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Strategic Brand Proliferation: Monopoly vs. Duopoly

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Abstract

This paper investigates the possibility that market competition may encourage a firm to proliferate its brand. To show this, using the multi-product firm (MPF) model, we compare the incentive in monopoly with the one in duopoly. We find that if firms compete in quantity and provide *close substitutes*, the incentive may be greater in duopoly than in monopoly. This is because the MPF brands may act as *complements* to each other in oligopoly, whereas they always act as substitutes in monopoly. This result is contrary to that of Judd [RAND J. Econ 16 (1985) 153].

JEL classification: L13; D43

Keywords: brand proliferation, multi-product firm (MPF), product substitution, substitutes and complements, Cournot competition

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1 Introduction

This paper investigates the possibility that in differentiated duopoly, the existence of rival firms may encourage one of the firms to proliferate its brand (providing a new differentiated good). As such, our analysis is motivated by the following theoretical expectations. First, (i) a brand proliferating firm bears "a cost" in addition to the initial investment cost. In the context of industrial organization and management (or business), this is called the *cannibalization effect*, which acts as an intra-firm competition. This is a factor that leads firms to hesitate to proliferate their brands.[1] However, (ii) in duopoly, the new goods provided by firms may increase their own shares because of business-stealing effects. This effect is positive for duopolists, but for monopolists. Thus, it is clear that *brand proliferation* by a duopolist plays a different role than that by a monopolist.

The topic of *strategic* brand proliferation has been of great interest to researchers of industrial organization since Schmalensee (1978). For example, an empirical study by Kadiyali et al. (1998) shows that brand proliferation by a duopolist in the yogurt industry yields market expansion and therefore both firms raise their demands. Similarly, Draganska and Jain (2005) extend the argument of Kadiyali et al. (1998) and indicate that brand proliferation by oligopolists in the yogurt industry may mitigate price competition. These findings highlight the fact that brand proliferation acts as a strategic tool to yield the proliferating firm a profit, although the cannibalization effect hurts the firm. In spite of such empirical arguments, it is not clear whether the incentive is greater in oligopoly (duopoly) than in monopoly. This paper provides a theoretical attempt to clarify this.

In previous works, brand proliferation has been mainly discussed in the context of entry deterrence. As implied by Schmalensee (1978), this is because if the market size is constant, brand proliferation fills niches and makes entry harder. Although brand proliferation as a tool of entry deterrence has been broadly studied, not many studies have compared the incentives between the case in which market competition exists and the case where it does not. Judd's (1985) study, one of the few on the subject, discusses the credibility of preemption by crowding the product spectrum. Using a spatial differentiated duopoly model, Judd (1985) asserts that the implication by Schmalensee (1978) that incumbents' preemption by crowding the product spectrum may act as an entry barrier is not persuasive. That is, an established firm cannot threaten the rival entrant with proliferating its brands in advance, since providing multiple goods is no longer beneficial if the rival enters at least one of the markets. Thus, Judd (1985) implies that *market competition discourages the firms from proliferating*

[1] The *cannibalization effect* is the only characteristic of the MPF model that provides multiple differentiated goods. Because of product substitutability between the firm's own goods, those goods compete with each other. Hence the demand for each good is likely to be smaller in the case of MPF than in the case of the single-product firm (SPF) model, which provides only one variety even if there is no rival firm. Thus, a monopolist does *not always* want to proliferate its brands. See also the studies on MPFs, including Lin (2004), Grossmann (2007), and Eckel and Neary (2010).

their brands. This conclusion has been widely supported in the literature on brand proliferation and entry deterrence.[2]

Although it seems that Judd's (1985) suggestion is very persuasive, it overlooks one fact. In differentiated duopoly with single-product firms (SPFs), as mentioned by Dixit (1979), an increase in the degree of product substitutability decreases both profits *monotonically* as long as there is no cost difference.[3] It operates positively on the incentive since the profit difference between MPFs and SPFs becomes greater if the profit of MPF is constant. In contrast, in differentiated duopoly with at least one MPF, it is not clear whether or not an increase in competition decreases the profit of the MPF. This is because the strategic interaction between the two firms becomes complex. That is, if all the degrees of product substitutability are not identical, such interactions bring about asymmetric business-stealing effects between the two firms. Hence, any one of the firms may *benefit* from an increase in competition. This point plays an important role in brand proliferation in duopoly since it means the extension of the profit difference between MPFs and SPFs. Combining these two points, we show that Judd's (1985) suggestion is not valid in the case of MPF duopoly with asymmetric product substitutabilities.

