GCOE Discussion Paper Series

Global COE Program

Human Behavior and Socioeconomic Dynamics

Discussion Paper No. 371

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May 2015

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Fiscal Policy under Long-run Stagnation: A New Interpretation of the Multiplier Effect*

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May 20, 2015

*This is a significantly revised version of ISER Discussion Paper, No. 779, titled "A Reinterpretation of the Keynesian Consumption Function and Multiplier Effect". An earlier version of this paper was presented at the third Graz Schumpeter Summer School 2011 held in the University of Graz, a seminar in Tokyo Institute of Technology and the conference on Economic Analysis of Dynamics and Preferences. Helpful comments from H. Kurz and the participants of the three seminars are gratefully acknowledged. We are grateful to financial supports from the Joint Usage/Research Center, the Ministry of Education, Culture, Sports, Science and Technology. In addition, Ono's research is financially supported by the Grants-in-Aid for Scientific Research, the Japan Society for the Promotion of Science.

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Abstract

We develop a Keynesian cross analysis with a dynamic optimization setting that explains long-run stagnation caused by aggregate demand deficiency. We show that an increase in government purchases boosts GDP through a multiplier process, but the implication is quite different from the conventional Keynesian one. It works not through an increase in disposable income but through moderation of deflation. Thus, countries that have lapsed into long-run stagnation should expand government spending that directly creates employment in order to reduce the deflationary gap.

Keywords: Aggregate Demand, Consumption Function, Keynesian Cross, Multiplier Effect, Persistent Unemployment

JEL Classification Codes: E12, E24, E62

1 Introduction

When countries fall into economic depression, their governments tend to increase spending in order to expand aggregate demand and reduce unemployment. In this context policy makers mostly have the Keynesian multiplier theory in mind. However, this theory has been criticized because the Keynesian consumption function lacks microeconomic foundations. In response, many economists have analyzed the multiplier effect in various frameworks with microeconomic foundations. Recently, in particular, it has actively been studied because of the Great Recession. For example, using a New Keynesian DSGE model with no unemployment, Christiano et al. (2011) find that a large multiplier effect appears under zero interest rates. Eggertsson and Krugman (2012) analyze a short-run deficiency of aggregate demand due to a borrowing constraint and show the existence of the multiplier effect. Monacelli et al. (2010) examine the multiplier effect in the presence of unemployment arising because of matching frictions, not aggregate demand deficiency.[1] In contrast with these studies, we analyze the multiplier effect that appears when aggregate demand deficiency occurs and creates unemployment in the long run.

Recently, long-run stagnation caused by deficiency of aggregate demand has attracted attention from economists. This is known as "secular stagna-

[1]Empirical studies have also been expanding and various results have been obtained. For instance, Ilzetzki et al. (2013) find that the magnitude of the multiplier depends on the degree of development, openness to trade and so on. Jha et al. (2014) conclude that in developing Asia tax cuts may be more effective as a countercyclical policy than government spending increases. According to Hong and Li (2015), the multipliers of public works investment and consumption vouchers implemented in Taiwan were 1.94 and 1.47, respectively.

tion", originally advocated by Alvin Hansen and recently revived by Lawrence Summers (see, e.g., Eggertsson and Mehrotra (2014) for details). Summers (2014) considers the US economy since the Lehman shock to be in secular stagnation, and suggests that increasing aggregate demand is a way of boosting the economy. Several economists have attempted to theoretically model secular stagnation. For example, Eggertsson and Mehrotra (2014) develop an overlapping generations model with a borrowing constraint and show that a persistent deleveraging shock leads to a persistent liquidity trap where aggregate demand deficiency and unemployment occur. Moreover, they show that the borrowing constraint, which makes Ricardian equivalence invalid, yields a large multiplier effect when government spending is financed by issuing bonds. Michaillat and Saez (2014) construct a job search model where wealth holdings yield direct utility. In their model the marginal utility of wealth becomes constant, which plays a crucial role in creating persistent stagnation.

However, prior to these studies, Ono (1994, 2001) presents long-run stagnation in a dynamic general equilibrium model with optimizing agents.[2] He shows that if a desire to save money is insatiable (i.e., the marginal utility of money stays positive), aggregate demand deficiency and involuntary unem-

[2]Recently, Ono's model has been extended in various analyses. For example, Matsuzaki (2003) and Hashimoto (2004) consider heterogeneous agents in the model and explore the effects of redistribution. Johdo (2006) combines the model with a spatial model and investigates the relationship between geographical space and stagnation. Johdo (2008a) introduces monopolistic competition into the model and analyzes the effects of production subsidies. Johdo (2009) incorporates habit formation into the model and examines the relationship between habit formation and stagnation. Ono (2006, 2014), Johdo (2008b), Johdo and Hashimoto (2009) and Hashimoto (2011) extend the model to open-economy models and examine the international spill-over effects of various macro- and microeconomic policies. Using the model, Hashimoto and Ono (2011) study pro-population policies.

ployment arise in the steady state. Thus, the approach of Michaillat and Saez (2014) is somewhat similar to that of Ono (1994, 2001). Furthermore, Ono (2010) discusses the mechanism of Japan's long-lasting stagnation since the early 1990s in his framework. Using a similar model, Murota and Ono (2012) comprehensively explain various phenomena observed in the Great Depression and Japan's stagnation, including involuntary unemployment, deflation, zero interest rates and excess bank reserves.

