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One-Leader and Multiple-Follower Stackelberg Games with Private Information

Tomoya Nakamura

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GCOE Secretariat
Graduate School of Economics
OSAKA UNIVERSITY
1-7 Machikaneyama, Toyonaka, Osaka, 560-0043, Japan

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Tomoya Nakamura*

Osaka University, ISER

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Abstract

This study analyzes one-leader and multiple-follower Stackelberg games with demand uncertainty. We demonstrate that the weight on public information regarding a follower's estimation of demand uncertainty determines the strategic relationship between the leader and each follower. When the relationship is strategic complement, the leader can exit from a market. The threshold is determined by the intensity of Cournot competition among the followers.

Keywords: Stackelberg and Cournot games; First-mover and second-mover advantages; Public and private information

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*E-mail address: nakamura@iser.osaka-u.ac.jp

1 INTRODUCTION

Recent literature on information economics includes discussions on the interrelationships between strategic behavior and public information, for example, Morris and Shin (2002), Angeletos and Pavan (2007) and Ui and Yoshizawa (2013). The studies consider simultaneous-move games with uncertainty that we call *horizontal* competitions. Moreover, the studies assume existence of exogenous public information.

In contrast, this study analyzes sequential-move games with endogenously generating public information by using one-leader and multiple-follower Stackelberg games. We call sequential-move games *vertical* competitions. It is well known that the Stackelberg leader has a first-mover advantage because he/she can commit a quantity of supply. However, if the leader intends to gain a profit from the advantage of a first move, then his action is exposed to all followers. By the observation of the leader's actions, each follower can infer the leader's private information. Consequently, the followers have an information advantage because the followers can estimate unknown demand more correctly using his own private information as well as that of the first mover. In addition, the leader's private information endogenously becomes public information if there exist multiple followers. The public information is a focal point for the actions of followers in followers' horizontal competition. This suggests that our model can extend the literature to analyze the interrelationships between strategic behavior and public information, for both horizontal and vertical competitions, which endogenously generate public information.

We demonstrate that the strategic relationship in vertical competition is determined by the weight on public information regarding followers' estimations of uncertainty. If the weight is sufficiently high (low), then the relationship is a strategic substitute (complement), and the leader has a first-mover (dis)advantage because the commitment (information) advantage dominates the information (commitment) advantage, respectively. On the other hand, horizontal competition is a strategic substitute regardless of the weight on public information because of the fundamental structure of Cournot competition. As Cournot competition among followers becomes intense, the output of each follower decreases, and the total output of followers increases, similar to deterministic Cournot competitions. However, vertical strategic relationships change the degree of output reduction of followers because the action of leader affects followers' actions as a focal point. In the case of vertically strategic substitutability (complementarity), followers strongly (weakly) decrease their output; and the total output of followers increases weakly (strongly). Particularly, in the case of vertically strategic complementarity, the leader can exit from a market if the weight on public information, or the intensity of competition among followers, is sufficiently high.

2 THE MODEL

A market consists of $n+1$ firms indexed by $i \in \{0, 1, \ldots, n\}$. Firm i chooses the quantity of production $q_i \geq 0$. The inverse demand function is given by $p = a + u - Q$, where $a > 0$, p is the market price, $Q \equiv \sum_{i=0}^{n} q_i$ is the aggregate production, and u is a random variable with mean $\theta > 0$ and variance $1/\gamma$, $\gamma > 0$. No firm can directly observe the realized value of the prior random variable u. Payoff function of firm i is defined as $\pi_i(q, x) \equiv p \cdot q_i = (a + u - Q)q_i$.

Assume that one of the firms acquires a chance to move first. The firm is denoted by $i = 0$ without loss of generality. Firm 0 receives private information x_0 on u. Then, x_0 satisfies that $E(x_0|u) = u$ and $Var(x_0|u) = 1/\alpha$, $\alpha > 0$. We further assume that the other firms $i \neq 0$ produce the goods after observing the output of firm 0. They also observe the private signal x_i on u. Here, x_i satisfies $E(x_i|u) = u$ and $Var(x_i|u) = 1/\beta$, $\beta > 0$. We restrict our attention to the posterior expectation of u and the conditional expectation for x_i given $x_{j \neq i}$ that satisfy linearity. Some combinations of prior and posterior distributions, for example, the combination of Gamma-Poisson, Beta-Binomial, and Normal-Normal distributions, satisfy following linearity. The first two combinations satisfy non-negativity. If we assume $a+\theta > 3\gamma^{-1/2}$ in Normal-Normal distributions, then $a + u$ is positive with a probability nearly 1. More detailed discussions are found in DeGroot (1970), Gal-Or (1987), Shinkai (2000), and Cumbul (2014).

