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Inflation Inertia and Optimal Delegation of Monetary Policy

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Abstract

This paper analyzes the relationship between the optimal weight on output gap in the central bank's loss function and the degree of inertia in a hybrid version of New Keynesian model with a pure discretionary inflation targeting. I present the policy recommendations as to the weight on output gap in the presence of endogenous persistence in inflation dynamics. Especially, I show that under endogenous persistence of inflation dynamics, even in discretionary monetary policy regime, a Rogoff's (1985) conservative central banker does not necessarily improve social welfare.

Keywords: hybrid New Keynesian model; inflation targeting; policy weight

JEL Classification: E52; E58

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1 Introduction

In recent years, hybrid New Keynesian models have been widely used for macroeconomic analyses. Departing from the standard New Keynesian models, the hybrid New Keynesian models introduce inertia into dynamics of aggregate demand (AD) and aggregate supply (AS) relations. In this paper, I investigate the relationship between the optimal weight on output gap in the central bank's loss function and the parameters contained in a hybrid New Keynesian model when the central bank conducts a discretionary monetary policy. This issue has not been fully discussed in the existing literatures. Among the parameters, I focus on the degree of inertia in a hybrid New Keynesian Phillips curve. This is because the recent empirical studies such as Galí et al. (2005) support the presence of inertial inflation dynamics.

In the literature, the seminal work of Rogoff (1985) suggests that to reduce the additional social loss generated by the inflation bias under discretionary optimal monetary policy, it is optimal to appoint a central banker who places a higher weight on inflation than the society. While Rogoff (1985) uses a traditional Lucas type of Phillips curve, Clarida et al. (1999) use a basic New Keynesian model and shows that the Rogoff's (1985) result holds under exogenous inflation persistence which a serial correlation of cost shocks gives rise to. [1]

In this paper, I analyse the relation ship between the degree of inflation inertia and the optimal policy weight on output gap in the central bank's loss function and show that under endogenous persistence of inflation dynamics, even in discretionary monetary policy regime, a Rogoff's (1985) conservative central banker does not necessarily improve social welfare. [2]

[1] For details, see section 2.4.

[2] Along the line of Rogoff (1985), a central banker is called conservative if λ^c is small, that is, she places a large relative weight on inflation.

2 Inflation Inertia and Optimal Policy Weight

2.1 Hybrid New Keynesian Model

To analyze the relation between inflation inertia and optimal policy weight, I use the hybrid version of New Keynesian models introduced by Galí and Gertler (1999). The model does not have any theoretically rigorous micro foundations but it is very simple, useful and sufficient for the purpose of the paper.

A hybrid New Keynesian model consists of the hybrid versions of IS relation and New Keynesian Phillips curve (HNKPC) respectively given by

$$x_t = (1-\phi)E_t x_{t+1} + \phi x_{t-1} - \sigma(i_t - E_t \pi_{t+1}) + u_t, \tag{1}$$

$$\pi_t = (1-\psi)\beta E_t \pi_{t+1} + \psi \pi_{t-1} + \kappa x_t + v_t, \tag{2}$$

together with a monetary policy rule. Here, x_t, i_t, π_t, u_t and v_t denote output gap, nominal interest rate, inflation rate, demand shock and cost shock in period t, respectively. Parameters σ, β and κ are positive constants, where β is the discount factor, σ is the intertemporal elasticity of substitution and κ is the impact of one unit of output gap on inflation. Constant parameters $\phi \in [0,1]$ and $\psi \in [0,1]$ represent the degrees of inertia in AD and AS relations. If $\phi = \psi = 0$, then the model is identical to a basic New Keynesian model. I assume that $\{u_t\}_{t=0}^{\infty}$ and $\{v_t\}_{t=0}^{\infty}$ follow AR(1) processes. That is, dynamics of u_t and v_t are given by

$$u_{t+1} = \rho_u u_t + \varepsilon_{t+1}^u,$$

$$v_{t+1} = \rho_v v_t + \varepsilon_{t+1}^v,$$

where $\rho_u \in [0,1)$, $\rho_v \in [0,1)$, $\varepsilon_{t+1}^u \sim N(0,\theta_u^2)$ and $\varepsilon_{t+1}^v \sim N(0,\theta_v^2)$.

To measure social welfare, I adopt the traditional social loss function such that

$$L = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda^s x_t^2), \tag{3}$$

where parameter λ^s is the relative weight that the representative household places on output gap relative to inflation.

