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Thankworthy Parental Altruismand Children's Habit-Formation

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Thankworthy Parental Altruism and Children's Habit-Formation*

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Abstract

We propose a model of parental altruism in relation with child habit formation, where children are unaware of their developing habits while young, and become cognizant of them only on growing up. We show that an altruistic mother (i) maintains the amount of income transferred to her child lower than the child would desire and (ii) reduces further income transfer upon an exogenous increase in the child's performance of a particular habit. The child, when grown, may end up grateful for the small income transfer if the mother is sufficiently rich and altruistic: when evaluated by the realized habitual preferences, a small income transfer leads to greater child welfare than the greater income transfer preferred by the child would have generated. This implies that parents from richer families, ceteris paribus, tend to guard against their children's profligacy.

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1. Introduction

Parents discipline their young children by keeping income transfer (e.g., allowances) to the children lower than what the children themselves desire. This causes conflict between the parents, who do so from an altruistic motive, and the children, who believe that they will be better off with more money. When the children grow up, some come to understand that their parents had altruistic intentions and feel grateful; others remain resentful of their parent's thriftiness. Why do altruistic parents not transfer as much income to their young children as the latter would like? Conversely, why do grown children not appreciate their parents' altruism? Although it is important to understand how parent-child interactions affect the consumption/saving behavior and related preference formation of the family members, economics has not yet examined these issues.

To fill this knowledge gap, the present paper aims to address two questions:

(i) Why do altruistic parents keep income transfers to young children lower than what the children desire?

(ii) Whether, and under which conditions does the parents' behavior of restricting income transfers enhance the children's welfare and is, therefore, thankworthy?

To do so, we consider an altruistic parent's behavior toward her habit-forming child (Becker, 1974; Barro, 1974): the parent obtains utility from his or her own consumption, as well as the child's utility. In line with Weinberg (2001), and Bhatt and Ogaki (2012), the altruistic parent—a mother, for example—can determine the consumption level of her young child—a son, in this case—by controlling the amount of income transferred to him. A unique feature of our model is

that the child is habit-forming. By controlling income transfer to her young son, the mother in our model can influence his consumption habits, and hence, his future consumption behavior.

We posit two key assumptions. First, as is the case of actual children, the young child is assumed to be unaware that consumption is habit-forming, whereas his mother is assumed to know this. The son thus tends to consume excessively. His altruistic mother has an incentive to keep her income transfer low, so as to restrict the son's excess consumption. The model of misperceived habit formation is reasonable because the psychological literature provides empirical support for the importance of parental intervention in children's habits, in contexts as diverse as food consumption (e.g., Baumrind, 1991), passive leisure such as TV viewing (Walsh et al., 1998), and saving (Webley and Nyhus, 2006).[1] Moreover, studies on the Easterlin paradox have provided empirical evidence of people experiencing welfare loss because they failed to incorporate the effects of habit formation (e.g., Easterlin, 1974).

We also assume that, after reaching adulthood, the child becomes aware of the true mechanisms of habit formation. We are interested in how the grown-up child re-evaluates his mother's strict upbringing retrospectively, by using his realized habitual preferences. In particular, we regard the mother's income transfer to her young son as thankworthy from the (grown-up) son's perspective if—when evaluated according to his true preferences—greater welfare was attained under the transfer than would have been attained under the greater income transfer that he sought in his youth.

For problem (i), we show analytically that an altruistic parent keeps income transfers to his or

[1] However, there has been some controversy regarding the empirical validity of habit formation models (see, e.g., Dynan, 2000).

her child lower than he wants to give. Thus, our model describes a conflict between altruistic parents who raise their children strictly, and the children who complain about it. It is crucial to remember that children's habitual preference formation is known to their parents, but not to the children themselves.

Our solution to problem (ii) may be more important, and will reveal whether the parent's income transfer to a child will induce gratitude in the child, to him as his parent is sufficiently rich and altruistic or not. This is because reducing a marginal unit of income transfer to a child has two countervailing effects. First, it directly reduces the child's felicity throughout the period. Second, it weakens the child's consumption habit in the following period, and mitigates excess consumption that comes from the child being unaware of his or her habit formation. The parental behavior of keeping income transfer low enhances the child's welfare only when the second positive effect dominates the first negative effect. Note, however, that unless parental income, as well as the degree of parental altruism are large enough—and hence, unless the income transfer level is large enough—the marginal utility of income transfer is necessarily so high that the first negative effect dominates the second positive effect.

As a casual observation, some rich parents indulge their children by giving them too much, while some poor parents strictly discipline their children by controlling their consumption/saving behavior. Depending on our results regarding the thankworthiness of parents' altruistic interventions, the difference could be attributable to whether the parents themselves experienced thankworthy transfers or not. Even when a mother is, at present, poor, she could be strict about her children's profligacy if she comes from a rich family and appreciates her own parents' strict rearing. In contrast, a rich mother may spoil her son if she is

from a poor family, and does not appreciate her own strict childhood.

We also show that when the child's habit parameter exogenously shifts upward, the altruistic parent necessarily reduces the unexpectedly low income transfer further, so as to mitigate the child's over-consumption. The result is comparable with parents' "tough love" behavior that Ogaki and Bhatt (2012) derive by specifying an endogenous time-discounting function. They show that an altruistic parent reduces income transfer in response to an exogenous rise in the child's time preference. Since an increase in the habit parameter is shown to raise the child's time preference induced by the consumption habit, our comparative static result may be considered a version of the tough love result. However, there are two key differences. First, in the present model, time preference is endogenously generated by habit formation, rather than by an exogenously given time preference formation. Second and more importantly, our tough love is thankworthy to the child if his parent is sufficiently altruistic and rich. This contrasts with the Ogaki and Bhatt model, wherein a mother's tough love necessarily reduces her son's welfare, because the mother evaluates her son's future utility by using a different time preference from that of the child.[2]

Our study is also related to that of Weinberg (2001). By using a static principal-agency model of the child-parent interaction, he explains the positive relationship between parental income and frequency of using pecuniary punishment, rather than corporal punishment. Unlike his

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[2] Ogaki and Bhatt (2012) also examine an "endogenous altruism model," in which the parent evaluates the child's future utility by using the same time discounting function that the child uses. In this case, the income transfer that the altruistic parent chooses necessarily equals what the child expects to receive. Therefore, even when the parent reduces income transfer due to a shock, it is not "tough" but always reasonable, even to the young child. This contrasts with our case, in which a reduction in the parent's income transfer after a shock always seems harmful, and hence tough, to the child at that time, because he is then unaware of the habit effect. It is, therefore, only reasonable and thankworthy to the child after reaching adulthood.

model with parent-child information asymmetry, however, our dynamic model supposes cross-generation asymmetry in cognitive power to identify future habit effects. We emphasize parental cognitive power that corrects children's habitual over-consumption.[3]

The results of this model jointly imply that it is important for the parents to control children's impulses and habitual consumption, because it is beneficial to children's academic, economic, and health outcomes in the long run. A series of empirical studies support this conclusion (see, for example, Shoda, Mischel and Peake, 1990; Heckman, 2006; Chabris et al., 2008; and Moffitt et al., 2011).

