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A model of urban demography

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Abstract

This paper develops an overlapping generations model that involves endogenous determination of fertility and explicit city structure. We provide conditions under which there exists a unique steady state, which can replicate spatial features of demography observed in Japanese cities. We also provide comparative steady state analysis by calibration.

JEL Classification: J11, R11, R23.

Keywords: city structure, land rent, fertility, demography

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1 Introduction

It has long been recognized that city structure has relationship with demography. We can easily find statements on this issue here and there: for example, the National Institute of Population and Social Security Research, Japan [7] surveyed the number of children that couples are going to have and the number of children that couples wish to have under ideal conditions. Its table 78 reports the reasons why couples are going to have fewer children than the ideal. The table shows that whereas 16.8 percent of couples who live in Densely Inhabited Districts (DIDs) choose the unaffordability of having a sufficiently spacious house as one of the reasons, this figure is 5.4 percent for those who live in non-DIDs. These figures indicate that the city structure, via high land rent, may have significant impacts on fertility in cities.

However, full-fredged analyses from a viewpoint of economics have been scarce until recently. An noticeable exception is Shultz [11], which provided empirical results that support the interdependence of city structure and fertility by showing that advances in urbanization reduces national fertility.

Very recently, we observe that several studies uncovered possible nonnegligible interactions between city structure and demography by using a framework of economics. As to empirical evidences, Galor [5] presented stylized facts that imply (i)that urbanization and economic development started simultaneously, and (ii)that in early phases of urbanization and economic development, population growth rate increased, but it declined in succeeding years. Simon and Tamura [12] showed the existence of a negative cross-sectional correlation between the price of living space as measured by rent per room and fertility for the Consolidated Metropolitan Statistical Areas in the United States over the period 1940–2000. The results described in these studies imply that city structure that includes land and housing markets can play a major role in determining demographic features in cities.

Theoretically, several studies used reduced form models to analyze the demographic impacts of urbanization (Zhang [14]; Sato [8]; Sato and Yamamoto [9]; and Sato and Yamamoto [10]).[1] Zhang [14] and Sato and Yamamoto [10] examined impacts of urbanization caused by better opportunities for earnings and education in cities on demography. Sato [8] and Sato and Yamamoto [9] investigated how urbanization and demographic transition interrelate with each other *via* merits of population concentration (agglomeration economies) and demerits of it (congestion diseconomies). All these studies showed that urbanization is accompanied by declines in fertility, which is consistent with the stylized facts described in Galor [5]. However, because the models developed in these studies have no explicit spatial structure, their analysis does not shed any light on spatial features of demography within cities.

In this paper, we contribute this literature by developing an overlapping generations model of endogenous fertility that involves explicit spatial structure. Especially, we focus on the relationship between the city size, the spatial patterns of fertility within a

[1] Eckstein et al [3] developed an overlapping generations growth model that involves land as a production input, and examined the impacts of the limitation of land availability on economic growth. However, their model does not deal with land consumption nor city structure.

city, and land consumption.

For this purpose, we fist provide an overview of basic stylized facts on demography of metropolitan areas in Japan. We use data on three largest Metropolitan Areas (MAs), which are Tokyo MA, Osaka MA, and Nagoya MA.[2] Tokyo, Osaka, and Nagoya are the first, second, and third largest cities in Japan. Figure 1 shows the city size in terms of population for the three largest MAs from 1950 to 2007.

[Insert Figure 1 around here]

These three cities have grown steadily, while keeping the order of size unaltered. Although they are different in size, they have a common spatial feature of fertility. In order to make this visible, Table 1 shows the total fertility rate for prefectures that consist of each MA during the past half century.

[Insert Table 1 around here]

We readily observe the following two facts: (i) that fertility is lower for a larger city, (ii) that fertility is lower for prefectures considered as the central cities than for those considered as suburbs. As is well known, the land rent/price is higher for a larger city and the center of a city has higher land rent/price than suburbs (see Table 2, for example).

[2] Each MA consists of several prefectures: Tokyo MA=Tokyo+Kanagawa+Saitama+Chiba, Osaka MA=Osaka+Kyoto+Hyogo+Nara, and Nagoya MA=Aichi+Gifu+Mie. This definition is often used in analysis by Ministry of Internal Affairs and Communications, Japan.

[Insert Table 2 around here]

Put differently, the level of land rent is negatively associated to fertility both across cities and within each city. As is also well known, the level of land consumption is positively associated to the level of land rent. Hence, this also implies that land consumption and fertility have positive correlation.

In order to analyze how city structure and demography can interact, we develop an overlapping generations model of endogenous fertility that involves the monocentric city structure à la Alonso [1]. In considering fertility decision, we adopt the view of Becker [2], which regards having children as consumption, not as investment. Each household obtains utility from numeraire consumption, land consumption, and the number of children. The key assumption here is the complementarity between land consumption and the number of children: one needs a certain amount of land in order to rear a child, and obtains utility from land consumption over the required level for child rearing. Population changes arise not only from changes in the number of children but also from migration into/out of the city. In such a model, the land rent is higher in the central part of the city, leading to lower land consumption and fewer number of children. Moreover, as the city grows, the land rent gets higher and land consumption and fertility decreases. These features are consistent with the stylized facts described above and in Simon and Tamura [12]. Comparative steady state analysis provides impacts of changes in city structure on demography in the city.

The remainder of the paper is organized as follows. Section 2 presents a basic model.

