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Why Hierarchy? Communication and Information Acquisition in Organizations

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Abstract

In most firms, if not all, workers are divided asymmetrically in terms of authority and responsibility. In this paper, we view the asymmetric allocations of authority and responsibility as essential features of hierarchy and examine why hierarchies often prevail in organizations from that perspective. The focus of attention is on the tradeoff between costly information acquisition and costless communication. When the agency problem concerning information acquisition is sufficiently severe, the contractual arrangement which allocates responsibility asymmetrically often emerges as the optimal organizational form, which gives rise to the chain of command pertaining to hierarchical organizations. This explains why hierarchies often prevail in firms since a relatively fixed group of members must confront with new problems and come up with solutions on the day-to-day basis, and hence the agency problem is an issue to be reckoned with.

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1 Introduction

It is very rare, if ever, to find a firm where all of its workers are treated just equally. In most cases, some workers, typically those higher up in the hierarchy, are conferred more authority and hence carry an asymmetrically larger weight in the decision-making process than others. At the same time, those with more authority are also given more stringent incentives and moreover held responsible for a wider range of outcomes, such as the firm's or their respective division's overall performances. While the degree of centralization or worker empowerment differs across firms as well as over time, workers are by and large divided asymmetrically in terms of both authority and responsibility. In this paper, we view these asymmetric allocations of authority and responsibility as essential features of hierarchical organizations and examine why hierarchies often prevail in organizations from that perspective.

At a glance, the asymmetric allocation of responsibility seems to be a straightforward consequence of that of authority: if a worker is entitled to make a decision, he should be held accountable for any consequences brought by that decision. In this line of reasoning, it is the allocation of authority which subsequently determines the allocation of responsibility, so that the causation runs from authority to responsibility. What is implicit in this argument is therefore that the principal can allocate (formal) authority at her own discretion, e.g., by granting or restricting access to critical resources, as often assumed in the literature (e.g., Aghion and Tirole, 1997; Rajan and Zingales, 2001; Dessein, 2002; Hart and Moore, 2005). This may not always be the case, however, because there may not exist such critical resources that are indispensable for production or because there may be no realistic way to control access to those resources. Since the presence of hierarchy is quite ubiquitous in firm organizations, it seems worthwhile to explore the nature of hierarchy in an environment where, due to some technological or informational constraints, the principal has no direct control over the allocation of authority.

To this end, we consider an organization with a principal (the contract designer) and two agents. Each agent privately chooses a task to implement, which stochastically leads to some observable output. The productivity of each task is not known *ex ante*, and each agent

must hence acquire information about which task is the most productive. This information-acquisition stage involves two rounds. In the first round, each agent privately exerts effort to acquire information (or produce an "idea") about the productivity of each task. This is followed by the second round where each agent sends a costless message to share this information (idea), if any, with the other agent. The problem is that the principal cannot observe each agent's task choice nor how that decision is reached (who had the idea and whose idea was adopted); the only way for the principal to control the agents is to allocate responsibility via incentive contracts contingent on the outputs. With no feasible way to allocate authority, no party can force an agent to take actions that are not in his best interest, and every decision to be made must be incentive-compatible under the agreed contracts. The chain of command (i.e., who orders and who obeys) then arises endogenously as an optimal response to the given structure of incentives.

An incentive contract in this setup is subject to two constraints: one for information acquisition (whether to exert effort to acquire information) and the other for truthful communication (whether to reveal truthfully the acquired information). The problem is that the constraint for information acquisition generally calls for competition between the agents while that for communication inherently calls for cooperation between them. These two constraints are therefore at odds with each other, and the optimal contract must achieve just the right balance between these two concerns. A hierarchy emerges when one agent (the superior) orders the other agent (the subordinate) what to do, and the subordinate has an incentive to follow the order. Under this contractual arrangement, the flow of information is restricted to be unilateral, always from the superior to the subordinate, and the superior is given a disproportionately large weight in the decision-making process *ex ante*. As an alternative to this arrangement, the incentive contracts may be designed to place no restriction over the flow of information. In this case, information flows from the informed party to the uninformed in an unspecified direction, and each agent is given an equal weight *ex ante* as in a committee. With the addition of the benchmark case, we consider and compare the following three organizational forms summarized as below.[1]

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[1]While there are many other contractual arrangements in this setup, we can show that it suffices to consider

1. Independent Production (the benchmark): Each agent independently exerts effort to acquire the information without ever communicating with each other. No interactions take place between the agents.

2. Committee (the symmetric contracts): Both of the agents are induced to exert effort and then communicate with each other. Communication is bilateral where each agent carries the same weight *ex ante* in the decision-making process.

3. Hierarchy (the asymmetric contracts): Only one of the agents, the superior, is induced to exert effort while the other, the subordinate, is not. Communication is unilateral where the superior orders the subordinate what to do.

We then show that the optimal contractual arrangement is often asymmetric, where only one agent is motivated to acquire the information, and Hierarchy thus emerges as the optimal organization form. Our analysis centers around the tradeoff between costly information acquisition and costless (cheap-talk) communication. To illustrate this, we start with the benchmark case of Independent Production which makes no use of communication. The optimal contract under this arrangement generically takes the form of relative performance evaluation, where the agents are compensated based on the difference in the outputs. This is the most efficient way to provide incentives if the principal's only concern is to motivate them to exert effort. Independent Production has a clear weakness, however, because the constraint for truthful communication cannot be satisfied under relative performance evaluation. With no information flowing between the agents, therefore, it may leave some of the existing knowledge unutilized: in this sense, the level of competition between the agents is often excessive under Independent Production, which impedes cooperation that is equally critical for efficient production. Given that information is free to disseminate once it is acquired, it is clearly *ex post* optimal to share any useful information between the agents. Since it is not known *ex ante* who actually ends up with a good idea, this can be done most effectively by removing any restrictions on the flow of information.

those three cases. See Appendix B for more detail.

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Given this result we now turn to Committee where both of the agents are induced to exert effort and no restriction whatsoever is placed on the extent of communication (bilateral communication). Under this arrangement, each agent *ex ante* carries the same weight in the decision-making process, and information flows from a party with an idea to a party without it. A virtue of Committee is then evident: it can make the best use of the existing knowledge within the organization by fully exploit the benefit of costless communication. As it turns out, though, this *ex post* optimal arrangement is not *ex ante* optimal, as it raises the agency cost of inducing costly effort. The analysis identifies three channels through which bilateral communication entails *ex ante* inefficiency.

Incentive provision: To facilitate communication, the agents must be held jointly accountable, which is a less efficient way to provide incentives.

Freeriding: When communication is informative, there arises an incentive to freeride on the other agent's effort.

Coordination: In the absence of communication, the agents can increase the chance of coordinating their task choices by acquiring the information. This effect is absent because the agents can always coordinate via communication.

