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Abstract

We construct an endogenous growth model with productive public capital and government debt when government debt is adjusted to the target level. We examine how reducing public debt in an economy with a large public debt affects the transition of the economy and welfare. We find that the government faces the following trade off when reducing its debt. In the short run, public investment is reduced to decrease the debt and this has a negative effect on welfare. However, as the interest payments on the debt decrease, public investment begins to increase. Eventually, the government can increase the amount of public investment by more than before. This has a positive effect on welfare, implying that reducing the debt is welfare improving. Furthermore, we find that the adjustment speed of reductions in debt affects welfare crucially. The relationships between the welfare gains and the adjustment speed are U-shaped in many cases. However, they are decreasing monotonically when the tax rate is low and the initial debt–GDP ratio is sufficiently large.

JEL classification: E62, H54, H63

Keywords: Reductions in public debt, Debt policy rule, Public capital, Endogenous Growth

1 Introduction

Recently, many developed countries have been suffering from large levels of government debt. In Europe, Greece has experienced severe government financial failure. Public debt as a share of GDP in Greece equaled 142.8 percent in 2010. The average level of public debt in the European Union (EU) was close to 80 percent of GDP in 2010.[1] In the United States (USA), the ratio was around 62 percent of GDP in 2010. In Japan, it is 225.8

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[1]The source of these data is Eurostat, which is owned by the European Commission. The debt–GDP ratios of the main countries of the EU are as follows: Austria, 72.3%; Belgium, 96.8%; Denmark, 43.6%; Finland, 48.4%; France, 81.7%; Germany, 83.2%; Hungary, 80.2%; Ireland, 96.2%; Italy, 119%; Netherlands, 62.7%; Poland, 55%; Portugal, 93%; Spain, 60.1%; and United Kingdom, 80%.

percent. If no adjustments are made and public debt keeps increasing, these countries may face a situation similar to that of Greece.

In the EU, there is a constraint on the size of government debt to reduce the chance of government financial failure. The Maastricht Treaty states that EU Member States are required to ensure that their government debt is below 60 percent of GDP. However, many countries in the EU have violated this criterion. In response, the European Commission has given the Member States projections for fiscal consolidation. In *Public finances in EMU – 2011*, published in 2011, the European Commission states that the public deficits must decrease faster and also that public debts should hit their inflection points in 2012, and then begin to fall as a share of GDP from then on. In September 2011, President Barack Obama sent a blueprint to Congress on how the USA can reduce its public deficit and pay down public debt. He presented a plan that aims to realize more than three trillion in net deficit reduction over the next 10 years. In his statement, the debt under this plan would be declining as a share of GDP over the next decade, falling from a high of 77 percent of GDP in 2013 to 73 percent of GDP in 2021. When considering the restructure of public finances in many developed countries, it is important to investigate how a reduction in public debt affects economic growth and welfare.

There are two main ways to reduce the debt levels. One is to increase tax rates, while the other is to cut spending. Some countries insist that spending cuts should be chosen, and others insist on a mix of spending cuts and tax increases. In the USA, the president plans to use both. On the other hand, in the EU, the European Commission proposes to use not tax increases but spending cuts for fiscal consolidation because evidence from the past indicates that expenditure-based consolidations (spending cuts) tend to have greater success. In November 2011, Germany, the Netherlands, Romania and the United Kingdom implemented spending cuts whereas France, Italy, Ireland, Greece, Portugal and Spain unveiled tax increases in addition to spending cuts.

This paper examines how reducing public debt affects economic growth and welfare when the government implements spending cuts, which are being discussed widely in USA and many European countries. When a government reduces debt levels by decreasing public spending, it faces the following trade off. In the short run, a decline in public spending worsens welfare levels because it implies a reduction in the amount of public services or infrastructure necessary for economic activities. However, a decline in public debt reduces interest payments and hence enables the government to increase public spending in the long run, thus improving welfare levels. Therefore, it is important to investigate whether reducing public debt by cutting public spending is really welfare improving. If it is welfare improving, should the government reduce its debt at a slower pace or at a more rapid pace? If the pace of the reduction is slow, the initial decline in public spending may be small. However, because the government takes a longer time to reduce its debt to the target level, the decline in public spending may be prolonged. On the other hand, a rapid reduction in debt may result in large initial declines in public spending. However, these large declines may be short lived because the government will take less time to reduce its debt level. Elucidation of this relationship between welfare and the speed of debt reduction would be helpful to the policy makers of countries with large public debt.

