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abstract

The demand for goods like seasonal fashion apparel is uncertain but the lead time needed for production is long, and so it is necessary to set the production quantity before the demand is fully known. Once sale begins, if demand is less than anticipated, the price will be low. In a futile attempt to avoid losses themselves, a competitive retail industry selling such merchandise will order too little, which will diminish the producer profit. A returns system is one response but it has problems also. Under a returns system in which retailers are fully reimbursed by the producer for any unsold merchandise, retailers will set their order quantities at the highest level allowed, which is also sub-optimal. So what to do? A slightly more sophisticated returns system is the answer. We show that a returns system with rebates implements the optimal production and sales strategy, attaining maximum expected profit in the channel.

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1 Introduction

In Japan, the returns system is common for various items including apparel, books, and magazines. Under this system, retailers may return any unsold merchandise to the producers and be fully reimbursed. The production quantities of soon out-of-date magazines, seasonal fashion apparel and the like have to be determined before the actual state of demand is fully known. To avoid losses from selling at a low price, competitive independent retailers of such merchandise will order small quantities. Then, if the producer matches its output to the retail industry order quantity, it will not in general attain maximum expected profit. In contrast, under a returns system, rather than sell to demanders at a low price, retailers can return the merchandise to the producer, for full reimbursement. In this way, even when demand is low, retailers need not sell at a loss-resulting low price, and so knowing this, they will order the maximum amount they believe they can sell, under the most optimistic demand forecast. Therefore, under a simple returns system the manufacturer rather than the retailers must determine the production quantity to maximize its own expected profit.

This sort of returns system is the subject of previous research. Flath and Nariu (1989) considered a case with zero marginal cost and linear demand with uncertain slope, and showed that the producers could attain maximum expected profit by implementing a returns system.[1] Nariu (1996) showed that the same result (maximum possible expected profit) attains under general demand and cost functions, if production and price are both set before the realization of demand. In both papers, the returns price (buy-back price) is the same as the shipping price and the retail price does not vary with the state of demand. This amounts to the same thing as

1Other research on returns systems but using different assumptions include Pellegrini (1986), Marvel and Peck (1992), and Flath and Nariu (2000). Also, Marvel and Wang (2007) consider the economic significance of returns systems for inventory adjustment.

manufacturer-stipulated minimum retail prices, known as resale price maintenance.[2] This sort of fixed price system is actually used in Japan for books and magazines, but not for apparel. For apparel, if the demand is low, price discounting is common. It is also frequently the case that the buy-back price for unsold apparel is rather less than the original shipping price. And, too, manufacturers often pay rebates to retailers. For instance publishers of books and magazines in Japan pay rebates to retailers under a system known as "nyuukin houshou sei" –reward deposit system.

This paper analyzes a case like that of apparel in which the retail price can be adjusted after the realization of demand, and in which rebates are used together with a returns system. In this case, the optimal retail price is always attained, given the actual state of demand. An important result is that for general demand and cost functions, a producer who implements this system attains maximum possible expected profit.

2 Model

Let us suppose that a risk-neutral producer faces the uncertain market demand

$$D(p,x), \qquad D_p < 0,\ D_x > 0,\ D(0,x) = \infty \qquad , \tag{1}$$

where D is quantity demanded, p is the retail price, and x is a random variable of known distribution that parameterizes the demand uncertainty. Denote the distribution function of x as $F(x)$. Producers of goods such as apparel with long production lead times have to produce before the true state of demand is known. The actual ultimate sales quantity S is thus constrained by the production quantity Q, and depends on the state of demand x and retail price p as represented here:

2Research concerning resale price maintenance has a long history. Refer for example to Telser (1960), Gould and Preston (1965), Matheson and Winter (1983, 1984) and Deneckere, Marvel and Peck (1997).

$$S(p,x,Q) = \min\{D(p,x), Q\} \quad . \tag{2}$$

The cost of producing is given by the function $C(Q)$, which we assume to be concave $C'(Q) > 0$ 、 $C''(Q) > 0$. We further assume that the producer and retailers are risk neutral.

In this section, we examine the optimal production and sales of a producer that is vertically integrated with the retail industry. The firm must produce before knowing the true state of demand x. Later, after learning the true state x, the firm sets the retail price $p(x)$.