Section 3 provides comparative steady state analysis. Section 5 concludes and suggests future research directions.

2 Basic model

2.1 Individuals

Consider a linear space, on which there is one Central Business District (CBD), i.e., we assume a linear monocentric city. We approximate the CBD by a point and assume that all workers commute to the CBD. Without loss of generality, we index the location of the CBD as 0, and describe each location by the distance x between it and the location of the CBD. Time is discrete and each individual lives for two periods; a childhood and an parenthood. Each individual has a single parent. In the parenthood, each individual is endowed with one unit of time, which she spends on working and on child rearing. At the beginning of period t, she decides on goods and land consumption (c_t and d_t) and her number of children (n_t). She exits the economy at the end of period t. N_t individuals in the parenthood live in the city in period t. This implies that $n_t N_t$ children are born in period t and grow to be parents in period $t+1$. In this model, n_t represents the total fertility rate.

We assume that Individuals have an identical utility function of the Cobb-Douglas form and the utility of each individual depends on the level of goods and land consumption and the number of children:

$$U_t = \alpha \ln c_t + \beta \ln n_t \left(d_t - \varepsilon n_t\right),$$

where α, β and ε are positive constants and satisfy $\alpha + 2\beta = 1$. There is only one kind of goods in this economy, which we treat as a numeraire. In order to rear a child, one needs a certain amount of land and we represent it as ε. An individual obtains utility from land consumption over the required level for child rearing. This complementarity between land consumption and the number of children brings forth the demographic characteristics in a city.

In order to have n_t children, each individual must spend bn_t units of time, where b is a positive constant. This assumption requires that n_t must satisfy $0 \leq n_t \leq 1/b$. Because each working individual is endowed with one unit of time, she spends $1 - bn_t$ units of time for working. The budget constraint for a working individual who resides at x distant from the CBD is given by

$$(1 - bn_t)I - \tau x = c_t + r_t(x)d_t,$$

where I denotes the wage income per unit of time and is a positive constant. τ represents the commuting cost per unit distance and $r_t(x)$ is the market land rent at x distant from the CBD. We assume that land is owned by absentee landlords.

The utility maximization gives the following demand functions:

$$\begin{aligned} c_t(x) &= \alpha (I - \tau x), \\ n_t(x) &= \frac{\beta}{bI + \varepsilon r_t(x)} (I - \tau x), \\ d_t(x) &= \beta \left[\frac{1}{r_t(x)} + \frac{\varepsilon}{bI + \varepsilon r_t(x)} \right] (I - \tau x). \end{aligned} \tag{1}$$

This leads to the indirect utility as follows:

$$V_t(x) = A + \ln (I - \tau x) - \beta \ln r_t (x) [bI + \varepsilon r_t (x)]. \tag{2}$$

where A is defined as $A \equiv \alpha \ln \alpha + 2\beta \ln \beta$.

We can see from (1) that a rise in wage income has two effects on the total fertility rate n_t. One is the positive income effect that is represented in the numerator of the right hand side. The other is the negative substitution effect that raises the opportunity cost of rearing children. This is described by the denominator of the right hand side. These relationships are relevant in the later section. More importantly, a higher land rent leads to a smaller number of children and to smaller land consumption. This is because when $r_t(x)$ is high, land requirement for child rearing enforces individuals to have less children.

2.2 City structure and location equilibrium

We assume that children live with their parents and a working individual can move freely within a city. The location equilibrium is attained in each period and it requires that the indirect utility is the same for all locations in a city:

$$V_t(x) = \overline{V}_t, \qquad \forall x \in (0, \overline{x}_t], \tag{3}$$

where $\overline{x}_t$ denotes the city fringe in period t. $\overline{x}_t$ represents the spatial size of the city.

We follow the well-established tradition of urban economics in determining the land rent in a city by using the concept of the "bid rent."[3] The bid rent is the maximum land rent at location x that each individual is willing to pay in order to reach her equilibrium utility level. We normalize the land rent outside of the city to one. This implies that the land rent at the city fringe $r_t(\overline{x}_t)$ is equal to 1. From (2) and (3), we obtain the bid rent at x by solving $A + \ln(I - \tau x) - \beta \ln R(bI + \varepsilon R) = V_t(\overline{x}_t)$ with respect to R, which yields

$$R = \frac{1}{2\varepsilon}\left[-bI + \sqrt{(bI)^2 + 4\varepsilon(bI + \varepsilon)\left(\frac{I - \tau x}{I - \tau \overline{x}_t}\right)^{1/\beta}}\right].$$

The market land rent is then given by

$$\begin{aligned} r_t(x) &= \max[R, 1] \\ &= \begin{cases} R & \text{if} \quad x \in (0, \overline{x}_t] \\ 1 & \text{if} \quad x > \overline{x}_t \end{cases}. \end{aligned} \tag{4}$$

We readily obtain $r_t'(x) < 0$ and $\partial r_t / \partial \overline{x}_t > 0$ for $x \in (0, \overline{x}_t]$, which we summarize in the following lemma.

Lemma 1 *Within the city, the land rent is lower for a location more distant from the CBD ($r_t'(x) < 0$). As the city becomes larger, the land rent in the city rises ($\partial r_t / \partial \overline{x}_t > 0$).*

[3] The usage of the bid rent is very standard in urban economics. See Kanemoto [6] and Fujita [4] for a comprehensive discussion on the bid rent in monocentric city models.