Assumption 1. $E(u|x_0, x_{i\neq 0}) = E(x_{j\neq 0,i}|x_0, x_{i\neq 0}) = \frac{\alpha x_0+\beta x_i+\gamma\theta}{\Delta}$, $E(u|x_0) = E(x_{i\neq 0}|x_0) = \frac{\alpha x_0+\gamma\theta}{\alpha+\gamma}$, $E(u|x_{i\neq 0}) = E(x_0|x_{i\neq 0}) = \frac{\beta x_i+\gamma\theta}{\beta+\gamma}$, $E(u^2) = E(u\cdot x_i) = E(x_i\cdot x_{j\neq i}) = \frac{1}{\gamma}+\theta^2$, $E\left(x_0^2\right) = \frac{1}{\alpha}+\frac{1}{\gamma}+\theta^2$, $E\left(x_{i\neq 0}^2\right) = \frac{1}{\beta}+\frac{1}{\gamma}+\theta^2$, *where* $\Delta\equiv\alpha+\beta+\gamma$.

We denote pure strategy space by $\mathbb{R}^+$ and support of a private signal by X_i. Firm 0 chooses its quantity of supply depending on its private information x_0. Its strategy can be denoted by $q_0 = H(x_0)$, where $H : X_0 \to \mathbb{R}^+$. Firm $i\neq 0$ chooses its quantity of output depending on its private information x_i and the leader's realized output q_0. Their strategy can be written as $q_i = G(x_i, q_0)$, where $G : X_i\times\mathbb{R}^+ \to \mathbb{R}^+$. Importantly, $H(\cdot)$ and $G(\cdot,\cdot)$ may have many types of functional forms in equilibrium as discussed in Gal-Or (1987), Shinkai (2000), and Cumbul (2014). However, we assume that $H(\cdot)$ and $G(\cdot,\cdot)$ are affine transportations. Formally, we derive the equilibrium strategy profile that satisfies following equations: $\forall x_0\in X_0$, $q_0^* = H(x_0) = \arg\max_{q_0\in\mathbb{R}^+} E[\pi_0(q_0, G(x_{i\neq 0}, q_0^*), u)|x_0] = A_0 + A_1x_0 \geq 0$, and $\forall x_{i\neq 0}\in X_i$, $\forall H(x_0) = q_0\in\mathbb{R}^+$, $q_{i\neq 0}^* = G(x_i, q_0) = \arg\max_{q_i\in\mathbb{R}^+} E[\pi_i(q_0, q_i, G(x_{j\neq 0,i}, q_0), u)|\ x_i, q_0] = B_0 + B_1x_i + B_2q_0 \geq 0$, where $A_0, A_1, B_0, B_1, B_2\in\mathbb{R}$.

Timing of the game is following. At $t = 0$, nature draws the unknown demand u and each firm receives x_i. At $t = 1$, firm 0 chooses q_0 given x_0. At $t = 2$, firms $i\neq 0$ decide q_i given x_i and q_0.

3 DERIVATION OF THE EQUILIBRIUM

First, we ignore the non-negativity constraint regarding q_i and solve the model by backward induction. Next, we check whether the derived equilibrium strategy satisfies the non-negativity constraint.

3.1 Without a non-negativity constraint

Second movers At stage 2, each follower chooses q_i to maximize $E[(a+u-Q)q_i|x_i, q_0]$, for any $i \neq 0$. The first-order conditions are: $2q_i^* = a + E[u|x_i, q_0] - q_0 - E[\sum_{j\neq i,0} q_j|x_i, q_0]$. Note that, in the equilibrium, followers can correctly infer x_0 from q_0^*: $x_0 = (q_0^* - A_0)/A_1$. Then, we can rewrite the first-order conditions as $2q_i^* = a + \left(\frac{\gamma\theta}{\Delta} - \frac{\alpha A_0}{\Delta A_1}\right) - (n-1)\left[B_0 + \left(\frac{\gamma\theta}{\Delta} - \frac{\alpha A_0 B_1}{\Delta A_1}\right)\right] + [1-(n-1)B_1]\frac{\beta}{\Delta}x_i + \left\{\frac{\alpha}{\Delta A_1} - 1 - (n-1)\left(\frac{\alpha B_1}{\Delta A_1} + B_2\right)\right\} q_0$. Comparing the coefficients of both sides in this equation, we have