This paper explores what fiscal policy is effective for stimulating an economy falling into such long-run stagnation. For this purpose, we examine the multiplier effect in the framework of Ono (1994, 2001). Whereas Ono (1994, 2001) does not consider the multiplier effect, we derive a consumption function from household optimizing behavior and establish an alternative Keynesian cross model. In the model, an increase in government purchases affects consumption and GDP through a multiplier-like process, but a tax cut (or a transfer increase) has no effect on either of them. The influences of various parameters such as liquidity preference, potential output and wage flexibility on the magnitude of the multiplier are also investigated.

The multiplier analysis in this paper is quite different from the conventional Keynesian or New Keynesian models in the following respects. First, it considers persistent unemployment resulting from aggregate demand deficiency. Second, our consumption function represents not the conventional Keynesian relationship between disposable income and consumption but the effect of an increase in output on consumption through mitigation of deflation—i.e., an increase in actual output relative to potential output narrows the deflationary gap and mitigates deflation, which makes holding

money more costly and thereby stimulates consumption. Third, in contrast with Eggertsson and Krugman (2012) and Eggertsson and Mehrotra (2014), Ricardian equivalence holds in this paper and yet the multiplier effect of government purchases arises.[3] The magnitude of the multiplier effect is independent of the means of financing: collecting taxes or issuing bonds. Finally, since our consumption function is founded on household optimizing behavior, changes in technology and preference parameters affect the form of the consumption function and vary the magnitude of the multiplier effect.

2 The Model

We start with a brief summary of the model, which is based on Ono (1994, 2001). The government finances government purchases g and interest payments $r_t b_t$, where r_t is the real interest rate on government bonds b_t, by collecting a lump-sum tax τ_t and issuing new bonds $\dot{b}_t$. Thus we have

$$g + r_t b_t = \tau_t + \dot{b}_t,$$

where τ_t denotes a lump-sum transfer if it is negative. Note that b_t and τ_t are adjusted so that the no-Ponzi-game condition is satisfied. The nominal money supply M_t is kept constant at $\overline{M}$, for simplicity, and hence the rate of change in real money balances m_t $(= \overline{M}/P_t)$, where P_t is the commodity price, is given by

$$\frac{\dot{m}_t}{m_t} = -\pi_t, \tag{1}$$

[3]Using overlapping generations models, Bénassy (2007a, b) argues that non-Ricardian equivalence is important for the appearance of the multiplier effect. Galí et al. (2007) develop a New Keynesian model with non-Ricardian consumers, and show that the presence of such consumer causes the multiplier effect to arise.

where π_t ($\equiv \dot{P}_t/P_t$) is the inflation rate.

The household sector maximizes the following lifetime utility:

$$\int_0^\infty [u(c_t) + v(m_t)] \exp(-\rho t) dt,$$

subject to

$$\dot{a}_t = r_t a_t + w_t n_t - c_t - R_t m_t - \tau_t,$$

where $u(c_t)$ is the utility of consumption c_t, $v(m_t)$ is the utility of real money holdings m_t, ρ is the subjective discount rate, a_t ($= b_t + m_t$) is real total assets, w_t is the real wage and R_t ($= r_t + \pi_t$) is the nominal interest rate. As usual, we assume that the first derivatives of $u(c_t)$ and $v(m_t)$ are positive and that the second derivatives are negative. The household inelastically supplies its labor endowment $\overline{n}$. However, as shown below, it may not be fully employed. Therefore, employment n_t is given by the short side of labor demand n_t^d and labor supply $\overline{n}$:

$$n_t = \min\left\{n_t^d, \overline{n}\right\}. \tag{2}$$

The optimality condition for this utility-maximization problem is

$$\rho + \eta(c_t)\frac{\dot{c}_t}{c_t} + \pi_t = R_t = \frac{v'(m_t)}{u'(c_t)}, \tag{3}$$

where $\eta(c_t) \equiv -[u''(c_t)c_t]/u'(c_t)$. The first equality in (3) indicates the Ramsey equation and the second implies portfolio choice between bonds and money.