Furthermore, although there may be many types of government spending that can be cut, we discuss cuts in public investment spending in this paper. This is because in some countries, public investment is cut to reduce debt. In a paper published by

European Union, Rubianes (2010) writes, "*In the event that the necessary adjustment of public finance is made at the expense of public investment, it could even fall below the current level and compromise competitiveness and future gains of productivity over the medium and long run.*" In addition, when the government holds too much debt, private investment in the economy is crowded out severely. In this situation, public investment may not contribute adequately to growth if there is complementarity in goods production between private and public capital.[2]. Then, inefficient public investment financed by public debts may only increase the interest payments on the debt, resulting in an increase in outstanding debt. Therefore, cuts in public investment to reduce the debts may be necessary to improve the efficiency of public investment.

We construct an endogenous growth model with public debt finance, where the growth engine is productive public capital as in Futagami et al. (1993) and Turnovsky (1997). In order to investigate the welfare effects of reducing debt, we introduce the debt policy rule introduced by Futagami, Iwaisako and Ohdoi (2008). Under this rule, the government sets a target level of the debt-to-private-capital ratio and reduces its debt gradually to the target level. Public debt is reduced by spending cuts in public investment.

In this paper, we obtain the following three main results from the numerical analyses. First, we obtain the transitional behavior of the economy when the government reduces public debt. In the short run, reducing public debts decreases the growth rate of public capital and this enables more resources to be allocated to private investment. Therefore, the growth rate of private capital increases initially. As a result, the return to private capital (the interest rate) decreases and then the growth rate of consumption declines in the early stages of the transition. However, as the level of public debt becomes small, public investment begins to increase and this, in turn, crowds out private investment. Then, the growth rate of private capital begins to decline. This decline of private investment increases the interest rate and then the growth rate of consumption begins to recover. In the long run, the economy can attain higher growth rates of public capital, private capital and consumption than those in the state before the government reduced debt. The reason for this is as follows. Because government debt is reduced, the interest payments become smaller than in the initial state. Hence, public investment becomes larger than before the government began to reduce its debt, which makes it possible to attain higher rates of economic growth in the long run.

Second, we find that reducing debt at the expense of public investment improves welfare. In the short run, households change their consumption level immediately after the policy change. Whether this short-run effect becomes positive or negative depends on how expected future income changes because of the policy change. Then, when the government reduces its debt, the growth rate in consumption decreases in the early stage of the transition, as we stated above. This has a negative welfare effect during the transition. However, in the long run, the growth rate increases above the initial level, which has a positive welfare effect. Moreover, countries whose initial debt–GDP ratio is larger can achieve larger welfare gains. This is mainly because when the initial debt–GDP ratio is large, the growth rate of consumption in the new steady state is much higher than that in the initial steady state. In addition, although governments must make larger reductions in public debt and the decline of public investment is larger, more resources are released

[2]Futagami, Morita and Shibata (1993) and Turnovsky (1997) apply complementarity between private and public capital in goods production in their analysis.

to the private sector and households can consume more.

Third, the adjustment speed of the reduction in debt is an important determinant of welfare. The extent of welfare gains from a decrease in government debt as the government increases the speed of debt reduction depends on the tax rate and on how much the debt–GDP ratio exceeds the target level initially. In many cases, (i) the relationships between the welfare gains and adjustment speed are U-shaped. However, (ii) the welfare gains decrease monotonically with respect to the adjustment speed when the tax rate is low and the initial debt–GDP ratio is sufficiently large. If the government increases the speed of public debt reduction, it faces the following trade-off. On the one hand, the initial decline of public investment increases. On the other hand, the growth of consumption recovers sooner. When the tax rate is low and the initial debt–GDP ratio is large, the initial decline of public investment is too large when the government increases the adjustment speed. Then, the welfare gains decrease monotonically with respect to the adjustment speed. In contrast, when the tax rate is high or the initial debt–GDP ratio is small, the initial decline of public investment is small. Then, when the adjustment speed is somewhat high, the positive effect of the earlier recovery of the growth of consumption dominates the negative effect of the initial decline of public investment. Thus, the relationship between the welfare gain and the adjustment speed is U-shaped.