These are very standard results in the literature of monocentric models (Fujita [4]). From (4), we obtain the level of utility in the city as

$$\overline{V}_t = A + \ln\left(I - \tau \overline{x}_t\right) - \beta \ln\left(bI + \varepsilon\right). \tag{5}$$

Therefore, for a given number N_t of individuals in the parenthood, we can determine all other endogenous variables once the city fringe $\overline{x}_t$ is determined. The city fringe $\overline{x}_t$ is given by the land market clearing condition for a given N_t:

$$\int_0^{\overline{x}_t} \frac{D}{d_t(x)} dx = N_t. \tag{6}$$

D represents the land supply for each location that is exogenous in our model. Letting $\overline{x}_t(N_t)$ denote $\overline{x}_t$ that is determined by (6), we can examine $\overline{x}_t'(N_t)$, that is, how the city population affects the city fringe:

Lemma 2 *An increase in the number of individuals in the city enlarges the city area ($\overline{x}_t'(N_t) > 0$).*

Proof. See Appendix A. ■

From Appendix A, we also know that $\overline{x}_t(0) = 0$ and $\lim_{N_t \to \infty} \overline{x}_t(N_t) = I/\tau$: there is no city area if no one is in the city, and the city fringe can at most reach I/τ if city population explodes.

Combined with Lemmas 1 and 2, (1) yields the following proposition:

Proposition 1 *An individual residing more distant from the CBD consumes more land ($d_t'(x) > 0$). If land requirement for child rearing is sufficiently large ($\varepsilon \geq \beta bI/\alpha$), she has more children ($n_t'(x) > 0$).*

Proof. See Appendix B. ■

A location of an individual has two effects on land consumption and on the number of children. On the one hand, when an individual lives more distant from the CBD, she must bear higher commuting costs, which reduces the income net of commuting costs. This has an effect of reducing land consumption and the number of children. On the other hand, the more distant from the CBD the location is, the lower the land rent is. A lower land rent enables an individual to consume larger land. This also induces her to have more children because a lower land rent implies lower payments for land requirement for child rearing. With respect to the land consumption, the latter effect dominates the former, and one who lives farther away from the CBD always consumes more land. This also holds true regarding the fertility rate if the land requirement for child rearing is sufficiently large.

Equation (1) shows that $\partial d_t/\partial r_t < 0$ and $\partial n_t/\partial r_t < 0$, which, combined with Lemmas 1 and 2, prove the following proposition.

Proposition 2 *The land rent is higher in a larger city, where an individual consumes less land and has less children (*$\partial r_t/\partial N_t > 0$*,* $\partial d_t/\partial N_t < 0$ *and* $\partial n_t/\partial N_t < 0$*).*

These relationships are consistent with the stylized facts presented in Introduction and in Simon and Tamura [12].

2.3 Population dynamics

We assume that there is migration into or out of the city depending on the utility difference between inside and outside of the city. This migration happens just before each period starts, and $M\left(\overline{V}_t - \overline{v}\right)$ individuals who are ready to become adults flows into the city just before period $t+1$ starts, where $\overline{v}$ (>0) is the utility level of people outside of the city and M represents the adjustment speed of migration.[4] We assume that $\overline{v}$ and M are positive constants. This migration function represents that the city attracts people if people there enjoyed higher utility than people outside of the city in the previous period, and the city loses people otherwise. We further assume that $\overline{V}_t\big|_{\overline{x}_t=0} > \overline{v}$. This implies that the land rent becomes sufficiently low for people to flow into the city if there are few people in the city.[5][6]

In period t, an individual residing at x has $n_t(x)$ children, who are grown up to be parents in period $t+1$. Therefore, the law of motion of population is given by

$$N_{t+1} = M\left(\overline{V}_t - \overline{v}\right) + \int_0^{\overline{x}_t} \frac{Dn_t(x)}{d_t(x)}dx. \tag{7}$$

The first term represents the flow into/out of the city and the second term is the total number of children in the previous period.

The following proposition establishes the existence of the steady state equilibrium

[4] For the discussion on the stability of spatial equilibrium under this type of migration function, see Tabuchi and Zeng [13].

[5] Remind that $\overline{x}_t(0) = 0$.

[6] These assumptions are equivalent to $A + \ln(I) - \beta \ln(bI + \varepsilon) > \overline{v}$, which is satisfied when the income I is sufficiently high.

and the sufficient condition of its uniqueness and stability.

Proposition 3 *There exists a steady state equilibrium of the model. It is unique and stable when* b *and* β *are sufficiently small.*

Proof. See Appendix C. ■

3 Numerical analysis

In this section, we provide comparative steady state analysis by calibration. In so doing, we fix parameter values so that the model can replicate demographic characteristics of Tokyo Metropolitan Area in Japan for the past half century. As shown in Introduction, population of Tokyo MA have grown steadily during the past half century. Because our model does not have growth factor, it would be natural to eliminate trend components from the population data when fitting our model to the data.

Although Tokyo MA have grown steadily during the past half century, its population size has shown slight fluctuations, which is confirmed by decomposition into trend components and cyclical components. We use Hodrick-Prescott filter for the decomposition.[7]

[Insert Figure 2 around here]

Figure 2 shows the trend and cyclical components of ln(population). By eliminating

[7] We set the multiplier $\lambda = 400$, which is often used for annual data.

the trend components, we can observe that city size slightly fluctuates for Tokyo MA. We use the cyclical components of ln(population) of Tokyo MA.