The remainder of this paper is organized as follows. Section 2 introduces the model. Section 3 describes the expected transfer by the child. Section 4 compares the parental income transfers and the child's welfare. Section 5 discusses the endogenous time preference and tough love. Section 6 concludes the paper.

2. The Model

Consider a family consisting of a parent and a child, both of whom live for three periods. The parent (a mother) gives birth to a child (a son) in her second period, which therefore overlaps with the child's first period.

The Child's Problem

In the child's first period, he receives transfer T from the parent. In the second period, the

[3] Despite the differences, our result—suggesting that income transfers of richer parents are likely to be thankworthy to children—seems consistent with the positive relationship that Weinberg (2001) reports between parental income and the frequency of using pecuniary punishment.

child receives an exogenous income y_2.[4] Variables c_i $(i = 1,2,3)$ denote the child's consumption in period i. The child is assumed to face borrowing constraints in period 1: $c_1 \leq T$. We assume that his disposable income T is small enough that the borrowing constraint is binding:

$$c_1 = T, \tag{1}$$

His intertemporal budget in periods 2 and 3 is given by

$$c_3 = R(y_2 - c_2). \tag{2}$$

where R is the gross interest rate.

The child forms consumption habits. His preferences are given by

$$U = u(c_1) + \beta u(c_2 - \theta_1 h_2) + \beta^2 u(c_3 - \theta_2 h_3), \tag{3}$$

$$\text{where} \quad h_2 = c_1 \text{ and } h_3 = c_2,$$

which captures the habit effects of consumption in the previous period. Parameters $\theta_1, \theta_2 > 0$ represent the strength of habit formation, and $\beta > 0$ denotes the discount factor.

Equation (3) implies that while the existence of habit formation raises the marginal utility of consumption, it also reduces the levels of period utilities. However, the child does not realize until his third period that his consumption in his first and second periods was habit-forming:

Assumption 1. The child is unaware of his habit in his first and second periods. In his third period, he becomes aware of the habit.

[4] We assume away the first-period income y_1 to the child. This, however, does not affect our main results below.

Under Assumption 1, the child's consumption behavior is assumed to proceed as follows. In the first period, the child naively maximizes his lifetime utility

$$u(c_1) + \beta u(c_2) + \beta^2 u(c_3). \tag{4}$$

In the second period, the child notices that the utility is actually different from what was expected in the previous period. However, instead of realizing the true mechanism of habit formation, he incorrectly understands it to be caused by a permanent preference shock[5] and maximizes the utility

$$u(c_2 - \theta_1 c_1^*) + \beta u(c_3 - \theta_1 c_1^*), \tag{5}$$

where c_1^* denotes the child's actual consumption level in the first period. In his third (the last) period, the child becomes aware of his true preferences with habit formation as in (3).

The Parent's Problem

Since the parent can affect the child's consumption—and, thereby, welfare—though income transfer T, we focus on her decision in her second period. In the second period, she receives endowment income y_p, and maximizes the sum of the second and third period utilities by choosing consumption basket $(c_{2,p}, c_{3,p})$ and transfer T to the child. Her budget constraint is given by

$$c_{3,p} = R(y_p - c_{2,p} - T). \tag{6}$$

Assumption 2. The parent knows that her child is unaware of his own habit formation until his last period.

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[5] For the case where the misperception disappears, see Appendix A4.

The parent is altruistic toward her child. We specify her utility function as a convex sum of the felicity from her own consumption ($u(c_{2,p}) + \beta u(c_{3,p})$) and her child's true utility (U):

$$V = (1-\gamma)\left[u(c_{2,p}) + \beta u(c_{3,p})\right] + \gamma U, \quad [6] \tag{7}$$

where $\gamma \in [0,1]$ denotes the degree of parental altruism.

She is aware that her child, who is ignorant of the effect of habit formation, will consume too much in his first and second periods. The altruistic parent therefore has an incentive to keep her child's consumption level lower by restricting the amount of income transfer.

In order to obtain closed-form solutions, we specify the period utility function as follows:

Assumption 3. $u(x) = \frac{x^{1-\alpha}}{1-\alpha}$ $(\alpha > 0)$.

We guarantee that the arguments of the utility functions are positive by assuming the following:

Assumption 4. When $\theta_2 < 1, (\beta R)^{-1/\alpha} < 1,\ y_p < \frac{R y_2}{\theta_1(1+R)}$;

when $\theta_2 \geq 1, (\beta R)^{-1/\alpha} < \frac{1}{\theta_2},\ y_p < \frac{[1-\theta_2(\beta R)^{-1/\alpha}] R y_2}{\theta_1(\theta_2+R)[1-(\beta R)^{-1/\alpha}]}$. [7]

Intuitively, the above assumption ensures that the child's consumption in the second and third periods is sufficiently low compared to its value in the previous period. This is achieved by

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[6] Without loss of generality, we assume that the parent's period utility function is the same as the child's.

[7] When the child gets the first-period income y_1, the corresponding assumption to Assumption 4 should be as follows: when $\theta_2 < 1$, $(\beta R)^{-1/\alpha} < 1$, $y_p + y_1 < \frac{R y_2}{\theta_1(1+R)}$; when $\theta_2 \geq 1$, $(\beta R)^{-1/\alpha} < \frac{1}{\theta_2}$, $y_p + y_1 < \frac{[1-\theta_2(\beta R)^{-1/\alpha}] R y_2}{\theta_1(\theta_2+R)[1-(\beta R)^{-1/\alpha}]}$.

restricting parameters, such that (i) the sum of the parent's income and the child's first-period income is sufficiently is lower than the child's second-period income and (ii) the child's discount factor is sufficiently high.