Besides these main results, there is also a possibility that the economy will fail to develop sustainably. When the initial level of public capital is sufficiently small relative to private capital, if the financing of public spending relies largely on issuing debt, the economy may fall into a development trap. In this trap, there exists no path the representative households with rational expectation and perfect foresight can follow. This occurs because tax revenue is too low to pay interest payments on debt because of scarce public capital. We show that the economy can avoid this situation by selecting tax financing rather than debt financing in its development process.

In the theoretical literature, many studies examine the fiscal policy of productive public spending with debt financing; for example, see Greiner and Semmler (2000), Ghosh and Mourmouras (2004) and Yakita (2008).[3] However, these studies deal not with the debt size criterion, but rather with the deficit constraint in the Maastricht treaty.[4] In contrast to these studies, Futagami et al. (2008) introduce a target rule for public debt size as we stated above, whereby public debt is issued to finance productive public flow services such as in Barro (1990). They show that there exist two steady states, and the transition paths are indeterminate.[5] Because the reduction of public debt generates transitional dynamics, the occurrence of indeterminacy of transition paths makes welfare analysis difficult. Therefore,

[3]Greiner and Semmler (2000) study how debt-financed fiscal policy affects growth in the long run. They introduce the golden rule of public finance (GRPF). Under this rule, public debt is issued only for public investment and government consumption, and the repayment of the principle and interest payments must be financed by tax revenue while the deficits are assumed to be a constant percentage of GDP. They show that a less strict policy rule does not necessarily promote growth. Ghosh and Mourmouras (2004) show the GRPF can avoid over-accumulation of public capital and be more efficient than the standard constraint of government. Yakita (2008) calculates the threshold that determines whether government debt is sustainable or not in an overlapping generations model.

[4]Under the Maastricht treaty, EU Member States are required to ensure their public deficits do not exceed three percent of GDP.

[5]Minea and Villieu (2012) find that if the government does not set the debt target, defined as the ratio of bonds to GDP, equal to the value of private capital as in Futagami et al. (2008), there exists a unique steady state and the indeterminacy is removed.

they do not investigate the welfare effects of reductions in public debt. We solve this problem by replacing the flow of public services with the stock of public capital. In this paper, the economically meaningful steady state is unique and thus the transition path is determined uniquely. Therefore, we obtain interesting results regarding the welfare effects and transitional responses of debt reduction.

The remainder of this paper is organized as follows. Section 2 presents the basic model. Section 3 derives the equilibria and transition dynamics of the economy. Section 4 conducts the comparative statics analyses of the equilibria and analyzes the policy effects on economic growth and public investment. Section 5 examines the transitional responses when the government lowers the target level of debt. Section 6 presents the policy effects on welfare. Section 7 concludes the paper.

2 The Model

Our model is based on the model of Futagami et al. (2008). The only difference between their model and ours is that productive government inputs used in the production of goods are public capital not public flow services. We consider an economy populated by an infinitely long-lived representative household with an infinite planning horizon and perfect foresight. Time is denoted as $t \geq 0$. Without any loss of generality, we assume that there is no population growth and the population size is normalized to unity.

2.1 Production structure

A single final good, Y_t, is produced with private capital, K_t, public capital, $K_{g,t}$, and labor, L_t. The production function takes the Cobb-Douglas form as follows:

$$Y_t = AK_t^{\alpha}(K_{g,t}L_t)^{1-\alpha}, \quad \alpha \in (0,1). \tag{1}$$

The first-order conditions for profit maximization are given by:

$$r_t = \alpha A k_{g,t}^{1-\alpha}, \tag{2}$$

$$w_t = (1-\alpha) A k_{g,t}^{1-\alpha} K, \tag{3}$$

where r_t and w_t denote the interest rate and the wage rate, and $k_{g,t}(\equiv K_{g,t}/K_t)$ is the ratio of public capital to private capital.

2.2 Households

The utility function of the representative household is specified as:

$$U_0 = \int_0^{\infty} (\ln C_t) e^{-\rho t} dt, \tag{4}$$

where C_t and $\rho(>0)$ denote consumption and the subjective discount rate, respectively.