While the commodity price P_t is perfectly flexible, the adjustment of the nominal wage W_t is assumed to be sluggish as follows:

$$\frac{\dot{W}_t}{W_t} = \alpha\left(\frac{n_t^d}{\overline{n}} - 1\right), \tag{4}$$

where α (> 0) denotes flexibility of the adjustment, in order to take into account the possibility of unemployment due to demand deficiency. See Ono and Ishida (2014) for a microeconomic foundation of this wage adjustment.[4] It is noteworthy that recently studied Phillips curves, such as the New Classical Phillips curve, the New Keynesian Phillips curve and the hybrid of the forward- and backward-looking Phillips curves, are not appropriate for the analysis of persistent stagnation due to aggregate demand deficiency because the possibility of market disequilibrium is not allowed from the beginning or because the inflation–deflation rate cumulatively expands as long as market disequilibrium exists.[5] Thus, the possibility of unemployment in a steady state, which we focus on, is intrinsically eliminated under these Phillips curves.

The firm sector has linear technology:

$$y_t = \theta n_t, \tag{5}$$

where y_t is output, θ is labor productivity, which is constant, and n_t is labor input. Since the production function is linear in labor, the firm sector decides

[4]They apply various fairness concepts to the mechanism of nominal wage setting and obtain nominal wage movements that depend on the unemployment rate if unemployment exists. They first obtain the dynamics of fair wages and find that with unemployment, firms set wages to be the same as the fair wages so as to urge their employees to work efficiently. In this setting $1/\alpha$ is the average duration of employment because wage adjustments are due to alternation of incumbent workers, whose fair wages depend on their past and their rivals' wages, by new recruits who have no preconceptions about fair wages.

[5]See Woodford (2003) for the properties of these Phillips curves.

labor demand n_t^d and commodity supply y_t to be

$$\begin{aligned} n_t^d = \infty, \quad y_t = \infty \quad &\text{if} \quad \theta > W_t/P_t, \\ 0 < n_t^d < \infty, \quad 0 < y_t < \infty \quad &\text{if} \quad \theta = W_t/P_t, \\ n_t^d = 0, \quad y_t = 0 \quad &\text{if} \quad \theta < W_t/P_t. \end{aligned}$$

Since W_t can only adjust sluggishly according to (4) while P_t is flexible, P_t instantaneously rises if $\theta < W_t/P_t$ because commodity supply is zero, and drops if $\theta > W_t/P_t$ because firms try to expand their shares by undercutting the price. Consequently, P_t takes the following value:[6]

$$\theta = \frac{W_t}{P_t} \left(\equiv w_t\right), \tag{6}$$

which straightforwardly implies

$$\pi_t = \frac{\dot{W}_t}{W_t}. \tag{7}$$

When (6) holds, y_t satisfies

$$c_t + g = y_t = \theta n_t, \tag{8}$$

where n_t is a positive finite value satisfying (2): $n_t = \min\{n_t^d, \overline{n}\}$.

In the following analysis we focus on the case where unemployment occurs, i.e., $n_t = n_t^d < \overline{n}$. In this case, (4), (5) and (7) yield

$$\pi_t = \frac{\dot{W}_t}{W_t} = \alpha\left(\frac{n_t}{\overline{n}} - 1\right) = \alpha\left(\frac{y_t}{\overline{y}} - 1\right), \tag{9}$$

where $\overline{y}$ denotes full-employment (or potential) output:

$$\overline{y} \equiv \theta\overline{n}.$$

[6]From (6), the profits and the firm value are zero, which implies that the household sector's total assets a_t consist of only m_t and b_t, as mentioned about household behavior.

Note that the dynamic behavior of the economy is characterized by differential equations for consumption and real money balances.[7]

3 The Consumption Function and the Multiplier Effect

In this section, we first show long-run stagnation with aggregate demand deficiency and unemployment and then propose a new analysis of the multiplier effect.

3.1 The Stagnation Steady State

Following Ono (1994, 2001), we assume that the marginal utility of money has a positive lower bound β:

$$\lim_{m\to\infty} v'(m) = \beta > 0, \tag{10}$$

which creates the Keynesian liquidity trap. In fact, from the second equality of (3):

$$R = v'(m)/u'(c),$$

we find that R approaches a positive lower bound $\beta/u'(c)$ even when money demand m diverges to infinity.

[7] From (1), (3), (8) and (9), we obtain $\dot{c}_t$ and $\dot{m}_t$ as functions of c_t and m_t, respectively:

$$\dot{c}_t = \frac{c_t}{\eta(c_t)}\left[-\alpha\left(\frac{c_t+g}{\overline{y}}-1\right)+\frac{v'(m_t)}{u'(c_t)}-\rho\right],$$
$$\dot{m}_t = -\alpha m_t\left(\frac{c_t+g}{\overline{y}}-1\right),$$

which yields $\{c_t\}_{t=0}^{\infty}$ and $\{m_t\}_{t=0}^{\infty}$ and consequently the sequences of all variables, including

$$R_t = \frac{v'(m_t)}{u'(c_t)}, \quad \pi_t = \alpha\left(\frac{c_t+g}{\overline{y}}-1\right), \quad r_t = R_t - \pi_t.$$

See in detail Ono (1994, 2001) for the present dynamics.