To show the welfare implications of the altruistic transfer, we distinguish and compare two types of the income transfers to the child: (i) the actual transfer that the parent determines by maximizing her altruistic utility (i.e., (7)), and (ii) the income transfer that is expected by the child. Because the parent knows of the child's habit formation, but the child does not, the two transfers will differ.

The parent understands her child's habit-formation, and considers it when she determines the transfer to her child. In this context, the parent indirectly steers her child away from falling into unintended excessive consumption due this habit effect. The child, however, does not know of the parent's altruistic intervention.

3. The Income Transfer Expected by the Child

We consider how much income transfer the child expects to receive from his parent. The expected level of income transfer could be a reference point for us to evaluate the parent's intervention through income transfer: it enables us to (i) show that, consistent with daily observations, the parent's income transfer is unexpectedly small for her young child, and (ii) examine whether the unexpectedly small transfer is appreciable to the grown-up child.

Consider the child's consumption choice prior to receiving an income transfer from the parent in his first period. Because he does not foresee the transfer level perfectly, it is a sort of expected consumption choice based on his expectations regarding parental income transfer. The

obtained solutions for expected consumption, c_i^e $(i = 1,2,3)$, will not be realized—first because the parent's income transfer differs from he expects, and second, because his preferences would shift due to the habit effect of the transfer in the first period.

With an expected level of the parent's income transfer T^e being given, the c_i^e are obtained by maximizing the child's lifetime utility (4) subject to budget constraints (1) and (2). The optimality conditions are given by

$$c_1^e = T^e, \tag{8}$$

$$u'(c_2^e) = \beta R u'(c_3^e). \tag{9}$$

Equation (8) determines c_1^e. Budget constraints (1) and (2) and first-order condition (9) jointly determine c_2^e and c_3^e. Note that c_2^e and c_3^e are independent of the expected parent's transfer T^e ($\frac{\partial c_2^e}{\partial T} = 0$, $\frac{\partial c_3^e}{\partial T} = 0$), because the child does not incorporate the habit effect from Assumption 1.

Given that the expected consumption behavior c_i^e is conditional on parental income transfer, the child, in turn, computes expected income transfer T^e by anticipating his parent's utility-maximizing behavior. When anticipating the parent's behavior, however, he does not incorporate his own habit from Assumption 1. From the anticipated utility-maximizing behavior of the parent, the expected transfer T^e is determined jointly with expected parent's consumption ($c_{2,p}^e$, $c_{3,p}^e$) by the budget constraint

$$c_{3,p}^e = R(y_p - c_{2,p}^e - T^e)$$

and the first-order conditions

$$(1-\gamma)[u'(c_{2,p}^e) - \beta R u'(c_{3,p}^e)] = 0, \tag{10}$$

$$\beta R(1-\gamma)u'\left(c_{3,p}^{e}\right)=\gamma u'(T^{e}), \tag{11}$$

where we set $\frac{\partial c_2^e}{\partial T}=0$ and $\frac{\partial c_3^e}{\partial T}=0$ in (11) from Assumption 1.

4. The Altruistic Intervention of the Parent

4.1. The Actual Transfer Decided by the Parent

The parent behaves differently from the child's expectations because she knows that her child's preferences will shift due to the effect of habit formation, even though the child himself does not recognize it in his early periods. Hence, when deciding an income transfer level for her young son, the mother takes into account the habit effect that the transfer will have on his consumption in his second and third periods. Formally, the parent chooses the transfer level to her child by maximizing her altruistic utility (7), which incorporates the child's true utility function (3). We denote the actual income transfer level that the parent decides optimally by T^*.

In the child's second period, he takes the habit as a permanent preference shock. Maximizing (5) leads to the first-order condition of the child in the second period:

$$u'[c_2-\theta_1 c_1^*]=\beta R u'[c_3-\theta_1 c_1^*], \tag{12}$$

where the first-period consumption is given by the borrowing constraint

$$c_1^*=T^*. \tag{13}$$

Under budget constraints (13) and

$$c_3^*=R(y_2-c_2^*),$$

the actual consumption levels c_2^* and c_3^* are determined by (12). Because the first-period consumption c_1^* depends on the income transfer level from (13), the actual consumption levels c_2^* and c_3^* are functions of the income transfer T from (12).

With the function c_2^* and c_3^* being given, the parent chooses $(c_{2,p}, c_{3,p}, T^*)$ so as to maximize (7). The first-order conditions are

$$(1-\gamma)\left[u'\left(c_{2,p}^*\right) - \beta R u'\left(c_{3,p}^*\right)\right] = 0, \tag{14}$$

$$\beta R(1-\gamma)u'\left(c_{3,p}^*\right) = \gamma[u'(T^*) + \beta u'(c_2^* - \theta_1 c_1^*)\left(\frac{\partial c_2^*}{\partial T^*} - \theta_1 \frac{\partial c_1^*}{\partial T^*}\right)$$

$$+\beta^2 u'(c_3^* - \theta_2 c_2^*)\left(\frac{\partial c_3^*}{\partial T^*} - \theta_2 \frac{\partial c_2^*}{\partial T^*}\right).$$

(15)

First-order conditions (14) and (15) and budget constraint (6) jointly determine the parental income transfer level. We denote it by $T^*\left(\alpha, \gamma, \beta, R, y_p, \theta_1, \theta_2, y_2\right)$.

In the following subsections, we characterize the income transfer T^* of the altruistic parent by comparing it with the young child's expectation on income transfer T^e. By substituting functions $c_i^*(T)$ into (3), we obtain the child's indirect utility as a function of income transfer T, $U(T)$. Notice that there exists a maximum of $U(T)$. We denote the transfer level that maximizes the child's utility by $\bar{T}$. By definition, it satisfies $U'(\bar{T}) = 0$. Using Assumption 3, we express $\bar{T}$ as $\bar{T}(\alpha, \beta, R, \theta_1, \theta_2, y_2)$.

Since the child's utility is maximized at the point of $\bar{T}$, too much income transfer (more than $\bar{T}$) harms him. Note that the parent necessarily chooses income transfer level T^* such that it is

smaller than $\bar{T}$: $T^* < \bar{T}$. This choice is made because if T^* were higher than $\bar{T}$, the parent could enhance both her own welfare and the child's welfare by reducing the income transfer. However, the expected transfer T^e can be either smaller or larger than $\bar{T}$, depending on the parent's income level y_p. To compare the actual and expected transfers (T^* and T^e) and the corresponding welfare levels, it would, therefore, be helpful to separate two cases: (i) $y_p > \bar{T}$ and (ii) $y_p \leq \bar{T}$, which represent the rich-parent and poor-parent cases, respectively.