The household's budget constraint is given by:

$$\dot{W}_t = (1-\tau)(r_t W_t + w_t) - C_t, \tag{5}$$

where W_t denotes assets and $\tau \in [0, 1)$ is the income tax rate, which is assumed to be time invariant, as in Futagami et al. (2008). Taking the interest rate, wage rate and tax rate as given, the household chooses its consumption path so as to maximize (4) subject to (5). Intertemporal maximization yields the following condition:

$$\frac{\dot{C}_t}{C_t} = (1 - \tau)r_t - \rho. \tag{6}$$

In addition, the following transversality condition must hold:

$$\lim_{t\to\infty} C_t^{-1} W_t \exp(-\rho t) = 0. \tag{7}$$

2.3 Government

The government in this economy imposes income taxation, and issues bonds, B_t, to finance public capital investment, G_t. For simplicity, we assume that the depreciation rate of public capital is zero. The evolution of public capital is given by:

$$\dot{K}_{g,t} = G_t. \tag{8}$$

The budget constraint of the government is:

$$\dot{B}_t = r_t B_t + G_t - \tau(r_t W_t + w_t). \tag{9}$$

Following Futagami et al. (2008), we assume that the government adjusts its bonds gradually to a target level. In this paper, we gauge the size of the economy by the level of private capital, K_t, as in Futagami et al. (2008). We assume the government adjusts $b_t \equiv B_t/K_t$ according to the following rule:

$$\dot{b}_t = -\phi(b_t - \bar{b}), \tag{10}$$

where $\bar{b}$ and $\phi(> 0)$ denote the target level of government bonds and the adjustment coefficient of the rule, respectively.[6] We consider the case of $\bar{b} > 0$. The government must adjust the level of government spending G_t at every point in time, given this bond issuance rule and the tax revenue. When the government reduces the target, $\bar{b}$, it has to reduce debt according to the difference between the current and the target level of b: $b_t - \bar{b}$. If the adjustment coefficient, ϕ, takes a large value, the government adjusts b_t to the target level rapidly. When ϕ is relatively small, the government adjusts b_t slowly to the target level. Under this debt policy rule, the reduction of $\bar{b}$ accompanies the cut in public investment, G, in the short run in order to satisfy (9). We see this clearly in the next section. Then, the expenditure-based reduction of public debt is formulated as following the discussion of fiscal consolidation in *Public finances in EMU – 2011* and Rubianes (2010).

[6] We can set the ratio of debt to GDP as the target. By using $\bar{b}$, we can calculate the ratio of debt to GDP because $B/Y = (B/K)(K/Y) = \bar{b}/(A(k_g)^{1-\alpha})$.

3 Equilibrium

3.1 Dynamic system

In this section, we first derive the dynamic system of the economy, and then examine the existence and the stability of the steady-state equilibria in Subsections 3.2 and 3.3.

We derive the equilibrium of this economy by using market equilibrium conditions. The labor market equilibrium condition is $L_t = 1$ because the population size is unity and each household supplies one unit of labor inelastically. The asset market clears as $W_t = K_t + B_t$. Substituting these into (9) and using (1), (2) and (3), we obtain:

$$\dot{B}_t = (1-\tau)r_t B_t - (\tau Y_t - G_t). \tag{11}$$

We assume that the depreciation rate of private capital is zero for simplicity. Then, the goods market equilibrium condition is given by:

$$\dot{K}_t = Y_t - C_t - G_t. \tag{12}$$

Let us define $c_t \equiv C_t/K_t$ and $g_t \equiv G_t/K_t$. By substituting (2) into (6), and using (1) and (12), we obtain:

$$\dot{c}_t = [c_t + g_t - \{1-(1-\tau)\alpha\} Ak_{g,t}^{1-\alpha} - \rho]c_t. \tag{13}$$

By using (1), (8) and (12) we have:

$$\dot{k}_{g,t} = (1+k_{g,t})g_t - Ak_{g,t}^{2-\alpha} + c_t k_{g,t}. \tag{14}$$

By using (11) with (1), (2) and the definition of b_t, we can rewrite (10) as:

$$\dot{b}_t = -\phi(b_t - \bar{b}) = \left\{(1-\tau)\alpha Ak_{g,t}^{1-\alpha} - \frac{\dot{K}_t}{K_t}\right\} b_t - \tau Ak_{g,t}^{1-\alpha} + g_t. \tag{15}$$