α is set to be 0.62, which comes from the share of consumption expenditure net of transportation and education costs in the disposable income for the year 2007 (Annual Report on the Family Income and Expenditure Survey, Ministry of Internal Affairs and Communications, Japan). This implies that $\beta = 0.19$. We set the level of nominal income I to be 3693.1, which is made as follows. First, we obtain the per household income of Tokyo MA from 1955 to 2005 for every 5 years by using Gross Prefectural Domestic Income (Prefectural Accounts, Cabinet Office, Japan) and the number of households (Population Census, Ministry of Internal Affairs and Communications, Japan). I is then calculated as the geometric mean of the series of the per household income.[8] Other parameters are set so that they can satisfy the following conditions: (i) $\epsilon \geq \beta b I/\alpha$, under which ${n_t}'(x) > 0$ for all $x < \overline{x}_t$, (ii) $-1 < \Psi < 0 < \Phi < 1$ (see Appendix C for the definitions of Ψ and Φ), which ensure the existence and stability of the steady state, (iii) the average share of commuting expenditure in income ($= \tau x/I$) does not exceed 0.1, which comes from the fact that the share of transport related expenditure in the disposable income is around 0.06 (Annual Report on the Family Income and Expenditure Survey, Ministry of Internal Affairs and Communications, Japan), (iv) we assume that each generation takes ten years and the series of simulated population for six generations can replicate the observed variance of the cyclical components of ln(population)

[8]justification to be written

of Tokyo Metropolitan Area from 1950 to 2007.[9]

We normalize the land supply D of each location to be one. The time and land requirements b and ε for child rearing are set to equal to 0.19 and 750, respectively. These values ensure (i) and (ii) to hold true. (iii) requires the per distant commuting cost τ to be 0.05. In order for (iv) to hold true, we set the utility level $\overline{v}$ of the outside of the city and the adjustment speed M of migration to 5.75 and 69.405, respectively.

The following figure shows the simulated fluctuations in ln(population) around the steady state value of it for six generations. The initial city population N_0 is 7.4175, which is ln(population) of Tokyo MA for the year 1950.[10]

[Insert Figure 3 around here]

We see from Figure 5 that the economy almost converges to the steady state at $t = 6$. Moreover, from generation 0 to 6, the city population fluctuates around the steady state, and the degree of fluctuation is decreasing. These features are consistent with the cyclical components of ln(population) of Tokyo MA.

Some comparative steady states are provided below. Table 3 summarizes the results.

[Insert Table 3 around here]

An increase in the share α of numeraire consumption expenditure decreases the fertility rate n_t. Still, the city grows both in terms of population N_t and space $\overline{x}_t$

[9] The variance of the cyclical components is 1.6×10^{-5}.

[10] The simulated steady state value of $\ln(N_t)$ is 1.993.

because of increases in the migration into the city. Accordingly, the land rent r_t rises and land consumption d_t decreases.

An improvement in the city transportation system (a decline in τ) increases the migration into the city and enlarges the city size (N_t and $\overline{x}_t$). In the neighborhood of the CBD, r_t declines, and n_t and d_t increase although the opposite holds true in the suburbs.

A reduction in the land requirement ε for child rearing increases d_t but decreases n_t and the migration into the city, leading to a smaller city size and lower r_t.

An improvement in the environment of child rearing (a decline in b) increases n_t and migration into the city, which enlarges the city size and higher r_t.

Although a higher income I reduces n_t, a better job opportunity induces more people to move into the city. The latter effect dominates the former, leading to a larger city size. This raises r_t to reduce d_t.

An increase in the land supply D for each location increases both n_t and migration into the city, which leads to a larger city population. However, a larger land supply enable more people to reside at each location, and the city fringe $\overline{x}_t$ declines, implying decreases in the city size in terms of space. In accordance with this, r_t decreases and d_t increases.

A decrease in the utility level v outside of the city and an increase in the adjustment speed M of migration have qualitatively the same effects. They decrease n_t but increase the migration into the city. The latter dominates the former, leading to a larger city size, a higher r_t, and lower d_t.

4 Concluding remarks

This paper provided a model with which we can analyze the interaction between city structure and demographic factors. In the developed model, we suppose the monocentric city structure and the complementarity between land consumption and having children. We showed conditions that ensure the existence and uniqueness of a steady state, and characterized the steady state. We further provided comparative steady state analysis by calibration.

It's worth pointing out an important possible direction of future research. Since our model does not have any growth factor, it cannot replicate the steady growth of metropolitan areas in Japan. Including growth factors such as human capital accumulation or technological progress would enables us to uncover possible impacts of city structure on demography in such a growth path.

References

[1] Alonso, W., 1964, *Location and Land Use*, Cambridge, MA, Harvard University Press.

[2] Becker, G. S., 1965, A Theory of the allocation of time, *Economic Journal* 75, 493-517.

[3] Eckstein, Z., S. Stern and K. I. Wolpin, 1988, Fertility choice, land, and the Malthusian hypothesis, *International Economic Review* 29, 353-361.

[4] Fujita, M., 1989, *Urban economic theory: Land use and city size,* Cambridge University Press, Cambridge.

[5] Galor, O., 2005, From Stagnation to Growth: Unified Growth Theory, in the Handbook of Economic Growth (P. Aghion and S. Durlauf eds.).

[6] Kanemoto, Y., 1980, *Theories of urban externalities*, North-Holland, Amsterdam.

[7] The National Institute of Population and Social Security Research, Japan, 11th National Fertility Survey (Dai 11-kai Syussei Doukou Kihon Chosa), 2003.