4.2. Comparison of Actual and Expected Transfers

The parent is aware of her child's habit formation and incorporates it in her decision making, while the child does not. Thus, the actual transfer T^* deviates from the expected one T^e. Proposition 1 states the relationship between T^* and T^e.

Proposition 1. For any degree of parental altruism $\gamma \in (0,1)$, the parental income transfer is unexpectedly small to the young child, that is, $T^* < T^e$.

Proof. See Appendix A1.

Proposition 1 implies that, insofar as the parent is not perfectly selfish or perfectly altruistic, she keeps the transfer level to her child lower than he expects. The parent knows that more transfer leads the child to form deeper habits, and hence consume more excessively in the next period. Hence, she intervenes in the child's behavior by keeping the transfer low.

Note that when the parent is perfectly selfish ($\gamma = 0$) or perfectly altruistic ($\gamma = 1$), T^* can equal T^e. In particular, when $\gamma = 0$, we have trivially $T^* = T^e = 0$ (i.e., purely selfish parents do not give, and are not expected to give, transfers to their children).

When $\gamma = 1$, we have $T^* = \bar{T}$ if the parent is poor, $y_p \leq \bar{T}$. Since the poor parent cannot afford to transfer $\bar{T}$, she just gives her whole income to her child, as he expects. In the poor parent case, therefore, we have $T^* = T^e = y_p$. When the parent is rich, $y_p > \bar{T}$; however, the parental transfer T^* is smaller than what the child expects and again, $T^* < T^e$. This is because T^* maximizes the child's utility, $T^* = \bar{T}$, whereas the naïve child expects the purely altruistic parent to transfer her whole income to him, $T^e = y_p (> \bar{T}$ by constuction).

4.3. The Welfare of the Child

As shown in the previous section, the parental income transfer is unexpectedly small in the eyes of the young child. Our interest here is to evaluate it from the viewpoint of the child's welfare; we shall show that the unexpectedly small transfer could make the child better off.

We compare utility values of actual transfer T^* $(U(T^*))$ and expected transfer T^e $(U(T^e))$ by employing the true utility function (3). Because the child is incognizant of the true utility function $U(\cdot)$ until his third period, the utility function $U(\cdot)$ can be regarded as a retrospective welfare measure of income transfer T.[8]

Of particular interest is whether the parental income transfer generates higher welfare of the child than the child's expected transfer does, when evaluated by the retrospective measure. If it does, the grown-up child is thankful to his parent for receiving the unexpectedly small transfer. Hence, we include the following definition.

Definition 1. When $U(T^*) > U(T^e)$, parental income transfer T^* is referred to as *thankworthy* to the child.

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[8] Pollack (1978) recommends "unconditional welfare ordering", which incorporates the effects of endogenous preference formation—we used the same welfare criterion here.

We first consider the case of a poor parent, where $y_p \leq \bar{T}$ (Figures 1 and 2). In this case, the parent cannot provide the child with a transfer exceeding $\bar{T}$. As Figure 1 shows, both T^* and T^e are less than y_p, and hence are less than $\bar{T}$. Recall from Proposition 1 that $T^* < T^e$ for $\gamma \in (0,1)$. Since the child's welfare increases with transfers in the region of $T < \bar{T}$, we have $U(T^*) < U(T^e)$ for $\gamma \in (0,1)$ (i.e., the parental income transfer is not thankworthy to the child).[9]

<Figure 1> & <Figure 2>

When the parent is rich, $y_p > \bar{T}$, the child's welfare first increases and then decreases with the income transfer level. As Figure 3 shows, the rich parent can provide the child with more transfer than $\bar{T}$, but she maintains transfer level T^* lower than this. However, the expected transfer T^e can be more than $\bar{T}$. Too much transfer (more than $\bar{T}$) causes the child's welfare U to decrease. Therefore, the retrospective utility of the expected transfer $U(T^e)$ can be lower than that of actual transfer $U(T^*)$.

Recall that when $\gamma = 0$, $T^* = T^e = 0$, and that when $\gamma = 1$, $T^* = \bar{T}$, which is an interior solution, and $T^e = y_p$, which is a corner solution (Figure 4). As γ increases from 0 to 1, the actual transfer T^* increases from 0 to $\bar{T}$, and the expected transfer T^e increases from 0 to y_p. The parent has to balance between her own utility and the child's utility. Under the condition of the rich parent, the child's welfare increases with the actual transfer T^*, and therefore also

[9] When the parent is purely selfish ($\gamma = 0$) or purely altruistic ($\gamma = 1$), we have $U(T^*) = U(T^e)$ (Figure 2).

increases with the degree of parental altruism γ. However, the child's retrospective utility level of T^e decreases with T^e when it exceeds $\bar{T}$. Therefore, there exists a certain degree of parental altruism, which equalizes $U(T^*)$ and $U(T^e)$.

Lemma 1. When the parent is rich, there exists a degree of parental altruism $\tilde{\gamma} \in (0,1)$ such that $U(T^*) = U(T^e)$.

Proof. See Appendix A2.

For all degrees of parental altruism higher than $\tilde{\gamma}$, the actual transfer leads to higher welfare of the child, and hence, the actual transfer is thankworthy for the child.

<Figure 3> & <Figure 4>

Proposition 2. When the parent is rich ($y_p > \bar{T}$), the parental income transfer T^* is thankworthy for the child, that is, $U(T^*) > U(T^e)$, if the parent is so altruistic that $\gamma > \tilde{\gamma}$.

Proof. See Appendix A2.

Reducing parental income transfer (i) directly lowers the child's utility in each period and (ii) weakens the child's consumption habit. When the parent's income and the degree of parental altruism are large enough, and hence the income transfer is large enough, the second negative effect dominates the first positive one. Therefore, the unexpectedly low transfer makes the child better off. In this context, in his third period, the child becomes aware of the habit and thanks the parent for having given him less transfer than he expected.

5. An Increase in the Habit Parameter

As shown in Proposition 1, the altruistic parent keeps income transfer to the child lower than what the child expects to receive. When the child's consumption habit and his consumption propensity are exogenously enhanced, the parent would further reduce the income transfer so as to mitigate the resulting increase in the child's over-consumption.

We can show the validity of this conjecture by examining the effect of an exogenous increase in the child's habit parameter in his second period θ_1.

Proposition 3. A higher degree θ_1 of the child's habit in the second period implies a lower level of parental income transfer T^*, that is, $\frac{dT^*}{d\theta_1} < 0$.