Due to these problems, Committee is often less profitable, especially when the agency problem regarding information acquisition is sufficiently severe. This does not necessarily mean, though, that we must give up the benefit of communication entirely; it rather means that going all the way from no communication to bilateral communication, which places no restriction on the extent of communication, is often excessive. We argue that there is a way to exploit the benefit of communication while keeping its cost minimum. This can be done by the asymmetric allocation of responsibility where one agent, the superior, is offered high-powered (team) incentives while the other, the subordinate, is offered low-powered (individual) incentives. This asymmetric contract yields two beneficial effects. First, Hierarchy reduces the total agency cost as it only needs to motivate one agent. In an environment where information acquisition is costly while communication is free, this is a thrifty way to acquire information as a group. Second, the asymmetric and concentrated allocation of

responsibility eliminates the freeriding incentive as the superior can no longer rely on the subordinate's information. We show that this asymmetric contractual arrangement is optimal for a wide range of circumstances, as it achieves the right balance of competition and cooperation. Under the asymmetric allocation of responsibility, the flow of information is restricted to be unilateral, always from the superior to the subordinate, which endogenously gives rise to the chain of command pertaining to hierarchical organizations.

The results obtained in this paper suggest that whether Hierarchy or Committee is optimal depends crucially on the severity of the agency problem. Hierarchy, which allocates responsibility asymmetrically, outperforms Committee when costly information acquisition is the main concern. We argue that this corresponds to typical firm organizations where a relatively fixed group of members must confront with and find solutions for new problems on the day-to-day basis. In such a case, Hierarchy is often optimal, lending a support for the view that most firms are hierarchical. This draws contrast to committees which typically consist of experts who are well informed in the first place. In committees, members are in many cases selected for specific problems and information acquisition is hence rarely an issue. In such a case, there is no reason to treat members asymmetrically, and Committee with the more symmetric allocation of authority and responsibility emerges as the better option to cope with the problem at hand.

The paper is related to several strands of literature. The paper is more closely related to a growing literature on committees with endogenous information. As often emphasized in the literature (Li, 2001; Li and Suen, 2009), one of the important aspects of the model is that the *ex post* efficient rule is not necessarily *ex ante* efficient. The freeriding problem in a committee is pointed out by Li (2001) where each member must independently acquire information.[2] In that environment, he shows that a super-majority rule, the one that biases against the *ex ante* preferred option, can be used to mitigate the freeriding incentive. The current model provides yet another example of this situation, illuminating the tradeoff between information acquisition and communication. A point of departure is that we introduce

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[2]Persico (2004) considers a similar problem where costly information acquisition is followed by voting. The optimal voting mechanism with costly information acquisition is also considered by Gerardi and Yariv (2007) and Gershkov and Szentes (2009).

incentive contracts into the model and suggest a different solution to this problem.[3] With the explicit consideration of incentive contracts, we also show that information acquisition in a committee entails different types of inefficiency – incentive provision and coordination.

Second, there is also a vast literature on hierarchy, too numerous to list them all. Among them, the paper is particularly related to works on the optimal allocation of authority. Aghion and Tirole (1997) is close in spirit to ours, as they shed light on the impact of the allocation of authority on the incentive to acquire information with its focus on the distinction between real and formal authority.[4] As emphasized earlier, in Aghion and Tirole and others, the starting point of the analysis is the allocation of authority which can be made by the principal at her own discretion. They then analyze how this allocation affects the agents' incentive to acquire information through the tradeoff between the loss of initiative and the loss of incentive. Instead, we consider a situation where the principal can only allocate responsibility, which in turn determines the allocation of authority: that is, the allocation of authority is at the end point of the analysis.[5] We view our analysis as complementary to the existing literature, as it approaches the same issue from the opposite angle.

Finally, the current model builds on cheap-talk communication, put forth by Crawford and Sobel (1982). There are now many applications of this insightful idea, and this paper belongs to this strand as well. In most applications, however, the payoff structure is exogenously given, despite the fact that the degree of preference incongruence is almost always the center of attention in cheap-talk models. Instead, we introduce incentive contracts which allow us to endogenize the degree of preference incongruence. By writing a contract, the principal can either facilitate or restrict communication. Of course, since the principal can only observe each agent's output and the incentive for information acquisition cannot hence be separated

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[3]The contrast between hierarchies and committees is also depicted in Sah and Stiglitz (1988) with a totally different approach. They compare three different decision-making protocols – committees, hierarchies and polyarchies – with its particular emphasis on the tradeoff between the type I and type II errors.

[4]Another approach to explore the link between hierarchy and knowledge acquisition is provided by Garicano (2000). There, the main focus is on the tradeoff between communication and knowledge acquisition costs with no regard to incentive issues.

[5]It is not our intention to argue that firms do not allocate formal authority: they may or may not. Our point is rather that for the allocation of formal authority to be meaningful, it must be incentive-compatible, at least to some extent, for the subordinate to follow the superior's order. Our specification thus represents an extreme point of the spectrum while the conventional approach represents the other end.

out from that for communication, any contract inevitably affects the process of information acquisition, leading to a tradeoff which lies at the core of the analysis.

2 Model

2.1 The setup

Consider an organization in which a risk-neutral principal (e.g., a firm owner) hires two risk-neutral agents (e.g., employees), each denoted by $i \in \{1, 2\}$. Each agent independently chooses a task $x_i \in \{L, R\}$ to implement, which yields the output $y_i \in \{0, 1\}$. Each agent's task choice is his private information, which cannot be observed by either the other agent or the principal. This means that there is no way to force an agent to take an action which is not in his best interest.

The output is either high ($y_i = 1$) or low ($y_i = 0$), depending on the state of nature represented by $(s, a) \in \{L, R\} \times \{G, B\}$. The state of nature is two-dimensional, where s indicates the relative productivity of each task while a the aggregate productivity. When $a = G$, the (aggregate) state is "good" and the output is always high regardless of the task choice. When $a = B$, the state is "bad" and the output is high if and only if the right task, i.e., $x_i = s$, is implemented. Letting $p(x_i, s, a) := \text{prob}\{y_i = 1 \mid x_i, s, a\}$, we thus have

$$\begin{cases} p(L, s, G) = p(R, s, G) = 1 \text{ for } s = L, R, \\ p(L, L, B) = p(R, R, B) = 1 \text{ and } p(L, R, B) = p(R, L, B) = 0. \end{cases}$$

2.2 Information Acquisition

The state of nature is not directly observable. The prior distribution of the state is given by

$$\begin{cases} \text{prob}\{s = L\} = \text{prob}\{s = R\} = 0.5, \\ \text{prob}\{a = G\} = \lambda \text{ and } \text{prob}\{a = B\} = 1 - \lambda, \end{cases}$$

where s and a are independent of each other. λ is one of the key parameters of the model, which measures the salience of the common stochastic shock. We mostly focus on a case where λ is strictly positive but relatively small.

While the state of nature is not freely observable, each agent may observe the relative productivity s with some probability by incurring some private cost. Let $\hat{s}_i \in \{L, R, \phi\}$ denote

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the private signal of s. Each agent may fail to observe any relevant information with some probability, and this event is denoted by $s_i = \phi$. If $\hat{s}_i \neq \phi$, then the signal accurately reflects the true state, i.e., $\hat{s}_i = s$.[6] For expositional purposes, we say that an agent is "informed" when $s_i \neq \phi$ and "uninformed" when $s_i = \phi$.

The probability that an agent becomes informed depends on how hard he works on acquiring it: it should be noted that what we mean by information acquisition in this context is rather broad, including not just the cost of literally gathering or collecting information but also that of processing and interpreting available evidence or generating ideas, insights and visions. Define $e_i \in \{0, 1\}$ as the effort level chosen by agent i, where $e_i = 1$ means that the agent chooses to collect evidence of his own. The probability of becoming informed is then given by

$$\text{prob}\{\hat{s}_i \neq \phi \mid e_i\} = \text{prob}\{\hat{s}_i = s \mid e_i\} = re_i, \; r \in (0, 1).$$

The cost of effort is given by $c > 0$.