Substituting (12) into (15) and solving for g_t, we obtain the ratio of public investment to private capital, $g_t \equiv G_t/K_t$, as follows:

$$g_t = \frac{1}{1+b_t}\left\{\tau(1+\alpha b_t)Ak_{g,t}^{1-\alpha} - \alpha b_t Ak_{g,t}^{1-\alpha} - \phi(b_t - \bar{b}) + (Ak_{g,t}^{1-\alpha} - c_t)b_t\right\}. \tag{16}$$

The first term of the right-hand side (RHS) denotes the tax revenue of the government and shows that, other things being equal, g_t increases as τ rises. The second term represents the interest payments on the government debt. Other things being equal, g_t increases as b_t decreases. The third and the last terms are given by the debt policy rule (10). The third term shows that given b_t, if the government reduces the target level of its debt $\bar{b}$, the volume of public investment falls in the short run. When ϕ takes a larger value, decreases in public investment in the short run are also larger because the government must reduce its debt more rapidly and hence its budget becomes tight. However, as b_t decreases steadily because of a decrease in $\bar{b}$ under the rule given by (10), the interest payments on government debt decrease gradually. This enables the government to spend more of its revenue on public investment. Finally, the last term implies that a higher ratio of public to private capital, k_g, because of a high growth rate of K_t, has a positive effect on public investment.[7] When K_t grows at a high rate, the government requires little effort to reduce $b_t \equiv B_t/K_t$ to $\bar{b}$. Then, high growth of K_t enables the government to increase public investment.

Substituting (16) into (13) and (14), respectively, we obtain the following dynamic system with respect to c_t and $k_{g,t}$.

$$\dot{c}_t = \frac{1}{1+b_t}[c_t - \zeta(k_{g,t}, \tau, b_t) - \phi(b_t - \bar{b})]c_t, \tag{17}$$

$$\dot{k}_{g,t} = \frac{1}{1+b_t}\left\{(k_{g,t} - b_t)c_t - \eta(k_{g,t}, \tau, b_t) - \phi(b_t - \bar{b})(1 + k_{g,t})\right\}, \tag{18}$$

where

$$\zeta(k_{g,t}, \tau, b_t) \equiv (1-\tau)(1-\alpha)Ak_{g,t}^{1-\alpha} + \rho(1+b_t),$$
$$\eta(k_{g,t}, \tau, b_t) \equiv [(1-\tau)(1+\alpha b_t)k_{g,t} - \{[1-(1-\tau)\alpha]b_t + \tau\}]Ak_g^{1-\alpha}.$$

Equations (10), (17) and (18), together with the initial values $k_{g,0}$ and b_0, and the transversality condition (7), characterize the dynamics of the economy.

3.2 Steady states

Now we derive the steady state of the economy where c_t, $k_{g,t}$ and b_t become constant over time. Setting $\dot{c}_t = 0$ and $\dot{k}_{g,t} = 0$ and $b_t = \bar{b}$ in (17) and (18), respectively, results in:

$$c = \zeta(k_g, \tau, \bar{b}), \quad \text{and} \quad (k_g - \bar{b})c = \eta(k_g, \tau, \bar{b}). \tag{19}$$

We omit the time index t in the above because c and k_g become constant over time in the steady state. By eliminating c from the two equations of (19), we have:

$$[(1-\tau)\alpha k_g - \tau]Ak_g^{1-\alpha} = \rho(k_g - \bar{b}). \tag{20}$$

This equation determines the steady-state value k_g^*. Substituting k_g^* into the first equation of (19), we obtain the steady-state value c^*. Let us denote the left-hand side (LHS) of (20) as $\Lambda(k_g)$. As shown in Figure 1, $\Lambda(k_g)$ is a convex function of k_g that equals zero when $k_g = 0$ and $k_g = \frac{\tau}{(1-\tau)\alpha}$ (see Appendix A). We denote the RHS of (20) as $\Pi(k_g)$. Apparently, $\Pi(k_g)$ is a straight line whose slope is ρ, and equals zero when $k_g = \bar{b}$ holds. Let us define $\tilde{k}_g$ by $\Lambda'(\tilde{k}_g) = \rho$. As shown in Appendix A, there exists a unique $\tilde{k}_g > 0$. Note that at $k_g = \tilde{k}_g$, both sides of (20) have the same slope.