Proof. See Appendix A3.

Proposition 3 shows that the parent penalizes the child for an increase in habitual consumption by reducing income transfer. This strict behavior, however, comes from the parent's altruistic motive.

Note that the apparently strict yet altruistic behavior of the parent is similar to the parents' "tough love" behavior that Bhatt and Ogaki (2012) derived. By assuming an endogenous time discounting function, they show that, in response to an exogenous rise in a child's future time preference, his altruistic parent reduces income transfer so as to mitigate the increase in his degree of impatience. Our result in Proposition 3 is comparable to this tough love result, because an increase in the habit parameter would raise the child's time preference.

Formally, following the literature on time preference (e.g., Obstfeld, 1990), we define the

child's pure subjective rate of time preference in his second period as follows:

$$\chi_{2,3}(c) = \log MRS(c_2, c_3)\,|_{c_2=c_3=c}$$
$$= \rho + \log\left[\frac{u'(c-\theta_1 h_2)}{u'(c-\theta_2 h_3)} - \frac{\theta_2}{1+\rho}\right], \tag{16}$$

where $\rho = {}^{1}/_{\beta} - 1$ is the subjective discount rate of the child. It is then is easy to see that when degree of the habit formation in the child's second period θ_1 exogenously increases, the child's time preference function $\chi_{2,3}(c)$ shifts upward. Because the parent reduces income transfer T^* (see Proposition 3) upon the increase in θ_1, the parent's behavior above seems to duplicate the tough love behavior discussed by Bhatt and Ogaki (2012).

Our discussion is distinct in two aspects. First, in the present model, time preference is endogenously generated by habit formation rather than by an exogenously given time preference formation. Second and more importantly, our tough love is thankworthy to the child if his parent is altruistic and rich enough. This contrasts with the case explored in the Ogaki and Bhatt model, wherein a mother's tough love necessarily reduces her son's welfare, because the mother evaluates her son's future utility by using different time preference than that of the child.

6. Concluding Remarks

By employing a parental altruism model where the child is ignorant of his own habit until he reaches adulthood, we show that the parental income transfer seems unexpectedly low to the young child. However, the unexpectedly low parental income transfer can generate higher welfare for the child than the expected transfer, when evaluated retrospectively. In other words, when the parent is rich and sufficiently altruistic, the parent's income transfer is thankworthy to

the grown-up child. The predictions of this model are consistent with daily observations.

This research may have important implications for understanding how parent-child interactions affect the consumption/saving behavior and related preference formation of family members. For example, our result is consistent with the observation that richer parents, or parents from richer families, tend to be stricter than poorer parents (or those from poorer families) when safeguarding against their children's profligacy.

Appendix

A1. Proof of Proposition 1

When $\forall\gamma \in (0,1)$, we compare the actual transfer T^* with the expected transfer T^e by dividing the first order condition (11) with the first order condition (15). By substituting first-order conditions (9) and (14) as well as Assumption 1 we have

$$\left(\frac{y_p - T^*}{y_p - T^e}\right)^{-\alpha} = \left(\frac{T^*}{T^e}\right)^{-\alpha}$$
$$- \frac{\beta\theta_1(R+1)(\beta R)^{-\frac{1}{\alpha}}[Ry_2(\beta R)^{-\frac{1}{\alpha}} - \theta_1(R+1)(\beta R)^{-\frac{1}{\alpha}}(T^*)]^{-\alpha}}{[1 + R(\beta R)^{-\frac{1}{\alpha}}]^{1-\alpha}(T^e)^{-\alpha}}$$
$$- \frac{\beta^2\theta_2(\theta_2+R)\left[1-(\beta R)^{-\frac{1}{\alpha}}\right]\left[Ry_2(1-\theta_2(\beta R)^{-\frac{1}{\alpha}}) - \theta_1(\theta_2+R)(1-(\beta R)^{-\frac{1}{\alpha}})(T^*)\right]^{-\alpha}}{[1 + R(\beta R)^{-\frac{1}{\alpha}}]^{1-\alpha}(T^e)^{-\alpha}}. \quad \text{(A1)}$$

Equation (A1) implies that $\frac{T^*}{T^e} < 1$. Therefore, we have $\forall\gamma \in (0,1),\ T^* < T^e$.

A2. Proof of Lemma 1 and Proposition 2

The child's utility U is continuous in the degree of parental altruism γ. When $T < \bar{T}$, U is increasing in T; and when $T > \bar{T}$, U is decreasing in T. The income transfers T^* and T^e are both continuous and increasing in γ.

When $y_p > \bar{T}$, we obtain $\lim_{\gamma\to 1} T^* = \bar{T}$, which is an interior solution and that $\lim_{\gamma\to 1} T^e = y_p$, which is a corner solution. We also have $\lim_{\gamma\to 1} U'(T^*) = 0$ and $\lim_{\gamma\to 1} U'(T^e) < 0$. Hence, in the neighborhood of $\gamma = 1,\ U(T^*) > U(T^e)$.

When γ decreases from 1, both T^* and T^e decrease, as does $U(T^*)$. However, $U(T^e)$ increases until T^e equals to $\bar{T}$. Hence, there exists a certain $\tilde{\gamma}$ such that $U(T^*) = U(T^e)$.

When $\gamma > \tilde{\gamma}$, we have $U(T^*) > U(T^e)$.

A3. Proof of Proposition 3

First-conditions (10) and (11) and budget constraint (4) jointly determine the optimal transfer of the parent $T^*(\alpha, \gamma, \beta, R, y_p, \theta_1, \theta_2, y_1, y_2)$.