2.3 Contracts

The principal designs and offers a contract to each agent to maximize her expected profit. The output y_i is the only contractible variable in this environment. A feasible contract can thus be written as $W_i := (w_i^{11}, w_i^{10}, w_i^{01}, w_i^{00})$ where $w_i^{y_i y_j}$, $i \neq j$, denotes the wage to be received by agent i contingent on the outputs y_i and y_j. We impose a limited liability condition $w_i^{y_i y_j} \geq 0$, so that any wage payment must be nonnegative. The contracts offered are publicly observable, so that each can observe the other agent's contract.

The nature of a contract is determined largely by how each agent's compensation depends on the other agent's output. For expositional purposes, we use the following terminologies:

- Independent performance evaluation (IPE): $(w_i^{11}, w_i^{01}) = (w_i^{10}, w_i^{00})$;

- Relative performance evaluation (RPE): $(w_i^{11}, w_i^{01}) \leq (w_i^{10}, w_i^{00})$;

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[6]This means that when an agent has some information about the state of nature, he knows it with precision. We make this assumption to avoid a purely technical problem which arises when two imperfectly informed parties communicate with each other. See Ishida and Shimizu (2009) on this problem.

- Joint performance evaluation (JPE): $(w_i^{11}, w_i^{01}) \geq (w_i^{10}, w_i^{00})$.

2.4 Communication and the Task Choice

Upon observing a signal, each agent communicates with each other to share the acquired information and chooses the task. More precisely, each agent simultaneously sends a message $m_i \in \{L, R, \phi\}$ to the other: for clarity, we consider the message space which coincides with the signal space. The message is costless and unverifiable, so this message game belongs to the class of cheap talk. Given the message and his own observed signal, each agent then chooses his task $x_i \in \{L, R\}$.

2.5 Preferences and the Timing

Both the principal and the agents are risk-neutral, where the principal maximizes the expected profit (the expected output minus the expected wage costs) while each agent maximizes the expected wage minus the effort cost. The timing of the model is summarized as follows:

1. The state of nature $(a, s) \in \{G, B\} \times \{L, R\}$ is randomly drawn.

2. The principal offers a contract W_i to each agent.

3. Each agent determines the effort level $e_i \in \{0, 1\}$.

4. Each agent observes a signal $\hat{s}_i \in \{L, R, \phi\}$.

5. Upon observing $\hat{s}_i$, each agent sends a message $m_i \in \{L, R, \phi\}$ to the other.

6. Upon observing $\hat{s}_i$ and m_j, each agent chooses the task $x_i \in \{L, R\}$.

7. The outputs $(y_1, y_2) \in \{0, 1\}^2$ are realized.

3 The Communication Stage

3.1 The Optimal Task Choice

In the communication stage, each agent sends a message m_i conditional on the observed signal. Let $M_i(\hat{s}_i)$ denote agent i's message strategy. For the cases we consider, we focus on

fully revealing (truth-telling) strategies where $M_i(\hat{s}_i) = \hat{s}_i$.[7] To obtain conditions for truth telling, however, we must first characterize the optimal task choice contingent on the message and the observed signal, under the premise that each agent reports truthfully.

First, if $\hat{s}_i \neq \phi$, the state is known to be $s = \hat{s}_i$. Letting $\sigma_j := \text{prob}\{y_j = 1 \mid \hat{s}_i, m_j\}$, the agent chooses $x_i = \hat{s}_i$, regardless of the message, if

$$\sigma_j w_i^{11} + (1 - \sigma_j)w_i^{10} \geq \sigma_j w_i^{01} + (1 - \sigma_j)w_i^{00}.$$

This condition holds for any σ_j if $w_i^{11} \geq w_i^{01}$ and $w_i^{10} \geq w_i^{00}$. Since this is generally satisfied by any optimal contract as we will see shortly, communication virtually plays no role when the agent is informed.

Communication may matter, on the other hand, when the agent is uninformed. Suppose first that $m_j \neq \phi$. In this case, the optimal task choice is to follow the other agent's recommendation if

$$w_i^{11} \geq w_i^{01}.$$

which is again satisfied by any optimal contract. If $m_j = \phi$, neither agent has any clue and hence no preference over either task. In this case, we allow the agents to coordinate any way they can to achieve *ex ante* Pareto-efficient outcomes, if any, when communication is technically feasible. We say that an agent has the incentive to coordinate (on the same task) if

$$w_i^{11} + w_i^{00} \geq w_i^{10} + w_i^{01}, \tag{1}$$

whereas an agent has the incentive to differentiate if this is not satisfied. If (1) is satisfied for both of the agents, it is Pareto-efficient for them to coordinate their task choices ($x_1 = x_2$). If it is satisfied for neither, it is then Pareto-efficient to differentiate ($x_1 \neq x_2$). Finally, if it is satisfied only for one of the agents, there is no Pareto-efficient outcome, and the agents must randomize and choose each task with equal probability.[8]

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[7] There are some exceptional cases where it is not possible to achieve full separation. We do not consider these cases because they are not optimal under the maintained assumptions. See Appendix B for more detail.

[8] This last case would not arise, however, under the three cases we consider.

3.2 Strategic Information Disclosure

We now explore conditions under which an agent has an incentive to reveal truthfully. Since an informed agent cannot be influenced by any message, we can focus on the case where the other agent is uninformed. Given that $\hat{s}_j = m_j = \phi$, there are two cases we need to consider.

Case 1 ($\hat{s}_i = \phi$): The agent has no information and hence has no preference over the task choice. If there exists a Pareto-efficient outcome (both agents have the incentive to coordinate or differentiate), it is weakly optimal to report truthfully since the agent is totally indifferent between any messages. This means that under any symmetric contract, an uninformed agent has no incentive to lie.

If one agent has the incentive to coordinate while the other has the incentive to differentiate, on the other hand, truth telling cannot be induced on the equilibrium path. To see this, if an agent reports truthfully, the agents end up with no information. In this case, they can never agree on who chooses which task and are hence forced to randomize over the tasks. If the agent misrepresents and claim either $m_i = L$ or $m_i = R$, however, he can induce the other agent to choose $x_j = m_i$, which allows the agent to coordinate or differentiate. No information can therefore be conveyed in this case.

Case 2 ($\hat{s}_i \neq \phi$): This is the case where communication becomes strategic. If the agent reports truthfully, then the other agent yields the high output for sure using that information. This may or may not be in his best interest, depending on the type of contract he faces. To see this, suppose that $\hat{s}_i = L$ without loss of generality. The agent has an incentive to lie only when he would like the other agent to choose $x_j = R$; the best way to achieve this is to claim $m_i = R$. Given this, the agent truthfully discloses his observation iff

$$w_i^{11} \geq w_i^{10}, \tag{2}$$

which we refer to as the condition for truth telling. If (2) does not hold, no message can be taken seriously by the other agent, and no information can hence be conveyed (a babbling equilibrium).

4 Optimal Organizations

4.1 Equilibrium with No Communication: a Benchmark

We start the analysis with a benchmark case where any form of communication is not feasible between the agents, possibly for some technological reasons. Each agent thus chooses the task independently without ever communicating with the other, as if there exists only one agent. This also precludes the possibility of task coordination when both of the agents are uninformed. For expositional purposes, we refer to this scheme as Independent Production. This benchmark case is instrumental in illuminating the role of communication in the current setup.