When a money-in-the-utility-function model is adopted,[8] it is almost always assumed that the marginal utility of money eventually decreases to zero as money holdings increase.[9] However, Ono et al. (2004) empirically find that the assumption (10) is better supported than the zero lower bound. Theoretically, Murota and Ono (2011) show that it remains positive if money is a status symbol, and Murota and Ono (2012) show that it reaches a positive lower bound when nominal interest rates are zero in a model that incorporates both money and deposits into a utility function. Camerer et al. (2005) mention the possibility that the utility of money has little association with consumption. If this is true, it may be possible that the marginal utility of money, in contrast to that of consumption, does not decline to zero. Furthermore, Michaillat and Saez (2014) consider the case where the marginal utility of wealth consisting of money and bonds always becomes positively constant in a model where accumulating wealth is an end in itself or a way of gaining social status.

We further assume that $\overline{y}$ is so large, ρ is so small or β is so large as to satisfy

$$\rho < \frac{\beta}{u'(\overline{y} - g)}, \tag{11}$$

which implies that from (3) the marginal benefit of money (the liquidity premium) exceeds that of consumption (the time preference rate ρ) if steady-

[8]The dominant view of money in contemporary economics is that people do not derive utility directly from money. However, Camerer et al. (2004, 2005) argue that money may directly provide utility on the ground of neuroscientific evidence that money and various reinforcers, i.e., attractive faces, funny cartoons, cultural objects and drugs, activate the same dopaminergic reward circuitry of the brain.

[9]Devoe et al. (2013) present evidence that may conflict with the assumption of the decreasing marginal utility of money. They find that individuals who earn more money from labor view money as more important.

state consumption c is at the full-employment level $\overline{y}-g$. Therefore, in order for the optimality condition (3) to be satisfied, c must be set to be lower than $\overline{y}-g$, which leads to $n=n^d<\overline{n}$ and $\pi<0$ from (2), (8) and (9). Intuitively, the household prefers saving money rather than consuming enough to attain full employment, which gives rise to aggregate demand deficiency, involuntary unemployment and deflation. In this case, from (1), (2), (3), (8) and (9), we obtain the following stagnation steady state:

$$\begin{aligned} &c+g=y<\overline{y}, \quad n=n^d<\overline{n}, \quad \pi=\alpha\left(\frac{y}{\overline{y}}-1\right)<0, \\ &\frac{\dot{m}}{m}=-\pi>0, \quad \rho+\pi=R=\frac{\beta}{u'(c)}, \quad r=\rho. \end{aligned} \tag{12}$$

From the first, third and fifth properties of (12), steady-state consumption c satisfies

$$\Phi(c)\equiv\rho+\alpha\left(\frac{c+g}{\overline{y}}-1\right)-\frac{\beta}{u'(c)}=0,$$

as shown by Ono (1994, 2001). From (11), one obtains

$$\Phi(\overline{y}-g)=\rho-\frac{\beta}{u'(\overline{y}-g)}<0.$$

Therefore, in order that c uniquely exists within $(0,\overline{y}-g)$, it must be valid that

$$\begin{aligned} &\Phi(0)=\rho+\alpha\left(\frac{g}{\overline{y}}-1\right)\geq\rho-\alpha>0, \\ &\Phi'(c)=\frac{\alpha}{\overline{y}}+\frac{\beta u''(c)}{\left[u'(c)\right]^2}<0, \end{aligned} \tag{13}$$

where the condition that $\rho-\alpha>0$ is required so that $\Phi(0)>0$ even when $g=0$. Given c, we uniquely obtain y and all the other endogenous variables.

In what follows we present a new interpretation of the multiplier effect, which is not examined by Ono (1994, 2001). Consumption c is given as a

function of output y, as in the Keynesian consumption function, and the determination of GDP (output y) is expressed in an alternative way, as in the 45-degree diagram. An increase in government purchases boosts GDP through a new multiplier process, and the magnitude of the multiplier is influenced by various parameters.