2.2 Inflation Targeting under Discretion (ITD)

Suppose that the central bank pursues an inflation targeting under pure discretion. In this regime, in each period, the central bank minimizes a discounted sum of the current and future loss with future inflaiton and output gap given. I set the period-loss function in such a way that

$$L_t = \frac{1}{2}(\pi_t^2 + \lambda^c x_t^2),\ \forall t \geq 0,$$

where λ^c is the relative weight selected by the central bank that may be different from λ^s. Since the state variables in period t are π_{t-1} and v_t, the Bellman equation for the central bank's optimization problem is

$$V(\pi_{t-1}, v_t) = \min_{i_t,\pi_t,x_t} \left\{\frac{1}{2}(\pi_t^2 + \lambda^c x_t^2) + \beta E_t V(\pi_t, v_{t+1})\right\}, \tag{4}$$

$$\text{s.t.} \quad x_t = (1-\phi)E_t x_{t+1} + \phi x_{t-1} - \sigma(i_t - E_t\pi_{t+1}) + u_t, \tag{5}$$

$$\pi_t = (1-\psi)\beta E_t\pi_{t+1} + \psi\pi_{t-1} + \kappa x_t + v_t. \tag{6}$$

Assuming that the nonnegativity constraint of nominal interest rate i_t is not binding, the equation (5) does not bind as a constraint since the Lagrangian for the optimization problem of the right-hand-side of (4) is linear in i_t. [3] Thus, the necessary condition for an optimum can be obtained as [4]

$$\pi_t = \frac{\lambda^c}{\kappa}(\beta\psi E_t x_{t+1} - x_t),\ \forall t \geq 0. \tag{7}$$

The equations (6) and (7), the hybrid New Keynesian Phillips curve and the optimal monetary policy rule, determine the equilibrium dynamics of the model economy as the sequences of inflation and output gap.

[3] As usual, the nonnegativity constraint of nominal interest rate is ignored in optimal monetary policy analyses in the New Keynesian framework.

[4] Substituting (6) into the right-hand-side of (4), the first order condition with respect to x_t is

$$\alpha\pi_t + \lambda^c x_t + \beta E V_1(\pi_t, v_{t+1})\kappa = 0.$$

The envelope condition is

$$V_1(\pi_{t-1}, v_t) = -\frac{\lambda^c}{\kappa}\psi x_t.$$

Eliminating the value function, we have the first order condition (7).

Table 1: Baseline Parameter Value

λ^s	β	ρ_u	ρ_v	κ	ϕ	σ	θ_u	θ_v
0.25	0.99	0	0	0.05	0.5	0.67	0.15	0.15

2.3 Simulation

Since the optimal sequences of inflation and output gap, $\{(\pi_t, x_t)\}_{t=0}^{\infty}$, depend on λ^c, in equilibrium, the social loss (3) can be expressed as a function of parameter λ^c. Therefore, the optimal policy weight (denoted by λ^*) can be selected by minimizing the social loss in equilibrium with respect to λ^c.

Moreover, note that from (7) the social loss in equilibrium depends on ψ, so that the selected λ^* depends on ψ as well. Hence, I can obtain the relationship between the degree of inertia, ψ, and the optimal policy weight, λ^*. In a mathematical formation, the optimal policy weight is

$$\lambda^*(\psi) \in \operatorname*{argmin}_{\lambda^c} \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \Big(\pi_t(\lambda^c, \psi)^2 + \lambda^s x_t(\lambda^c, \psi)^2 \Big),$$

where $\pi_t(\lambda^c, \psi), x_t(\lambda^c, \psi)$ are equilibrium inflation rate and output gap in period t when the policy weight and degree of inflation inertia are λ^c and ψ respectively.

To calculate the optimal policy weight on output gap, I reset the social loss function (3) as

$$\lim_{\beta \to 1} 2(1-\beta)L = V[\pi] + \lambda^s V[x],$$

where $V[\pi]$ and $V[x]$ are the asymptotic variances of inflation and output gap.

I analyze the relation between λ^* and ψ numerically. In so doing, I set the baseline pameter value in Table 1.[5]

The numerical analysis reveals the following fact:

Result 1 *For a set of parameters with plausible magnitudes, there is ψ^* such that λ^* decreases with ψ for $\psi \in [0, \psi^*]$ and increases with ψ for $\psi \in [\psi^*, 1]$.*

[5] The values of parameters are the same as in Jensen (2002). I conduct the numerical calculation by use of the algorithm in Söderlind (1999).

Figure 1 illustrates Result 1 based on our numerical example. Result 1 is intuitively plausible. A conservative central banker tries to stabilize inflation actively. Endogenous persistence in inflation dynamics helps such a stabilizing action directly, because a part of future inflation is controlled by current inflation through the economic agents' partially backward-looking behaviors. Thus, appointing a more conservative central banker improves a trade-off between inflation and output gap as long as the degree of inflation inertia is not too high. This is because the cost of stabilizing inflation generated by an expansion of output gap is relatively small. However, when the degree of inertia is sufficiently high, stabilizing inflation yields a large output gap. Hence, in this case stronger conservatism leads to a worse trade-off between inflation and output gap. As a consequence, there is a turning point ψ^* in the relation between the degree of inflation inertia ψ and the optimal weight λ^*.

