not incur trade costs.

3.5 The equilibrium path from *Regime NN* to another regime

In the previous subsections, I explored the dynamic behaviors in *Regime NN*, *Regime NS*, and *Regime SS*, and showed the boundary conditions among Regimes. Now, I integrate the three phase diagrams of these regimes into one. When the inequality, $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, holds, the economy will transit through three regimes, that is, *Regime NN*, *Regime NS*, and *Regime SS*. I can depict the phase diagram for the system on the (n, w_N) plane. Figures 1, 3, 4, and 5 are integrated into Figure 7. In Figure 7, the arrow *ABCD* shows the traditional dynamics. These results are stated as the following proposition. **Proposition 2.** Suppose that the economy is initially in Regime NN. When $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, and $\frac{\alpha a \rho}{1-\alpha} < L^N$ holds, there exists an equilibrium path along which the economy enters into Regime SS, passing through Regime NS.



Figure 7: Phase diagram when $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$ and $\frac{\alpha a \rho}{1-\alpha} < L^N$

Suppose that the initial number of firms is sufficiently small. In this economy, the firms have rational expectation. The firms expect that the economy become Regime SS and the economy follows the stable saddle path in Regime SS in the future. To follow the stable saddle path in Regime SS, the economy follows the divergent path in Regime NS to reach the intersection point C of the stable saddle path with the boundary line between Regime NS and Regime SS. Then, to follow the divergent path in Regime NS, the economy follows the divergent path in Regime NS to reach the intersection point Regime NN to reach the intersection point B of the divergent path in Regime NS.

In order to get some clear results, I show the equilibrium path when $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$ numerically. In this example, the labor supply in the North is $L_N = 2.5$. The subjective discount rate is $\rho = 0.05$ and the parameters of utility are $\alpha = 0.5$ and $\mu = 0.49$, respectively. The parameters of production function are A = 1.1, $\beta = 0.7$, and $\gamma = 0.4$ respectively. The parameter of R&D sector is a = 0.2. I assume that the parameters of management costs and trade costs are $\epsilon = 4$ and $\tau = 1.2$ to hold $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$. Figure 8 represents the number of final goods firms in the equilibrium path and Figure 9 represents the wage rate in the North in the equilibrium path. The points A, B, and C in Figures 8 and 9 correspond to the points in Figure 7. The horizontal lines of both Figures represent time. In the *Regime NN*, both the number of final goods firms and the wage rate in the North raise moderately because at initial time, the number of final good firms is small and the innovative activities are not vigorous in this Regime. As the economy reaches to the point B, the final good firms obtain their intermediate goods from South. The labor supply to R&D sector increases and innovative activities become vigorous. Therefore, both number of final goods firms and the wage rate in the North increases sharply. In Regime SS, the economy follows the stable path and both the number of final good firms and the wage rate in the North increases sharply.



Figure 8: The numerical example of the number of final goods firms when $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$ rate in the North when $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$

On the other hand, when the inequality, $\tau \ge A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, the economy will transit through two regimes, *Regime NN* and *Regime SS*. I can depict the phase diagram similarly and Figures 1, 3, 4, and 6 are integrated into Figure 10. In Figure 10, the arrow *EFG* shows the traditional dynamics. These results are stated as the following proposition.

Proposition 3. When $\tau \geq A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$, and $\frac{\alpha a \rho}{1-\alpha} < L^N$ holds, there exists an equilibrium path along which the economy evolves from Regime NN to Regime SS.

Suppose that the initially number of firms is sufficient small. The final goods firms expect that the economy becomes Regime SS and the economy follows the stable saddle path in Regime SS in the future. To follow the stable saddle path in Regime SS, the economy follows the divergent path in Regime NN to reach the intersection point F of the stable saddle path with the boundary line between Regime NN and Regime SS.

I show the numerical example when $\tau \ge A^{\frac{1}{\Psi}} e^{\frac{1-\Psi}{\Psi}}$. In this case, I assume that the trade costs are larger than the former case, that is $\tau = 2$. Figure 11 represents the number



Figure 10: Phase diagram when $\tau \ge A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$ and $\frac{\alpha a \rho}{1-\alpha} < L^N$

of final goods firms in the equilibrium path and Figure 12 represents the wage rate in the North in the equilibrium path. The points E and F in Figures 11 and 12 correspond to the points in Figure 10. The horizontal lines of both Figures represent time. In the *Regime NN*, both the number of final goods firms and the wage rate in the North rise moderately. As the economy reaches to the point F, the Regime changes from *Regime NN* to *Regime SS*. The labor supply to R&D sector increases sharply and innovative activities become vigorous. Then, the economy follows the stable path and both the number of final good firms and the wage rate in the North rise become vigorous.