Optimal Production and Sales Strategy（Optimal Solution）

Given the previously produced quantity Q and actual state of demand x, the optimization problem of the producer is to maximize revenue R, subject to constraint:

$$\max_p R(p,x,Q) = p(x)D(p,x), \quad s.t. \quad D(p,x) \le Q \, . \tag{3}$$

Here, let us define the price that maximizes revenue in state x, unconstrained by requirement that sales quantity not exceed production quantity, as $\hat{p}(x)$, and define the highest price that results in the entire production quantity being sold in state x as $\hat{p}(x,Q)$, or

Definition 1: $\hat{p}(x) \equiv \arg\max_p p(x)D(p,x)$

Definition 2: $\hat{p}(x,Q) \equiv \{p \mid D(p,x) = Q\}$

The latter is the revenue maximizing price only when the constraint that sales quantity cannot exceed the production quantity is binding. Also, let us define the implied true state of demand $X(Q)$ such that the production quantity Q is just binding at the retail price $\hat{p}(x)$. That is

Definition 3: $X(Q) \equiv \{x | D(\hat{p}(x), x) = Q\}$

From this definition, it follows that if $x < X(Q)$, then the production quantity Q is not a binding constraint on the choice of revenue-maximizing price. Using these definitions, the following lemma characterizes the optimal sales strategy contingent on production quantity Q.

Lemma 1

If $x < X(Q)$, then the optimal retail price is $\hat{p}(x)$, and the implied sales quantity is $S(x) = D\{\hat{p}(x), x\} \leq Q$, and the resulting revenue is $\hat{R}(x) = \hat{p}(x)D\{\hat{p}(x), x\}$. If on the contrary $x \geq X(Q)$, then the optimal retail price is $\hat{p}(x, Q)$, and the implied sales quantity is $S(x, Q) = D\{\hat{p}(x, Q), x\} = Q$, and resulting revenue is $\hat{R}(x, Q) = \hat{p}(x, Q)Q$.

Proof:

The Lagrangean objective function for revenue maximization subject to constraint is

$$L = p(x)D(p, x) - \lambda\{D(p, x) - Q\},$$

which leads to first-order condition for maximum

$$\frac{dL}{dp(x)} = D(p, x) + p\frac{dD}{dp(x)} - \lambda\frac{dD}{dp(x)} = 0, \qquad (4\text{-}1)$$

$$\frac{dL}{d\lambda} = -\{D(p, x) - Q\} \geq 0, \quad \lambda\frac{dL}{d\lambda} = 0 \quad \text{and} \quad \lambda \geq 0. \qquad (4\text{-}2)$$

Here if $\lambda = 0$ (that is, if the constraint that sales quantity not exceed production quantity is not binding), so that $x < X(Q)$, then equation (4-1) becomes $D(p,x) + p\dfrac{dD}{dp(x)} = 0$, and the optimal (revenue maximizing) retail price is $\hat{p}(x)$. If to the contrary $\lambda > 0$ (so that $x > X(Q)$), then equation (4-2) becomes $D(p,x) = Q$, and the optimal (revenue-maximizing) retail price is $\hat{p}(x,Q)$. Q.E.D.

Here, notice that from definition 3,

$$\text{If } x = X(Q), \text{ then } \hat{R}(x) = \hat{R}(x,Q). \tag{5}$$

The following lemma is also useful for comparative static analysis of the optimal retail price.

Lemma 2

Assuming that the second-order condition for maximization ($pD_{pp} + 2D_p < 0$) is met, if $D_x + pD_{px} > 0$, then

$$\frac{\partial \hat{p}(x)}{\partial x} > 0, \quad \frac{\partial \hat{p}(x)}{\partial Q} = 0, \quad \text{and}$$
$$\frac{\partial \hat{p}(x,Q)}{\partial x} > 0, \quad \frac{\partial \hat{p}(x,Q)}{\partial Q} < 0.$$

That is, if the actual demand state is good then the optimal retail price is high, and if production quantity is great then the optimal retail price $\hat{p}(x,Q)$ is low. (proof omitted).