2.4 Discussion

The results of Clarida et al. (1999) claim that in a basic New Keynesian model without inertia, if the cost shock $\{v_t\}_{t=0}^{\infty}$ is not serially correlated as related literatures often assumes, then the optimal weight on output gap in ITD is identical to the social preference λ^s. In other words, appointing a central banker sharing the social preference is optimal. In fact, since that optimal weight λ^* is given by[6]

$$\lambda^* = (1 - \beta\rho)\lambda^s, \tag{8}$$

$\rho = 0$ implies that $\lambda^* = \lambda^s$. However, Fact 1 demonstrates that this policy implication does not hold when inflation dynamics has endogenous persistence. In this case, it raises social welfare to appoint a more conservative central banker. This is because, in addition to the *expectations effect*, there is an *inertia effect* mentioned in the previous section.[7]

[6] See Vestin (2006).

[7] If $\rho > 0$, the future values of the cost shocks can be partially forecast. Hence, the rational agents, who know that a conservative central banker react to the cost shocks harder, expect stable future inflation. This behavior contributes to stabilizing current inflation, which may be called the expectations effect. It disappears if $\rho = 0$. Note that the expectations effect is generated through a mechanism which is different from the inertia effects.

Equation (8) also means that in the absence of endogenous persistence of inflation dynamics, if $\{v_t\}_{t=0}^{\infty}$ is serially correlated, the optimal weight on output gap is lower than λ^s and it monotonically decreases with the degree of exogenous persistence ρ. However, when the inflation persistence is endogenous, by the mechanism mentioned above, there is a critical value ψ^* after which λ^* increases with ψ: see Figure 1. That is, under inertial inflation dynamics, the behavior of the optimal policy weight is not monotone, so that stronger inflation persistence does not necessarily require a more conservative central banker: the central bank should place a higher weight on the loss from income fluctuation when inflation inertia is intense enough. Besides, the former result, $\lambda^* < \lambda^s$, can be reversed if inflation dynamics exhibits very strong inertia. In this case, since inflation behaves stably by itself, the gain of stabilizing income flactuation more actively is relatively large. Hence, the central bank should place a higher weight on output gap than society, that is, $\lambda^* > \lambda^s$.

It is important to study how the critical point ψ^* varies as the other parameters change. Figures 2 and 3 respectively depict the relations between ψ^* and two key parameters, ρ_v and κ. It is easy to interpret those graphs: a rise in ρ_v increases both the expectations effect explained in footnote 6 and the relevance of reducing income fluctuation so that ψ^* is lowered, while a larger κ requires to reduce output gap and, hence, ψ^* again decreases.

Result 2 *For a set of parameters with plausible magnitudes, the critical point ψ^* decreases with ρ_v and κ.*

Result 1 and 2 suggests that under inertial inflation dynamics, the policy implication on the optimal weight on output gap in the central bank's loss function is not simple as the literature claims. The parameter value should be considered more carefully in the face of monetary policy delegation problem.

3 Concluding Remark

One of the remaining problems is to find the degree of inertia in inflation dynamics on which researchers majoring in monetary economics reach a consensus. According to Fact 1, the concrete policy implication about the optimal weight for the real economy depends mainly on the true degree of inertia. Rudebusch (2002) estimates $\psi = 0.71$ for the U.S. data. Galí et al. (2005) estimate ψ by three methods and the values of the estimators are 0.349, 0.374 and 0.260.[8] Fuhrer (1997) demonstrates that the case $\psi = 1$ can not be rejected. Thus, there has not been a general agreement with the value of ψ.

[8]In Galí et al. (2005), they use real marginal costs in place of output gaps of HNKPC.

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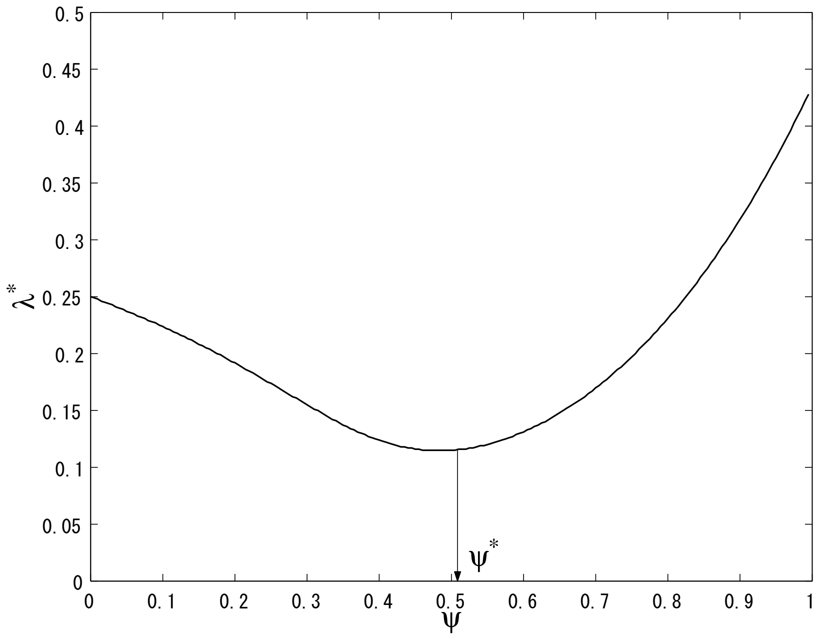