Figure 11: The numerical example of the Figure 12: The numerical example of the number of final goods firms when $\tau >$ wage rate in the North when $\tau > A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}} A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$

4 Conclusion

This paper constructs a North-South endogenous growth model to investigate how the production location of final goods firms change. The final goods firms have four strategies, based on the regions in which they produce the final goods and from which obtain the intermediate goods. This paper shows why the location choice of the final goods firms change as the economy develops and why the final goods firms did not adopt a strategy of obtaining intermediate goods from abroad until a few decades ago. Suppose that, initially, the final goods firms obtain the intermediate goods from Northern firms and produce the final goods in the North. When trade costs are sufficiently low and management costs are sufficiently high, as the economy develops, final goods firms produce the final goods in the North and purchase the intermediate goods from the Southern firms. Then, as the economy develops further, the final goods firms produce the final goods in the South and obtain the intermediate goods from the Southern firms. However, when trade costs are sufficiently high, the final goods firms do not choose the strategy that they produce the final goods in the North and obtain the intermediate goods from Southern firms like Japanese cotton spinning companies before World War II. Then, as the economy develops, the final goods firms produce the final goods in the developing countries obtaining the intermediate goods from firms in the developing countries.

In this paper, we assume that the R&D sector is located only in North. However, in these days, innovative activities also are conducted in developing countries. In addition, multinational firms in developed countries outsource R&D to the developing countries. I can extend this model and innovative activities can be operated in both two countries or multinational firms outsource innovation activities to the developing countries. This is future research problems.

A Appendix

A.1 Derivation of the profit functions

First, I focus on the case where the final goods firms are in the North. Then, I consider the profit-maximization problem of the final goods producers in the North. Thus, the final goods producer i maximizes profits as follows:

$$\pi_{N,k}(i) = X_{N,k}^{\mu-\alpha} x_{N,k}(i)^{\alpha} - w_N^{1-\beta} (p_{N,k}^m)^{\beta} x_{N,k}(i), \ k \in \{N, S\}, \ i \in [0, n],$$
(A.1)

where $\pi_{N,k}$ is the operating profit of the final goods firms in the North when they obtain the intermediate goods from country k. The profit-maximizing output is given by:

$$x_{N,k}(i) = \alpha^{\frac{1}{1-\alpha}} X_{N,k}^{\frac{\alpha-\mu}{\alpha-1}} w_N^{\frac{1-\beta}{\alpha-1}} (p_{N,k}^m)^{\frac{\beta}{\alpha-1}}, k \in \{N, S\}.$$
 (A.2)

By substituting (A.2) into (3), I obtain:

$$X_{N,k} = n^{\frac{1-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} w_N^{\frac{-(1-\beta)}{1-\mu}} (p_{N,k}^m)^{\frac{-\beta}{1-\mu}}, k \in \{N, S\}.$$
 (A.3)

Using (A.2) and (A.3), I can obtain the following output and the profit functions when the final goods firms produce the final goods in the North:

$$x_{N,k}(i) = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} w_N^{\frac{-(1-\beta)}{1-\mu}} (p_{N,k}^m)^{\frac{-\beta}{1-\mu}}, \qquad (A.4)$$

$$\pi_{N,k} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu(1-\beta)}{1-\mu}} (p_{N,k}^m)^{\frac{-\beta\mu}{1-\mu}} (1-\alpha), \ k \in \{N,S\}, \ i \in [0,n].$$
(A.5)

The profits of the final goods producers decrease with the wage rate in the North w_N and the number of the final goods firms n.

Next, I turn to the case where the final goods firms are in the South and derive the profit function of the final goods firms similarly to the above case. The final goods producer i maximizes the following profits:

$$\pi_{S,k}(i) = X_{S,k}^{\mu-\alpha} x_{S,k}(i)^{\alpha} - A(\epsilon w_N)^{\gamma(1-\beta)} (p_{S,k}^m)^{\beta} x_{S,k}, \ k \in \{N, S\}, \ i \in [0, n].$$
(A.6)

The profit-maximizing output is given by:

$$x_{S,k}(i) = \left(\frac{A}{\alpha}\right)^{\frac{1}{\alpha-1}} X_{S,k}^{\frac{\alpha-\mu}{\alpha-1}} (\epsilon w_N)^{\frac{\gamma(1-\beta)}{\alpha-1}} (p_{S,k}^m)^{\frac{\beta}{\alpha-1}}, k \in \{N, S\}.$$
(A.7)