Substituting first-order condition (10) into (11) and taking the total derivative of the optimal transfer by the parent T^* with respect to the habit parameter θ_1 leads to

$$
\begin{aligned}
&\{\frac{(1-\gamma)\alpha\beta R^{1-\alpha}\left(y_p - T^*\right)^{-\alpha-1}}{\gamma\left[1+R(\beta R)^{-\frac{1}{\alpha}}\right]^{-\alpha}} + \alpha(T^*)^{-\alpha-1} \\
&+\frac{\alpha\beta(\beta R)^{-\frac{1}{\alpha}}\theta_1(1+R)[Ry_2(\beta R)^{-\frac{1}{\alpha}} - \theta_1(1+R)(\beta R)^{-\frac{1}{\alpha}}(T^*)]^{-\alpha-1}}{[1+R(\beta R)^{-\frac{1}{\alpha}}]^{1-\alpha}} \\
&+\frac{\alpha\beta^2\theta_1\theta_2(\theta_2+R)^2[1-(\beta R)^{-\frac{1}{\alpha}}]^2[Ry_2\left(1-\theta_2(\beta R)^{-\frac{1}{\alpha}}\right) - \theta_1(\theta_2+R)(1-(\beta R)^{-\frac{1}{\alpha}})(T^*)]^{-\alpha-1}}{[1+R(\beta R)^{-\frac{1}{\alpha}}]^{1-\alpha}}\}\frac{dT^*}{d\theta_1} \\
&= -\frac{\beta(\beta R)^{-\frac{1}{\alpha}}(1+R)[Ry_2(\beta R)^{-\frac{1}{\alpha}} - \theta_1(1+R)(\beta R)^{-\frac{1}{\alpha}}(T^*)]^{-\alpha-1}}{[1+R(\beta R)^{-\frac{1}{\alpha}}]^{1-\alpha}} \\
&\cdot\{\left[Ry_2(\beta R)^{-\frac{1}{\alpha}} - \theta_1(1+R)(\beta R)^{-\frac{1}{\alpha}}(T^*)\right] + (1+R)(\beta R)^{-\frac{1}{\alpha}}(T^*)\} \\
&-\frac{\alpha\beta^2\theta_2(\theta_2+R)^2[1-(\beta R)^{-\frac{1}{\alpha}}]^2[Ry_2\left(1-\theta_2(\beta R)^{-\frac{1}{\alpha}}\right) - \theta_1(\theta_2+R)(1-(\beta R)^{-\frac{1}{\alpha}})(T^*)]^{-\alpha-1}}{[1+R(\beta R)^{-\frac{1}{\alpha}}]^{1-\alpha}} \\
&\cdot(\theta_2+R)(T^*),
\end{aligned}
\tag{A2}
$$

where, from Assumption 4, the coefficient of $\frac{dT^*}{d\theta_1}$ is positive; and the right hand side is negative.

Therefore, we have $\forall\gamma(0,1)$, $\frac{dT^*}{d\theta_1} < 0$.

A4. When Misperception Disappears in the Child's Second Period

Here, we show that our main results are robust even when the child is assumed to become aware of his own habit formation in the second period, rather than in the third period.

In order to guarantee that the arguments of the utility functions are positive, we assume, the following, instead of Assumption 1:

Assumption A1. $y_p < \frac{Ry_2}{\theta_1(R+\theta_2)}$.

We denote optimal solutions in the present case by using an apostrophe, instead of an asterisk in the text.

In the second period, the misperception of the child disappears and the child maximizes the utility $u(c_2 - \theta_1 c_1') + \beta u(c_3 - \theta_2 c_2)$. The first-order condition is given by

$$u'[c_2 - \theta_1 c_1'] = \beta(R + \theta_2)u'[c_3 - \theta_2 c_2]. \tag{A3}$$

With c_1' being given by the parent's income transfer T', (A3) and budget constraint (3) jointly determine the consumption levels c_2' and c_3'.

The parent's first-order conditions of the parent are

$$(1-\gamma)\left[u'\left(c_{2,p}'\right) - \beta R u'\left(c_{3,p}'\right)\right] = 0, \tag{A4}$$

$$\beta R(1-\gamma)u'\left(c_{3,p}'\right) = \gamma[u'(T') + \beta u'(c_2' - \theta_1 c_1')\left(\frac{\partial c_2'}{\partial T'} - \theta_1 \frac{\partial c_1'}{\partial T'}\right)$$

$$+\beta^2 u'(c_3' - \theta_2 c_2')\left(\frac{\partial c_3'}{\partial T'} - \theta_2 \frac{\partial '}{\partial T'}\right). \tag{A5}$$

First-order conditions (A4) and (A5) and budget constraint (6) jointly determine the parental income transfer T'. By noting that the expected transfer T^e is the same as in the text, we obtain the following result:

Proposition A1. For any degree of parental altruism $\gamma \in (0,1)$, the parental income transfer is unexpectedly small to the young child, that is, $T' < T^e$.

Proof.

By substituting (9) into (10), (A4) into (.5), and dividing (A5) with (10), we have

$$\left(\frac{y_p - T'}{y_p - T^e}\right)^{-\alpha} = \left(\frac{T'}{T^e}\right)^{-\alpha}$$
$$-\frac{\beta^2\theta_1(R+\theta_2)[Ry_2 - \theta_1(R+\theta_2)(T')]^{-\alpha}}{\left\{1 + (R+\theta_2)[\beta(R+\theta_2)]^{-\frac{1}{\alpha}}\right\}^{-\alpha}(T^e)^{-\alpha}} \tag{A6}$$

Equation (A6) implies that $\frac{T'}{T^e} < 1$. Therefore, we have $\forall\gamma \in (0,1),\ T' < T^e$.

We can also show that there is a critical degree of parental altruism $\hat{\gamma} \in (0,1)$, such that $U(T') = U(T^e)$. As in Proposition 2, we have:

Proposition A2. When the parent is rich ($y_p > \bar{T}$), the parental income transfer T' is thankworthy for the child, that is, $U(T') > U(T^e)$, if the parent is so altruistic that $\gamma > \hat{\gamma}$.

Proof.

The child's utility U is continuous in the degree of parental altruism γ. When $T < \bar{T}$, U is increasing in T; and when $T > \bar{T}$, U is decreasing in T. The income transfers T' and T^e are both continuous and increasing in γ.

When $y_p > \bar{T}$, we obtain $\lim_{\gamma \to 1} T' = \bar{T}$, which is an interior solution and that $\lim_{\gamma \to 1} T^e = y_p$, which is a corner solution. We also have $\lim_{\gamma \to 1} U'(T') = 0$ and $\lim_{\gamma \to 1} U'(T^e) < 0$. Hence, in the neighborhood of $\gamma = 1$, $U(T') > U(T^e)$.

When γ decreases from 1, both T' and T^e decrease, and $U(T')$ also decreases. However, $U(T^e)$ increases until T^e equals to $\bar{T}$. Hence there exists a certain $\tilde{\gamma}$ such that $U(T') = U(T^e)$. When $\gamma > \tilde{\gamma}$, we have $U(T') > U(T^e)$.