To show this, in the next section, we first construct and analyze two basic models, a monopoly model and a duopoly model, and then compare the incentive to proliferate the brand in monopoly with that in duopoly. The timings are as follows. A monopolist (a duopolist), Firm 1, decides whether or not to proliferate its brand in Stage 1, then decides the quantity (competes in quantity with Firm 2) in Stage 2. To compare the two cases and exclude the effect of cost advantage, we assume that there is no asymmetry of marginal costs. In addition, suppose that in the case of duopoly, there exist asymmetries in the degree of product substitutability.[4] That is, we divide one parameter that measures the cross-price effect into two different parameters measuring the *competition effect* and the cannibalization effect, respectively. The competition effect indicates the strategic interaction between the firms. Under the above settings, we ascertain which incentive is stronger: monopoly or duopoly.

We first show that the profit of a duopolist may increase when the competition effect becomes larger (Proposition 1). This result is similar to that of Zanchettin (2006). Zanchettin's (2006) study, an important study of industrial organization, indicates the possibility that an increase in the degree of product substitutability between firms (competition effect) increases the profit of an *efficient* firm. However, our result is different from that of Zanchettin (2006) in that our result does not need any asymmetry of the marginal costs, although Zanchettin (2006) *necessarily* requires an asymmetry of the costs between two

[2]For instance, Belleflamme and Peitz (2010, p. 417), one of the latest textbooks on industrial organization, states, "If the incumbent can withdraw its product at sufficiently low cost from a segment in which it faces a direct competitor, brand proliferation is not a credible strategy for entry deterrence." This is the essence of Judd (1985).

[3]Zanchettin (2006) implies that in case of asymmetric costs, a more efficient firm may benefit from an increase in competition effect.

[4]The asymmetry means the insight of consumers that *"Goods A and B are not alike, but are similar to C."*

SPFs. Second, in contrast to Judd (1985), we suggest that there exists a case in which the incentive of a firm to provide another brand is greater in duopoly than in monopoly if the competition effect is sufficiently large but the cannibalization effect is sufficiently small (Proposition 2). The intuition behind the result is as follows. A relatively high competition effect affects both firms negatively, but a relatively small cannibalization effect mitigates only the effect on the proliferating firm, Firm 1. Accordingly, Firm 1 may *benefit* from an increase in the degree of the competition effect. Namely, Firm 1 gains the power of successive predation. Such a result is caused because the goods provided by an MPF may *not* act as *substitutes*, but as *complements* in duopoly, although they *always* act as substitutes in monopoly. This implies, for instance, the existence of brand loyalty, consumers' cost of moving among multiple stores, or any other *shopping costs* (Klemperer, 1992). That is, consumers loyal to Firm 1 prefer buying from Firm 1 over Firm 2. In addition, if consumers' cost of moving among multiple stores is very high, they may want to buy similar goods at a single store rather than at multiple stores. The consumers' subjective costs are called *shopping costs* which is a type of lump *switching costs*. If there exist shopping costs, the power of predation by the proliferating firm may become substantially stronger. Hence, we can see that Judd's suggestion is *not* always true.

We extend the basic model to a generalized analysis after the main study. That is, the asymmetry of the option of brand proliferation assumed in the previous section will be relaxed. By this, we find that there exist two asymmetric equilibria that hold the main proposition of this paper even if all firm conditions are symmetric: when the fixed cost for brand proliferation is relatively high, one of the firms proliferates its brand and the other does not. This is simply because of the *strategic substitute* interaction between the firms.

There are several related studies in addition to the above papers. First, in contrast to Judd (1985), Ishibashi (2003) suggests that preemption by a monopolist may operate as a tool for a credible entry barrier. Nevertheless, according to Ishibashi (2003), if an entrant *can* enter the market, the incumbent no longer wants to proliferate its brands. Ishibashi (2003) also implies that Judd's (1985) suggestion holds if there exists at least one *established* rival firm. However, this paper shows that an established firm may want to introduce a new brand more aggressively if there is a rival in the industry. Second, Jing and Zhang (2011) study endogenous product line selection and price promotions in duopoly. Using a vertical differentiation model with MPFs, Jing and Zhang (2011) show that there is an equilibrium in which one of the firms offers two different types of product, high-end and low-end, while the other offers only one type. The MPF with vertical differentiated goods benefits from providing multiple goods since the positive effect of price discrimination dominates the negative effect of cannibalization. This is similar to our paper in that the positive effect of product line extension dominates the negative effect of cannibalization and yields a competitive advantage. However, Jing and Zhang (2011) analyze a case of vertical differentiation in price competition, which stands in contrast with our study of horizontal differentiated quantity competition.