Figure 1: relation between ψ and λ^* ($\rho = 0$, $\kappa = 0.05$, $\lambda^s = 0.25$)

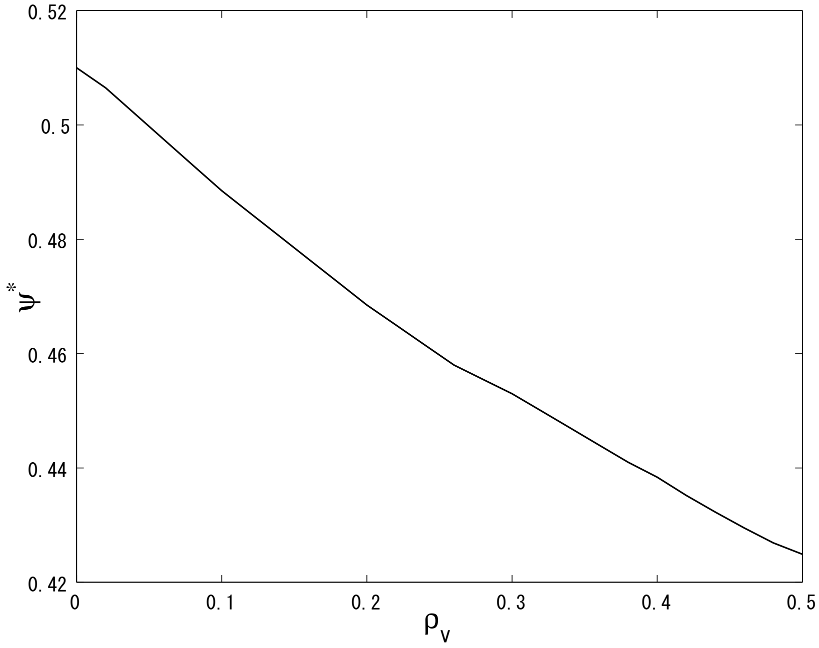

Figure 2: relation between ρ_v and ψ^* in Fig.1 ($\kappa = 0.05$)

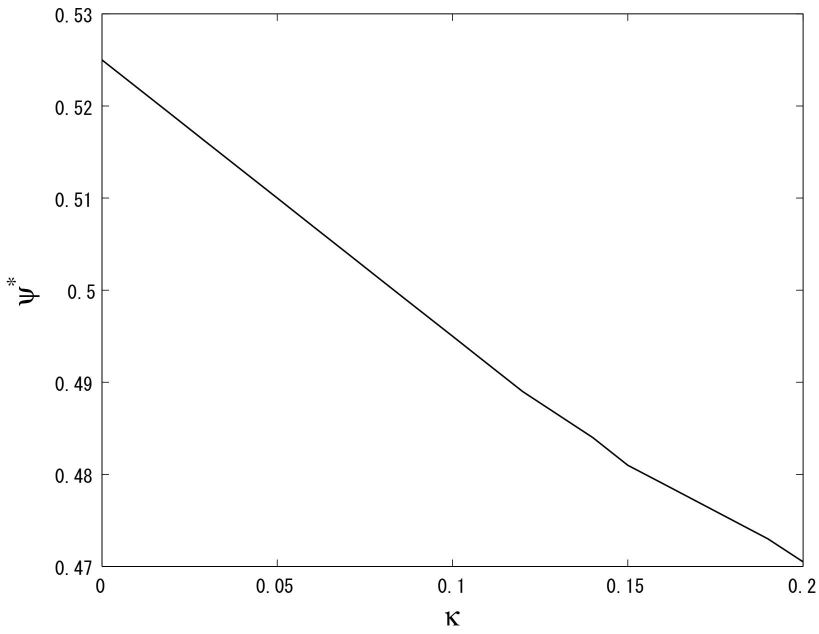

Figure 3: relation between κ and ψ^* in Fig.1 ($\rho = 0$)