[8] Sato, Y., 2007, Economic geography, fertility, and migration, *Journal of Urban Economics* 61, 372-387.

[9] Sato, Y. and K. Yamamoto, 2005, Population concentration, urbanization, and demographic transition, *Journal of Urban Economics* 58, 45-61.

[10] Sato, Y. and K. Yamamoto, 2005, Urbanization and poverty trap, DEE Discussion Paper No.05-2, Graduate School of Environmental Studies, Nagoya University.

[11] Schultz, T. P., 1985, Changing world prices, women's wages, and the fertility transition: Sweden, 1860-1910, *Journal of Political Economy* 93, 1126-1154.

[12] Simon, C. J. and R. Tamura, 2009, Do higher rents discourage fertility? Evidence from U.S. cities, 1940-2000, *Regional Science and Urban Economics* 39, 33-42.

[13] Tabuchi, T. and D.-Z. Zeng, 2004, Stability of spatial equilibrium, Journal of Regional Science 44, 641–660.

[14] Zhang, J., 2002, Urbanization, population transition, and growth, *Oxford Economic Papers* 54, 91-117.

Appendix A: Proof of Lemma 2.

Let $\Omega(\overline{x}_t)$ denote the left hand side of (6). $\Omega(\overline{x}_t)$ has the following properties:

$$\begin{aligned} \Omega(0) &= 0, \\ \lim_{\overline{x}_t \to I/\tau} \Omega(\overline{x}_t) &= \infty, \\ \Omega'(\overline{x}_t) &= \frac{D}{d_t(\overline{x}_t)} + \int_0^{\overline{x}_t} \frac{D}{d_t(x)^2} \left(-\frac{\partial d_t}{\partial r_t} \right) \frac{\partial r_t}{\partial \overline{x}_t} dx > 0. \end{aligned}$$

Hence, (6) determines $\overline{x}_t$ once N_t is given as described in the following figure.

[Figure A1 around here]

From this figure, we readily know that

$$\begin{aligned} \overline{x}_t'(N_t) &> 0, \\ \overline{x}_t(0) &= 0, \\ \lim_{N_t \to \infty} \overline{x}_t(N_t) &= \frac{I}{\tau}. \end{aligned}$$

Appendix B: Proof of Proposition 1.

Substituting (4) into (1) and differentiating them with respect to x, we have that for $x \in (0, \overline{x}_t]$,

$$\begin{aligned} d_t'(x) &= \frac{\tau (bI)^2 (1-\beta) + 2\alpha\varepsilon\tau (bI+\varepsilon) \left(\frac{I-\tau x}{I-\tau \overline{x}_t}\right)^{1/\beta}}{(bI+\varepsilon) \left(\frac{I-\tau x}{I-\tau \overline{x}_t}\right)^{1/\beta} [bI + 2\varepsilon r_t(x)]} > 0, \\ n_t'(x) &= \frac{\varepsilon\tau (1-2\beta) r_t(x) - \beta\tau bI}{[bI + \varepsilon r_t(x)] [bI + 2\varepsilon r_t(x)]}. \end{aligned}$$

The latter equation yields

$$n_t'(x) > 0 \qquad \Leftrightarrow \qquad r_t(x) > \frac{\beta\tau bI}{\varepsilon\tau (1-2\beta)}.$$

From the fact that $r_t(x) \geq 1$, this leads to

$$n_t'(x) > 0 \qquad \Leftarrow \qquad 1 > \frac{\beta\tau bI}{\varepsilon\tau (1-2\beta)} \qquad \Leftrightarrow \qquad \varepsilon > \frac{\beta bI}{\alpha}.$$

Appendix C: Proof of Proposition 3.

Note here that we can determine all the other variables in period t once we fix N_t and that the law of motion of population (7) determines N_t for a given N_{t-1}. Therefore, we have a steady state equilibrium if there exists a steady state value of N_t. It is, if any, given by the intersection of $N_{t+1} = N_t$ and (7).

(7) is rewritten as

$$\begin{aligned} N_{t+1} &= \Lambda(N_t), \\ \Lambda(N_t) &\equiv M \left(\overline{V}_t - \overline{v}\right) + \int_0^{\overline{x}_t} \frac{D r_t(x)}{bI + 2\varepsilon r_t(x)} dx. \end{aligned}$$

From (5), (4) and the results that $\overline{x}_t(0) = 0$ and $\lim_{N_t\to\infty} \overline{x}_t(N_t) = I/\tau$, we have

$$\begin{aligned}\Lambda(0) &= M\left(\overline{V}_t\big|_{\overline{x}_t=0} - \overline{v}\right) > 0,\\ \lim_{N_t\to\infty} \Lambda(N_t) &= -\infty.\end{aligned}$$

These establish the existence of at least one intersection of $N_{t+1} = N_t$ and (7).