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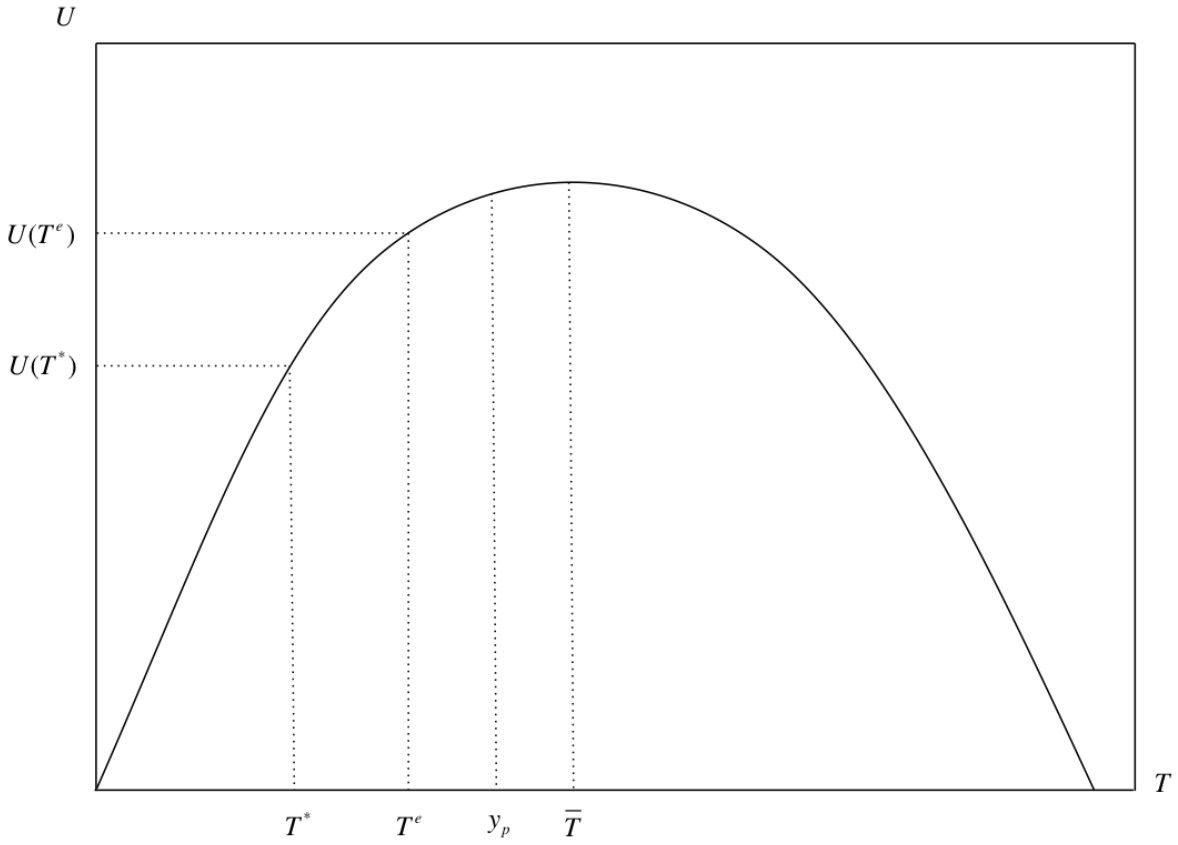