Next, we analyze the choice of production quantity. The producer in implementing the optimal strategy and producing before the true state of demand is known, chooses the production quantity that maximizes his own expected profit. Under this condition he confronts the following problem

$$\max_{Q} E\pi = \int^{X(Q)} [\hat{p}(x) D\{\hat{p}(x), x\}] dF(x) + \int_{X(Q)} [\hat{p}(x,Q) Q] dF(x) - C(Q).$$

Here, substituting from equation (5), leads to the following condition for maximization:

$$\frac{dE\pi}{dQ} = \int_{X(Q)} \left\{ \hat{p}(x,Q) + \frac{\partial \hat{p}}{\partial Q} Q \right\} dF(x) - \frac{dC}{dQ} = 0 \quad . \qquad (6)$$

This equation has the simple interpretation that the optimal production quantity is such that "marginal expected revenue = marginal expected cost".

Here, let us define the demand state X^* such that production quantity Q^* just attains the unconstrained maximum revenue with optimal pricing. That is,

Definition 4: $X^* = X(Q^*)$

The following proposition characterizes the optimal production and sales strategy.

Proposition 1

The producer, before learning the true state of demand, based on equation (6) sets production quantity Q^*. Later, when the true state of demand is realized he sets the retail price as follows

$$p^*(x) = \hat{p}(x), \qquad \text{if } x < X^*$$

$$p^*(x, Q^*) = \hat{p}(x, Q^*), \quad \text{if } x \geq X^*,$$

and the implied sales quantity is

$$S^*(x) = D\{p^*(x), x\}, \quad \text{if} \quad x < X^*$$
$$S^*(x, Q^*) = Q^*, \qquad \text{if} \quad x \geq X^*$$

If $x \geq X^*$ then all merchandise is sold, and if $x < X^*$ then the quantity $Q^* - S^*(x)$ remains unsold.

3 Dealer System

In the following, we presume that a risk-neutral producer sells through a competitive industry of risk-neutral independent retailers. In this section, we analyze the terms of optimal dealer contracts between the manufacturer and retailers, and show that in general these fail to attain as great an expected profit as is possible under vertical integration. We posit a game with the following timing. In the first stage, when the state of demand is still uncertain, the producer sets a shipping price. Both the producer and the retailers share common knowledge of the probability distribution of the demand parameter $F(x)$. At the second stage, the competitive independent retailers set their order quantities at the announced shipping price, and the producer chooses an output level. After production, the actual state of demand is revealed. Then at the third stage, the retailers' competitive behavior determines the retail sales price $p(x)$. We analyze the subgame perfect solution of this game, by backward induction.

First we analyze the third stage, in which the retailers set their selling price. At this stage, the selling price $\hat{p}(x, Q)$ follows from the production quantity Q, demand state parameter x, and supply-demand equilibrium condition（$D(p, x) = Q$. Anticipating this result, in the second stage the retailers set their order quantities, taking as given the shipping price set by the producer. Certainly, it is in the interest of all retailers to constrict their total quantity ordered. But each individual retailer chooses its own order quantity so as to maximize its own expected profit, so the total orders in fact surpass the amount that would maximize the retail industry expected profit. Furthermore, competition among retailers leads to industry order Q^M that entails zero expected profit (that is it leads to an expected final retail price equal to the shipping price the retailers pay the producer):

$$Ey(x) = \int \{\hat{p}(x,Q)Q\} dF(x) - wQ = 0 \Rightarrow \int \hat{p}(x,Q^M) dF(x) = w \quad . \tag{7}$$

One implication of this is that if the demand state turns out to be low, the actual retail price will fall below the shipping price the retailers will have already paid to the producer, and so the retailers will have suffered a loss. Here, recalling that $\frac{d\hat{p}(x,Q)}{dQ} < 0$, we have that

$$\frac{dQ^M}{dw} = \frac{1}{\int \left\{ \frac{d\hat{p}(x,q)}{dQ} \right\} dF(x)} < 0 \quad , \tag{8}$$

which means the higher is the shipping price the smaller is the total quantity ordered by the retailers.

With this sort of competitive behavior of retailers in mind, in the first stage, the producer sets the shipping price so as to maximize his own expected profit. The producer's decision problem is thus the following:

$$\max_w \pi = wQ^M(w) - c\{Q^M(w)\} \quad .$$

The condition for maximization is

$$\frac{d\pi}{dw} = Q^M(w) + \left(w - \frac{dC}{dQ} \right) \frac{dQ^M}{dw} = 0 \quad , \tag{9}$$

which determines the shipping price w^M. Rearranging the above equation yields

$$\frac{dC}{dQ} = w^M + Q^M \cdot \frac{dQ^M}{dw},$$

and recalling that $\frac{dQ^M}{dw} < 0$, we deduce that $\frac{dC}{dQ} < w^M$. That is, the shipping price is set above marginal cost.

