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Preemptive Investment Game with Alternative Projects

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Abstract

This paper derives a preemptive equilibrium in strategic investment in alternative projects. The problem is formulated in a real options model with a multidimensional state variable that represents project-specific uncertainty. The proposed method enables us to evaluate the value of potential alternatives. The results not only extend previous studies with a one-dimensional state variable but also reveal new findings. Preemptive investment takes place earlier and the project value becomes lower if the numbers of both firms and projects increase by the same amount. Interestingly, a strong correlation among profits from projects, unlike in a monopoly, plays a positive role in moderating preemptive competition.

JEL Classifications Code: C73, G13, G31.

Keywords: strategic real options, preemption, alternative projects, stopping game.

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1 Introduction

The global financial crisis that began in 2007 has increased uncertainty about future market demand in industries throughout the world. It is becoming increasingly important for firm project management to take into account uncertainty and flexibility in the future. The real options approach, in which option pricing theory is applied to capital budgeting decisions, better enables us to find an optimal investment strategy and project valuation involving such uncertainty and flexibility than the Net Present Value (NPV) method could (see [4]).

Although the early literature on real options investigated monopolists' investment decisions, recent studies have investigated the problem of several firms competing in the same market from a game theoretic approach. Many studies, such as [6, 9, 16], analyze the preemptive equilibrium in a duopoly investment game.[1] Their main result, that competition among firms accelerates investment in a project, has been supported by empirical papers such as [14].

Most studies of strategic real options assume one-dimensional Geometric Brownian Motion (GBM) to be the stochastic process (called the state variable) representing the future cash flow from a project. This is because explicit results are more appealing due to the difficulty of model calibration in many real options models. Although such simplification could be justified for a problem concerning a single investment project, a problem involving several projects should be modelled by a multidimensional state variable instead of a one-dimensional state variable. In fact, several papers have investigated a monopolist's investment decision involving several projects in a model with a bidimensional state variable. For example, [5] investigated land development timing with an alternative land use choice and [11] investigated timing in switching methods of nuclear waste disposal.[2]

To the best of my knowledge, however, there are no papers that investigate preemptive investment involving several projects with a multidimensional state variable.[3] The contribution of the paper is to first clarify the preemptive equilibrium in an investment game by several firms with alternative projects, using a multidimensional state variable. This paper shows several properties of the investment region and the option value in a

[1] In [7, 10] derived the equilibrium strategies in a Cournot–Nash framework instead of the preemption game. The competitive equilibrium where the output price moves between upper and lower barriers has also been investigated in [4, 17]. On the other hand, [8, 13, 15] investigated the agency problem in a single firm by the method of mechanism design.

[2] These studies apply the results of financial options for multiple assets (see Chapter 6 in [3]) to capital budgeting. Although in several papers a problem with a bidimensional state variable is reduced to a one-dimensional case by homogeneity, such cases are very restrictive.

[3] One paper, [1], conducted a case study on the preemptive competition in the textile industry with three types of uncertainty, but the preemptive game is essentially modelled on the one-dimensional state variable. So, theoretically, their paper is no different from the previous papers [6, 9, 16].

model where firms optimize both investment time and project choice among remaining projects that have not been chosen by the leading competitors.

The use of this model is also motivated by the following practical issue. When we evaluate the value of a project by the real options method, we are often puzzled by the question of which value in monopolistic and strategic models is reliable. Indeed, the difference is likely to be quite large because the theoretical models with a one-dimensional state variable calculate the extreme values. This paper provides us with a useful criterion toward solving such a problem. That is, we should evaluate the value of considering a potential alternative in a strategic model with a multidimensional state variable. I find that the strategic option values with a symmetric alternative are $40\% \sim 60\%$ of monopoly with two alternative projects, or equivalently, $70\% \sim 80\%$ of monopoly with a single project.

Furthermore, I show that preemptive investment takes place earlier and the option value becomes lower if the numbers of both firms and projects increase by the same amount. It is intuitively explained that in the preemptive equilibrium all the firms are dragged into a scenario with the worst project. Taking into account the fact that the number of competitors is likely to increase with the number of alternatives, the result seems consistent with empirical studies on strategic real options such as [14].

Another new finding is that preemptive competition is moderated by the correlation among profits from projects. This contrasts with the monopoly situation where strong correlation among cash flows decreases the value of project choice. Thus, the sensitivity of the correlation with project value in an oligopoly depends on a trade-off between moderation of the preemptive competition (positive effect) and a decrease in the value of project choice (negative effect). In particular, when there are as many projects as firms, the competition deprives firms of the value of project choice and hence a strong correlation increases the option value.

Finally, let me mention several applications of the model in this paper. As mentioned above, the model is suitable for strategic investment involving several alternatives. An example is a war among firms opening new stores. A follower must open a store in a different place or of a different type from that of the leader. In the situation where big firms fight for market share in emerging countries, an alternative to preemptive entry into the market in India might be preemption into the market in the Republic of South Africa.

The model also applies to M&A struggles. For instance, in the pharmaceutical industry large corporations strategically acquire venture businesses that develop new drugs. Because many M&As take place by private negotiation rather than through a public bidding process, it is necessary for a firm to preempt the competitors. In the pharmaceutical industry numerous potential targets generate a low correlation in gains in takeovers, and then severe preemptive competition occurs.

The paper is organized as follows. Section 2 introduces the setup and the preliminary results in three cases; a monopoly with a single project, a duopoly with a single project, and a monopoly with two alternative projects. Section 3 describes the new results. In particular I present the details in a duopoly with two projects, though the results can be extended to an oligopoly of n firms with m projects in Section 3.3. The results of the investment region and the project value contrasts a duopoly or oligopoly with a monopoly. Section 4 concludes the paper.

2 Preliminaries

Consider a risk-neutral[4] firm that has an option to invest in a project. Consider two kinds of projects denoted by $i = 1, 2$. When a firm conducts project i at time t with sunk cost $I_i(> 0)$, it receives a temporary profit $X_i(t)$.[5] Assume that the profit $X_i(t)$ follows a continuous diffusion process:

$$\mathrm{d}X_i(t) = \mu_i(X_i(t), t)\mathrm{d}t + \sigma_i(X_i(t), t)\mathrm{d}B_i(t), \tag{1}$$

where $(B_1(t), B_2(t))$ is a two-dimensional Brownian Motion (BM) with correlation coefficient ρ. Mathematically, the model is built on the filtered probability space $(\Omega, \mathcal{F}, P; \mathcal{F}_t)$ generated by $(B_1(t), B_2(t))$ as usual. The set $\mathcal{F}_t$ means the available information set to time t, and a firm optimizes its investment strategy under this information. Let $r(> 0)$ and $T(> 0)$ denote the constant risk-free rate and maturity of the option throughout the paper. We may take $T = \infty$ when we consider a perpetual option, as in many real options models.