Third, Klemperer and Padilla (1997) argue for brand proliferation as a tool

of predation which indicates the possibility that in the case that there exist consumers having shopping costs, an established firm benefits from brand proliferation even if the new brand is independent of the other brands. Fourth, adopting asymmetric parameters of product substitutability, Xing and Zhao (2008) imply that asymmetries of the degrees of product substitutability among three goods provided by an SPF and an MPF themselves bring about demand shifts between the firms. This effect is the most important feature of Xing and Zhao (2008) and a similar effect also occurs in our paper. However, Xing and Zhao (2008) do not compare the monopoly case with the duopoly case.

Fifth, there is an investigation into whether competition makes firms proliferate brands in Hotelling's spatial model. Tabuchi (2012) finds that in the case of three or more firms, firms proliferate brands since the negative aspect of brand proliferation dominates the positive aspect that is the effect of reducing consumers' cost of moving dominates the effect of intensifying price competition, although it does not hold in duopoly. Thus, Tabuchi (2012) indicates the possibility that intensifying competition encourages firms to proliferate their brands. This point is common to our paper. However, Tabuchi (2012) definitively differs from our paper, as we compare the case of no competitor with the case of a competitor. Sixth, Raubitschek (1987) provides a fundamental study of brand proliferation by MPFs. This study, using a model of monopolistic competition with MPFs, examines the relation between the number of firms and brands in an industry, and concludes that the former is inversely proportional to the latter. In contrast, we conclude that competition may encourage a firm to provide an additional variety of a good. In other words, the number of goods in an industry may be larger in oligopoly than in monopoly.

The remainder of this paper is as follows. In Section 2, we present the basic model and analyze the equilibrium in monopoly and duopoly. In Section 3, we compare the firm's incentives to provide another good in monopoly and duopoly. In Section 4, we extend the basic model to a symmetric option case, and then generalize the main result. Finally, we conclude this paper.

2 The Basic Model

In this section, we present a model to determine sure whether a firm's incentive to provide another differentiated good is stronger in duopoly than in monopoly. First, we set up a two-stage monopoly model in which the monopolist, Firm 1, chooses whether to become a multi-product firm (MPF) by paying fixed cost F or remain a single-product firm (SPF) in the first stage, and then decides the quantity level in the second stage. Second, we provide a differentiated duopoly model where there exists a rival firm, Firm 2. Then, by comparing the former with the latter, we derive the condition to accomplish the purpose of the analysis. For simplicity, we assume that only Firm 1 has the option to provide a new variety of good and become an MPF in this section (this assumption will be relaxed in Section 4). This assumption can be interpreted as a situation in which Firm 2 faces some financial restriction or has no technology to create

a new variety. Firm 1 provides goods A and B. Let q_A and q_B denote the quantities of good A and B respectively. Similarly, q_2 indicates the quantity of good 2 provided by Firm 2. The inverse demand functions are as follows.[5]

$$p_A = a - q_A - \gamma_{AB} q_B - \gamma_{A2} q_2, \tag{1}$$

$$p_B = a - q_B - \gamma_{AB} q_A - \gamma_{B2} q_2, \tag{2}$$

$$p_2 = a - q_2 - \gamma_{A2} q_A - \gamma_{B2} q_B, \tag{3}$$

where a is the intercept of the inverse demand for each demand. $\gamma_{ij} \in (0, 1)$ indicates the degree of product substitutability between goods i and j for $i, j \in \{A, B, 2\}$, $i \neq j$. If there exists no rival, $q_2 = 0$. Similarly, $q_B = 0$ in the single-product firm case.

2.1 Monopoly Case

First, this paper examines the monopoly case. After solving two profit maximization problems of a monopolist (second stage), Firm 1 compares both profits and chooses the more profitable one (first stage).