Based on the previous discussion, the equilibrium under the dealer system is as follows. The producer sets the shipping price w^M according to equation (9). In light of this, the retail industry orders a total quantity $Q^M = \left\{ Q \middle| \int \hat{p}(x,Q) dF(x) = w^M \right\}$, as expressed by equation (7). The entire order quantity is shipped, and ultimately sells at the retail price $p^M(x) = \hat{p}(x, Q^M)$, which is contingent on the actual demand state. There is no unsold merchandise.

Comparing the production quantity under the dealer system with the optimal production quantity yields the following propositions.

Proposition 2

Under the dealer system, the producer cannot in general achieve the optimal solution.

Proof:

From equation (7) and equation (9), the production quantity under the dealer system fulfills the condition

$$\frac{dC}{dQ} = w^M + Q^M \cdot \frac{dQ^M}{dw} = \int \left\{ \hat{p}(x, Q^M) + Q^M \frac{d\hat{p}(x, Q^M)}{dQ} \right\} dF(x),$$

while, from equation (6), the optimal production quantity is determined by

$$\frac{dC}{dQ} = \int_{X^*} \left\{ \hat{p}\left(x,Q^*\right) + Q^* \frac{\partial \hat{p}\left(x,Q^*\right)}{\partial Q} \right\} dF(x)$$
.

Because $Q^M = Q^*$, it must be the case that $X^* = \min x$. Under this condition, the production quantity equals the quantity demanded at the revenue maximizing price in the worst demand state, which is in general sub-optimal, compared with the vertically integrated outcome. Q.E.D.

4 Returns System with Rebates

In this section we investigate the optimal features of a regime in which the producer implements a returns system with rebates. The timing of the game is as follows. In the first stage, the producer sets the shipping price w, buy-back price t, and state-contingent rebate $r(x)$. The rebate is an amount per unit of merchandise sold to final demanders that the producer pays to the retailer contingent on the actual demand state x. In the second stage, given the shipping price, buy-back price and rebate, the competitive retailers each choose their order quantities and the producer fills the orders. The actual demand state is observed only after production. Then in the third stage, the retailers set their selling price $p(x)$. The solution of the game is subgame perfect and we solve it by induction.

First we analyze the third stage in which the retailers set their selling price. It depends on the rebate per unit sold $r(x)$, which depends on the demand state x, and depends on the buy-back price t for unsold merchandise. In general, the retail price equals the buy-back price net of rebate $p(x) = t - r(x)$.

Next consider the producer's determination of its buy-back price t^* and rebate $r^*(x)$. These are as follows:

$$t^*=p^*\left(X^*,Q^*\right), \tag{10}$$

$$r^*(x)=t^*-p^*(x), \quad \text{if } x<X^* \tag{11}$$
$$=0, \qquad \text{if } x>X^* .$$

That is to say, if the constraint that sales quantity not exceed the production quantity is not binding, so that exactly Q^* units are sold, then the per-unit rebate equals the buy-back price minus the revenue-maximizing retail price. If on the other hand the constraint is binding, then the per-unit rebate is zero.

For the moment presuming that the retail industry order quantity is Q^*, and presuming also that the demand state is $x<X^*$, denote the implied final sales quantity $S^*(x)$, and denote the optimal solution by $\{p^*(x), S^*(x)\}$. If $x\geq X^*$, then the retail price is $p^*\left(x, Q^*\right)$, the sales quantity equals the production and order quantity Q^*, and the solution is $\{p^*\left(x,Q^*\right), Q^*\}$. Here note that when the retail industry order quantity is Q^* and the demand state is x then the implied retail profit per unit $v(x)$ is

$$v(x)=p^*(x)+r^*(x)=t^*, \quad \text{if } x<X^*$$
$$= p\left(x,Q^*\right)>t^*, \qquad \text{if } x\geq X^*.$$

Now consider the second stage in which the retailers choose their order quantity. In this case, competition leads to inflation of orders so that in equilibrium, expected retail profit is zero.