2.1 Monopoly with a single project

As a benchmark, we consider a firm that has a monopolistic option to invest in a single project, i. It is well known that the option value at time $t(\leq T)$ with the state variable $X_i(t) = x_i$ is equal to the value function of the following optimal stopping problem:

$$V_i^1(x_i, t) = \sup_{\tau \in \mathcal{T}_t} \mathbb{E}_t^{x_i}[\mathrm{e}^{-r(\tau - t)}(X_i(\tau) - I_i)1_{\{\tau \leq T\}}], \tag{2}$$

where $\mathcal{T}_t$ denotes the set of all stopping times τ satisfying $\tau \geq t$ and $E_t^{x_i}[\cdot]$ is the expectation conditional on $X_i(t) = x_i$.[6] Throughout the paper, the superscript and the subscript on V_i^1 represent the number of firms and available project(s), respectively; that is, V_i^1 in (2) means the value function in a monopoly with a single project i.

Many diffusions satisfy the following properties.

[4]Generally we can assume risk-adjusted profit dynamics (1) rather than the risk-neutrality assumption.
[5]The profit can be interpreted as the discounted cash flow during the lifetime of the project.
[6]We do not consider $1_{\{\tau \leq \infty\}}$ but $1_{\{\tau < \infty\}}$ in the case of $T = \infty$ throughout the paper.

Assumption (i) The value function $V_i^1(\cdot, t)$ is a (finite) continuous increasing function.

Assumption (ii) There exists a finite investment trigger $x_i^1(t)$ such that the optimal stopping time $\tau_i^1(t)$ of problem (2) is written as the threshold strategy:

$$\tau_i^1(t) = \inf\{s \geq t \mid X_i(s) \in S_i^1(s) = [x_i^1(s), \infty)\}. \tag{3}$$

We restrict our attention to a continuous diffusion $X(t)$ satisfying the assumptions above. In addition, as in the related papers, we assume nonnegativeness of $X(t)$ as follows.

Assumption (iii) $X_i(t)$ is nonnegative. If $X_i(s) = 0$ for any s, $X_i(t) = 0$ for all $t \geq s$.

The assumptions are not restrictive. In fact, we can take a wide range of diffusions including a GBM, i.e., (1) with $\mu_i(X_i(t), t) = \mu_i X_i(t)$, $\sigma_i(X_i(t), t) = \sigma_i X_i(t)$ where $\mu_i(< r)$ and $\sigma_i(> 0)$ are constant, and a mean-reverting process (1) with $\mu_i(X_i(t), t) = \eta(\bar{X} - X_i(t))$, $\sigma_i(X_i(t), t) = \sigma_i X_i(t)$ where $\eta, \bar{X}$ and σ_i are positive constants.

Note that for a GBM with $T = \infty$, $V_i^1(x_i, t)$ is explicitly derived independently from time t (see [4]). In fact, the option value $V_i^1(x_i)$ is expressed as:

$$V_i^1(x_i) = \begin{cases} \left(\dfrac{x_i}{x_i^1}\right)^{\beta_i} (x_i^1 - I_i) & (0 \leq x_i < x_i^1) \\ x_i - I_i & (x_i \geq x_i^1). \end{cases} \tag{4}$$

Here, x_i^1 is the constant investment trigger defined by:

$$x_i^1 = \frac{\beta_i}{\beta_i - 1} I_i, \tag{5}$$

where β_i is the positive characteristic root:

$$\beta_i = \frac{1}{2} - \frac{\mu_i}{\sigma_i^2} + \sqrt{\left(\frac{\mu_i}{\sigma_i^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_i^2}}\ (> 1).$$

2.2 Duopoly with a single project

This subsection considers two symmetric firms that struggle to take a single project i. The following outcome, called "preemptive investment", is well known. For details, refer to [6, 9, 16]. Assume that the initial value satisfies $X_i(0) \leq I$.

We can solve the game between the firms backward. We begin by supposing that one of the firms (called the leader) has first invested at time $t(\leq T)$ with $X_i(t) = x_i$, and we find the optimal decision of the other (called the follower). Because the follower's opportunity to invest is completely lost, the follower's profit is 0. On the other hand, the leader's profit is $x_i - I_i$. In the situation where neither firm has invested, each firm attempts to preempt the other in order to obtain the leader's payoff if $X_i(t) - I_i > 0$. As a result, in the preemptive equilibrium, both firms attempt to invest at the zero-NPV time:

$$\tau_i^2 = \inf\{t \geq 0 \mid X_i(t) - I_i = 0\} \tag{6}$$

and gain no project value:

$$V_i^2(x_i,t) = 0. \tag{7}$$

Recall that the superscript 2 and the subscript i represent duopoly with a single project i.

Strictly speaking, both firms' investment strategy at (6) proves to be a Nash equilibrium in the stopping game formulated under the appropriate assumption.[7] The outcome can be interpreted to mean that the leading firm invests at (6), but the follower cannot conduct a project. The leader's profit is also zero because of investing too early. This is a well-known preemptive equilibrium in the strategic real options literature (refer to [9]).

2.3 Monopoly with two alternative projects

This subsection considers a firm that has a monopolistic option to invest a single project among projects $1,2$. The model applies to the situation where a firm cannot execute both projects for a reason such as budget constraint. The problem has been essentially investigated in [5] and Section 6 in [3]. In contrast, [2] investigated investment with different scales under a one-dimensional state variable, i.e., the case where $\rho = 1, X_1(0) \neq X_2(0)$ and $I_1 \neq I_2$.

The option value at time $t(\leq T)$ with $X_i(t) = x_i$ is equal to the value function of the optimal stopping problem as follows:

$$V_{1,2}^1(x,t) = \sup_{\tau \in \mathcal{T}_t} \mathbb{E}_t^x[\mathrm{e}^{-r(\tau-t)} \max_{i=1,2}(X_i(\tau) - I_i)1_{\{\tau \leq T\}}]. \tag{8}$$

Recall that $V_{1,2}^1$ in (8) means the value function in monopoly with projects $1,2$.

The optimal stopping time $\tau_{1,2}^1$ in problem (8) becomes:

$$\tau_{1,2}^1(t) = \inf\{s \geq t \mid X(s) \in \mathcal{S}_{1,2}^1(s)\}, \tag{9}$$

where the stopping region $\mathcal{S}_{1,2}^1(s)$ is defined by:

$$\mathcal{S}_{1,2}^1(s) = \{x \in \mathbb{R}_+^2 \mid V_{1,2}^1(x,s) = \max_{i=1,2}(x_i - I_i)\}. \tag{10}$$

The stopping region $\mathcal{S}_{1,2}^1(t)$ proves to be the union of two disjoint convex sets corresponding to the immediate investment region of each project when $X(t)$ follows a GBM (refer to Section 6 in [3] and Figure 3 in Section 3.2).