2.1.1 Second Stage: Quantity Decision

In case of the single-product firm, the profit function of Firm 1 is

$$\pi_m^S(q_A) = (a - q_A - c) q_A, \tag{4}$$

where $c > 0$ is the constant marginal cost. The equilibrium quantity and profit of single-product firm are

$$q_m^S = \frac{a - c}{2}, \quad \pi_m^S = (\frac{a - c}{2})^2. \tag{5}$$

On the other hand, in case of the multi-product firm, the profit function is

$$\pi_m^M(q_A, q_B) = (a - q_A - \gamma_{AB} q_B - c) q_A + (a - q_B - \gamma_{AB} q_A - c) q_B - F, \tag{6}$$

where F is the fixed cost for the entry of good B. γ_{AB} implies the degree of product substitutability between goods A and B, which measures the *cannibalization effect.* The MPF's equilibrium quantities and profit are

$$q_A = q_B = \frac{a - c}{2(1 + \gamma_{AB})}, \tag{7}$$

$$\pi_m^M(\gamma_{AB}) = \frac{2}{1 + \gamma_{AB}} (\frac{a - c}{2})^2 - F. \tag{8}$$

By eq. (8), it is clear that the MPF's quantities and profit are the decreasing function of γ_{AB}.

[5] This type of inverse demand function is also used in Zhao and Xing (2006, 2008).

2.1.2 First Stage: MPF or SPF?

In the first stage, Firm 1 chooses whether to create an additional differentiated good:

$$\begin{cases} SPF & \text{if} \quad \pi_m^M(\gamma_{AB}) - \pi_m^S < 0, \\ \textit{indifferent} & \text{if} \quad \pi_m^M(\gamma_{AB}) - \pi_m^S = 0, \\ MPF & \text{if} \quad \pi_m^M(\gamma_{AB}) - \pi_m^S > 0. \end{cases}$$

These conditions can be rewritten as

$$\begin{cases} SPF & \text{if} \quad F_m(\gamma_{AB}) < F, \\ \textit{indifferent} & \text{if} \quad F_m(\gamma_{AB}) = F, \\ MPF & \text{if} \quad F_m(\gamma_{AB}) > F, \end{cases} \tag{9}$$

where $F_m(\gamma_{AB}) \equiv \frac{1-\gamma_{AB}}{1+\gamma_{AB}}(\frac{a-c}{2})^2$. $F_m(\gamma_{AB})$ also decreases in γ_{AB}.

2.2 Duopoly Case

Next, the case where there is a rival firm, Firm 2, is presented. For simplicity, we assume that only Firm 1 has the option to become an MPF.

2.2.1 Second Stage: Cournot Competition

The profit functions are

$$\pi_1^S = (a - q_A - \gamma_{A2}q_2 - c)q_A, \tag{10}$$

$$\pi_2^S = (a - q_2 - \gamma_{A2}q_A - c)q_2, \tag{11}$$

where γ_{A2} denotes the degree of product substitutability between goods A, provided by Firm 1, and good 2, provided by Firm 2.[6] γ_{A2} measures the *competition effect* between the firms. The equilibrium quantities and profits are

$$q_A = q_2 = \frac{a-c}{2+\gamma_{A2}}, \tag{12}$$

$$\pi_1^S = \pi_2^S = (\frac{a-c}{2+\gamma_{A2}})^2. \tag{13}$$

Similarly, if Firm 1 becomes a multi-product firm, the profit functions are

$$\begin{aligned} \pi_1^M =& (a - q_A - \gamma_{AB}q_B - \gamma_{A2}q_2 - c)q_A \\ &+ (a - q_B - \gamma_{AB}q_A - \gamma_{B2}q_2 - c)q_B - F, \end{aligned} \tag{14}$$

$$\pi_2^M = (a - q_2 - \gamma_{A2}q_A - \gamma_{B2}q_B - c)q_2, \tag{15}$$

[6] If $\gamma_{A2} = 0$, these two firms are the different market monopolists. In addition, if $\gamma_{A2} = 1$, the firms provide homogeneous goods.

where γ_{B2} indicates the degree of product substitutability between good B and good 2. In the following, to focus on the relation between the degree of the *competition effect* (γ_{A2} and γ_{B2}) and that of the *cannibalization effect* (γ_{AB}), we assume $\gamma_{A2} = \gamma_{B2}$ and redefine these parameters as γ_{12}. That is, $\gamma_{A2} = \gamma_{B2} \equiv \gamma_{12}$.[7] This redefined parameter implies the degree of product substitutability between the goods of Firm 1 and those of Firm 2.