If the total retail industry order quantity is Q^*, then the expected profit of the competitive retail industry is

$$Ey(x)= E\left[v(x)S(x)+t^*\left(Q^*-S(x)\right)-wQ^*\right].$$

The first term in the right-hand side of the above equation $v(x)S(x)$ is the total revenue including rebate, the second term $t^*\{Q^*-S(x)\}$ is the revenue from return of unsold merchandise to the producer, and the last term wQ^* is the retail stocking expenditure.

Rearranging the equation leads to the following:

$$E\left[\left\{v(x)-t^*\right\}S(x)\right]-\left(w-t^*\right)Q^*$$

$$=\int^{X^*}\left[\left(t^*-t^*\right)S^*(x)\right]dF(x)+\int_{X^*}\left[\left\{p^*\left(x,Q^*\right)-t^*\right\}Q^*\right]dF(x)-\left(w-t^*\right)Q^*$$

$$=\int_{X^*}\left[\left\{p^*\left(x,Q^*\right)-t^*\right\}Q^*\right]dF(x)-\left(w-t^*\right)Q^*.$$

The shipping price that implies retail expected profit is zero is thus

$$w^*=t^*+\int_{X^*}\left\{p^*\left(x,Q^*\right)-t^*\right\}dF(x) \quad . \tag{12}$$

If the competitive retail industry orders Q^*, then its expected profit is zero. Here, if $x \geq X^*$ then $p^*\left(x,Q^*\right)\geq t^*$ and $w^*>t^*$. That is, the shipping price is greater than the buy-back price.

In the discussion thus far, if the producer buy-back price is given by equation (10), its per-unit rebate by equation (11) and its shipping price by equation (12), then the competitive retail industry order quantity equals the optimal production quantity Q^*. Furthermore、because in demand state x the optimal retail price $p^*(x)$ attains, the optimal solution is assured, in which expected channel profit is the maximum possible. Because the competitive retail industry gains zero expected profit, this describes a first-best outcome for the producer.

Finally, we address cases in which the retail industry total order quantity is different from the production quantity Q^*. We maintain the same equations (10)-(11) which characterize the buy-back price t^*, rebate per-unit sold $r^*(x)$, and shipping price w^*. Here, if the buy-back price equals t^*, then exactly Q units are sold in demand state $X\left(t^*\right)$ by definition. That is,

Definition 5: $X(t^*) \equiv \{x | D(t^*, x) = Q\}$.

Here consider the case in which the total order quantity is $Q < Q^*$. In this instance, by the above definition we have that $X(t^*) < X^*$. This entails the competitive retail industry profit per-unit sold as follows

$$
\begin{aligned}
v(x) &= p^*(x) + r^*(x) = t^*, && \text{if } x < X^* \\
v(x) &= \hat{p}(x, Q) + r^*(x) > t^*, && \text{if } X(t^*) \le x < X^* \\
v(x) &= \hat{p}(x, Q) > \hat{p}(x, Q^*), && \text{if } X^* \le x.
\end{aligned}
$$

That is, if $x < X(t^*)$, the retail price is $p^*(x)$ and sales are $S^*(x)$, then the total quantity of unsold goods returned to the producer is $Q - S^*(x)$. Therefore if $x > X(t^*)$, then the gross retail profit-per-unit is greater than the optimal retail price. Referring to equation (12), in this condition the expected retail profit is

$$
\begin{aligned}
Ey(x) = &\int^{X(t^*)} \{(t^* - t^*) D\{p^*(x), x\}\} dF(x) + \int_{X(t^*)}^{X^*} \{\hat{p}(x, Q) + r^*(x) - t^*\} Q dF(x) \\
&+ \int_{X^*} \{\hat{p}(x, Q) - t^*\} Q dF(x) - (w^* - t^*) Q \\
= &\int_{X(t^*)}^{X^*} \{\hat{p}(x, Q) + r^*(x) - t^*\} Q dF(x) + \int_{X^*} \{\hat{p}(x, Q) - t^*\} Q dF(x) - (w^* - t^*) Q \\
= &\int_{X(t^*)}^{X^*} \{\hat{p}(x, Q) + r^*(x) - t^*\} Q dF(x) + \int_{X^*} \{\hat{p}(x, Q) - \hat{p}(x, Q^*)\} Q dF(x).
\end{aligned}
$$

Here substituting the following,

$$
\begin{aligned}
&\hat{p}(x, Q) > t^* \text{ and } r^*(x) > 0, && \text{if } X < x < X^* \\
&\hat{p}(x, Q) > \hat{p}(x, Q^*), && \text{if } Q < Q^*,
\end{aligned}
$$

we have that $Ey(x) > 0$. That is, given w^*, t^* and $r^*(x)$, the total order quantity is less than the optimal production quantity Q^*, and retailers' expected profit is positive. In this case the competitive retail industry would have an incentive to increase its orders; it is not an equilibrium.