$$2B_0 = a + \left(\frac{\gamma\theta}{\Delta} - \frac{\alpha A_0}{\Delta A_1}\right) - (n-1)\left[B_0 + \left(\frac{\gamma\theta}{\Delta} - \frac{\alpha A_0 B_1}{\Delta A_1}\right)\right], \tag{1}$$

$$2B_1 = [1-(n-1)B_1]\frac{\beta}{\Delta}, \tag{2}$$

$$2B_2 = \frac{\alpha}{\Delta A_1} - 1 - (n-1)\left(\frac{\alpha B_1}{\Delta A_1} + B_2\right). \tag{3}$$

First mover At $t = 1$, firm 0 chooses its supply depending on x_0. The objective function is $E\pi_0 = E\left[(a+u-Q)\,q_0|x_0\right]$. The first-order condition is $\frac{\partial E\pi_0}{\partial q_0} = a + E(u|x_0) - E\left(\sum_{j\neq 0} q_j|x_0\right) - 2q_0^* - q_0^* \sum_{i\neq 0}\frac{\partial q_i}{\partial q_0} = 0$. Note that $\frac{\partial q_{i\neq 0}}{\partial q_0} = B_2$. Then, the first-order condition can be rewritten as $a + \frac{\gamma}{\alpha+\gamma}\theta - 2A_0 - n\left(2A_0B_2 + B_0 + B_1\frac{\gamma}{\alpha+\gamma}\theta\right) - \left[2A_1 - \frac{\alpha}{\alpha+\gamma} + n\left(2A_1B_2 + B_1\frac{\alpha}{\alpha+\gamma}\right)\right] x_0 = 0$. This equation should satisfy for any realization of x_0. Hence,

$$a + \frac{\gamma}{\alpha+\gamma}\theta - 2A_0 - n\left(2A_0B_2 + B_0 + B_1\frac{\gamma}{\alpha+\gamma}\theta\right) = 0, \tag{4}$$

$$2A_1 - \frac{\alpha}{\alpha+\gamma} + n\left(2A_1B_2 + B_1\frac{\alpha}{\alpha+\gamma}\right) = 0. \tag{5}$$

Define $\rho \equiv \frac{\alpha+\gamma}{\Delta}$ that represents weight on public information regarding followers' estimation of u. Then, solving the system of five equations (1), (2), (3), (4), and (5), we obtain the following result:

$$A_0 = \frac{1+\rho+(1-3\rho)n}{2[1+\rho+(1-\rho)n]}\left(a + \frac{\gamma}{\alpha+\gamma}\theta\right), \quad A_1 = \frac{1+\rho+(1-3\rho)n}{2[1+\rho+(1-\rho)n]}\left(\frac{\alpha}{\alpha+\gamma}\right)$$

$$B_0 = \frac{1-\rho}{1+\rho+(1-\rho)n}a, \quad B_1 = \frac{1-\rho}{1+\rho+(1-\rho)n}, \quad B_2 = \frac{3\rho-1}{1+\rho+(1-3\rho)n}.$$

Non-negativity constraint We can easily check $q^*_{i\neq 0} > 0$. On the other hand, q^*_0 is not always non-negative. q^*_0 can be written as follows.

$$q^*_0 = \frac{1+\rho+(1-3\rho)n}{2[1+\rho+(1-\rho)n]}\left(a + \frac{\gamma}{\alpha+\gamma}\theta + \frac{\alpha}{\alpha+\gamma}x_0\right)$$

$2[1+\rho+(1-\rho)n]$ and the value in the blanket is strictly positive because we assume $a, \theta > 0$ and (almost all) the support of x_0 is positive. Therefore, $Z(\rho, n) \equiv 1+\rho+(1-3\rho)n$ determines the sign of q^*_0. Then, we have the following non-negativity condition.

Proposition 1. *If $Z(\rho, n) > 0$, then $q^*_0 > 0$. If $Z(\rho, n) \leq 0$, then $q^*_0 = 0$.*

Observation 1 in Gal-Or (1987) shows that the leader always chooses a positive output. Similarly, in our model, if $n = 1$, then $Z(\rho, n) > 0$ always holds and the leader always chooses a positive output. However, departing from her observation, if $n \geq 2$, then $Z(\rho, n)$ can be non-positive in the case of $\rho > \bar{\rho} \equiv \frac{n+1}{3n-1} \in (1/3, 1)$. This suggests that, in contrast to Gal-Or (1987), horizontal competition by followers can drive out the leader from the market if $\rho \in [\bar{\rho}, 1)$.