Moreover, the uniqueness and stability is ensured if $-1 < \Lambda'(N_t) < 1$. We readily obtain

$$\begin{aligned}\Lambda'(N_t) &= \Delta \overline{x}_t'(N_t),\\ \Delta &\equiv \frac{D}{bI + 2\varepsilon} - \frac{\tau M}{I - \tau\overline{x}_t} + \int_0^{\overline{x}_t} \Omega dx,\\ \Omega &\equiv \frac{DbI}{\left[bI + 2\varepsilon r_t(x)\right]^2} \frac{\partial r_t(x)}{\partial \overline{x}_t}.\end{aligned}$$

Note that

$$\begin{aligned}0 &\leq \Omega \qquad\qquad (A1)\\ &= \frac{\tau bID\left(bI+\varepsilon\right)\frac{(I-\tau x)^{1/\beta}}{(I-\tau\overline{x}_t)^{1+1/\beta}}}{\beta\left[(bI)^2 + 4\varepsilon\left(bI+\varepsilon\right)\left(\frac{I-\tau x}{I-\tau\overline{x}_t}\right)^{1/\beta}\right]^{3/2}}\\ &< \frac{\tau bID\left(I-\tau\overline{x}_t\right)^{1/(2\beta)-1}}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}\left(I-\tau x\right)^{1/(2\beta)}}\\ &\leq \frac{\tau bID}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}\left(I-\tau\overline{x}_t\right)}.\end{aligned}$$

Note next that (6) yields

$$\begin{aligned}0 &< \overline{x}_t'(N_t) \qquad\qquad (A2)\\ &= \left\{\frac{D\left(bI+\varepsilon\right)}{\beta\left(I-\tau\overline{x}_t\right)\left(bI+2\varepsilon\right)} + \int_0^{\overline{x}_t} \frac{2\varepsilon^2 D\left[r_t(x)\right]^2 + bID\left[bI+2\varepsilon r_t(x)\right]}{\beta\left(I-\tau x\right)\left[bI+2\varepsilon r_t(x)\right]^2}\frac{\partial r_t(x)}{\partial \overline{x}_t}dx\right\}^{-1}\\ &< \frac{\beta\left(I-\tau\overline{x}_t\right)\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)}.\end{aligned}$$

Consider the case in which $\Delta \geq 0$. From Lemma 2, we have

$$\begin{aligned}
0 &\leq \Lambda'(N_t) \\
&< \left[-\frac{\tau M}{I-\tau\overline{x}_t}+\frac{D}{bI+2\varepsilon}+\int_0^{\overline{x}_t}\frac{\tau bID}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}\left(I-\tau\overline{x}_t\right)}dx\right]\overline{x}_t'(N_t) \\
&< \left[-\frac{\tau M}{I-\tau\overline{x}_t}+\frac{D}{bI+2\varepsilon}+\frac{\tau bID\overline{x}_t}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}\left(I-\tau\overline{x}_t\right)}\right]\frac{\beta\left(I-\tau\overline{x}_t\right)\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)} \\
&= \frac{\beta}{bI+\varepsilon}\left\{I+\left(bI+2\varepsilon\right)\left[\frac{bI^2}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}}-\frac{\tau M}{D}\right]\right\}.
\end{aligned}$$

Consider next the case in which $\Delta < 0$. In this case, we obtain

$$\begin{aligned}
0 &> \Lambda'(N_t) \\
&> \left(\frac{D}{bI+2\varepsilon}-\frac{\tau M}{I-\tau\overline{x}_t}\right)\frac{\beta\left(I-\tau\overline{x}_t\right)\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)} \\
&= \frac{\beta}{bI+\varepsilon}\left[I-\tau\overline{x}_t-\frac{\tau M\left(bI+2\varepsilon\right)}{D}\right] \\
&\geq -\frac{\beta\tau M\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)}.
\end{aligned}$$

Let Φ and Ψ denote

$$\begin{aligned}
\Phi &\equiv \frac{\beta}{bI+\varepsilon}\left\{I+\left(bI+2\varepsilon\right)\left[\frac{bI^2}{\beta\left(4\varepsilon\right)^{3/2}\left(bI+\varepsilon\right)^{1/2}}-\frac{\tau M}{D}\right]\right\}, \\
\Psi &\equiv -\frac{\beta\tau M\left(bI+2\varepsilon\right)}{D\left(bI+\varepsilon\right)}.
\end{aligned}$$

We have $-1 < \Lambda'(N_t) < 1$ if $-1 < \Psi < 0 < \Phi < 1$. From the facts that $\lim_{(b,\beta)\to(0,0)}\Phi = \lim_{(b,\beta)\to(0,0)}\Psi = 0$, we have Proposition 3.

Tokyo MA				
	Central city	Suburbs		
year	Tokyo	Saitama	Chiba	Kanagawa
1930	3.51	5.33	5.05	4.34
1950	2.73	3.92	3.59	3.25
1960	1.7	2.16	2.13	1.89
1970	1.96	2.35	2.28	2.23
1980	1.44	1.73	1.74	1.7
1990	1.23	1.5	1.47	1.45
2000	1.07	1.3	1.3	1.28

Osaka MA				
	Central city	Suburbs		
year	Osaka	Kyoto	Hyogo	Nara
1930	3.21	3.59	3.94	4.39
1950	2.87	2.8	3.08	3.08
1960	1.81	1.72	1.9	1.87
1970	2.17	2.02	2.12	2.08
1980	1.67	1.67	1.76	1.7
1990	1.46	1.48	1.53	1.49
2000	1.31	1.28	1.38	1.3

Nagoya MA				
	Central city	Suburbs		
year	Aichi	Mie	Gifu	
1930	4.6	5.01	5.47	
1950	3.27	3.33	3.55	
1960	1.9	1.95	2.04	
1970	2.19	2.04	2.12	
1980	1.81	1.82	1.8	
1990	1.57	1.61	1.57	
2000	1.44	1.48	1.47	

Table 1: Total fertility rate in three largest metropolitan areas in Japan.