Under this scheme, an agent has no choice but to choose the task randomly when he is uninformed. The expected payoff as a function of the effort choices under this scheme, denoted by π_i^I, is then obtained as

$$\pi_i^I(e_i, e_j) = r^2 e_i e_j w_i^{11} + r e_i(1 - r e_j)\left(\lambda w_i^{11} + \frac{(1-\lambda)(w_i^{11} + w_i^{10})}{2}\right)$$
$$+ r e_j(1 - r e_i)\left(\lambda w_i^{11} + \frac{(1-\lambda)(w_i^{11} + w_i^{01})}{2}\right)$$
$$+ (1 - r e_i)(1 - r e_j)\left(\lambda w_i^{11} + \frac{(1-\lambda)(w_i^{11} + w_i^{10} + w_i^{01} + w_i^{00})}{4}\right) - c e_i.$$

Taking the other agent's effort choice e_j as given, each agent chooses to exert effort iff $\pi_i^I(1, e_j) \geq \pi_i^I(0, e_j)$, which can be written as

$$r e_j \frac{w_i^{11} - w_i^{01}}{2} + (1 - r e_j)\frac{w_i^{11} + w_i^{10} - w_i^{01} - w_i^{00}}{4} \geq \frac{c}{r(1 - \lambda)}. \tag{3}$$

Lemma 1 *The optimal contract in the absence of communication is given by*

$$w_i^{10} = \frac{4c}{r(1 - r)(1 - \lambda)}, \ w_i^{11} = w_i^{01} = w_i^{00} = 0.$$

PROOF: See Appendix A.

As is well known, in the presence of common stochastic shocks, the optimal contract in this case takes the form of RPE where an agent is compensated based on the difference in

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the outputs. RPE is generically superior in inducing costly effort as it filters out common stochastic shocks.[9] The expected profit under this optimal contract is given by

$$\Pi^I = 2\lambda + (1-\lambda)(1+r) - \frac{(1-\lambda)(1-r)(1+r)}{4} \frac{8c}{r(1-r)(1-\lambda)}$$
$$= 2\lambda + (1-\lambda)(1+r) - \frac{2(1+r)c}{r}.$$

We assume that this is larger than the expected profit when the agents are not induced to exert effort. This is the case if

$$\Pi^I \geq 1 + \lambda \iff r \geq \frac{2(1+r)c}{r(1-\lambda)}.$$

In what follows, we assume that c is sufficiently small, so that this condition holds.

4.2 Decision Making by Committee: Bilateral Communication

In the presence of common stochastic shocks, the optimal contract typically takes the form of RPE where the agents are compensated based on the difference in the outputs. One drawback of this type of contract is that by having the agents compete with each other, it necessarily impedes cooperation between them. In this particular context, even when one agent is informed, there is no incentive to reveal truthfully his own observation under this type of contract: the optimal contract in the absence of communication cannot satisfy the condition for truth telling (2). The lack of truthful communication entails an efficiency loss since information is typically a public good whose value does not depend on the number of people who use it. When the cost of communication is negligibly small, it is clearly *ex post* efficient to share all the relevant information within the organization.

Here, we seek for an equilibrium where both of the agents exert effort to acquire the information and then truthfully report what is observed. We refer to this situation as decision making by Committee, where both agents carry the same weight in the decision-making process *ex ante* with no restriction on the flow of information (bilateral communication). An apparent virtue of Committee is that it allows the information to flow from an informed party to an uninformed party, making the best use of the existing knowledge within the organization.

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[9]IPE, RPE and JPE are all equivalent when $\lambda = 0$; for any $\lambda > 0$, RPE is strictly better.

In order to facilitate communication between the agents, the condition for truth telling becomes an additional constraint to be satisfied. Once (2) is satisfied, however, the condition for information acquisition also needs to be modified because an uninformed agent may now receive the information via communication. Suppose for now that the agents have the incentive to coordinate (which needs to be verified later). The expected payoff, denoted by π_i^C, is then obtained as

$$\pi_i^C(e_i, e_j) = (re_i + re_j(1 - re_i))w_i^{11} + (1 - re_i)(1 - re_j)\frac{(1 + \lambda)w_i^{11} + (1 - \lambda)w_i^{00}}{2} - ce_i.$$

The condition for information acquisition is given by

$$r(1 - re_j)(w_i^{11} - w_i^{00}) \geq \frac{2c}{1 - \lambda}, \tag{4}$$

while the condition for truth telling is given by (2). The optimal contract must also satisfy (1) so that the agents indeed have the incentive to coordinate.

Lemma 2 *The optimal contract under Committee is given by*

$$w_i^{11} = \frac{2c}{r(1 - r)(1 - \lambda)}, \; w_i^{10} = w_i^{01} = w_i^{00} = 0.$$

PROOF: Given that each agent have an incentive to coordinate when uninformed, the principal's problem under Committee is formulated as

$$\max_{W_i} \quad \frac{2\lambda + (1 - \lambda)(2 - (1 - r)^2)}{2}w_i^{11} + \frac{(1 - \lambda)(1 - r)^2}{2}w_i^{00},$$

subject to (4) and (2). It follows that $w_i^{10} = w_i^{01} = w_i^{00} = 0$. Solving (4) then yields the proposed contract, which can easily be verified to satisfy (1).

Q.E.D.

The expected output is maximized under Committee as the output is low only when both fail to acquire the information. This does not necessarily raise the principal's profit, however, since facilitating communication has its own costs. There are three reasons for this First, to facilitate communication, the agents must be held jointly accountable (JPE), but JPE is generically a less efficient way to provide incentives. Second, when communication

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is informative, there arises an incentive to freeride on the other's information acquired via costless communication. Finally, in the absence of communication, the agents can increase the chance of coordinating their task choices by acquiring the information. This effect is absent because the agents can always coordinate via communication.

This indicates that the *ex post* efficient rule is not necessarily *ex ante* efficient as is often emphasized in the literature. The expected profit is

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\begin{aligned}
\Pi^C &= 2 - (1-\lambda)(1-r)^2 - \frac{2-(1-\lambda)(1-r)^2}{2}\frac{4c}{r(1-r)(1-\lambda)} \\
&= 2 - (1-\lambda)(1-r)^2 - \frac{2(2-(1-\lambda)(1-r)^2)c}{r(1-r)(1-\lambda)},
\end{aligned}
$$

Committee becomes increasingly less profitable as λ or r gets larger: the cost of adopting JPE is more salient when λ is relatively large, and the freeriding incentive intensifies when r is large.

4.3   Decision Making by Hierarchy: Unilateral Communication

As we have seen, it is apparently *ex post* efficient to facilitate communication between the agents because communication is free in the current setup. The problem is that it is not *ex ante* efficient because communication raises the agency cost of inducing costly effort. There may be a better way to exploit the benefit of communication than the symmetric contract obtained under Committee. The key is the nature of information: when communication is informative, only one agent needs to be informed. To minimize the cost arising from communication, we now let the conditions for information acquisition and truth telling hold for only one agent, say agent 1, so that information only flows from agent 1 to 2 but not the other way around. The chain of command is now hierarchical in that one orders the other what to do (note that the task choice is his private information).