3.2 With a non-negativity constraint

If $Z(\rho, n) \leq 0$, the leader chooses $q^*_0 = 0$. Then, we assume that the followers play a Cournot competition without firm 0. Then, an observation of firm $i \neq 0$ is only x_i. Therefore, we assume linear equilibrium such that $q^c_i = G^c(x_i) = B^c_0 + B^c_1 x_i, \forall x_i \in X_i$. The payoff of followers is $\pi_i = E[(a+u-\sum_{j\neq 0} q_j)q_i|x_i]$, and the first order condition is $2q_i = a + E(u|x_i) - E(\sum_{j\neq 0,i} q^c_j|x_i)$, where $E(\sum_{j\neq 0,i} q^c_j|x_i) = (n-1)[B^c_0 + B^c_1(\frac{\beta}{\beta+\gamma}x_i + \frac{\gamma}{\beta+\gamma}\theta)]$. Hence, q^c_i can be rewritten as $2q^c_i = a + \frac{\gamma}{\beta+\gamma}\theta - (n-1)\left(B^c_0 + B^c_1\frac{\gamma}{\beta+\gamma}\theta\right) + [1-(n-1)B^c_1]\frac{\beta}{\beta+\gamma}x_i$ Using a method of undetermined coefficient, we have $B^c_0 = \frac{1}{n+1}\left\{a + \frac{2\gamma}{(1+n)\beta+2\gamma}\theta\right\}$, and $B^c_1 = \frac{\beta}{(1+n)\beta+2\gamma}$. Summing up the results, we have following proposition.

Proposition 2 (A linear equilibrium of the game). *(i) If $Z(\rho,n)>0$, then*

$$q_0^* = \frac{1}{2}\left\{1-\frac{2n\rho}{2[1+\rho+n(1-\rho)]}\right\}[a+E(u|x_0)]\,,$$
$$q_{i\neq 0}^* = \frac{1}{2}\left\{1-\frac{n(1-\rho)}{2[1+\rho+n(1-\rho)]}\right\}\left[a+\frac{2(1-\rho)}{1+\rho}x_i+\frac{3\rho-1}{1+\rho}E(u|x_0)\right]. \tag{6}$$

(ii) If $Z(\rho,n)\leq 0$, then $q_0^c=0$ and $q_{i\neq 0}^c=\frac{1}{1+n}\left\{a+\frac{(1+n)\beta}{(1+n)\beta+2\gamma}x_i+\frac{2\gamma}{(1+n)\beta+2\gamma}\theta\right\}$.

Corollary 1 (Ex ante expected production). *(i) If $Z(\rho,n)>0$, then*

$$Eq_0^* = \frac{a+\bar{\theta}}{2}\left[\frac{1+\rho+(1-3\rho)n}{1+\rho+(1-\rho)n}\right], \qquad Eq_i^* = \frac{a+\bar{\theta}}{2}\left[\frac{1+\rho}{1+\rho+(1-\rho)n}\right],$$
$$E\sum_{i\neq 0} q_i^* = n\frac{a+\bar{\theta}}{2}\left[\frac{1+\rho}{1+\rho+(1-\rho)n}\right], \qquad E\sum_{i} q_i^* = \frac{a+\bar{\theta}}{2}\left[\frac{2(1-\rho)n+1+\rho}{(1+\rho)+(1-\rho)n}\right].$$

(ii) If $X(\rho,n)\leq 0$, then $Eq_0^c=0$ and $Eq_{i\neq 0}^c=\frac{a+\theta}{n+1}$.