Suppose that the quantity already produced is $Q > Q^*$. In this case $X(t^*) > X^*$ and the profit per-unit-sold of the competitive retailers is

$$\begin{aligned}
&v(x) = p^*(x) + r^*(x) = t^*, && \text{if } x < X^* \\
&v(x) = \max\{\hat{p}(x,Q) + r^*(x), t^*\} = t^*, && \text{if } X^* \le x \le X(t^*) \\
&v(x) = \hat{p}(x,Q) > \hat{p}(x,Q^*), && \text{if } X(t^*) \le x.
\end{aligned}$$

That is, if the demand state is $x < X(t^*)$, the retail price is $p^*(x)$ and sales are $S^*(x)$, then $Q - S^*(x)$ units are unsold and returned to the producer. Further, if $X^* \le x < X(t^*)$, at the retail price $t^* - r^*(x)$, then the demand is $D(t^* - r^*(x), x)$, and unsold merchandise is returned. In this case the expected profit of the retail industry is

$$\begin{aligned}
Ey(x) = &\int^{X^*} \{(t^* - t^*) D\{p^*(x), x\}\} dF(x) + \int_{X^*}^{X(t^*)} (t^* - t^*) Q dF(x) \\
&+ \int_{X(t^*)} \{\hat{p}(x,Q) - t^*\} Q dF(x) - (w^* - t^*) Q \\
= &\int_{X(t^*)} \{\hat{p}(x,Q) - t^*\} Q dF(x) - (w^* - t^*) Q,
\end{aligned}$$

and from $w^* = t^* + \int_{x^*} \{p^*(x, Q^*) - t^*\} dF(x)$, and referring to lemma 2、

$$\hat{p}(x,Q) < \hat{p}(x,Q^*), \quad \text{if } Q > Q^*.$$

So the above equation becomes

$$Ey(x)=\int_{X(t^*)}\{\hat{p}(x,Q)-\hat{p}(x,Q^*)\}QdF(x)<0\,.$$

That is, in the case $Q>Q^*$, retail expected profit is negative, which is not an equilibrium; the retailers would tend to place smaller orders.

Conclude that given $\{w^*,t^*,r^*(x)\}$, retailers would order Q^*. Therefore、the optimal production and sales strategy attains. This discussion is summarized in the following proposition.

Proposition 3

Under a returns system with rebates, the producer chooses its production quantity, buy-back price and rebate to fulfill equations (10)-(12), and it thereby attains the optimal solution, capturing for itself the maximum possible expected profit.

5 Conclusion

This paper has analyzed a returns system implemented by a monopoly producer selling through an independent competitive retail industry. In our model the producer must decide on a production quantity while demand is still uncertain but can adjust the price after the true state of demand is known. In the marketing of goods such as apparel, under a returns system it is common to see price discounting if the state of demand is bad, and also common for producers to buy-back unsold merchandise at prices below the shipping prices already paid by the retailers. However, this kind of returns system has not been analyzed in the previous literature. In this paper, rebates paid to retailers by producers offset retailer losses from buy-back prices that may turn out to be less than shipping prices, and so retail prices are free to vary with the actual state of demand.

From analysis of our model, we conclude that for general demand function and general cost function, a returns system with rebates enables the producer to attain a first-best outcome, capturing maximum possible expected profit, just as it would under vertical integration. In the model of this paper, the expected profit of the competitive retail industry is zero. In other words, the retail price is just equal to the buy-back price paid by the producer for unsold merchandise net of rebates paid by the producer for actual sales. Consequently, the producer, by adjusting its shipping price can control retailers' order quantities, and after learning the true state of demand, by implementing its state-contingent rebate can control the retail price. From the standpoint of retailers, compared to a simple dealer system, the returns system with rebates enables them to recoup a portion of their initial outlays if the state of demand is bad, which pushes them to order more, tending to lower the average selling price and enlarge the expected channel profit. The returns system with rebates is thus unambiguously better for producers than the dealer system, when the premises of our model are true.