Since agent 2 exerts no effort, we only need to look at agent 1's incentives. Suppose for now that both of the agents have the incentive to coordinate when uninformed. The expected

payoff, denoted by π_i^{H}, is then given by

$$\pi_i^{\mathrm{H}}(e_i, e_j) = re_i w_i^{11} + re_j(1 - re_i)\frac{(1+\lambda)w_i^{11} + (1-\lambda)w_i^{01}}{2}$$
$$+ (1 - re_i)(1 - re_j)\frac{(1+\lambda)w_i^{11} + (1-\lambda)w_i^{00}}{2} - ce_i.$$

The agent exerts effort iff

$$w_i^{11} - re_j \frac{w_i^{11} + w_i^{01}}{2} - (1 - re_j)\frac{w_i^{11} + w_i^{00}}{2} \geq \frac{2c}{2r(1-\lambda)}. \tag{5}$$

We can then show the following.

Lemma 3 *The optimal contract under Hierarchy is*

$$w_1^{11} = \frac{2c}{r(1-\lambda)}, \ w_1^{10} = w_1^{01} = w_1^{00} = 0 \ and \ w_2^{11} = w_2^{10} = w_2^{01} = w_2^{00} = 0,$$

which implements $e_1 = 1$ and $e_2 = 0$.

PROOF OF LEMMA 3:   We again suppose that both of the agents have an incentive to co-ordinate. Since $e_2 = 0$, it is evident that the optimal contract for agent 2 is a trivial one where $w_2^{11} = w_2^{10} = w_2^{01} = w_2^{00} = 0$. For agent 1, the principal's problem under Hierarchy is formulated as

$$\max_{W_1} \ \frac{2\lambda + (1-\lambda)(1+r)}{2} w_1^{11} + \frac{(1-\lambda)(1-r)}{2} w_1^{00},$$

subject to (5) and (2). It follows that $w_i^{10} = w_i^{01} = w_i^{00} = 0$. Solving (5) then yields the proposed contract, which can easily be verified to satisfy (1).

<div align="right">Q.E.D.</div>

The expected profit under Hierarchy is given by

$$\Pi^{\mathrm{H}} = 1 + r + \lambda(1-r) - \left(\lambda + \frac{(1+r)(1-\lambda)}{2}\right)\frac{2c}{r(1-\lambda)}$$
$$= 1 + r + \lambda(1-r) - \frac{(1 + \lambda + r(1-\lambda))c}{r(1-\lambda)}.$$

Note that under Hierarchy, the allocation of responsibility is asymmetric in that agent 1 is given high-powered incentives and also held accountable for agent 2's output (team incentives) while agent 2 is given low-powered incentives and in principle held accountable only

for his own output (individual incentives).[10] There are two virtues of Hierarchy in this setup. First, under Hierarchy, only one agent needs to be motivated to exert effort. Although this raises the probability that neither agent is informed compared to Committee,[11] this is an efficient way to exploit the fact that information transmission via communication is free. Second, it also eliminates the freeriding incentive, which occupies a substantial part of the cost of communication under Committee. By allocating responsibility asymmetrically, agent 1 cannot rely on agent 2, which in turn raises agent 1's motivation to exert effort.

4.4 Comparison

We have thus far examined three distinct schemes: (i) Independent Production with no communication, (ii) Committee and (iii) Hierarchy.[12] We now compare each of these schemes and see which one yields the highest expected profit, especially when Hierarchy outperforms the other organizational forms.

When the outputs are positively correlated, RPE functions well by filtering out common stochastic shocks. It is thus clear that Independent Production, which employs RPE, emerges as the only profitable organizational form when λ is arbitrarily close to one. As λ decreases toward zero, on the other hand, the benefit of communication outweighs the cost, and Hierarchy outperforms Independent Production.

Proposition 1 *There exists a threshold*

$$\bar{\lambda}^{\mathrm{HI}} = \frac{1+r}{3+r},$$

such that Hierarchy outperforms Independent Production for $\bar{\lambda}^{\mathrm{HI}} \geq \lambda$.

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[10]In the current specification, agent 2 is actually given no incentives and held accountable for nothing, so that the optimal contract is a special case of IPE. With a simple twist to the basic setup, however, the optimal contract can be made IPE in the strict sense. Suppose that the production process requires costly individual effort $d_i \in \{0, 1\}$, and the output is given by $y_i = p(x_i, s, a)d_i$ so that the output is high iff the agent chooses to exert effort for production. In this case, the optimal contract for agent 2 is IPE where the cost of effort is compensated iff his own output is high.

[11]In fact, this potential cost is totally offset note that the expected output under Hierarchy is the same as that under Independent Production.

[12]These three schemes do not exhaust all the possible contract forms; however, other forms are rarely optimal under plausible assumptions. See Appendix for this point.

PROOF: It follows from above that

$$\Pi^H - \Pi^I = \frac{2(1-\lambda)(1+r)c - (1+\lambda+r(1-\lambda))c}{r(1-\lambda)}$$
$$= \frac{(1-\lambda)(1+r) - 2\lambda}{r(1-\lambda)}c.$$

Solving $\Pi^H - \Pi^I \geq 0$ for λ yields the result.

Q.E.D.

When λ is sufficiently small, communication works better to exploit the information which may be dispersed within the organization. The question is then the direction of the information flow. One way is to have the agents communicate only unilaterally in the fixed direction, as in the case of Hierarchy. When both agents exert effort, however, unilateral communication is not *ex post* efficient because it is *ex ante* not clear who has the useful information. By having both agents exert effort and communicate bilaterally, the expected output is in general maximized under Committee. When this benefit outweighs the higher agency cost, Committee becomes superior to Hierarchy.

Proposition 2 *If*

$$r^2(1-r)^2 > (2(1+r) - (1-r)^2)c,$$

there exists a threshold

$$\bar{\lambda}^{CH} = 1 + \frac{(1-r)c - \sqrt{(1-r)^2c^2 + 8r^2(1+r)c}}{2r^2(1-r)},$$

such that Committee outperforms Hierarchy for $\bar{\lambda}^{CH} \geq \lambda$. Otherwise, Hierarchy always outperforms Committee.

PROOF: It follows from above that

$$\Pi^H - \Pi^C = \frac{2(2 - (1-\lambda)(1-r)^2)c - (1-r)(1+\lambda+r(1-\lambda))c}{r(1-r)(1-\lambda)} - r(1-r)(1-\lambda)$$
$$= \frac{4 - (1-r)(3-\lambda-r(1-\lambda))}{r(1-r)(1-\lambda)}c - r(1-r)(1-\lambda).$$

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Note that this is strictly increasing in λ, so that $\Pi^H - \Pi^C \geq 0$ is least likely to hold at $\lambda = 0$. If

$$r^2(1-r)^2 > (2(1+r) - (1-r)^2)c,$$

there exists a threshold below which Committee outperforms Hierarchy; otherwise, Hierarchy outperforms Committee for any λ. Hierarchy outperforms Committee if

$$-r^2(1-r)^2(1-\lambda)^2 - (1-r)^2 c(1-\lambda) + 2(1+r)c \geq 0,$$

which can be written as

$$\frac{-(1-r)c + \sqrt{(1-r)^2 c^2 + 8r^2(1+r)c}}{2r^2(1-r)} \geq 1 - \lambda.$$

Q.E.D.