	Central city	Suburbs		
Tokyo MA	Tokyo	Saitama	Chiba	Kanagawa
	1081.7	525.3	417.3	796.9
Osaka MA	Osaka	Kyoto	Hyogo	Nara
	749.4	538.2	487.5	422.4
Nagoya MA	Aichi	Mie	Gifu	
	427.4	227	246.6	

Table 2: Value of land for housing per 3.3 ㎡ (in thousand yen) in three largest metropolitan areas in Japan for the year 2000.

parameters	changes in endogenous variables						
	N	Net migration	$\bar{x}$	c	d	n	r
α	+	+	+	+	−	−	+
ε	−	−	−	0	+	−	−
τ	−	−	−	−	C: − S: +	C: − S: +	C: + S: −
b	−	−	−	0	C: − S: +	−	−
D	+	+	−	0	+	+	−
I	+	+	+	+	−	−	+
M	+	+	+	0	−	−	+
v	−	−	−	0	+	+	−

Table 3: Comparative steady states.

+ and−describe that the variable increases and that it decreases, respectively.

C and S denotes the central city and the suburbs, respectively.

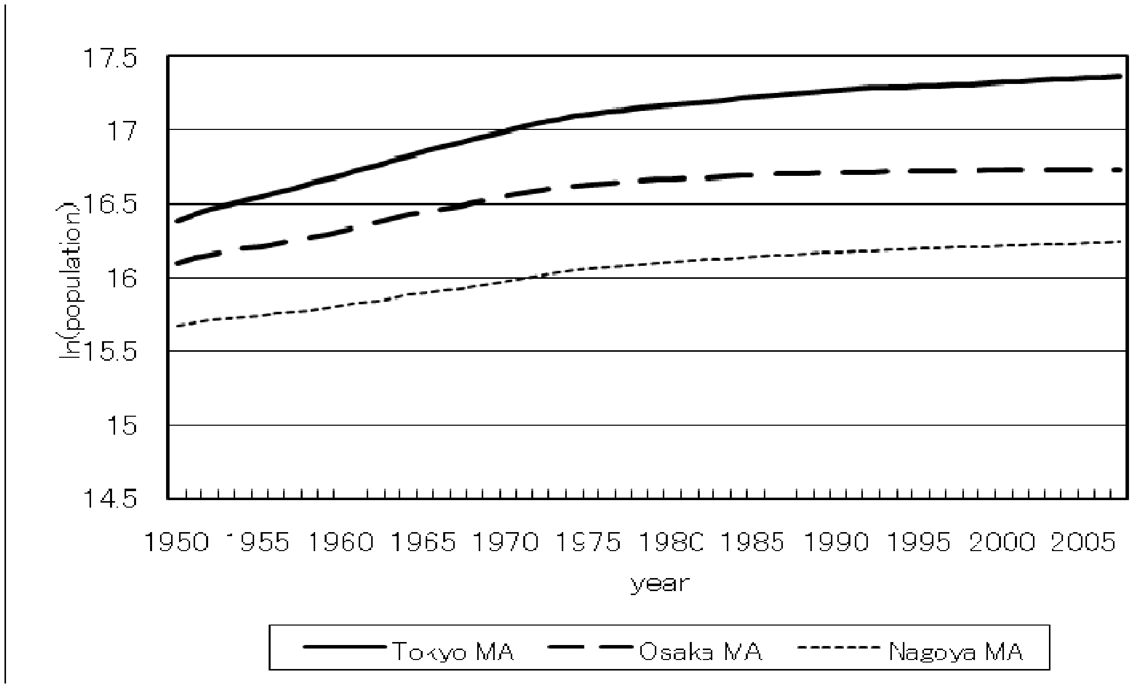

Figure 1: Population of three largest metropolitan areas in Japan.

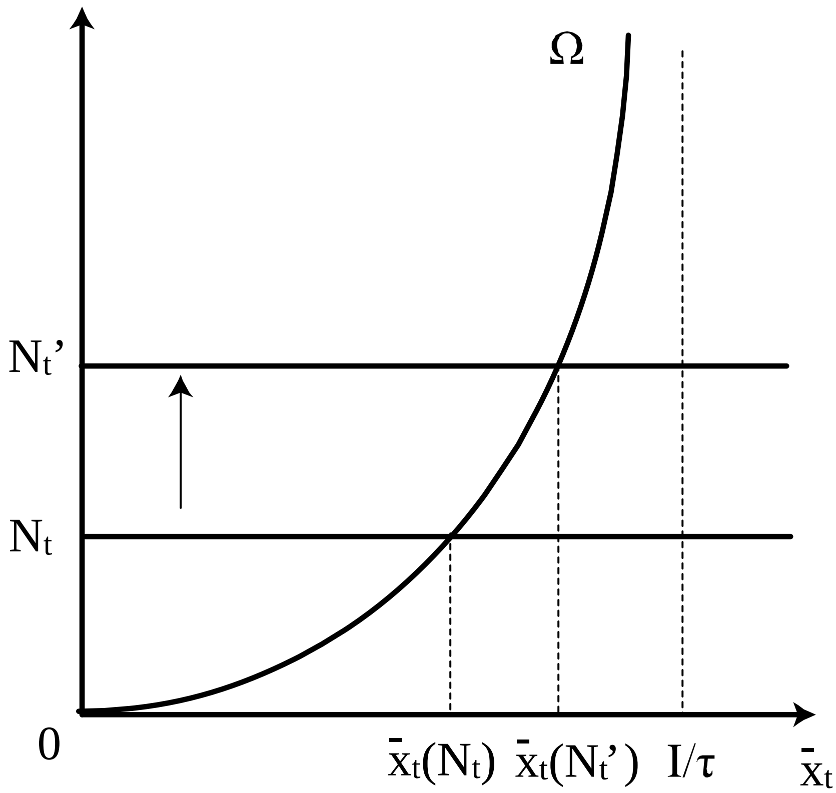

Figure 2: Determination of the city fringe $\overline{x}_t$ for a given number of individuals N_t.