Appendix: Concrete Example

In this appendix we use a concrete example to demonstrate the assertions of the text. To simplify, suppose that the uncertain market demand is given by $D(p,x) = x - p$. Here, the demand state is represented by the random variable x which is uniformly distributed on the closed interval $[x_L = u - d, x_H = u + d]$, where $u > d$. Further, suppose that the cost of production is $C = cQ$.

Optimal Solution

Suppose that the producer is vertically integrated with the retail sector. Having already produced the quantity Q, if the demand state is x, then, noting from definition 3 that $X(Q) \equiv 2Q$, the revenue maximizing sales strategy is the following

$$\hat{p}(x) = \frac{x}{2}, \qquad \hat{S}(x) = \frac{x}{2}, \qquad \hat{\pi}(x) = \frac{x^2}{4}, \qquad \text{if } x < X(Q)$$

$$\hat{p}(x,Q) = x - Q, \quad \hat{S}(x,Q) = Q, \quad \hat{\pi}(x,Q) = (x-Q)Q, \quad \text{if } X(Q) \le x.$$

Here, if $x = X(Q) = 2Q$, then $\hat{\pi}(x) = Q^2 = \hat{\pi}(x,Q)$.

Based on the preceding, the producer chooses its production quantity before learning the true demand state. The producer's decision problem amounts to the following

$$\max_Q E\pi(Q) = \int^{2Q} \left(\frac{x^2}{8d} \right) dx + \int_{2Q} \left\{ \frac{(x-Q)Q}{2d} \right\} dx - cQ$$

$$= \frac{1}{24d}\left(8Q^3 - 12x_H Q^2 + 6x_H{}^2 Q - x_L{}^3\right).$$

Consequently, from the first-order condition for maximum, $\dfrac{d\pi}{dQ} = \dfrac{1}{d}\left(\dfrac{x_H}{2} - Q\right)^2 - c = 0$, the optimal production quantity is $Q^*=\frac{x_H}{2}-(dc)^{\frac{1}{2}}$. Here, from definition 4, if $X^*=2Q^*=x_H-2(dc)^{\frac{1}{2}}$, then the optimal solution is the following:

$$p^*(x)=\frac{x}{2}, \qquad S^*(x)=\frac{x}{2}, \qquad \text{if } x<X^*=x_H-2(dc)^{\frac{1}{2}}$$

$$p^*(x,Q^*)=x+(dc)^{\frac{1}{2}}-\frac{x_H}{2}, \quad S^*(x,Q^*)=Q^*=\frac{x_H}{2}-(dc)^{\frac{1}{2}}, \quad \text{if } X^*\leq x.$$

Dealer system

Under the dealer system, given the production quantity Q and demand state x, the equilibrium retail price is $\hat{p}(x,Q)=x-Q$. In the second stage, orders by the competitive retail industry result in expected retail price equal to shipping price, or:

$$\int\left\{(x-Q)\frac{1}{2d}\right\}dx=\frac{1}{4d}(x_H-x_L)(x_H+x_L-2Q)=w.$$

Consequently、 $x_H-x_L=2d$ and

$$Q^M(w)=\frac{x_H+x_L}{2}-w=u-w.$$

Based on this, the producer profit is

$$\pi=(w-c)Q^M(w)=(w-c)(u-w)$$

and the shipping price is set to maximize producer profit. From the first-order condition ($\frac{d\pi}{dw}=u-c-2w=0$) it follows that

$$w^M = \frac{u+c}{2}.$$

The implied production quantity and retail price are[3]

$$Q^M = u - w^M = \frac{u-c}{2},$$

$$p^M(x) = x - Q^M = x - \frac{u-c}{2}.$$

Here comparing the production quantity Q^M with the optimal production quantity $Q^* = \frac{x^H}{2} - (dc)^{\frac{1}{2}} = \frac{u+d}{2} - (dc)^{\frac{1}{2}}$, we have that

$$Q^* \geq Q^M \Rightarrow d + c - 2(dc)^{\frac{1}{2}} = \left[d^{\frac{1}{2}} - c^{\frac{1}{2}} \right]^2 \geq 0.$$

Consequently, if $c = d$, then the dealer system is sub-optimal.