3.2 The Consumption Function and Keynesian Cross

We begin by expressing c as a function of y. From the third and fifth properties of (12), we obtain

$$\rho + \alpha \left(\frac{y}{\overline{y}} - 1 \right) = \frac{\beta}{u'(c)}, \tag{14}$$

which gives the consumption function in the present model:

$$c = u'^{-1} \left(\frac{\beta}{\rho - \alpha + \dfrac{\alpha}{\overline{y}} y} \right) \equiv c(y; \alpha, \overline{y}, \beta). \tag{15}$$

In (15) c does not depend on m, implying that the Pigou effect does not work when the economy is caught in the above-mentioned liquidity trap. This is the cause of the persistent aggregate demand deficiency. In contrast, in the usual money-in-the-utility-function model that does not consider the liquidity trap, from the second equality of (3) consumption is represented as a function of real money holdings and the nominal interest rate:

$$c = \phi(m, R).$$

Therefore, the Pigou effect works:

$$\frac{\partial c}{\partial m} = \frac{u'(c)v''(m)}{u''(c)v'(m)} > 0,$$

and c eventually reaches the full-employment level as deflation continues and m expands.

Differentiating (15) with respect to y yields

$$c_y \equiv \frac{\partial c(y;\alpha,\overline{y},\beta)}{\partial y} = -\frac{\alpha\left[u'(c)\right]^2}{\beta\overline{y}u''(c)} > 0. \tag{16}$$

From (13), (15) and (16), one obtains

$$c(0;\alpha,\overline{y},\beta) = u'^{-1}\left(\frac{\beta}{\rho-\alpha}\right) > 0, \quad 0 < c_y < 1. \tag{17}$$

These properties are mathematically the same as those of the Keynesian consumption function. If $u(c) = \log c$, for example, the consumption function represented by (15) becomes linear:

$$c = c_0 + c_y y, \quad \text{where} \quad c_0 = \frac{\rho-\alpha}{\beta} \quad \text{and} \quad c_y = \frac{\alpha}{\beta\overline{y}}, \tag{18}$$

which indeed looks like the textbook consumption function.

The implications of the properties given in (17) are, however, quite different from the conventional ones. $c(0;\alpha,\overline{y},\beta)$ does not imply autonomous consumption, nor does c_y represent the marginal propensity to consume. $c(0;\alpha,\overline{y},\beta)$ is simply the magnitude of consumption when $y = 0$ (and then $\pi = -\alpha$ from the third property of (12)). c_y indicates the effect on consumption c of an increase in output y through mitigation of deflation. An increase in output requires an increase in employment, which mitigates deflations in the nominal wage and price, as is clear from (9). This makes holding money more costly and thereby stimulates consumption ($dc/d\pi = -(u')^2/(\beta u'') > 0$ from the fifth property of (12)). Thus, consumption c depends not on disposable income $y - \tau$ but rather on output y.

The consumption function (15), which satisfies the properties in (17), is valid only under long-run stagnation in which aggregate demand deficiency and involuntary unemployment persist. In fact, the Keynesian consumption function was observed during the 1930s (the Great Depression) in the US (see, e.g., Davis, 1952; Shapiro, 1988; Emerson, 2011). Meanwhile, when full-employment output is realized ($y = \overline{y}$), we have the following linear relationship:

$$c = \overline{y} - g,$$

where its slope equals one ($dc/d\overline{y} = 1$) and its intercept takes a negative value. Shapiro (1988) and Emerson (2011) find these properties from the US data in the period after World War II.

Using the consumption function (15), we present an analysis similar to the Keynesian cross. By substituting (15) into the first property of (12), we obtain

$$c(y; \alpha, \overline{y}, \beta) + g = y, \tag{19}$$

where the left-hand side (LHS) denotes aggregate demand and the right-hand side (RHS) denotes aggregate supply. (11), (15) and (17) imply that

$$c(0; \alpha, \overline{y}, \beta) + g > 0, \quad c(\overline{y}; \alpha, \overline{y}, \beta) + g < \overline{y},$$

and that the LHS of (19) has a positive slope less than one. Therefore, steady-state GDP is uniquely determined. This is illustrated in figure 1, where it is given by y^*. Note that the figure presents the case of such a linear consumption function as (18)

As shown in the figure, GDP is determined mathematically in the same manner as the conventional Keynesian cross. However, the positively sloped

consumption function does not imply the Keynesian income effect on consumption. In the present framework y is output rather than disposable income. As output y increases, the deflationary gap shrinks and deflation declines. This decline in deflation stimulates household consumption, which leads to an increase in aggregate demand. We will discuss the multiplier effect generated by this process in the next subsection.

3.3 The Multiplier Effect

From (19), we obtain seemingly the same multiplier effect as that of the conventional Keynesian model:

$$\frac{dy}{dg} = \frac{1}{1 - c_y} > 1, \quad \frac{dc}{dg} = \frac{c_y}{1 - c_y} > 0. \tag{20}$$

However, the multiplier process substantially differs from the conventional one.[10] An increase in government purchases g by dg initially boosts output y by dg. It reduces the deflationary gap and moderates deflation, which urges households to increase consumption c by $c_y dg$. The increase in c additionally boosts y by $c_y dg$, which again moderates deflation and increases c by $(c_y)^2 dg$.[11] Such interactions between the moderation of deflation and the increase in consumption repeatedly occur, cumulatively increasing consumption and output, and eventually leading to (20).[12]

[10] Ono (2011) discusses the implication of the conventional multiplier effect and argues that the multiplier effect of a fiscal expansion may be seriously misunderstood. He shows that even in the conventional Keynesian framework, the true effect of fiscal spending depends not on the amount of spending but on the benefit directly generated by the spending.