The proposition shows that for any given c, Hierarchy is superior to Committee when r is close to either zero or one. There are two reasons for this. First, since the marginal increase in the output (from Hierarchy to Committee) is $r(1-r)(1-\lambda)$, the benefit diminishes as r approaches either end. Second, as r approaches one, the freeriding incentive intensifies and sharply increases the agency cost. Combined with Proposition 1, the result leads to a sufficient condition for Hierarchy to be the optimal organizational form.

Corollary 1 *Hierarchy is optimal if $\bar{\lambda}^{HI} \geq \lambda$ and r is sufficiently close to zero or one.*

We now explore conditions under which Committee becomes optimal. While Hierarchy surely dominates Independent Production for a sufficiently small λ, the same statement cannot be made for Committee.

Proposition 3 *If*

$$r(1-r)^2 > 4c,$$

there exists a threshold

$$\bar{\lambda}^{CI} = 1 + \frac{2c - 2\sqrt{c^2 + r^2 c}}{r^2(1-r)},$$

such that Committee outperforms Independent Production for $\bar{\lambda}^{CI} \geq \lambda$. Otherwise, Independent Production always outperforms Committee.

PROOF: It follows from above that

$$\Pi^{C} - \Pi^{I} = \frac{2(1-\lambda)(1-r)(1+r)c - 2(2-(1-\lambda)(1-r)^2)c}{r(1-r)(1-\lambda)} + r(1-r)(1-\lambda)$$

$$= \frac{4((1-\lambda)(1-r)-1)}{r(1-r)(1-\lambda)}c + r(1-r)(1-\lambda).$$

Note that this is strictly decreasing in λ, so that $\Pi^{C} - \Pi^{I} < 0$ is least likely to hold at $\lambda = 0$. If

$$r(1-r)^2 > 4c,$$

there exists a threshold below which Committee outperforms Independent Production; otherwise, Independent Production outperforms Committee for any λ. Committee outperforms Independent Production if

$$r^2(1-r)^2(1-\lambda)^2 + 4(1-\lambda)(1-r)c - 4c \geq 0,$$

which can be written as

$$1 - \lambda \geq \frac{-2c + 2\sqrt{c^2 + r^2c}}{r^2(1-r)}.$$

Q.E.D.

From propositions 2 and 3, we can now obtain a sufficient condition for Committee to be optimal. First, from proposition 2, $\lim_{c \to 0} \bar{\lambda}^{CH} = 1$, so that Committee outperforms Hierarchy for any λ. Second, from proposition 3,

$$\lim_{c \to 0} \bar{\lambda}^{CI} = \bar{\lambda}_0^{CI} := 1 - \frac{v(1+r)}{r(1-r)}.$$

The result implies that for any positive c, Committee cannot be optimal if r is close to one.

Corollary 2 *Committee is optimal if $\bar{\lambda}_0^{CI} \geq \lambda$ and c is sufficiently close to zero or one.*

In general, Committee works better when the cost of information acquisition is relatively small: this is intuitive because the cost of bilateral communication comes from the increase in the agency cost. This means that if an organization does not need to motivate its members to acquire information, Committee is often an efficient way to make decisions. After all, this is

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what we expect of a typical committee: in many cases, a committee consists of well informed experts to begin with, and information acquisition is rarely an issue. In such a situation, it is often less optimal to assign more weight on anyone's opinion by restricting the information flow; it thus makes more sense to treat its members more evenly.

5   Conclusion

This paper asks why hierarchies prevail in organizations in an environment where the principal can only allocate responsibility via incentive contracts but not authority. The focus of attention is placed on costly information acquisition and costless communication. In this setting, we show that the optimal incentive scheme is often asymmetric, where one agent is given high-powered team incentives while the other is given low-powered individual incentives. Under Hierarchy, the flow of information is restricted to be unilateral and this gives rise to the chain of command pertaining to hierarchial organizations. This asymmetric allocation of responsibility is the optimal way to acquire information and knowledge as a group when the agency cost is sufficiently severe.

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Appendix A: Proofs

PROOF OF LEMMA 1: Provided that it is optimal to implement $e_i = 1$, the principal's problem in the absence of communication is defined as

$$\min_{W_i} \quad \frac{4\lambda + (1-\lambda)(4r + (1-r)^2)}{4} w_i^{11} + \frac{(1-\lambda)(2r(1-r) + (1-r)^2)}{4}(w_i^{10} + w_i^{01})$$
$$+ \frac{(1-\lambda)(1-r)^2}{4} w_i^{00},$$

subject to (3). It directly follows from (3) that $w_i^{01} = w_i^{00}$ at the optimal solution. This leaves us two possibilities, either $w_i^{11} > w_i^{10} = 0$ (JPE) or $w_i^{10} > w_i^{11} = 0$ (RPE). A candidate contract under JPE, denoted by $\tilde{w}_i^{11}$, is

$$\tilde{w}_i^{11} = \frac{4c - 2rv}{r(1+r)(1-\lambda)}.$$

Similarly, a candidate contract under RPE, denoted by $\tilde{w}_i^{10}$, is

$$\tilde{w}_i^{10} = \frac{4c - 2rv}{r(1-r)(1-\lambda)}.$$

RPE yields higher profit than JPE if

$$\frac{4\lambda + (1-\lambda)(4r + (1-r)^2)}{4}\tilde{w}_i^{11} \geq \frac{(1-\lambda)(2r(1-r) + (1-r)^2)}{4}\tilde{w}_i^{10}.$$

This can be written as

$$\frac{4\lambda + (1-\lambda)(4r + (1-r)^2)}{1+r} \geq \frac{(1-\lambda)(2r(1-r) + (1-r)^2)}{1-r},$$

which is further simplified to

$$\lambda(1-r) \geq 0.$$

This shows that RPE is generically better than JPE (the two are equivalent only when $\lambda = 0$).

Q.E.D.

Appendix B: Other Contractual Arrangements

In the analysis, we examine only three contractual arrangements, which apparently do not exhaust all the possibilities. Since there are two incentive conditions, one for information acquisition (hereafter IA) and the other for truth telling (TT), there are more possible contract forms. Since the condition for truth telling matters only when the condition for information acquisition is satisfied, there are generically six possible contract forms.

1. IA and TT are satisfied for both agents (Committee).
2. IA is satisfied for both agents but TT is satisfied for one agent.
3. IA is satisfied for both agents but TT is satisfied for neither.
4. IA and TT are satisfied for one agent (Hierarchy).
5. IA is satisfied for one agent but TT is satisfied for neither.
6. IA is satisfied for neither.

Cases 1 and 4 are already analyzed. We have also seen that it is optimal to let the agents acquire the information in the absence of communication, which rules out cases 5 and 6. This implies that we only need to consider cases 2 and 3.

Case 2

In this case, both of the agents exert effort to acquire the information, but only one of them reports it truthfully. We refer to this situation as Delegated Hierarchy, since the flow of information is unilateral as in the case of Hierarchy but the subordinate acts on his own information whenever he has one. For the sake of the argument, suppose that TT is satisfied for agent 1 but not for agent 2. To derive the optimal contract within this class, we need to consider four distinct possibilities.