Let us now focus on two symmetric projects, i.e., $\mu_1 = \mu_2, \sigma_1 = \sigma_2$ and $I_1 = I_2$. In this case, the larger the correlation coefficient ρ, the more likely it is that profits $X_1(t)$ and $X_2(t)$ take close values. Then the option value $V_{1,2}^1$ decreases and the stopping region $\mathcal{S}_{1,2}^1(t)$ enlarges with the correlation. This can be explained in terms of a decrease of

[7] This assumption is that if two firms choose the same timing, one of the firms is chosen as the leader with probability 1/2. Most studies, including [6, 16], are built on this assumption.

diversification effects. In particular, in the case of the perfect correlation, i.e., $\rho = 1$, the option value $V_{1,2}^1$ and the investment time $\tau_{1,2}^1$ for $x_1 = x_2$, agree with those in a monopoly with a single project, i.e., V_i^1 and τ_i^1, respectively. The effect of a correlation will be compared in detail with that in a duopoly with two projects in Section 3.

The next section is the main contribution of the paper. Although the results can be extended in the case of n firms with m projects in Section 3.3, I first present the details of a duopoly with two projects in order to avoid unnecessary confusion.

3 Several firms with several alternative projects

3.1 Duopoly with two alternative projects

This subsection investigates two symmetric firms that compete for one of two projects $1, 2$. Assume that the one that first invests (the leader) can choose the better project while the other (the follower) loses the opportunity to invest in that project. The leader's advantage of being able to choose the better project brings about preemptive competition between the firms. As mentioned in Section 1, the model has a wide range of applications, such as preemption in the new market and M&A struggles. Relevant to this model, [12] investigated a duopoly with two projects following a one-dimensional state variable. Assume $X_i(0) \le I_i$ $(i = 1, 2)$.

As in Section 2.2, the problem can be solved in a reverse manner. Suppose that the leader has first invested in the better project $i(t)$ at time $t(\le T)$ with $X(t) = x$, where the function $i(t)$[8] is defined by:

$$i(t) = k \quad \text{if } X_k(t) - I_k = \max_{i=1,2}(X_i(t) - I_i). \tag{11}$$

Under this assumption, we find the optimal response of the follower. Because for $i \neq i(t)$ the follower has the monopolistic option to invest in a single project i, the option value and the optimal investment timing coincide with V_i^1, τ_i^1 (see (2) and (3)), respectively. On the other hand, the leader's payoff is equal to $\max_{i=1,2}(X_i(t) - I_i)$.

Let us return to the situation where neither firm has invested. The region $\mathcal{S}_{1,2}^{2F}(t)$ where the leader's profit dominates that of the follower is:

$$\mathcal{S}_{1,2}^{2F}(t) = \{x_1 - I_1 \ge V_2^1(x_2, t)\} \cup \{x_2 - I_2 \ge V_1^1(x_1, t)\}.$$

Each firm attempts to preempt the competitor as long as $X(t) \in \mathcal{S}_{1,2}^{2F}(t)$. In addition, one of the firms reluctantly invests $X(t) \in \mathcal{S}_1^1(t) \cup \mathcal{S}_2^1(t)$ if it knows that the other invests at time:

$$\tau_{1,2}^{2F} = \inf\{t \ge 0 \mid X(t) \in \partial\mathcal{S}_{1,2}^{2F}(t)\}, \tag{12}$$

[8]We do not have to be concerned about the value of $i(t)$ when $X_1(t) - I_1 = X_2(t) - I_2$.

where $\partial\mathcal{S}^{2F}_{1,2}(t)$ denotes the boundary of $\mathcal{S}^{2F}_{1,2}(t)$. This is because for $X(t) \in \mathcal{S}^1_1(t) \cup \mathcal{S}^1_2(t)$ immediate investment generates a higher profit than the option value to wait until $\tau^{2F}_{1,2}$ (this will be shown in the proof of Proposition 1). Therefore, the preemptive investment region $\mathcal{S}^2_{1,2}(t)$ becomes:

$$\mathcal{S}^2_{1,2}(t) = \{x_1 - I_1 \geq V^1_2(x_2,t)\} \cup \{x_2 - I_2 \geq V^1_1(x_1,t)\} \cup \mathcal{S}^1_1(t) \cup \mathcal{S}^1_2(t). \tag{13}$$

The preemptive investment takes place at:

$$\tau^2_{1,2} = \inf\{t \geq 0 \mid X(t) \in \partial\mathcal{S}^2_{1,2}(t)\}, \tag{14}$$

where $\partial\mathcal{S}^2_{1,2}(t)$ denotes the boundary of $\mathcal{S}^2_{1,2}(t)$ which consists of three parts, i.e.:

$$\begin{aligned}\partial\mathcal{S}^2_{1,2}(t) \quad = \quad & \underbrace{\{x_i \leq x^1_{i'}(t) - I_{i'} + I_i, x_i - I_i = V^1_{i'}(x_{i'},t)\}}_{(a)} \\ & \cup \underbrace{\{x_{i'} \leq x^1_{i'}(t), x_{i'} - I_{i'} = V^1_i(x_i,t)\}}_{(b)} \\ & \cup \underbrace{\{x_{i'} = x^1_{i'}(t), (V^1_i)^{-1}(x^1_{i'}(t) - I_{i'}) \leq x_i \leq x^1_{i'}(t) - I_{i'} + I_i\}}_{(c)},\end{aligned} \tag{15}$$

for i such that:

$$x^1_i(t) - I_i \geq x^1_{i'}(t) - I_{i'}, \tag{16}$$

where i' denotes project $i' \neq i$ throughout the paper.

Figure 1 illustrates the preemptive investment boundary $\partial\mathcal{S}^2_{1,2}(t)$. The first part (a) is the region where the leader's investment in project i generates the same value as the follower's option value to invest in project i'. In the second part (b), both firms are indifferent to being the leader with project i' and the follower with project i. In the last part (c), both firms prefer to be the follower with project i to being the leader with project i' due to $X(t) \notin \mathcal{S}^{2F}_{1,2}(t)$. However, one of the firms invests first if it knows that the other does not invest until $\tau^{2F}_{1,2}(t)$. It must be noted that, unlike the monopolist investment region, the preemptive investment boundary $\partial\mathcal{S}^2_{1,2}(t)$ is independent of the correlation coefficient ρ.

The option value (of the leader) at time $t(\leq \min(T, \tau^2_{1,2}))$ with $X(t) = x$ is written as:

$$V^2_{1,2}(x,t) = \mathbb{E}^x_t[\mathrm{e}^{-r(\tau^2_{1,2}-t)} \max_{i=1,2}(X_i(\tau^2_{1,2}) - I_i)]. \tag{17}$$

The leader's advantage of choosing the better project is completely lost by its earlier investment than the optimal timing. Furthermore, the leader's profit becomes less than that of the follower if and only if the process $X(t)$ hits part (c).