The equilibrium quantities and profits are

$$q_A^M = q_B^M = \frac{(2-\gamma_{12})(a-c)}{2(2+2\gamma_{AB}-\gamma_{12}^2)}, \quad q_2^M = \frac{(1+\gamma_{AB}-\gamma_{12})(a-c)}{2+2\gamma_{AB}-\gamma_{12}^2}, \tag{16}$$

$$\pi_1^M = \frac{(1+\gamma_{AB})(2-\gamma_{12})^2(a-c)^2}{2(2+2\gamma_{AB}-\gamma_{12}^2)^2} - F, \quad \pi_2^M = \frac{(1+\gamma_{AB}-\gamma_{12})^2(a-c)^2}{(2+2\gamma_{AB}-\gamma_{12}^2)^2}. \tag{17}$$

2.2.2 First Stage: MPF or SPF?

In the first stage, Firm 1 in the differentiated duopoly chooses whether to enter an additional differentiated good:

$$\begin{cases} SPF \quad if \quad \pi_1^M(\gamma_{12},\gamma_{AB}) - \pi_1^S(\gamma_{12}) < 0, \\ indifferent \quad if \quad \pi_1^M(\gamma_{12},\gamma_{AB}) - \pi_1^S(\gamma_{12}) = 0, \\ MPF \quad if \quad \pi_1^M(\gamma_{12},\gamma_{AB}) - \pi_1^S(\gamma_{12}) > 0. \end{cases}$$

These conditions can be rewritten as

$$\begin{cases} SPF \quad if \quad F_d(\gamma_{12},\gamma_{AB}) < F, \\ indifferent \quad if \quad F_d(\gamma_{12},\gamma_{AB}) = F, \\ MPF \quad if \quad F_d(\gamma_{12},\gamma_{AB}) > F, \end{cases} \tag{18}$$

where

$$F_d(\gamma_{12},\gamma_{AB}) \equiv \frac{(1-\gamma_{AB})(8+8\gamma_{AB}-\gamma_{12}^4)}{2(2+2\gamma_{AB}-\gamma_{12}^2)^2(2+\gamma_{12})^2}(a-c)^2.$$

3 Does Quantity Competition encourage a Duopolist to become an MPF?

In this section, we compare Firm 1's incentive to provide another variety between the monopoly case and the duopoly case. Since the purpose of this paper is to determine whether there exists a case where quantity competition encourages Firm 1 to provide another differentiated good, we combine two conditions (11) and (20). The condition where a duopolist, Firm 1 provides another differentiated good and becomes an MPF but monopolistic Firm 1 does not is

$$F_d(\gamma_{12},\gamma_{AB}) \geq F \geq F_m(\gamma_{AB}). \tag{19}$$

[7] In the sense that both goods A and B are produced by Firm 1 and only good 2 is produced by firm 2, γ_{A2} and γ_{B2} can be summarized in a competition effect.

This inequality implies the parameter condition that Firm 1's incentive to provide differentiated good is larger in duopoly than in monopoly. Likewise, since $F_d(0, \gamma_{AB}) = F_m(\gamma_{AB})$, the above condition can be rewritten as

$$F_d(\gamma_{12}, \gamma_{AB}) \geq F \geq F_d(0, \gamma_{AB}). \tag{20}$$

Hence, what we investigate hereafter is summarized as follows: *whether the existence of quantity competition enhances the profitability of MPF."* To verify this, we first partially differentiate $F_d(\gamma_{12}, \gamma_{AB})$ with respect to γ_{12}, then find the conditions of γ_{12} and γ_{AB} where the derivative is positive:

$$\frac{\partial F_d(\gamma_{12}, \gamma_{AB})}{\partial \gamma_{12}} > 0 \tag{21}$$

As a consequence, we obtain Proposition 1 and Corollary 1.

Proposition 1 *(i) For* $\gamma_{12} \in (2 - \sqrt{2(1-\gamma_{AB})}, 1)$ *and* $\gamma_{AB} \in (0, \frac{1}{2})$*, the profit of an MPF is monotonically increasing. In addition, (ii) the profit difference between MPF and SPF,* $F_d(\gamma_{12}, \gamma_{AB})$*, is also monotonically increasing.*

Proof of Proposition 1

$$\frac{\partial F_d(\gamma_{12}, \gamma_{AB})}{\partial \gamma_{12}} = \frac{\partial \pi_1^M}{\partial \gamma_{12}} - \frac{\partial \pi_1^S}{\partial \gamma_{12}}. \tag{22}$$