Case 2-1. Only agent 1 has the incentive to coordinate: In this case, fully separating communication cannot be implemented because there is no incentive to report truthfully when $\hat{s}_1 = \phi$. For communication to be meaningful, it is then necessary for agent 2 to follow agent 1's message when he is uninformed. Given that $\hat{s}_2 = \phi$, this condition can be written

as

$$re_1 w_2^{11} + (1 - re_1)\left(\lambda w_2^{11} + (1 - \lambda)\frac{w_2^{11} + w_2^{00}}{2}\right)$$
$$\geq re_1(\lambda w_2^{11} + (1 - \lambda)w_2^{01}) + (1 - re_1)\left(\lambda w_2^{11} + (1 - \lambda)\frac{w_2^{10} + w_2^{01}}{2}\right),$$

which is simplified to

$$(1 + re_1)(w_2^{11} - w_2^{01}) \geq (1 - re_1)(w_2^{10} - w_2^{00}). \tag{6}$$

The condition for truth telling for agent 1 is the same as before: the agent reports truthfully iff $w_1^{11} \geq w_1^{10}$.

Given that these conditions are satisfied, the expected payoff for agent 1 is obtained as

$$\pi_1(e_1, e_2) = re_1 w_1^{11} + re_2(1 - re_1)\frac{(1 + \lambda)w_1^{11} + (1 - \lambda)w_1^{01}}{2}$$
$$+ (1 - re_1)(1 - re_2)\frac{(1 + \lambda)w_1^{11} + (1 - \lambda)w_1^{00}}{2} - ce_1.$$

The agent exerts effort iff

$$w_1^{11} - re_2 w_1^{01} - (1 - re_2)w_1^{00} \geq \frac{2c}{r(1 - \lambda)}.$$

A candidate contract is

$$w_1^{11} = \frac{2c}{r(1 - \lambda)}, w_1^{10} = w_1^{01} = w_1^{00} = 0,$$

which also satisfies the condition for truth telling. Under this contract, the agent also has an incentive to coordinate.

Similarly, the expected payoff for agent 2 is obtained as

$$\pi_2(e_2, e_1) = re_1 w_2^{11} + re_2(1 - re_1)\frac{(1 + \lambda)w_2^{11} + (1 - \lambda)w_2^{10}}{2}$$
$$+ (1 - re_2)(1 - re_1)\frac{(1 + \lambda)w_1^{11} + (1 - \lambda)w_1^{00}}{2} - ce_2.$$

The agent exerts effort iff

$$(1 - re_1)(w_2^{10} - w_2^{00}) \geq \frac{2c}{r(1 - \lambda)}. \tag{7}$$

We have also assume that the agent has an incentive to differentiate, which means that

$$w_2^{10} - w_2^{00} > w_2^{11} - w_2^{01}. \tag{8}$$

The optimal contract within this class must satisfy (6), (10) and (8). Since $e_1 = 1$, all of these condition together imply

$$w_2^{10} - w_2^{00} > w_2^{11} - w_2^{01} \geq \frac{1-r}{1+r}(w_2^{10} - w_2^{00}) \geq \frac{2c}{r(1+r)(1-\lambda)}.$$

A candidate contract is thus

$$w_2^{11} = \frac{2c}{r(1+r)(1-\lambda)}, w_2^{10} = \frac{2c}{r(1-r)(1-\lambda)},\ w_1^{01} = w_1^{00} = 0.$$

Under this contract, the agent has no incentive to report truthfully, as assumed.

We now show that this contract cannot be optimal. The expected profit under this arrangement, denoted by Π^{P}, is

$$\Pi^{\mathrm{P}} = 2r + r(1-r) + \frac{(1-r)(2-r)(1+\lambda)}{2} - \left(r + \frac{(1-r)(1+\lambda)}{2}\right)\frac{2c}{r(1-r)(1-\lambda)}$$
$$- \left(r + \frac{(1-r)(1+\lambda)}{2}\right)\frac{2c}{r(1+r)(1-\lambda)} - \frac{r(1-r)(1-\lambda)}{2}\frac{2c}{r(1-r)(1-\lambda)}.$$

Let the expected profit under each arrangement be decomposed into the expected output Y^k and the expected wage cost W^k, such that $\Pi^k = Y^k - W^k$, $k = H, C, P$. Hierarchy outperforms Partial Delegation if $Y^{\mathrm{H}} - W^{\mathrm{H}} \geq Y^{\mathrm{P}} - W^{\mathrm{P}}$ while Committee outperforms Partial Delegation if $Y^{\mathrm{C}} - W^{\mathrm{C}} \geq Y^{\mathrm{P}} - W^{\mathrm{P}}$. Since

$$Y^{\mathrm{C}} - Y^{\mathrm{P}} = Y^{\mathrm{P}} - Y^{\mathrm{H}} = \frac{r(1-r)(1-\lambda)}{2},$$

Partial Delegation cannot be optimal if $2W^{\mathrm{P}} \geq W^{\mathrm{C}} + W^{\mathrm{H}}$, which can be written as

$$\frac{4(2r + (1-r)(1+\lambda))}{r(1-r)(1+r)(1-\lambda)} + 2 \geq \frac{4 - 2(1-\lambda)(1-r)^2}{r(1-r)(1-\lambda)} + \frac{1+\lambda+r(1-r)}{r(1-\lambda)}.$$

With some algebra obtain

$$1 \geq \frac{(1+r)(4 - 2(1-\lambda)(1-r)^2) + (1-r)(1+r)(1+\lambda) - 8r - 4(1-r)(1+\lambda)}{r(1-r)(1+r)(1-\lambda)}.$$

which is further simplified to

$$1 \geq \frac{(1+r)(3\lambda - 2r(1-\lambda) - 1) - 4\lambda}{r(1+r)(1-\lambda)} \quad \Leftrightarrow \quad (1+r)^2 \geq (4r + r^2 - 1)\lambda.$$

One can then easily verify that this condition holds for any $\lambda \in [0,1]$, which proves that Delegated Hierarchy cannot be optimal in this case.

Case 2-2. Only agent 1 has the incentive to differentiate: Again, in this case, fully separating communication cannot be implemented because there is no incentive to report truthfully when $\hat{s}_1 = \phi$. We thus need the same condition as above, i.e., it is necessary for agent 2 to follow agent 1's message when he is uninformed. Given that $\hat{s}_2 = \phi$, this condition can be written as

$$re_1 w_2^{11} + (1 - re_1)\left(\lambda w_2^{11} + (1-\lambda)\frac{w_2^{10} + w_2^{01}}{2}\right)$$
$$\geq re_1(\lambda w_2^{11} + (1-\lambda)w_2^{01}) + (1 - re_1)\left(\lambda w_2^{11} + (1-\lambda)\frac{w_2^{11} + w_2^{00}}{2}\right),$$

which is simplified to

$$(1 + re_1)(w_2^{11} - w_2^{01}) \geq (1 - re_1)(w_2^{00} - w_2^{10}). \tag{9}$$

The condition for truth telling for agent 1 is the same as before: the agent reports truthfully iff $w_1^{11} \geq w_1^{10}$.