By substituting (A.7) into (3), I obtain:

$$X_{S,k} = n^{\frac{1-\alpha}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-1}{1-\mu}} (\epsilon w_N)^{\frac{-\gamma(1-\beta)}{1-\mu}} (p_{S,k}^m)^{\frac{-\beta}{1-\mu}}, k \in \{N, S\}.$$
 (A.8)

Using (A.7) and (A.8), I can obtain the following output and the profit functions when the final goods firms are in the North:

$$x_{S,k}(i) = n^{\frac{\mu - \alpha}{\alpha(1 - \mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-1}{1 - \mu}} (\epsilon w_N)^{\frac{-\gamma(1 - \beta)}{1 - \mu}} (p_{S,k}^m)^{\frac{-\beta}{1 - \mu}},$$
(A.9)

$$\pi_{S,k} = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} (p_{S,k}^m)^{\frac{-\beta\mu}{1-\mu}} (1-\alpha), \ k \in \{N,S\}, \ i \in [0,n].$$
(A.10)

The profits of the final goods producers decrease with the wage rate in the North w_N and the number of the final goods firms n.

A.2 Derivation of the labor market equilibrium condition

To derive the labor demand of the final goods sector and the intermediate goods sector when the final goods firms are in the North, I apply Shepard's lemma on (12) and use (A.4), which yields:

$$L_{N,k}^{F} = \int_{0}^{n} \frac{\partial c_{N}(w_{N}, p_{N,k}^{m})}{\partial w_{N}} x_{N,k}(i) di$$

= $n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (1-\beta) \alpha^{\frac{1}{1-\mu}} w_{N}^{\frac{\beta\mu-1}{1-\mu}} (p_{N,k}^{m})^{\frac{-\beta\mu}{1-\mu}},$ (A.11)

$$L_{N,k}^{M} = \int_{0}^{n} \frac{\partial c_{N}(w_{N}, p_{N,k}^{m})}{\partial p_{N,k}^{m}} x_{N,k}(i) di$$

= $n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \beta \alpha^{\frac{1}{1-\mu}} w_{N}^{\frac{-\mu(1-\beta)}{1-\mu}} (p_{N,k}^{m})^{\frac{\mu(1-\beta)-1}{1-\mu}}, \ k \in \{N, S\}.$ (A.12)

Then, the production of one unit of the homogeneous good requires one unit of labor. Thus, production of the homogeneous good equals the labor demand. I can write the production of the homogeneous good by using the budget constraint, (4), (A.3), and (A.4).

$$y_{N,k} = E - \left[\int_{0}^{n} P(i)x(i)di\right]^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} = E - \left[\int_{0}^{n} x(i)^{\alpha} X_{N,k}^{\frac{\alpha(\mu-\alpha)}{\alpha-1}} di\right] = E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{\mu}{1-\mu}} w_{N}^{\frac{-\mu(1-\beta)}{1-\mu}} (p_{N,k}^{m})^{\frac{-\beta\mu}{1-\mu}}, \ k \in \{N, S\}.$$
(A.13)

On the other hand, to derive the labor demand of the final goods sector and the intermediate goods sector when the final goods firms are in the South, I apply Shepard's lemma on (13) and use (A.9), which yields

$$L_{S,k}^{F} = \int_{0}^{n} \frac{\partial c_{S}(w_{N}, p_{S,k}^{m})}{\partial (\epsilon w_{N})} x_{S,k}(i) di$$
$$= n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha \gamma (1-\beta) \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_{N})^{\frac{-\mu\gamma(1-\beta)}{1-\mu}-1} (p_{S,k}^{m})^{\frac{-\beta\mu}{1-\mu}}, \qquad (A.14)$$

$$L_{S,k}^{F,S} = \int_{0}^{n} \frac{\partial c_{S}(w_{N}, p_{S,k}^{m})}{\partial(w_{S})} x_{S,k}(i) di$$

= $n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha(1-\gamma)(1-\beta) \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_{N})^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} (p_{S,k}^{m})^{\frac{-\beta\mu}{1-\mu}},$ (A.15)

$$L_{S,k}^{M} = \int_{0}^{n} \frac{\partial c_{S}(w_{N}, p_{S,k}^{m})}{\partial p_{S,k}^{m}} x_{S,k}(i) di$$

= $n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha \beta \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_{N})^{\frac{-\mu(1-\beta)}{1-\mu}} (p_{S,k}^{m})^{\frac{-\beta\mu}{1-\mu}-1}, \ k \in \{N, S\}.$ (A.16)

Then, I can obtain the production of the homogeneous good by using the budget constraint, (4), (A.8), and (A.9).