[11] Note that this is not the actual adjustment process over time but the conceptual process, as is the case of the conventional multiplier effect. The economy, in fact, immediately jumps to a new steady state when g unexpectedly changes in the stagnation steady state.

[12] Mankiw (1988) also obtains a multiplier effect that is mathematically similar to the conventional Keynesian one and explicitly shows the multiplier process in a general equi-

In the present model, Ricardian equivalence holds and hence the magnitude of the multiplier effect does not depend on the means of financing: issuing government bonds or collecting the lump-sum tax τ. This is clear from the consumption function (15), where c does not depend on τ. Moreover, from (15), a change in τ affects neither consumption nor GDP:[13]

$$\frac{dc}{d\tau} = 0, \quad \frac{dy}{d\tau} = 0.$$

Therefore, in order to stimulate the economy by fiscal expansions, the government has to allocate the budget not to direct transfers or tax cuts but to commodity or service purchases that create new employment, because this increase in employment moderates deflations in the nominal wage and price.

Since the present multiplier effect works through moderation of deflation, it disappears in the typical Keynesian case where nominal wages and prices are fixed. In fact, from (15), if $\alpha = 0$ then c is constant:

$$c = u'^{-1}(\beta/\rho),$$

and hence neither g nor τ affects c. This result is the same as the conventional Keynesian case with a balanced budget. It is because Ricardian equivalence holds, which essentially leads to the same situation as that where the government adopts a balanced budget in the conventional Keynesian model.

librium model. However, his model is static, and neither aggregate demand deficiency nor unemployment exists. Moreover, imperfect competition among firms is crucial for creating the multiplier effect.

[13]Feldstein (2009) and Shapiro and Slemrod (2009) find that the 2008 tax rebate in the US was not very effective in increasing private consumption. According to Shapiro and Slemrod (2009, table 1), only one-fifth of households receiving the rebate planned to spend most of it while the remaining four-fifths planned to use it mostly to save or to pay off debt.

We have so far considered the case where g is wasteful. Let us briefly discuss the case where g increases productivity or utility. When the labor productivity θ is a function of g:

$$\theta = \theta(g), \quad \theta'(g) > 0,$$

an increase in g raises full employment output $\overline{y}$ $(= \theta(g)\overline{n})$, which expands the deflationary gap and worsens deflation. Thus, the effect of g on y via a change in θ is negative, making the multiplier effect lower (or even negative).

When the utility is given by $u(c, g)$, (14) is rewritten as

$$\rho + \alpha\left(\frac{y}{\overline{y}} - 1\right) = \frac{\beta}{u_c(c, g)} \quad \Longrightarrow \quad c = c(y; \alpha, \overline{y}, \beta, g),$$

from which we obtain

$$c_g = \frac{u_{cg}}{-u_{cc}}.$$

(19) is replaced by

$$c(y; \alpha, \overline{y}, \beta, g) + g = y,$$

and the multiplier effect on output given in (20) turns to be

$$\frac{dy}{dg} \equiv \frac{1 + c_g}{1 - c_y} = \frac{1 + \dfrac{u_{cg}}{-u_{cc}}}{1 - c_y},$$

i.e., it depends on u_{cg}. If c and g are complementary (i.e., $u_{cg}(c, g) > 0$), an increase in g encourages private consumption, which enhances the multiplier effect. If they are substitutes (i.e., $u_{cg}(c, g) < 0$), the multiplier effect is smaller (or may even be negative) because g is substituted for c. In particular, if c and g are perfect substitutes (i.e., $u(c, g) = u(c+g)$), $u_{cg} = u_{cc}$ and hence the multiplier is zero, implying that an increase in such government spending

completely crowds out private consumption. If $u_{cg}(c, g) = 0$, the multiplier effect is of the same magnitude as that in the case where g is wasteful, while the utility increases.