Although so far we intuitively see the preemptive outcome, to do a more precise derivation we formulate the following stopping game by two symmetric firms $j = 1, 2$. Define the action space of both firms as follows:

$$\mathcal{A} = \{(\tau, i) \mid \tau \in \mathcal{T}_0, i : \mathcal{F}_\tau \text{measurable random variable taking values in } \{0,1\}\}.$$

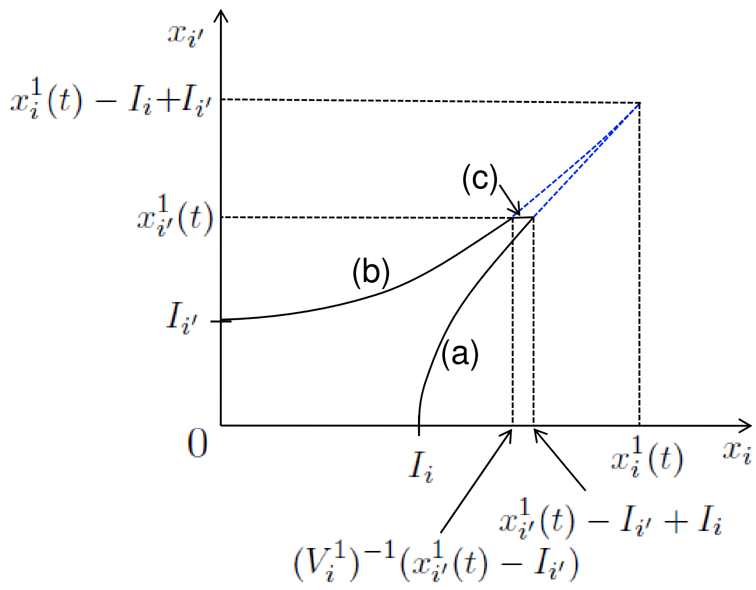

Figure 1: The preemptive investment boundary $\partial\mathcal{S}_{1,2}^2(t)$

Define the firm 1's payoff π_1 as:

$$\begin{aligned}\pi_1(\tau_1, i_1, \tau_2, i_2) \quad &= \quad \mathbb{E}[1_{\{\tau_1<\tau_2\}}\mathrm{e}^{-r\tau_1}(X_{i_1}(\tau_1) - I_{i_1}) + 1_{\{\tau_1>\tau_2\}}\mathrm{e}^{-r\tau_2}V_{i_2'}^1(X_{i_2'}(\tau_2), \tau_2) \\ &\quad + 1_{\{\tau_1=\tau_2\}}\frac{\mathrm{e}^{-r\tau_1}}{2}(X_{i_1}(\tau_1) - I_{i_1} + V_{i_2'}^1(X_{i_2'}(\tau_2), \tau_2))], \end{aligned} \tag{18}$$

where (τ_1, i_1) and (τ_2, i_2) in $\pi_1(\tau_1, i_1, \tau_2, i_2)$ denote the strategies of firm 1 and 2, respectively. The last term of (18) corresponds to the assumption in footnote 7. We also define the payoff of firm 2 as π_2 symmetrically.

We wish to find a Nash equilibrium in the stopping game, i.e., $(\tilde{\tau}_1, \tilde{i}_1, \tilde{\tau}_2, \tilde{i}_2) \in \mathcal{A} \times \mathcal{A}$ satisfying both:

$$\pi_1(\tilde{\tau}_1, \tilde{i}_1, \tilde{\tau}_2, \tilde{i}_2) = \max_{(\tau_1, i_1)\in\mathcal{A}} \pi_1(\tau_1, i_1, \tilde{\tau}_2, \tilde{i}_2), \tag{19}$$

and

$$\pi_2(\tilde{\tau}_1, \tilde{i}_1, \tilde{\tau}_2, \tilde{i}_2) = \max_{(\tau_2, i_2)\in\mathcal{A}} \pi_2(\tilde{\tau}_1, \tilde{i}_1, \tau_2, i_2). \tag{20}$$

Let $\tau_{1,2}^2(t)$ denote (14), replacing initial time 0 with t. We assume that the diffusion process $X(t)$ satisfies the following condition[9]:

Assumption (iv)

$$\max_{i=1,2}(x_i - I_i) \leq \mathbb{E}_t^x[\mathrm{e}^{-r(\tau_{1,2}^2(t)-t)} \max_{i=1,2}(X_i(\tau_{1,2}^2(t)) - I_i)] \quad (x \notin \mathcal{S}_{1,2}^2(t)).$$

[9]I do not know any proof, but the assumption is satisfied in many cases as far as I can judge from a wide range of computations. Even if Assumption (iv) is not satisfied, the violation is so small that we can regard the outcome as *an approximate equilibrium*.

The next proposition shows that the intuitive equilibrium above is indeed a Nash equilibrium in the stopping game.

Proposition 1 The pair of strategies $(\tau_{12}^2, i(\tau_{12}^2), \tau_{12}^{2F}, i(\tau_{12}^{2F}))$ is a Nash equilibrium in the stopping game, where the stopping times $\tau_{12}^2, \tau_{12}^{2F}$ are defined by (14),(12), and the functions $i(\tau_{12}^2), i(\tau_{12}^{2F})$ are defined by (11), respectively.

Proof To simplify the notations, let $\tilde{\tau_1} = \tau_{12}^2, \tilde{i_1} = i(\tau_{12}^2), \tilde{\tau_2} = \tau_{12}^{2F}, \tilde{i_2} = i(\tau_{12}^{2F})$. Recall that $\tilde{i_2}'$ denotes project $\tilde{i_2}' \neq \tilde{i_2}$. Take an arbitrary $(\tau_1, i_1) \in \mathcal{A}$. We calculate:

$$
\begin{aligned}
& \pi_1(\tau_1, i_1, \tilde{\tau_2}, \tilde{i_2}) \\
= \;& \mathbb{E}[1_{\{\tau_1<\tilde{\tau_2}\}} \mathrm{e}^{-r\tau_1}(X_{i_1}(\tau_1) - I_{i_1}) + 1_{\{\tau_1>\tilde{\tau_2}\}} \mathrm{e}^{-r\tilde{\tau_2}} V^1_{\tilde{i_2}'}(X_{\tilde{i_2}'}(\tilde{\tau_2}), \tilde{\tau_2}) \\
& + 1_{\{\tau_1=\tilde{\tau_2}\}} \frac{\mathrm{e}^{-r\tau_1}}{2}(X_{i_1}(\tau_1) - I_{i_1} + V^1_{\tilde{i_2}'}(X_{\tilde{i_2}'}(\tilde{\tau_2}), \tilde{\tau_2}))] \\
\leq \;& \mathbb{E}[1_{\{\tau_1<\tilde{\tau_2}\}} \mathrm{e}^{-r\tau_1} \max_{i=1,2}(X_i(\tau_1) - I_i) + 1_{\{\tau_1\geq\tilde{\tau_2}\}} \mathrm{e}^{-r\tilde{\tau_2}} \max_{i=1,2}(X_i(\tilde{\tau_2}) - I_i)] \qquad (21) \\
= \;& \sup_{\tau_1 \in \mathcal{T}_0, \tau_1 \leq \tilde{\tau_2}} \mathbb{E}[\mathrm{e}^{-r\tau_1} \max_{i=1,2}(X_i(\tau_1) - I_i)] \\
= \;& \mathbb{E}[\mathrm{e}^{-r\tilde{\tau_1}}(X_{\tilde{i_1}}(\tilde{\tau_1}) - I_{\tilde{i_1}})] \qquad (22) \\
= \;& \pi_1(\tilde{\tau_1}, \tilde{i_1}, \tilde{\tau_2}, \tilde{i_2})
\end{aligned}
$$

where (21) results from $V^1_{\tilde{i_2}'}(X_{\tilde{i_2}'}(\tilde{\tau_2}), \tilde{\tau_2}) = \max_{i=1,2}(X_i(\tilde{\tau_2}) - I_i)$ and (22) is proved as follows.

By Assumption (iv), immediate investment is not optimal for $X(t) = x \notin \mathcal{S}^2_{1,2}(t)$. On the other hand, immediate investment is optimal for $X(t) = x \in \mathcal{S}^2_{1,2}(t) \setminus \mathcal{S}^{2F}_{1,2}(t)$ (the triangle-like region in Figure 1). In fact, for any $\tau_1 < \tilde{\tau_2}$, $\max_{k=1,2}(X_k(\tau_1) - I_k) \leq V_i^1(X_i(\tau_1), \tau_1)$ $(i = 1, 2)$ because of $X(\tau_1) \notin \mathcal{S}^{2F}_{1,2}(\tau_1)$. Then, for i' satisfying (16) we have:

$$
\begin{aligned}
\sup_{\tau_1 \in \mathcal{T}_t, \tau_1 \leq \tilde{\tau_2}} \mathbb{E}_t^x[\mathrm{e}^{-r(\tau_1-t)} \max_{k=1,2}(X_k(\tau_1) - I_k)] \;&\leq\; \mathbb{E}_t^x[\mathrm{e}^{-r(\tau_1-t)} V^1_{i'}(X_{i'}(\tau_1), \tau_1)] \\
&\leq\; V^1_{i'}(x, t) \qquad (23) \\
&=\; x_{i'} - I_{i'},
\end{aligned}
$$

where (23) follows from the supermartingale property of the discounted price process $\mathrm{e}^{-rt}V^1_k(X_k(t), t)$. Thus, (22) (and hence (19)) has been proved. We can similarly show (20) under Assumption (iv). □

Proposition 1 includes the results in a duopoly with a single project. In fact, if $X_i(0) = x_i > X_{i'}(0) = 0$ the preemptive equilibrium in Proposition 1 agrees with that in Section 2.2. For most of the diffusion process $X_i(t)$, higher volatility σ_i brings about later investment τ_i^1 and higher option value V_i^1. In such a case, by (13) the preemptive investment region $\mathcal{S}^2_{1,2}$ becomes smaller, which leads to later investment $\tau^2_{1,2}$ and a higher option value $V^2_{1,2}$. That is to say, the effects of volatility σ_i in a duopoly are inherited from a monopoly.

If $X(t)$ follows a GBM and $T = \infty$, we have an explicit form of the time homogeneous investment boundary $\partial\mathcal{S}^2_{1,2}$ by (4), (5) and (15) .

Corollary 1 Assume that $T = \infty$, $\mu_i(X_i(t), t) = \mu_i X_i(t)$, and $\sigma_i(X_i(t), t) = \sigma_i X_i(t)$, where $\mu_i(< r)$ and $\sigma_i(> 0)$ are constant for $i = 1, 2$. For all $t > 0$, the preemptive investment boundary $\partial\mathcal{S}^2_{1,2}$ is:

$$\begin{aligned}\partial\mathcal{S}^2_{1,2} &= \left\{x_i \le x^1_{i'} - I'_i + I_i, x_i - I_i = \left(\frac{x_{i'}}{x^1_{i'}}\right)^{\beta_1}(x^1_{i'} - I_{i'})\right\} \\ &\cup \left\{x_{i'} \le x^1_{i'}, x_{i'} - I_{i'} = \left(\frac{x_i}{x^1_i}\right)^{\beta_1}(x^1_i - I_i)\right\} \\ &\cup \left\{x_{i'} = x^1_{i'}, (V^1_i)^{-1}(x^1_{i'} - I_{i'}) \le x_i \le x^1_{i'} - I_{i'} + I_i\right\},\end{aligned}$$

where i satisfies (16).

The explicit derivation of the investment boundary $\partial\mathcal{S}^2_{1,2}$ would be a big benefit in applications of the model. Although the option value $V^2_{1,2}$ (see (17)) becomes the solution of the corresponding partial differential equation with boundary $\partial\mathcal{S}^2_{1,2}$ instead of an explicit form, I would like to emphasize that the results are quite useful for applications.

For a general diffusion process $X(t)$ we can show the following properties of the investment region $\mathcal{S}^2_{1,2}$, the timing $\tau^2_{1,2}$, and the option value $V^2_{1,2}$.

Proposition 2 The following relationships hold.
Investment Region

$$\mathcal{S}^1_{1,2}(t) \subset \mathcal{S}^1_i(t) \subset \mathcal{S}^2_{1,2}(t), \tag{24}$$

Investment Timing

$$\tau^2_{1,2} \le \tau^1_i \le \tau^1_{1,2}, \tag{25}$$

Option Value

$$0 = V^2_i(x_i, t) \le V^2_{1,2}(x, t) \le V^1_i(x_i, t) \le V^1_{1,2}(x, t). \tag{26}$$

for all $i = 1, 2$.