Returns System with Rebates

Under this system, the shipping price, buy-back price and state-contingent rebate are, from equations (10)-(12) as follows:

$$w^* = t^* + c = \frac{x_H}{2} - (dc)^{\frac{1}{2}} + c,$$

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3 Because $p^M(x_L) = u - d - \frac{u-c}{2} > 0$, it follows from $u + c - 2d > 0$ that a necessary condition for maximum revenue is $d < \frac{u+c}{2}$.

$$t^* = \frac{x_H}{2} - (dc)^{\frac{1}{2}},$$

$$r^*(x) = \frac{x_H - x}{2} - (dc)^{\frac{1}{2}}, \quad \text{if } x < X^*$$

$$= 0, \qquad \text{if } x > X^*.$$

Furthermore, from definition 5, because

$$X(t^*) = t^* + Q = \frac{x_H}{2} + Q - (dc)^{\frac{1}{2}},$$

it follows that $r^*(X^*) = 0$. Yet further, noting that $X^* = x_H - 2(dc)^{\frac{1}{2}}$, we deduce that

$$X(t^*) \leq (\geq) X^*, \qquad \text{if } Q \leq (\geq) Q^* = \frac{x_H}{2} - (dc)^{\frac{1}{2}}.$$

Now consider the case in which the retail industry order quantity is less than the production quantity. In this instance if $X(t^*) < X^*$, competitive order behavior of retailers in the third stage results in[4]

$$v(x) = t^* = \frac{x_H}{2} - (dc)^{\frac{1}{2}} \text{ and } p(x) = \frac{x}{2}, \qquad \text{if } x < X^*$$

$$= x - Q + r^*(x) = \frac{x_H}{2} - (dc)^{\frac{1}{2}} + \left(\frac{x}{2} - Q\right) \text{ and } p(x) = \frac{x}{2} - Q, \quad \text{if } X(t^*) < x < X^*$$

$$= x - Q \quad \text{and} \quad p(x) = x - Q, \qquad \text{if } X^* < x.$$

and the expected profit of retailers is

4If $X(t^*) < x < X^*$ and $p(x) > 0$, then $v(x) > t^*$.

$$Ey(x) = \int_{X(t^*)}^{X^*} \{\hat{p}(x,Q) + r^*(x) - t^*\} QdF(x) + \int_{X^*} \{\hat{p}(x,Q) - \hat{p}(x,Q^*)\} QdF(x)$$

$$= \int_{X(t^*)}^{X^*} \left\{\frac{x}{2} - Q\right\} QdF(x) + \int_{X^*} \{Q^* - Q\} QdF(x)$$

.

Here, if $x > X(t^*) = \frac{x_H}{2} + Q - (dc)^{\frac{1}{2}}$ and $Q < Q^* = \frac{x_H}{2} - (dc)^{\frac{1}{2}}$, then $x > \frac{x_H}{2} + Q - (dc)^{\frac{1}{2}} = Q + Q^* > 2Q$, and it thus it follows that $Ey(x) > 0$. Conclude that this can not be an equilibrium; retail orders would increase.

If to the contrary $Q > Q^*$ (meaning that $X(t^*) > X^*$), then in the third stage, the competitive order behavior of retailers leads to

$$\begin{aligned} v(x) &= t^* = \frac{x_H}{2} - (dc)^{\frac{1}{2}} \text{ and } p(x) = \frac{x}{2}, \quad \text{if } x < X(t^*) \\ &= x - Q \text{ and } p(x) = x - Q, \qquad \text{if } X(t^*) < x, \end{aligned}$$

and the competitive retailers' expected profit is

$$Ey(x) = \int_{X(t^*)} \{\hat{p}(x,Q) - \hat{p}(x,Q^*)\} QdF(x) = \int_{X(t^*)} (Q^* - Q) QdF(x) < 0,$$

which is again not an equilibrium, in this instance leading to further reductions in order quantity.

In equilibrium the competitive retail industry attains zero expected profit, which requires that their order quantity equals the production quantity Q^*. Because $X^* = X(t^*)$ in the third stage, competitive behavior of the retails leads to

$$v(x) = t^* = \frac{x_H}{2} - (dc)^{\frac{1}{2}} \quad \text{and} \quad p(x) = \frac{x}{2}, \quad \text{if} \ x < X^*$$

$$= x - Q^* \quad \text{and} \quad p(x) = x - Q^*, \quad \text{if} \quad X^* < x.$$

This is the optimal production and sales outcome.

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