Corollary 2 (Ex ante expected profits). *(i) If $\rho<\bar{\rho}$,*

$$E\pi_0^* = \frac{[2(\alpha+\gamma)+\beta][2(1-n)(\alpha+\gamma)+\beta(n+1)]}{4[2(\alpha+\gamma)+\beta(n+1)]^2}\left[(a+\theta)^2+\frac{\alpha}{\gamma(\alpha+\gamma)}\right],$$
$$E\pi_{i\neq 0}^* = \frac{[2(\alpha+\gamma)+\beta]^2}{4[2(\alpha+\gamma)+\beta(n+1)]^2}\left[(a+\theta)^2+\frac{\beta^2(\alpha+4\gamma)+4(\alpha+\beta)(\alpha+\gamma)^2}{\gamma(\alpha+\gamma)[2(\alpha+\gamma)+\beta]^2}\right].$$

(ii) If $\rho<\bar{\rho}$, then $E\pi_0=0$ and $E\pi_{i\neq 0}=E\pi^c=\frac{(a+\theta)^2}{(n+1)^2}+\frac{\beta(\beta+\gamma)}{\gamma[(n+1)\beta+2\gamma]^2}$.

4 EQUILIBRIUM ANALYSIS

Strategic relationship in vertical competition is determined by $B_2=\frac{3\rho-1}{Z(\rho,n)}$ because it represents slope of followers' reaction function to leader's action. In the case of $Z(\rho,n)>0$, the strategic relationship between the leader and each follower is determined by the sign of $3\rho-1$.

Proposition 3. *If $\rho>1/3$, $\rho=1/3$ and $\rho<1/3$, then followers' reaction functions are upward,* *constant and downward sloping; thereby, the strategic relationship is complementary, neutral, and substitutive, respectively.*

This result is closely related to Gal-Or (1985, 1987). Gal-Or (1985) shows that, in the deterministic Stackelberg duopoly model, the first mover has an (dis)advantage if the reaction functions of the players are downwards (upwards) sloping. The slope in her model is determined by deterministic parameters. On the other hand, Gal-Or (1987) and our model show that, in the model of Stackelberg games with demand uncertainty, the slope of reaction functions can be upward sloping because of the conditions of uncertainties. Gal-Or (1987) assumes that the markets faced by each firm are partially segmented. She shows that, if the markets are *sufficiently segmented*, then the reaction function of follower can be upward sloping. On the other hand, we show that the reaction function of follower can be upward sloping despite a *perfectly integrated* market.

Horizontal competition is a strategic substitute because of the fundamental structure of Cournot competition. However, the degree of strategic substitution is affected by strategic relationships with the leader. From (6), in the case of vertically strategic substitutability (complementarity), the weight on the public signals are negative (positive), and then the degree of strategic substitutability among followers increases (decreases), respectively.

Comparative statics results of expected productions and expected profits are similar. Here, we concentrate on the analysis regarding expected productions.

Proposition 4 (Comparative statics)**.** *(i) If* $Z(\rho, n) > 0$*, then* $\frac{\partial}{\partial \rho} Eq_0^* \leq 0$*,* $\frac{\partial}{\partial n} Eq_0^* < 0$*,* $\frac{\partial}{\partial \rho} Eq_{i \neq 0}^* \geq 0$*,* $\frac{\partial}{\partial n} Eq_{i \neq 0}^* < 0$*,* $\frac{\partial}{\partial \rho} \sum_{i \neq 0} Eq_i^* \geq 0$*,* $\frac{\partial}{\partial n} \sum_{i \neq 0} Eq_i^* > 0$*,* $\frac{\partial}{\partial \rho} \sum_i Eq_i^* \leq 0$*, and* $\frac{\partial}{\partial n} \sum_i Eq_i^* > 0$*. (ii) If* $Z(\rho, n) \leq 0$*, then* $\frac{\partial}{\partial \rho} Eq_{i \neq 0}^c = 0$*,* $\frac{\partial}{\partial n} Eq_{i \neq 0}^c < 0$*, and* $\frac{\partial}{\partial n} E \sum_{i \neq 0} q_i^c > 0$*.*

Figure 1 shows the results of comparative statics regarding the weight on public signals, ρ, given the intensity of followers' horizontal competition. When $\rho \to 0$, the leader produces more than the followers. As ρ increases, the leader's production decreases, and each follower's production increases: $\partial Eq_0^* / \partial \rho < 0$ and $\partial Eq_i^* / \partial \rho > 0$. If $\rho = 1/3$, then the leader's output