Figure 1. The utility of the child as a function of T when the parent is poor

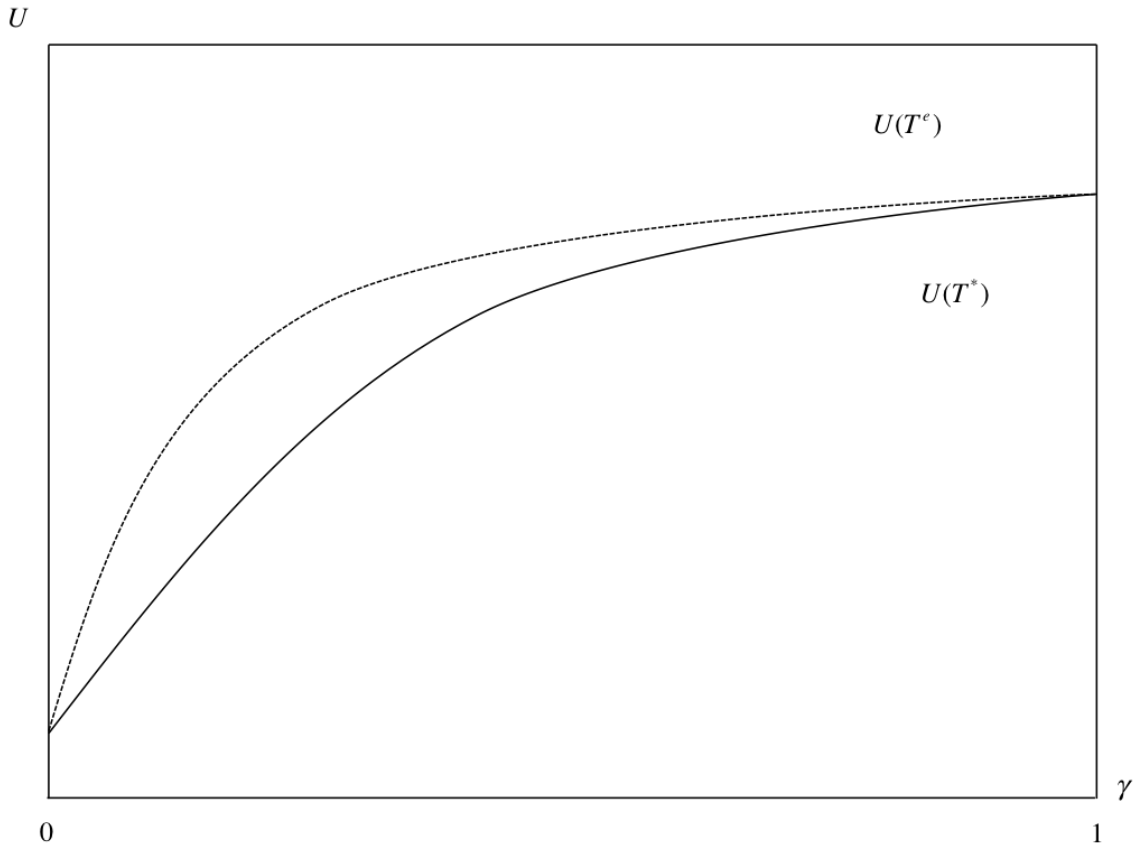

Figure 2. The utility of the child as a function of γ when the parent is poor

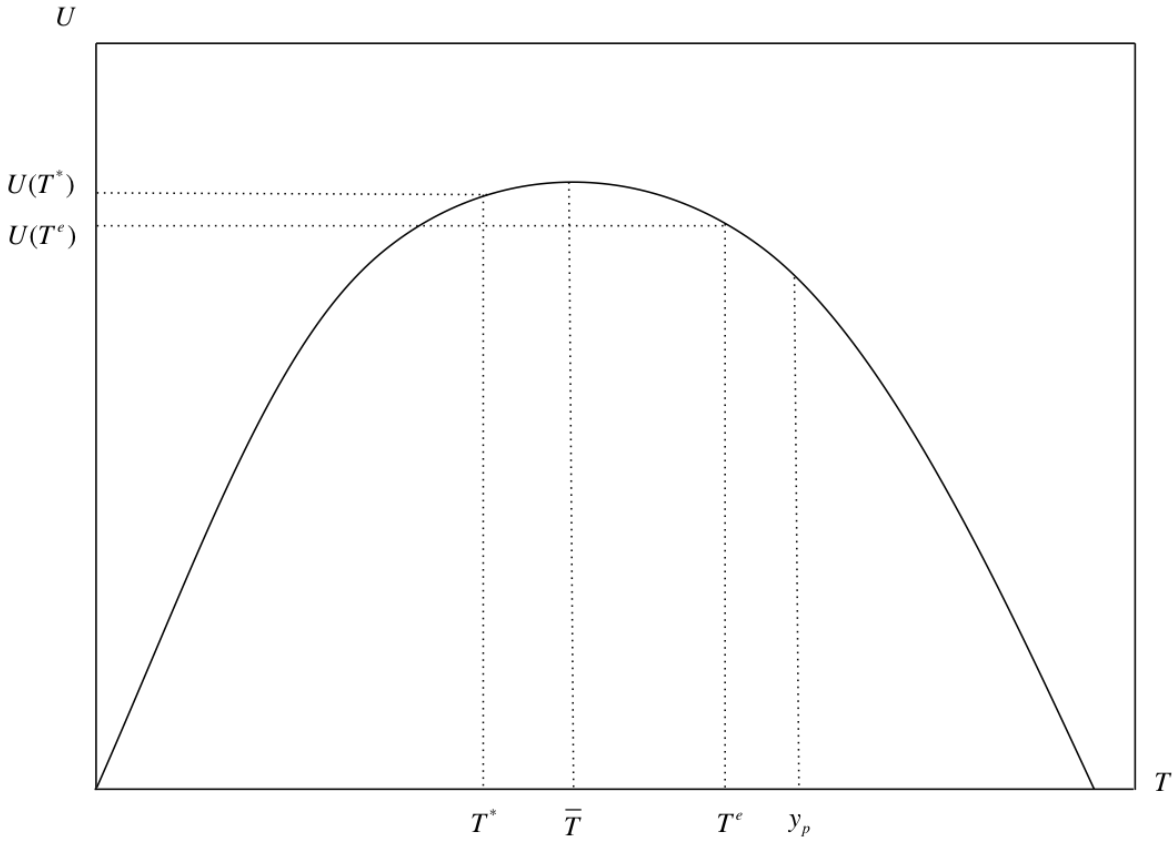

Figure 3. The utility of the child as a function of T when the parent is rich

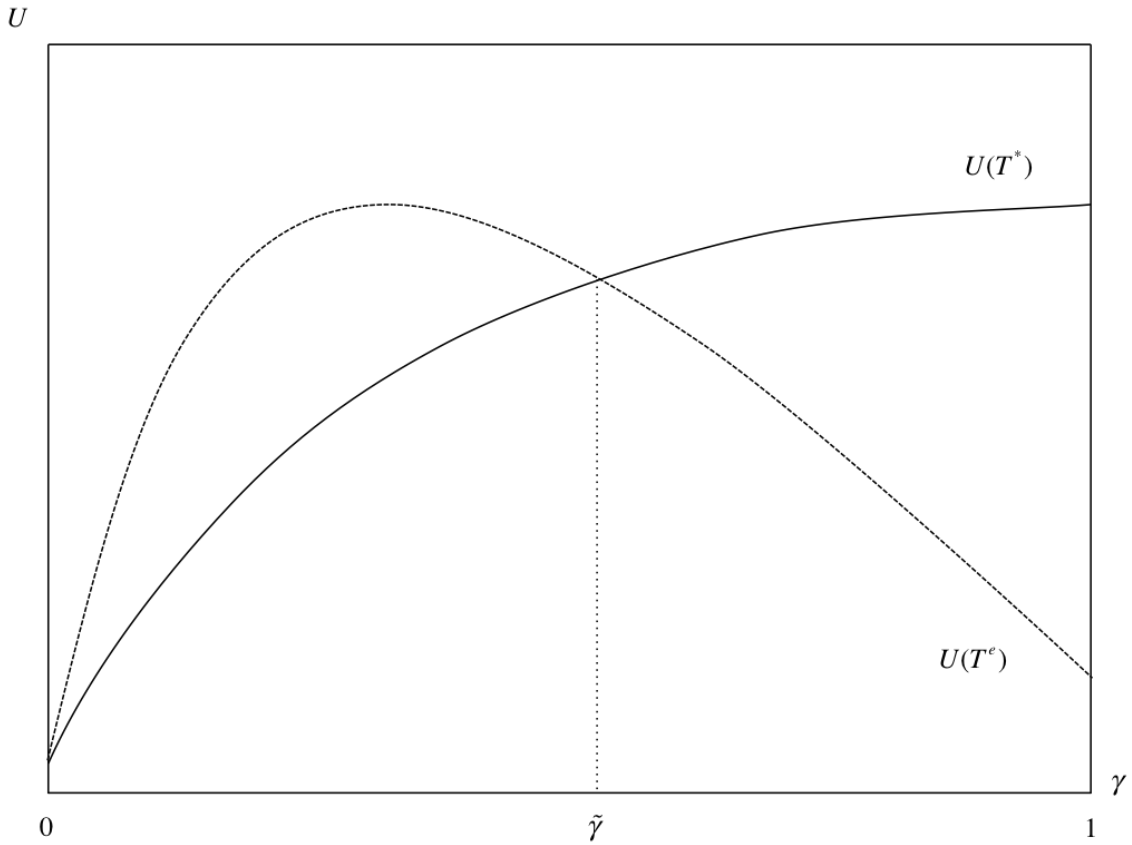

Figure 4. The utility of the child as a function of γ when the parent is rich