Proof I prove only $V^2_{1,2}(x, t) \le V^1_i(x_i, t)$ $(i = 1, 2)$ because the others are clear (from Figure 1). We have $\max_{k=1,2}(X_k(\tau^2_{1,2}) - I_k) \le V^1_i(X_i(\tau^2_{1,2}), \tau^2_{1,2})$ $(i = 1, 2)$, because of $X_i(\tau^2_{1,2}) \notin \mathcal{S}^{2F}_{1,2}(\tau^2_{1,2}) \setminus \partial\mathcal{S}^{2F}_{1,2}(\tau^2_{1,2})$. Then we calculate (17):

$$\begin{aligned}V^2_{1,2}(x, t) &\le \mathbb{E}^x_t[\mathrm{e}^{-r(\tau^2_{1,2} - t)} V^1_i(X_i(\tau^2_{1,2}), \tau^2_{1,2})] \\ &\le V^1_i(x_i, t),\end{aligned} \tag{27}$$

where (27) follows from the supermartingale property of the discounted price process $\mathrm{e}^{-rt} V^1_i(X_i(t), t)$. □

The point of Proposition 2 is that preemptive investment in a duopoly with two projects is less efficient than investment in a monopoly with a single project (needless to say, than that in monopoly with two projects). In other words, the preemptive competition becomes more severe if the numbers of both firms and projects increase by the same amount. This result is consistent with both the theoretical and empirical results in previous studies (cf. [7, 14]). We can say that the result extends previous finding in the sense that the model considers the follower's choice of an alternative project.

Let us consider two symmetric projects with the same initial value $x_1 = x_2$. We focus on the correlation coefficient ρ. In the sensitivity analysis in the model, this correlation is the most important because the previous strategic models with a one-dimensional state variable cannot reveal its effects. For example, what happens if the profits $X_1(t)$ and $X_2(t)$ are perfectly correlated, i.e., $\rho = 1$? In that case, no preemption occurs because the two projects generate the same profit. Indeed, the preemptive investment timing $\tau^2_{1,2}$ and the option value $V^2_{1,2}$ (see (14) and (17)) coincide with τ^1_i and V^1_i in monopoly with a single project, respectively. Taking this and (26) into account, we can easily show the following corollary.

Corollary 2 Consider the symmetric projects with $x_1 = x_2$. The following equalities hold for the correlation coefficient ρ:

$$\max_{\rho\in[-1,1]} V^2_{1,2}(x,t) = V^1_i(x_i,t) = \min_{\rho\in[-1,1]} V^1_{1,2}(x,t) \quad (i=1,2), \tag{28}$$

where $\rho = 1$ gives the maximum of $V^2_{1,2}(x,t)$ and the minimum of $V^1_{1,2}(x,t)$.

It should be noted that in a duopoly the option value $V^2_{1,2}(x,t)$, unlike the investment boundary $\partial\mathcal{S}^2_{1,2}(t)$ (see (15)), depends on the correlation coefficient ρ. Recall that in a monopoly a weaker correlation increases the option value by diversification. In contrast, in a duopoly a stronger correlation increases the strategic option value by moderation of the preemptive competition. The preemptive competition is moderated by a stronger correlation because the leader's advantage of project choice is reduced. This result is consistent with frequent takeovers in the pharmaceutical industry where there are uncorrelated potential targets.

3.2 Numerical examples

This subsection presents numerical examples of the results. Assume that $X(t)$ follows a symmetric GBM. I set the same base parameter values as [3]:

$$r = 6\%,\ \mu_1 = \mu_2 = 0\%,\ \sigma_1 = \sigma_2 = 20\%,\ I_1 = I_2 = 100,$$

which are also similar to those of [5]. All option values are computed for the initial point $x(t) = (100, 100)$.

Figure 2 illustrates the investment boundaries $\partial\mathcal{S}^2_{1,2}(t)$, 6 months, 1 year, 5 years, and 10 years before maturity. The investment boundary is composed of two parts (a) and (b) with a vertex on $(x^1_1(t), x^1_2(t))$ which is a pair of the investment triggers in a monopoly with a single project.[10] Needless to say, the investment region becomes larger as time to maturity. This implies that the option value increases with time to maturity. In fact, the option values 6 months, 1 year, 5 years, and 10 years before maturity are $V^2_{1,2} = 3.72, 5.15, 9.83$, and 12.16, respectively.

Let us now examine the effects of the correlation coefficient ρ, which is the most interesting feature in the model. Fix time to maturity as 1 year. Figure 3 depicts the investment boundary $\partial\mathcal{S}^2_{1,2}(t)$ in a duopoly with those of a monopoly with two projects, i.e., $\partial\mathcal{S}^1_{1,2}(t)$. The investment boundary in a duopoly, unlike that of a monopoly, is independent of the correlation. We see from Figure 3 that the investment region in a monopoly becomes smaller with the correlation. In other words, the monopolistic option value decreases with the diversification effects.

Table 1 presents the option values and percentages for a range of correlation coefficients ρ. The option value $V^2_{1,2}$ in a duopoly increases to $V^1_i = 7.15$ with ρ, while the option value $V^1_{1,2}$ in a monopoly drops to $V^1_i = 7.15$, as shown in the previous subsection. For a reasonable correlation $\rho = -0.2 \sim 0.8$ the option value in a duopoly is $40\% \sim 60\%$ of the monopolist with two projects, or equivalently $70\% \sim 80\%$ of the monopolist with a single project.

It should be noted that the results concerning the percentages $V^2_{1,2}/V^1_{1,2}, V^2_{1,2}/V^1_i$ are robust for time to maturity T, drift μ, and volatility σ. For example, for $\rho = 0$, the option value 10 years before maturity is $V^2_{1,2} = 12.16$, which is more than twice that of Table 1, while the percentages are $V^2_{1,2}/V^1_{1,2} = 42.72\%, V^2_{1,2}/V^1_i = 74.73\%$. The option value and the percentages for volatility $\sigma = 0.5$ and $\rho = 0$ are $V^2_{1,2} = 12.32$ and $V^2_{1,2}/V^1_{1,2} = 38.87\%, V^2_{1,2}/V^1_i = 69.99\%$, respectively.

In a valuation of a project by a real options approach, it sometimes occurs that a monopolistic model and strategic model generate polar valuations, namely, the value in the former is too high while that in the latter becomes too low. Then, a substantial problem for a practitioner arises. How can we judge the gap and which value is reliable? The model of the paper would provide us with a useful criterion in such a case. That is, we should evaluate the value of a project considering a potential alternative using the methodology of this paper.

[10] All computations in the paper use a bivariate version of the lattice binomial method with 500 time steps, and hence the discretization is rougher for longer times to maturity.

Table 1: Option values.