3.4 The Comparative Statics

Because the consumption function is derived from household optimizing behavior, we can analyze the effects on the consumption function of changes in various preference and technology parameters such as wage flexibility α, potential output $\overline{y}$ and liquidity preference β. From (15), they are

$$\begin{aligned} c_\alpha &\equiv \frac{\partial c(y;\alpha,\overline{y},\beta)}{\partial \alpha} = -\frac{[u'(c)]^2}{\beta u''(c)}\left(\frac{y}{\overline{y}} - 1\right) < 0, \\ c_{\overline{y}} &\equiv \frac{\partial c(y;\alpha,\overline{y},\beta)}{\partial \overline{y}} = \frac{\alpha y[u'(c)]^2}{\beta \overline{y}^2 u''(c)} < 0, \\ c_\beta &\equiv \frac{\partial c(y;\alpha,\overline{y},\beta)}{\partial \beta} = \frac{u'(c)}{\beta u''(c)} < 0. \end{aligned} \tag{21}$$

When y is given, an increase in α accelerates the decline in the nominal wage whereas an increase in $\overline{y}$ expands the deflationary gap, both of which aggravate deflation and thus urge the household to save money and reduce consumption. An increase in β straightforwardly induces the household to save more and consume less.

From (19) and (21), the effects of α, $\overline{y}$ and β on GDP are

$$\frac{dy}{d\alpha} = \frac{c_\alpha}{1 - c_y} < 0, \quad \frac{dy}{d\overline{y}} = \frac{c_{\overline{y}}}{1 - c_y} < 0, \quad \frac{dy}{d\beta} = \frac{c_\beta}{1 - c_y} < 0. \tag{22}$$

The first and second properties, respectively, show that more flexible wage adjustments lower GDP, which implies the "paradox of flexibility", and that an increase in potential output decreases GDP, which implies the "paradox

of toil", both of which are discussed by Eggertsson and Krugman (2012).[14] The third is the "evil of thrift" mentioned by Keynes (1936, p. 358): an increase in the household's desire to hold money reduces GDP.

We next explore the influences of α, $\overline{y}$ and β on the magnitude of the multiplier effect dy/dg. Because the multiplier effect is given by the first equation of (20) and from (15), (16) and (19) c_y is expressed as a function of α, $\overline{y}$ and β, one obtains

$$\frac{d}{di}\left(\frac{dy}{dg}\right) = \frac{1}{(1-c_y)^2}\left(\frac{\partial c_y}{\partial i} + \frac{\partial c_y}{\partial y}\frac{dy}{di}\right), \quad \text{where} \quad i = \alpha, \overline{y}, \beta. \tag{23}$$

The first term in parentheses on the RHS implies the direct effect of each parameter on the multiplier effect, whereas the second term represents the indirect effect through a change in the output level due to the parameter change. Therefore, we ignore the latter by assuming a logarithmic utility function ($u(c) = \log c$) and focus on the former.[15] In this case $\partial c_y/\partial y = 0$, as is clear from (18), and hence the second term disappears.

To examine the sign and implication of the first term $\partial c_y/\partial i$ in (23), using the third and fifth equations of (12), we decompose c_y into the moderation effect of an increase in y on deflation, $\partial\pi/\partial y$, and the stimulative effect of this moderation on consumption, $\partial c/\partial\pi$. With logarithmic utility, they are

$$c_y = \frac{\partial c}{\partial \pi}\cdot\frac{\partial \pi}{\partial y}, \quad \text{where} \quad \frac{\partial c}{\partial \pi} = \frac{1}{\beta} \quad \text{and} \quad \frac{\partial \pi}{\partial y} = \frac{\alpha}{\overline{y}},$$

[14] In Eggertsson and Krugman (2012), the paradox of flexibility means the case where a negative shock decreases output more if price flexibility increases. Eggertsson (2010) originally uses the paradox of toil to describe the case where the desire of everyone to work more results in decreasing aggregate employment, whereas Eggertsson and Krugman (2012) use it in the sense that an increase in potential output leads to a decrease in actual output.

[15] If $u(c)$ has a general form, the indirect effect depends on the third derivative of $u(c)$, as is clear from (16), and thus is ambiguous.

which yields

$$\frac{\partial c_y}{\partial \alpha} = \frac{1}{\beta \overline{y}} > 0, \quad \frac{\partial c_y}{\partial \overline{y}} = -\frac{\alpha}{\beta \overline{y}^2} < 0, \quad \frac{\partial c_y}{\partial \beta} = -\frac{\alpha}{\beta^2 \overline{y}} < 0.$$

A rise in α makes π more sensitive to a change in the output gap, which increases $\partial \pi / \partial y$ and hence c_y. As $\overline{y}$ is larger, an increase in y becomes less effective in narrowing the deflationary gap, causing $\partial \pi / \partial y$ to decline and c_y to be lower. As the liquidity preference is stronger (as β rises), the desire for money compared with that for consumption becomes the dominant factor in the consumption decision, making the role of deflation in deciding consumption less important. Therefore, a rise in β reduces $\partial c / \partial \pi$, while $\partial \pi / \partial y$ is intact because β has nothing to do with the wage–price adjustment. Consequently, c_y decreases. From (23), the magnitude of the multiplier effect varies in the same direction as c_y, namely,

$$\frac{d}{d\alpha}\left(\frac{dy}{dg}\right) > 0, \quad \frac{d}{d\overline{y}}\left(\frac{dy}{dg}\right) < 0, \quad \frac{d}{d\beta}\left(\frac{dy}{dg}\right) < 0. \tag{24}$$