Given that these conditions are satisfied, the expected payoff for agent 1 is obtained as

$$\pi_1(e_1, e_2) = re_1 w_1^{11} + re_2(1 - re_1)\frac{(1+\lambda)w_1^{11} + (1-\lambda)w_1^{01}}{2}$$
$$+ (1 - re_1)(1 - re_2)\frac{2\lambda w_1^{11} + (1-\lambda)(w_1^{10} + w_1^{01})}{2} - ce_1.$$

The agent exerts effort iff

$$re_2(w_1^{11} - w_1^{01}) + (1 - re_2)(2w_1^{11} - w_1^{10} - w_1^{01}) \geq \frac{2c}{r(1-\lambda)}.$$

Since $w_1^{10} + w_1^{01} \geq w_1^{11} + w_1^{00}$ (the incentive to differentiate), a necessary condition for this is

$$re_2(w_1^{11} - w_1^{01}) + (1 - re_2)(w_1^{11} - w_1^{00}) \geq \frac{2c}{r(1-\lambda)}.$$

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A candidate contract is

$$w_1^{11} = w_1^{10} = \frac{2c}{r(1-\lambda)}, \ w_1^{01} = w_1^{00} = 0,$$

which also satisfies the condition for truth telling.

Similarly, the expected payoff for agent 2 is obtained as

$$\pi_2(e_2, e_1) = re_1 w_2^{11} + re_2(1 - re_1)\frac{(1+\lambda)w_2^{11} + (1-\lambda)w_2^{10}}{2}$$
$$+ (1 - re_2)(1 - re_1)\frac{2\lambda w_1^{11} + (1-\lambda)(w_2^{10} + w_1^{01})}{2} - ce_2.$$

The agent exerts effort iff

$$(1 - re_1)(w_2^{11} - w_2^{01}) \geq \frac{2c}{r(1-\lambda)}. \tag{10}$$

Since the condition for truth telling must not hold for agent 2, a candidate contract is

$$w_2^{11} = w_2^{10} = \frac{2c}{r(1-r)(1-\lambda)}, \ w_1^{01} = w_1^{00} = 0.$$

These contracts are, however, clearly less profitable than the ones obtained in the previous case, implying that Delegated Hierarchy is not optimal in this case.

Case 2-3. Both agents have the incentive to coordinate: The condition for information acquisition is the same as in case 2-1. The condition for agent 1 is given by

$$w_1^{11} - re_2 w_1^{01} - (1 - re_2)w_1^{00} \geq \frac{2c}{r(1-\lambda)}.$$

while that for agent 2 is

$$(1 - re_1)(w_2^{10} - w_2^{00}) \geq \frac{2c}{r(1-\lambda)}.$$

Since full revelation is possible in this case, the incentive for agent 2 to follow agent 1's message is not necessary. Agent 2 must have the incentive to coordinate, however, and that requires

$$w_2^{11} + w_2^{00} \geq w_2^{10} + w_2^{01}.$$

Note that this restriction is more stringent than (6), so that the resulting profit must be less than that in case 2-1.

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Case 2-4. Both agents the incentive to differentiate: The condition for information acquisition is the same as in case 2-2. The condition for agent 1 is given by

$$re_2(w_1^{11} - w_1^{01}) + (1 - re_2)(2w_1^{11} - w_1^{10} - w_1^{01}) \geq \frac{2c}{r(1 - \lambda)}.$$

while that for agent 2 is

$$(1 - re_1)(w_2^{11} - w_2^{01}) \geq \frac{2c}{r(1 - \lambda)}.$$

Since full revelation is possible in this case, the incentive for agent 2 to follow agent 1's message is not necessary. Agent 1 must have the incentive to differentiate, however, and that requires

$$w_1^{10} + w_1^{01} \geq w_1^{11} + w_1^{00}.$$

Moreover, the condition for truth telling must not hold for agent 2. These additional requirements are the same as in case 2-2, leading to the same outcome.

Case 3

This situation is almost identical to Independent Production, except that communication is now feasible, which allows them to coordinate on the task choice while exchanging no information about the state of nature. Suppose first that each agent has the incentive to differentiate. We can then consider the following strategy for each agent.

- When $\hat{s}_i \neq \phi$, $m_i = L$ or $m_i = R$ with equal probability (a babbling message);

- When $\hat{s}_i = \phi$, $m_i = \phi$.

One can verify that neither agent has the incentive to deviate from this strategy. Under this strategy, the expected payoff, denoted by $\tilde{\pi}_i^I$, is

$$\tilde{\pi}_i^I(e_i, e_j) = r^2 e_i e_j w_i^{11} + re_i(1 - re_j)\left(\lambda w_i^{11} + \frac{(1 - \lambda)(w_i^{11} + w_i^{10})}{2}\right)$$
$$+ re_j(1 - re_i)\left(\lambda w_i^{11} + \frac{(1 - \lambda)(w_i^{11} + w_i^{01})}{2}\right)$$
$$+ (1 - re_i)(1 - re_j)\left(\lambda w_i^{11} + \frac{(1 - \lambda)(w_i^{10} + w_i^{01})}{2}\right) - ce_i.$$

The agent chooses to exert effort iff

$$\frac{w_i^{11} - w_i^{01}}{2} \geq \frac{c}{r(1-\lambda)}.$$

Since the agents must the incentive to differentiate, we must also have

$$w_i^{10} + w_i^{01} \geq w_i^{11} + w_i^{00}.$$

The optimal contract within this class is hence

$$w_i^{11} = w_i^{10} = \frac{2c}{r(1-\lambda)}, \ w_i^{01} = w_i^{00} = 0.$$

but this is clearly less efficient than Independent Production. To see this, note that the expected output under this contract is exactly the same as that under Independent Production because any task coordination when both agents are uninformed does not increase the expected output. Independent Production then dominates this if

$$\left(r^2 + r(1-r)(1+\lambda) + (1-r)^2\lambda + \frac{(1-r)(1-\lambda)}{2}\right)\frac{2c}{r(1-\lambda)} \geq \frac{(1+r)c}{r},$$

which can be shown to hold.

Now suppose that both of the agents have the incentive to coordinate. The same strategy considered above works, and under this strategy, the expected payoff is

$$\tilde{\pi}_i^I(e_i, e_j) = r^2 e_i e_j w_i^{11} + re_i(1 - re_j)\left(\lambda w_i^{11} + \frac{(1-\lambda)(w_i^{11} + w_i^{10})}{2}\right)$$
$$+ re_j(1 - re_i)\left(\lambda w_i^{11} + \frac{(1-\lambda)(w_i^{11} + w_i^{01})}{2}\right)$$
$$+ (1 - re_i)(1 - re_j)\left(\lambda w_i^{11} + \frac{(1-\lambda)(w_i^{11} + w_i^{00})}{2}\right) - ce_i.$$

The agent chooses to exert effort iff

$$re_j\frac{w_i^{11} - w_i^{01}}{2} + (1 - re_j)\frac{w_i^{10} - w_i^{00}}{2} \geq \frac{c}{r(1-\lambda)}.$$

Since the agents must the incentive to coordinate, we must also satisfy (1). The principal's problem is then defined as

$$\min_{W_i} \ \frac{2r^2 + (1-r^2)(1+\lambda)}{2}w_i^{11} + \frac{r(1-r)(1-\lambda)}{2}(w_i^{10} + w_i^{01})$$
$$+ \frac{(1-\lambda)(1-r)^2}{2}w_i^{00},$$

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subject to the incentive constraints. In the absence of (1), the optimal contract would have $w_i^{11} = 0$. The optimal contract within this class is thus given by

$$w_i^{11} = w_i^{10} = \frac{2c}{r(1-\lambda)}, \ w_i^{01} = w_i^{00} = 0.$$

which yields exactly the same profit as the one derived above (for the case where both agents have the incentive to differentiate).