$$y_{S,k} = E - \left[\int_0^n P(i)x(i)di \right]^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} = E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} (\epsilon w_N)^{\frac{-\mu(1-\beta)}{1-\mu}} (p_{S,k}^m)^{\frac{-\beta\mu}{1-\mu}}, \ k \in \{N, S\}.$$
(A.17)

A.3 Derivation of the differential equation (31)

I derive the differential equation for the wage rate in the North. I differentiate the Free entry condition (23) by t as follows:

$$v_{SS}^{\cdot} = \frac{aw_N n - aw_N \dot{n}}{n^2}.$$
(A.18)

Substituting (7) and (A.18) into (22), I obtain:

$$\rho \frac{aw_N}{n} = \pi_{SS} + \frac{a\dot{w_N}}{n} - \frac{aw_N}{n}\frac{\dot{n}}{n}.$$
(A.19)

Then, I substitute (18) and (30) into (A.19) and I can obtain the differential equation for the wage rate in the North as follows:

$$\dot{w_N} = \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1-\alpha}{a} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} \left(\epsilon w_N\right)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}}
- \frac{\alpha\gamma(1-\beta)}{a\epsilon} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} \left(\epsilon w_N\right)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}}
= \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1}{a} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} \left(\epsilon w_N\right)^{\frac{-\mu\gamma(1-\beta)}{1-\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (1-\alpha + \frac{\alpha\gamma(1-\beta)}{\epsilon}). \quad (A.20)$$

A.4 Proof of the stable saddle paths in *Regime NN* and *Regime NS*

First, I show the proof of the stable saddle path in *Regime NN*. Suppose that I define $z_{NN} \equiv n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} w_N^{\frac{-1}{1-\mu}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}}$. Then, differentiating z_{NN} with respect to time t and using (36) and (37) yields the following differential equation for z_{NN} :

$$\frac{z_{NN}}{z_{NN}} = \frac{\mu(1-\alpha)}{\alpha(1-\mu)} \left(\frac{\dot{n}}{n}\right) - \frac{1}{1-\mu} \left(\frac{\dot{w}_N}{w_N}\right)$$
$$= \left[\left(\frac{(\mu-\alpha)-\alpha\mu}{\alpha(1-\mu)}\right) \frac{L^N}{a} - \frac{\rho}{1-\mu} \right] + \frac{1-\mu(1-\alpha)}{1-\mu} \left(\frac{z_{NN}}{a}\right).$$

From $\mu < \alpha$, the first parentheses is negative. Then, $\frac{z\dot{N}N}{z_{NN}} = 0$ schedule is a straight line through the negative intercept with slope $\frac{\alpha}{1-\mu} > 0$ in Figure 13. There is a unique steady state at E'. The steady state value is as follows:

$$z_{NN}^* = \frac{(\alpha - \mu + \alpha\mu)L^N + a\alpha\rho}{\alpha \left\{1 - \mu(1 - \alpha)\right\}}.$$
(A.21)

Because the steady state E' is unstable and z_{NN} is jump variable, z_{NN} jumps to z_{NN}^* at the initial time. In the steady state, the relationship between the number of final goods, n, and the wage rate in the North, w_N , becomes as follows:

$$w_N = \left(\frac{A}{\alpha}\right)^{-\mu} \left(\frac{(\alpha - \mu + \alpha\mu)L^N + a\alpha\rho}{\alpha\left\{1 - \mu(1 - \alpha)\right\}}\right)^{-1 + \mu} n^{\frac{\mu(1 - \alpha)}{\alpha}}.$$
 (A.22)

Therefore, the dynamic equilibrium follows the stable saddle path shown by the solid locus with arrow (II) in Figure 3.



Figure 13: Phase diagram about z_{NN}

Next, I show the saddle path stability in *Regime NS* in the same way. Suppose I define $z_{NS} \equiv n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} w_N^{\frac{-1+\beta\mu}{1-\mu}} \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\mu}} \tau^{\frac{-\beta\mu}{1-\mu}}$. Then, differentiating z_{NS} with respect to time t and using (40) and (41) yields the following differential equation for z_{NS} :

$$\frac{\dot{z}_{NS}}{z_{NS}} = \frac{\mu(1-\alpha)}{\alpha(1-\mu)} \left(\frac{\dot{n}}{n}\right) - \frac{1-\beta\mu}{1-\mu} \left(\frac{\dot{w}_N}{w_N}\right)$$
$$= \left[\left(\frac{(\mu-\alpha)-\alpha\mu(1-\beta)}{\alpha(1-\mu)}\right) \frac{L^N}{a} - \frac{(1-\beta\mu)\rho}{1-\mu} \right] + \frac{\Gamma_{NS}z_{NS}}{a(1-\mu)},$$

where

$$\Gamma_{NS} \equiv (1 - \beta \mu)(1 - \alpha \beta) - \mu (1 - \alpha)(1 - \beta)$$