ρ	$V^2_{1,2}$	$V^1_{1,2}$	$V^2_{1,2}/V^1_{1,2}$	$V^2_{1,2}/V^1_i$
-0.4	4.99	13.41	37.27%	69.85%
-0.2	5.06	12.99	38.99%	70.78%
0	5.15	12.51	41.18%	72.01%
0.2	5.26	11.97	43.99%	73.6%
0.4	5.41	11.34	47.75%	75.7%
0.6	5.62	10.58	53.17%	78.64%
0.8	5.96	9.57	62.27%	83.33%
1	7.15	7.15	100%	100%

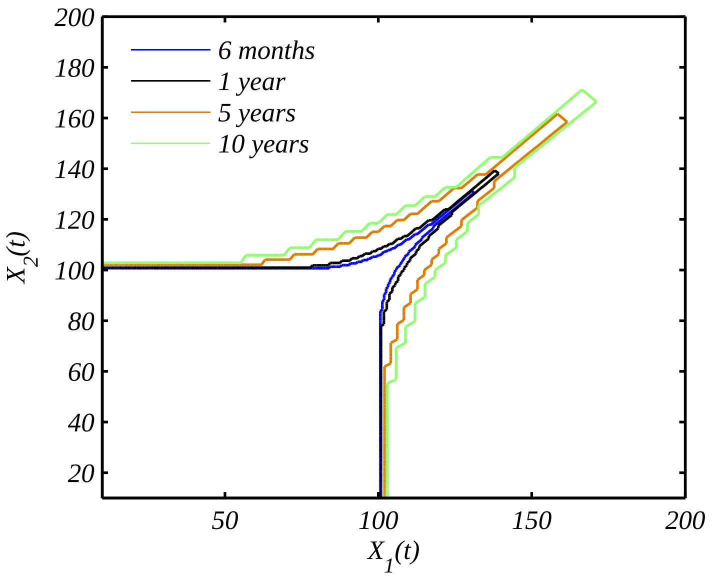

Figure 2: The preemptive boundary $\partial\mathcal{S}^2_{1,2}(t)$, $0.5, 1, 5$, and 10 years before maturity.

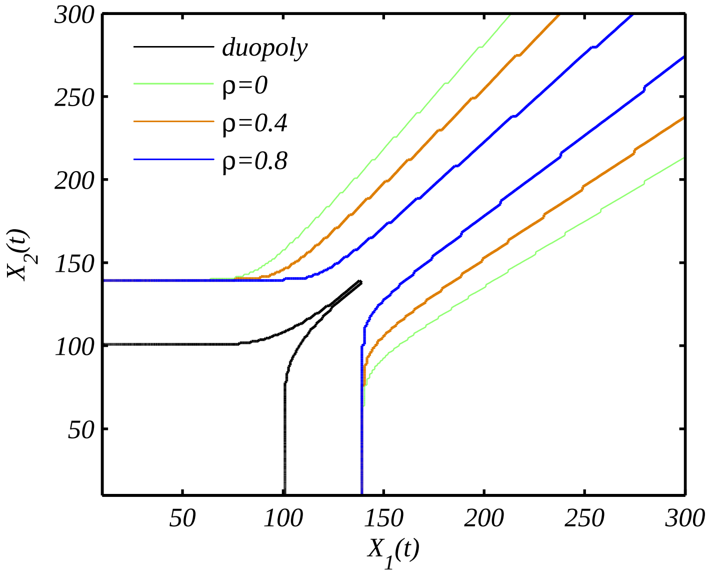

Figure 3: The investment boundary $\partial\mathcal{S}^2_{1,2}(t)$ and $\partial\mathcal{S}^1_{1,2}(t)$ for $\rho = 0, 0.4$, and 0.8.

3.3 n firms with m alternative projects

This subsection extends the results in Section 3.2 to an oligopoly of n firms with m alternative projects. Assume that:

$$X_i(0) \le I_i \quad (i = 1, 2, \ldots, m). \tag{29}$$

As in Section 3.2, we can solve the problem backward. We restrict our attention to the case of symmetric projects to avoid unnecessary confusion. For asymmetric projects the preemptive investment regions include the region corresponding to part (c) in (15), but the results below remain true.

Let us first look at a case where $n \le m$. Consider the last two firms' game among $m - n + 2$ projects. Assume that at time t with $X(t) = x$, $m - n + 2$ projects $i_{n-1}, \ldots, i_m$ remain. If one of the firms (the leader) invests in project i_k at time t, the follower's option value becomes $V^1_{i_{n-1},\ldots,\check{i_k},\ldots,i_m}(X(t), t)$, where $\check{i_k}$ denotes the exclusion of i_k. Recall that the superscript and the subscript represent the number of firms and the available projects, respectively.

Then, the region where the leader's payoff exceeds the follower's, denoted by $\mathcal{S}^2_{i_{n-1},\ldots,i_m}(t)$, is:

$$\mathcal{S}^2_{i_{n-1},\ldots,i_m}(t) = \cup_{k=n-1,\ldots,m}\{x_{i_k} - I_{i_k} \ge V^1_{i_{n-1},\ldots,\check{i_k},\ldots,i_m}(x, t)\}. \tag{30}$$

Under the assumption that:

$$X(t) = x \notin \mathcal{S}^2_{i_{n-1},\ldots,i_m}(t), \tag{31}$$

both firms attempt to invest at the preemptive time:[11]

$$\tau^2_{i_{n-1},\dots,i_m}(t) = \inf\{s \geq t \mid X(s) \in \partial\mathcal{S}^2_{i_{n-1},\dots,i_m}(s)\}$$

and gain the option value:

$$V^2_{i_{n-1},\dots,i_m}(x,t) = \mathbb{E}^x_t[\mathrm{e}^{-r(\tau^2_{i_{n-1},\dots,i_m}(t)-t)} \max_{k=n-1,\dots,m}(X_{i_k}(\tau^2_{i_{n-1},\dots,i_m}(t)) - I_{i_k})].$$

It is readily verified from (30) that the investment region $\mathcal{S}^2_{i_{n-1},\dots,i_m}(t)$ has the relationship:

$$\mathcal{S}^1_{i_{n-1},\dots,i_m}(t) \subset \mathcal{S}^1_{i_{n-1},\dots,\check{i_k},\dots,i_m}(t) \subset \mathcal{S}^2_{i_{n-1},\dots,i_m}(t)$$

for any k (cf. (24)). Thus we have:

$$\tau^2_{i_{n-1},\dots,i_m}(t) \leq \tau^1_{i_{n-1},\dots,\check{i_k},\dots,i_m}(t) \leq \tau^1_{i_{n-1},\dots,i_m}(t)$$

with respect to the timing (cf. (25)) and:

$$V^2_{i_{n-1},\dots,\check{i_k},\dots,i_m}(x,t) \leq V^2_{i_{n-1},\dots,i_m}(x,t) \leq V^1_{i_{n-1},\dots,\check{i_k},\dots,i_m}(x,t) \leq V^1_{i_{n-1},\dots,i_m}(x,t)$$

with respect to the value (cf. (26)).

Next let us turn back to the 3 firm game among projects $i_{n-2},\dots,i_m$ at time t with $X(t) = x$. If one of the firms (the leader) invests in project i_k at time t, the two followers' option value becomes $V^2_{i_{n-2},\dots,\check{i_k},\dots,i_m}(X(t),t)$ derived under assumption (31) earlier.