Apparently, the second term of RHS ($-\frac{\partial \pi_1^S}{\partial \gamma_{12}}$) is positive. Thus, the remaining problem is only the sign of the first term:

$$\begin{aligned} \frac{\partial \pi_1^M}{\partial \gamma_{12}} =& \frac{\partial \pi_1^M(q_1, q_2, \gamma_{12}, \gamma_{AB})}{\partial q_2} \frac{\partial q_2(\gamma_{12}, \gamma_{AB})}{\partial \gamma_{12}} \\ &+ \frac{\partial \pi_1^M(q_1, q_2, \gamma_{12}, \gamma_{AB})}{\partial \gamma_{12}} \end{aligned} \tag{23}$$

$$= 2q_1(-\gamma_{12} \frac{\partial q_2(\gamma_{12}, \gamma_{AB})}{\partial \gamma_{12}} - q_2), \tag{24}$$

where the first term is positive (Business-stealing effect) and the second term is negative (price down effect or market shrink effect). If the former effect dominates the latter, the sign of $\frac{\partial \pi_1^M}{\partial \gamma_{12}}$ becomes positive. It is a function of γ_{12} and γ_{AB}:

$$-2(\gamma_{12} \frac{\partial q_2}{\partial \gamma_{12}} + q_2) > 0 \quad \Leftrightarrow \quad -\gamma_{12}(\gamma_{12} - 4) - 2(1 + \gamma_{AB}) > 0. \tag{25}$$

Q.E.D.

The implications of Proposition 1 are as follows. (i) As the competition between the firms becomes more intense, MPF yields more profit. This result is similar to

that of Zanchettin (2006), who indicates that an intense competition may benefit an efficient firm. However, our result holds even if firms' costs are symmetric. (ii) There exists the possibility that the existence of quantity competition gives more benefit to a duopolist that provides another differentiated good.

Corollary 1 *The cross-derivative of the incentive in duopoly,* $F_d(\gamma_{12}, \gamma_{AB})$*, is negative:* $\frac{\partial^2 F_d(\gamma_{12},\gamma_{AB})}{\partial\gamma_{12}\partial\gamma_{AB}} < 0$.

Corollary 1 implies that the benefit from a marginal decrease in the degree of the cannibalization effect, γ_{AB}, is larger in duopoly than in monopoly. Accordingly, Proposition 1 and Corollary 1 yield a remarkable consequence: if the cannibalization effect is sufficiently low, *a higher competition effect makes the MPF more profitable.* Based on these, Proposition 2 is derived.

Proposition 2 *One of the duopolists wants to provide a differentiated good more aggressively in duopoly than in monopoly if the competition effect,* γ_{12}*, is sufficiently large but the cannibalization effect,* γ_{AB}*, is sufficiently small.*

Proof of Proposition 2 Take the limits $\gamma_{12} \to 1$ and $\gamma_{AB} \to 0$. Then, eq. (22) is satisfied in the neighborhood of $(\gamma_{12}, \gamma_{AB}) = (1, 0)$ since $F_d(\gamma_{12}, \gamma_{AB})$ and $F_m(\gamma_{AB})$ are continuous.

Q.E.D.

Further, Corollary 2 is derived from Proposition 2.

Corollary 2 *The brands provided by an MPF competing with a rival that provides a close substitute act as complements as long as Proposition 2 holds: nevertheless, they act as substitutes in monopoly.*

Proof of Corollary 2 Transform the inverse demand function of good A (B) into the demand function and differentiate it with respect to the price of good B (A). Then, in duopoly, $\frac{\partial q_i}{\partial p_j} < 0$, $i, j \in \{A, B\}, i \neq j$, if Proposition 2 holds. In contrast, in monopoly, the sign is always opposite.

Q.E.D.

The intuition behind Proposition 1 is as follows. If γ_{12} is sufficiently large, the driving out power becomes stronger. On the one hand, if γ_{AB} is small enough, Firm 1's goods A and B do not cannibalize each other too much. As a result, these two effects improve *only* Firm 1's position, and yield it higher profit (and reduce Firm 2's profit). Such an effect *never* occurs in monopoly. This implies that the brands provided by the MPF do not act as substitutes, but as complements to each other. This can explain the case in which a firm that competes with its rivals intensively attempts to provide multiple similar brands. For example, if Thomas can buy clothes by two apparel brands, 1 and 2, he may want to buy at one shop rather than at two. This is because he wants to get a complete set of brand 1's jeans. In this case, Thomas first chooses the

shop, then purchases the jeans.[8] Thus, the two types of jeans provided by brand 1 compete intensively with the similar one provided by brand 2. Of course, in this case, the competitivenesses between A and 2 and between B and 2 are the same.