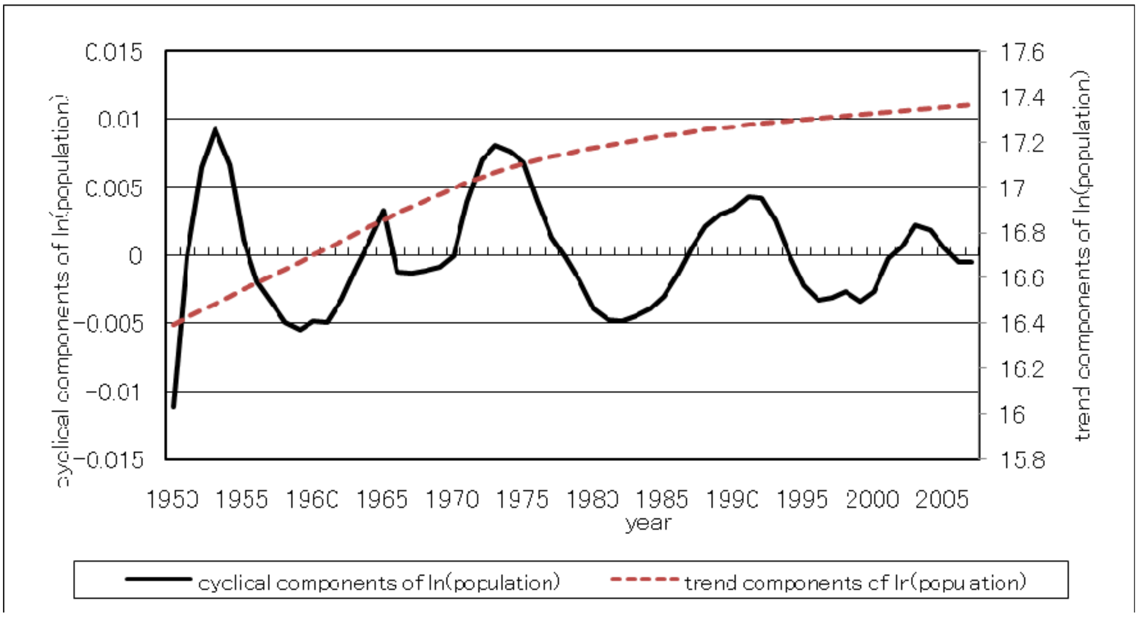

Figure 3: Trend and cyclical components of ln(population) in Tokyo MA in Japan.

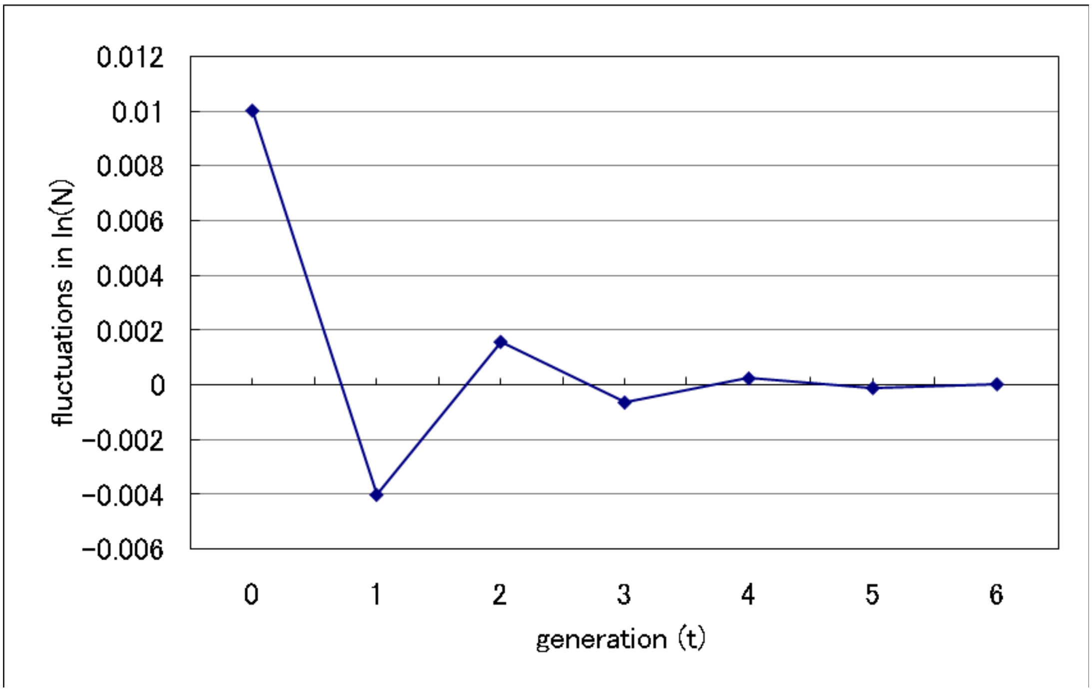

Figure 4: Simulated population.