Let us summarize the properties in (22) and (24). Increases in liquidity preference and potential output are definitely harmful to the stagnant economy. They not only decrease the level of GDP but also weaken the multiplier effect. This result may explain why Japan's stagnation since the early 1990s has seriously persisted and why an increase in government purchases was not as effective as expected (see Kameda (2014) and therein references for this ineffectiveness). In fact, the following phenomena associated with increases in β and $\overline{y}$ were observed during this stagnation. Money demand motivated by factors other than the transaction motive increased (Otani and Suzuki, 2008), and the government of Japan repeatedly implemented policies intended to

increase potential output, such as deregulation and privatization, despite the presence of the deflationary gap.[16] Meanwhile, an increase in wage flexibility strengthens the multiplier effect but reduces the level of GDP.[17] This result suggests that improving imperfection in the labor market may not necessarily be beneficial to the economy.

4 Conclusion

Long-run stagnation with aggregate demand deficiency occurs if intertemporally optimizing households have insatiable preferences for holding money. In this long-run stagnation, consumption is expressed as a function of output, as is the Keynesian consumption function, and an increase in government purchases boosts GDP through a multiplier process. However, this consumption function represents not the Keynesian relationship between income and consumption but the effect on consumption of an increase in output through moderation of deflation. Therefore, the multiplier effect of government purchases results from the repetition of the interactive process between moderation of deflation and an increase in consumption.

This multiplier of government purchases is larger than one although Ricardian equivalence holds. This is because an increase in government purchases of goods and services directly creates new employment, which mitigates deflations in the nominal wage and price. Meanwhile, a tax cut (or a transfer increase) has no effect on GDP because a tax cut in itself does not create new employment. Thus, direct creation of new employment is essential for

[16] See, e.g., Nishizaki et al. (2014) for Japan's deflationary gap.

[17] Using a DSGE model, Christiano et al. (2011) obtain a similar result in a short-run slump.

stimulating the economy. Moreover, we find that an increase in potential output reduces GDP and weakens the multiplier effect. These results lead us to the conclusion that expanding government purchases is effective, cutting a tax is ineffective and increasing potential output is harmful for economies falling into long-run stagnation with aggregate demand deficiency, probably such as the USA and Japan in recent years.

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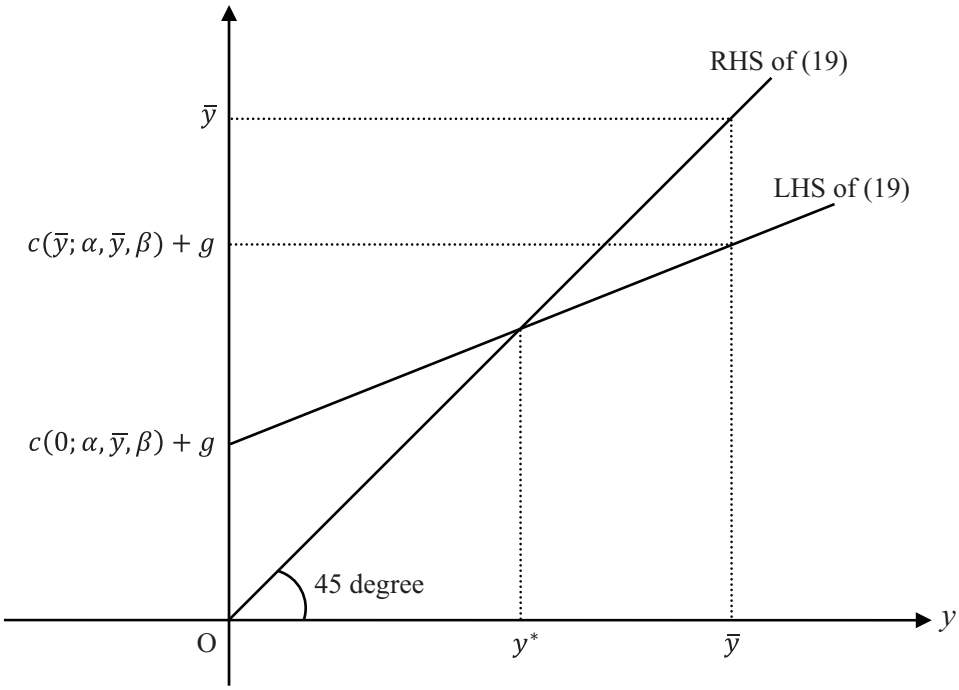

Figure 1: The determination of GDP