In addition, the following result is also derived from Proposition 2.

Corollary 3 *If the degree of the competition effect equals that of the cannibalization effect ($\gamma_{12} = \gamma_{AB}$), there exists no case that a duopolist wants to add another good more aggressively than a monopolist.*

This indicates the importance of the separation of the measure of product substitutability into the measures of two effects, the competition effect and the cannibalization effect.

4 Extension

In this section, we try to extend the basic model to a symmetric option case.

4.1 Symmetric option case

The assumption in which only one of the two firms has the option to provide another good is relaxed in this subsection. That is, each firm has the option. In this case, each firm chooses its strategy in the first stage (Table 1).

Firm 1 \ Firm 2	MPF	SPF
MPF	π_1^{MM}, π_2^{MM}	π_1^{MS}, π_2^{SM}
SPF	π_1^{SM}, π_2^{MS}	π_1^{SS}, π_2^{SS}

Table 1: MPF or SPF?

Note that the consequence of the asymmetric case is shown in the previous section. Therefore, the remaining problem in this subsection is the result of MPF vs. MPF (π_1^{MM} vs. π_2^{MM}).

If both firms choose to provide good A with good B in the first stage, the inverse demand functions that the two firms face in the second stage are given by

$$p_{1A} = a - q_{1A} - \gamma_{AB} q_{1B} - \gamma_{12}(q_{2A} + q_{2B}), \tag{26}$$

$$p_{1B} = a - q_{1B} - \gamma_{AB} q_{1A} - \gamma_{12}(q_{2A} + q_{2B}), \tag{27}$$

$$p_{2A} = a - q_{2A} - \gamma_{AB} q_{2B} - \gamma_{12}(q_{1A} + q_{1B}), \tag{28}$$

$$p_{2B} = a - q_{2B} - \gamma_{AB} q_{2A} - \gamma_{12}(q_{1A} + q_{1B}). \tag{29}$$

[8] Such an interpretation is common to the *shopping costs* mentioned in Klemperer (1992) and Klemperer and Padilla (1997).

The profit functions are

$$\pi_1^{MM} = (p_{1A} - c)q_{1A} + (p_{1B} - c)q_{1B} - F, \tag{30}$$

$$\pi_2^{MM} = (p_{2A} - c)q_{2A} + (p_{2B} - c)q_{2B} - F. \tag{31}$$

The equilibrium quantity and profit are, respectively,

$$q_{1A} = q_{1B} = q_{2A} = q_{2B} = \frac{a - c}{2(1 + \gamma_{AB} + \gamma_{12})}, \tag{32}$$

$$\pi_1^{MM} = \pi_2^{MM} = \frac{(1 + \gamma_{AB})(a - c)^2}{2(1 + \gamma_{12} + \gamma_{AB})^2} - F. \tag{33}$$

After that, in the first stage, each firm chooses its strategy (MPF or SPF) given the opponent's strategy.

Then, we likewise compare the incentive in duopoly with a rival MPF to the one in monopoly. The condition is described by an inequality as follows.

$$F_{MM}(\gamma_{12}, \gamma_{AB}) \geq F \geq F_{MM}(0, \gamma_{AB}) \tag{34}$$

where $F_{MM}(\gamma_{12}, \gamma_{AB}) = \pi_1^{MM} - \pi_1^{SM} + F$ and $F_{MM}(0, \gamma_{AB}) = F_m(\gamma_{AB})$. As a consequence, we obtain the following result.

Proposition 3 *If the rival firm also provides multiple goods (MPF vs. MPF), then the incentive to provide another variety is always less in duopoly than in monopoly.*

Proof of Proposition 3 Suppose $F_{MM}(\gamma_{12}, \gamma_{AB}) \geq \mathrm{F} \geq F_{MM}(0, \gamma_{AB})$. However, for $\gamma_{12}, \gamma_{AB} \in [0, 1]$, the maximum value of $F_{MM}(\gamma_{12}, \gamma_{AB}) - F_{MM}(0, \gamma_{AB})$ is 0 if $\gamma_{12} = 0$ or $\gamma_{AB} = 1$. Therefore, $\forall \gamma_{12}, \gamma_{AB} \in (0, 1)$, $F_{MM}(0, \gamma_{AB}) > F_{MM}(\gamma_{12}, \gamma_{AB})$. This contradicts the above inequality.