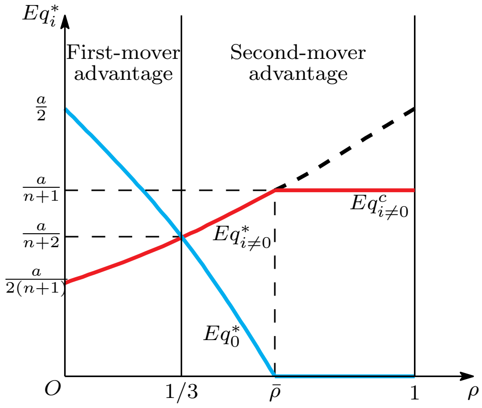

Figure 1: Effect of weight on public signals

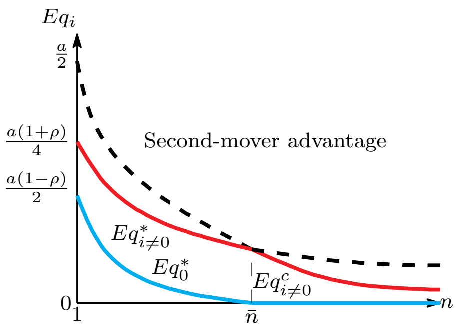

Figure 2: Effect of intensity of horizontal competition: $\rho > 1/3$

corresponds with the follower's output. This suggests that, if $\rho < 1/3$ ($\rho > 1/3$), the leader has a first-mover (dis)advantage. Intuition is related to the tradeoff between commitment advantage and information advantage. If a leader who has a low (high) precision of private signal reveals a low (high) precision of private information, then that leader's commitment advantage dominates (is dominated by) the followers' information advantage. Finally, if $\rho \geq \bar{\rho} \equiv \frac{n+1}{3n-1} \in (1/3, 1)$, the leader refrains from production; then, $\bar{\rho}$ decreases with n. This indicates that the more followers that exist, the leader stops production at smaller $\bar{\rho}$. This effect can also be examined by $\partial^2 Eq_0^*/\partial\rho\partial n < 0$ and $\partial^2 Eq^*_{i\neq 0}/\partial\rho\partial n > 0$. If n increases, slopes of Eq_0^* and $Eq^*_{i\neq 0}$ become steeper.

Next, we consider the results of comparative statics regarding the intensity of horizontal competition, n. All firms reduce output with respect to n that can be regarded as intensity of horizontal competition: $\partial Eq_0^*/\partial n < 0$ and $\partial Eq_{i\neq 0}/\partial n < 0$. However, the effect is changed by ρ. If $\rho \leq 1/3$, that is, the region of a first-mover advantage, then the output of firm 0 is no less than that of the follower's. Hence, firm 0 always chooses non-negative output because followers produce non-negative output. If $\rho > 1/3$, then the output of firm 0 is less than that of follower's. That is, if horizontal competition is sufficiently intense ($n \geq \bar{n} \equiv (\rho+1)/(3\rho-1)$), then firm 0 stops production and followers play a Cournot competition without firm 0. Therefore, a follower's production is kinked at $\bar{n}$ (see Figure 2). The reason why firm 0 stops production is that, unless an individual follower reduces output, the total output of followers increases with respect to n that can be regarded as the intensity of followers' horizontal competition: $\partial \sum_{i\neq 0} Eq_i/\partial n > 0$. Consequently, unless firm 0 has a first-mover commitment advantage, the intensity of followers' competition excludes firm 0 from the market.

5 Conclusion and future researches

In this study, we analyzed one-leader and multiple-follower Stackelberg games with demand uncertainty, and obtained the following results. First, the strategic relationship in vertical competition is determined by the weight on common signals regarding follower's estimation of uncertainty. If the weight is sufficiently high (low), then the relationship is a strategic substitute (complement), and the leader has a first-mover (dis)advantage. Second, in contrast to Gal-Or (1987), a first mover can exit from the market if the intensity of horizontal competition is sufficiently high, or if the weight on common signals is sufficiently high. These results connect two branches of the literatures. One branch is the classical literature on industrial organization. One of its main interests is first-mover (dis)advantages in vertical competition. Another branch is the recent literature on information economics. One of its main interests is the interaction between strategic behavior and public information. This study provides a simultaneous analysis of these two branches of scholarship.

There are still some open questions. First, we assume the precision of exogenous signals. Introducing the cost of acquiring signals, we could endogenize the equilibrium precision of signals as in Colombo and Femminis (2008), Colombo et al. (2013), Ui (2013), and Arato et al. (2014). Second, we assume the number of followers exogenously. Assuming the cost of entering a market, we could endogenize the number of followers. Third, it might be useful to consider the implications of analyzing the social surplus as in the study of Vives (2008, 2011). While our paper chose to emphasize strategic situations with the endogenous public information, these points merit attention in future research.

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