> $(1 - \alpha)(1 - \beta \mu - \mu (1 - \beta)) = (1 - \alpha)(1 - \mu) > 0$

From $\mu < \alpha$ and $1 - \alpha < 1 - \alpha\beta$, $\frac{z_{NS}}{z_{NS}} = 0$ schedule is a straight line through the negative intercept with a positive slope in Figure 14. There is a unique steady state at E''. The

steady state value is as follows:

$$z_{NS}^* = \frac{\left[(\alpha - \mu + \alpha\mu(1 - \beta))L^N + (1 - \beta\mu)a\alpha\rho\right]}{\alpha\Gamma_{NS}}.$$
(A.23)

Because the steady state E'' is unstable and z_{NS} is jump variable, z_{NS} jumps to z_{NS}^* at initial time. In the steady state, the relationship between the number of final goods, n, and the wage rate in the North, w_N , becomes as follows:

$$w_{NS} = \left(\frac{A}{\alpha}\right)^{\frac{-\mu}{1-\beta\mu}} (z_{NS}^*)^{\frac{-1+\mu}{1-\beta\mu}} \tau^{\frac{\beta\mu}{1-\beta\mu}} n^{\frac{\mu(1-\alpha)}{\alpha(1-\beta\mu)}}.$$
 (A.24)

Therefore, the dynamic equilibrium follows the stable saddle path shown by the arrow (III) in Figure 4.



Figure 14: Phase diagram about z_{NS}

A.5 Proof of proposition 1

Suppose that the boundary condition between *Regime NN* and *Regime SS* is bigger than the condition between *Regime NN* and *Regime NS* in terms of the wage rate in the North, w_N . Using (44) and (45), this inequality holds: $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$. Then, I compare the boundary condition between *Regime NN* and *Regime SS*, (45), to the condition between *Regime NS* and *Regime SS*, (45), as follows:

$$\begin{split} & A^{\frac{1}{\Psi-\beta}}\tau^{\frac{-\beta}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}} - A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}} \\ > & A^{\frac{1}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}} \left[A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}\right]^{\frac{-\beta}{\Psi-\beta}} - A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}} \\ = & A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}} \left[A^{\frac{1}{\Psi-\beta}}\epsilon^{\frac{\gamma}{1-\gamma}} \left\{A^{\frac{1}{\Psi}}\epsilon^{\frac{1-\Psi}{\Psi}}\right\}^{\frac{-\Psi}{\Psi-\beta}} - 1\right] \\ = & 0, \end{split}$$

where I derive the above inequality to use $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$. If this inequality holds, Regime of the economy is chosen from three regimes, that is *Regime NN*, *Regime NS*, and *Regime SS*.

On the other hand, if the inverse inequality holds, Regime of economy is chosen from two regimes, that is, *Regime NN* and *Regime SS*.

A.6 Proof of Lemma 2

When $\tau < A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$ holds, the boundary line is (44) and (46). Differentiating (46) with respect to τ and ϵ yields

$$\frac{\partial}{\partial \tau} \left(A^{\frac{1}{\Psi - \beta}} \tau^{\frac{-\beta}{\Psi - \beta}} \epsilon^{\frac{\gamma}{1 - \gamma}} \right) = \frac{\beta}{\Psi - \beta} A^{\frac{1}{\Psi - \beta}} \tau^{\frac{-\Psi}{\Psi - \beta}} \epsilon^{\frac{\gamma}{1 - \gamma}} < 0, \tag{A.25}$$

$$\frac{\partial}{\partial \epsilon} \left(A^{\frac{1}{\Psi - \beta}} \tau^{\frac{-\beta}{\Psi - \beta}} \epsilon^{\frac{\gamma}{1 - \gamma}} \right) = \frac{\gamma}{1 - \gamma} A^{\frac{1}{\Psi - \beta}} \tau^{\frac{-\beta}{\Psi - \beta}} \epsilon^{\frac{\gamma}{1 - \gamma} - 1} > 0.$$
(A.26)

When $\tau \ge A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}}$ holds, the boundary line is (45). Differentiating (45) with respect to ϵ yields

$$\frac{\partial}{\partial \epsilon} \left(A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}} \right) = \frac{1-\Psi}{\Psi} A^{\frac{1}{\Psi}} \epsilon^{\frac{1-\Psi}{\Psi}-1} > 0. \tag{A.27}$$

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