Q.E.D.

Then, we show that asymmetric equilibria can exist in the first stage. In the previous section, we suppose that only one of the firms has the option of brand proliferation. Nevertheless, in this subsection, because of the symmetry of the option, a symmetric equilibrium in which both firms proliferate their brands can be shown. However, at the same time, there can also exist two asymmetric equilibria because of the fixed costs. Thus, we now need to ascertain the parameter domain of the fixed cost that holds the asymmetric equilibria. If it is non-empty, we can see the generality of the consequence of the previous section.

As supposed in the previous section, we assume anew that $F_{MS}(\gamma_{12}, \gamma_{AB}) \geq F$ where $F_d(\gamma_{12}, \gamma_{AB}) = F_{MS}(\gamma_{12}, \gamma_{AB}) = \pi_1^{MS} - \pi_1^{SS} + F$. If so, the thing to be shown is whether an inequality

$$\pi_1^{MS} \geq \pi_1^{SS} \geq \pi_1^{SM} \geq \pi_1^{MM} \tag{35}$$

holds under $F_{MS}(\gamma_{12}, \gamma_{AB}) \geq F \geq F_m(\gamma_{AB})$. The first half of the inequality, $\pi_1^{MS} \geq \pi_1^{SS}$, is the incentive to proliferate the brand given the opponent's

strategy for an SPF. Likewise, the second half of the inequality, $\pi_1^{SM} \geq \pi_1^{MM}$, implies the disincentive to provide another variety B given Firm 2's strategy for an MPF. Then, $\pi_1^{SS} \geq \pi_1^{SM}$. We must now prove that the incentive condition ($F_{MS}(\gamma_{12}, \gamma_{AB}) \geq F$) is consistent with the disincentive condition ($F \geq F_{MM}(\gamma_{12}, \gamma_{AB})$). By Proposition 2 and Proposition 3, we obtain Proposition 4.

Proposition 4 *If the competition effect is sufficiently large and the cannibalization effect is sufficiently small, the asymmetric equilibria where one of the firms proliferates its brand and the other remains an SPF exist even if the rival firm also has the option of brand proliferation.*

Proof of Proposition 4 By Proposition 3, $\forall \gamma_{12}, \gamma_{AB} \in (0,1)$, $F_m(\gamma_{AB}) > F_{MM}(\gamma_{12}, \gamma_{AB})$. Hence, $F_{MS}(\gamma_{12}, \gamma_{AB}) \geq F \geq F_m(\gamma_{AB}) > F_{MM}(\gamma_{12}, \gamma_{AB})$ holds in the neighborhood of $(\gamma_{12}^*, \gamma_{AB}^*) = (1, 0)$.

Q.E.D.

In contrast, if the fixed cost is relatively low, the disincentive condition may not hold. In this case, the MPF strategy is the dominant strategy for both players, and hence (MPF, MPF) is the unique Nash equilibrium. In other words, if the cost is relatively high, eq. (35) holds and quantity competition may encourage one of the firms to provide an additional variety in the case of a symmetric option in addition to the asymmetric option in Section 3. Thus, from the above argument, we can conclude that our main proposition (Proposition 2) is persuasive if the fixed costs are relatively high. Thus, our result is still contrary to Judd (1985).

5 Concluding Remarks

In this paper, we investigate the incentive of a duopolist to provide another variety. Since the competition effect and the cannibalization effect have been described by an identical parameter in most previous works, an increase in the degree of product substitutability among the goods tends to mean that the incentive to provide another variety and become an MPF is less in duopoly than in monopoly. However, in this paper, it is seen that the effects are separated. As a result, we find that if the competition effect is sufficiently high and the cannibalization effect is sufficiently low, the power of predation increases since the two brands act as complements in duopoly but as substitutes in monopoly. Thus, in this case, the existence of a rival firm encourages one of the firms to provide another differentiated good and become an MPF. Such a result is contrary to that of Judd (1985).

This paper constrains the game of brand proliferation to be one-shot. However, in reality, the strategic interaction is continuous: the rival SPF ought to respond to the strategy. Thus expansion to a sequential proliferation model is a subject for future research.

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