The region where the leader has the advantage of preemption, denoted by $\mathcal{S}^3_{i_{n-2},\dots,i_m}(t)$, is:

$$\mathcal{S}^3_{i_{n-2},\dots,i_m}(t) = \cup_{k=n-2,\dots,m}\{x_{i_k} - I_{i_k} \geq V^2_{i_{n-2},\dots,\check{i_k},\dots,i_m}(x_{i_k},t)\} \tag{32}$$

Under the assumption that:

$$X(t) = x \notin \mathcal{S}^3_{i_{n-2},\dots,i_m}(t),$$

all the firms attempt to invest at the preemptive time:

$$\tau^3_{i_{n-2},\dots,i_m}(t) = \inf\{s \geq t \mid X(s) \in \partial\mathcal{S}^3_{i_{n-2},\dots,i_m}(s)\}$$

and gain the option value:

$$V^3_{i_{n-2},\dots,i_m}(t) = \mathbb{E}^x_t[\mathrm{e}^{-r(\tau^3_{i_{n-2},\dots,i_m}(t)-t)} \max_{k=n-2,\dots,m}(X_{i_k}(\tau^3_{i_{n-2},\dots,i_m}(t)) - I_{i_k})].$$

Note that the state variable $X(\tau^3_{i_{n-2},\dots,i_m}(t))$ satisfies (31) necessary at the initial point in the duopoly game.

It readily follows from (32) that the relationship:

$$\mathcal{S}^2_{i_{n-2},\dots,i_m}(t) \subset \mathcal{S}^2_{i_{n-2},\dots,\check{i_k},\dots,i_m}(t) \subset \mathcal{S}^3_{i_{n-2},\dots,i_m}(t)$$

[11] To be precise, similar assumptions to footnote 7 and Assumption (iv) are necessary.

hold for any k. By this we have:

$$\tau^3_{i_{n-2},\dots,i_m}(t) \le \tau^2_{i_{n-2},\dots,\check{i_k},\dots,i_m}(t) \le \tau^2_{i_{n-2},\dots,i_m}(t)$$

and

$$V^3_{i_{n-2},\dots,\check{i_k},\dots,i_m}(x,t) \le V^3_{i_{n-2},\dots,i_m}(x,t) \le V^2_{i_{n-2},\dots,\check{i_k},\dots,i_m}(x,t) \le V^2_{i_{n-2},\dots,i_m}(x,t).$$

By backward induction we can show the following results in n firms' game at time 0 with $X_i(0) = x_i < I_i$ ($i = 1, 2, \dots, m$). The preemptive investment region $\mathcal{S}^n_{1,\dots,m}(t)$, the preemptive timing $\tau^n_{1,\dots,m}$, and the option value $V^n_{1,\dots,m}(x,t)$ are:

$$\mathcal{S}^n_{1,\dots,m}(t) = \cup_{k=1,\dots,m}\{x_{i_k} - I_{i_k} \ge V^{n-1}_{1,\dots,\check{k},\dots,m}(x_k,t)\},$$

$$\tau^n_{1,\dots,m} = \inf\{t \ge 0 \mid X(t) \in \partial\mathcal{S}^3_{i1,\dots,m}(t)\},$$

and

$$V^n_{1,\dots,m}(x,t) = \mathbb{E}^x_t[\mathrm{e}^{-r(\tau^n_{1,\dots,m}-t)} \max_{k=1,\dots,m}(X_k(\tau^n_{1,\dots,m}) - I_k)],$$

respectively. The following relationships and inequalities hold:

$$\mathcal{S}^{n-1}_{1,\dots,m}(t) \subset \mathcal{S}^{n-1}_{1,\dots,\check{k},\dots,m}(t) \subset \mathcal{S}^n_{1,\dots,m}(t),$$

$$\tau^n_{1,\dots,m}(t) \le \tau^{n-1}_{1,\dots,\check{k},\dots,m}(t) \le \tau^{n-1}_{1,\dots,m}(t),$$

and

$$V^n_{1,\dots,\check{k},\dots,m}(x,t) \le V^n_{1,\dots,m}(x,t) \le V^{n-1}_{1,\dots,\check{k},\dots,m}(x,t) \le V^{n-1}_{1,\dots,m}(x,t).$$

The results are a generalization of Proposition 2. Again, the point is that the preemptive competition intensifies if the numbers of both firms and projects increase by the same amount. It is intuitively explained that in the preemptive equilibrium all the firms are dragged into a scenario with the worst project.

Let us focus on the symmetric projects with the same initial value. The perfect correlation gives $V^1_1(x,t)$ in a monopoly with a single project. For $n = m$, as in Corollary 2, $V^1_1(x,t)$ agrees with the maximum of $V^n_{1,\dots,n}(x,t)$. On the other hand, it does not necessarily hold for $n < m$. This is because the last firm's monopolistic value $V^1_{i_n,\dots,i_m}(x,t)$ decreases with the correlation. Generally, the sensitivity of the correlation in an oligopoly depends on a trade-off between moderation of the preemptive competition (positive effect) and a decrease in a value of project choice (negative effect).

In the case where the number of firms is larger than that of projects, i.e., $n > m$, it can be easily shown that at each stage all the remaining firms attempt to invest with the zero-NPV timing and hence obtain nothing. The outcome is precisely the same as that in Section 2.2.

4 Conclusion

This paper has investigated the preemptive equilibrium in a real options model with the multidimensional state variable, which represents potential alternative projects. The results are summarized as follows.

First, preemptive investment takes place earlier and the option value becomes lower if the numbers of both firms and projects increase by the same amount. The result can be regarded as extension of the previous results with a one-dimensional state variable as well as being consistent with empirical findings.

Second, the preemptive competition is moderated by the correlation among profits from projects. The effect contrasts with that in a monopoly where a strong correlation decreases the value of project choice. The sensitivity of the correlation to the project value in an oligopoly depends on a trade-off between moderation of the preemptive competition and a decrease in the value of project choice.

Third, the strategic option values with a symmetric alternative is 40% $\sim$ 60% of a monopoly with two alternative projects, or equivalently 70% $\sim$ 80% of a monopoly with a single project. This indicates the importance of the existence of a potential alternative. Although monopolistic and strategic models with a one-dimensional state variable tend to calculate extreme values, the method in this paper allows a reasonable valuation taking account of the follower's potential alternative investment.

Lastly, I should point out important but difficult topics for future research. The paper assumes that profits from the projects are not sensitive to a competitor's alternative investment. However, the leader's cash flow could be affected by the follower's initiation of a project even if it is an alternative project that is different from the leader's project. Also, the projects may have different maturity in some cases.

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