[Figure 1]

Figure 1 shows that if $\Lambda(0) > \Pi(0)$ and $\Lambda(\tilde{k}_g) < \Pi(\tilde{k}_g)$ hold, there exist two steady states. Because $\Lambda(0) \equiv 0$, $\Pi(0) \equiv -\rho\bar{b}$ and $\bar{b} > 0$, the first inequality is satisfied. The second inequality is equivalent to:

$$\bar{b} < \tilde{k}_g - \frac{[(1-\tau)\alpha\tilde{k}_g - \tau]A\tilde{k}_g^{\,1-\alpha}}{\rho}. \tag{21}$$

[7] Eq.(12) shows that given G_t, when the difference between $Y_t/K_t = Ak_{g,t}^{1-\alpha}$ and $C_t/K_t = c_t$ becomes large, the growth rate of private capital rises.

If $\bar{b}$ is larger than the RHS of the above inequality, there exist no paths that the rational representative households can choose.[8] Therefore, we henceforth assume (21). We denote the steady-state value of k_g in each steady state as $k^*_{g,H}$ and $k^*_{g,L}$ ($k^*_{g,H} > \tilde{k}_g > k^*_{g,L} > 0$). From (2) and (6), the long-run growth rate, $\gamma = \dot{C}_t/C_t = \dot{K}_t/K_t = \dot{K}_g/K_g$, is given by:

$$\gamma = (1-\tau)\alpha A(k^*_g)^{1-\alpha} - \rho, \tag{22}$$

where k^*_g is equal to $k^*_{g,H}$ or $k^*_{g,L}$. We can confirm immediately that the long-run growth rate increases with k^*_g. This is because the return on private capital increases with k^*_g and the households save more. From now on, we call the steady state with $k^*_{g,H}$ the high-growth steady state, whereas we call the steady state with $k^*_{g,L}$ the low-growth steady state. From the discussion so far, we obtain the following proposition.

Proposition 1

There exist two steady states, the high-growth and the low-growth steady states, if (21) *holds.*

As in Futagami et al. (2008), there exist two steady states in our model. In their model, both of the steady states are meaningful economically. In our model, only the high-growth steady state is meaningful economically, as we will show in the next subsection.

3.3 Stability

We next examine the stability of the steady states. The dynamic adjustment of b_t is determined autonomously by (10) and is always stable, whereas $(c_t, k_{g,t})$ evolves over time according to (17) and (18). When $\dot{b}_t = 0$ ($b_t = \bar{b}$) holds, from (17) and (18), the dynamics of c_t and $k_{g,t}$ are obtained as:

$$\dot{c}_t = \frac{1}{1+\bar{b}}[c_t - \zeta(k_{g,t}, \tau, \bar{b})]c_t, \tag{23}$$

$$\dot{k}_{g,t} = \frac{1}{1+\bar{b}}\left\{(k_{g,t} - \bar{b})c_t - \eta(k_{g,t}, \tau, \bar{b})]\right\}. \tag{24}$$

Using these, we can depict the movement of $(c_t, k_{g,t})$ in (c, k_g) space as shown in Figure 2. Depending on the value of $\bar{b}$, the $\dot{k}_g = 0$ locus takes different shapes. As a result, there are five different phase diagrams.[9] When $\bar{b}$ is relatively small, $\bar{b} < \frac{\tau}{(1-\tau)\alpha}$, and we obtain the phase diagram in Figure 2-(b). When $\bar{b}$ equals $\frac{\tau}{(1-\tau)\alpha}$, we have two types of diagrams represented in Figure 2-(a). Figure 2-(a)-(i) is the case when $k_g = \bar{b}$ is the high-growth steady state, while $x \equiv \left\{\frac{\rho(1+\frac{\tau}{(1-\tau)\alpha})}{A[1-(1-\tau)(1-\alpha)]}\right\}^{\frac{1}{1-\alpha}}$ is the low-growth steady state. Figure 2-(a)-(ii), in contrast, is the case where $k_g = \bar{b}$ is the low-growth steady state, while x is the high-growth steady state. Finally, when $\bar{b}$ is relatively large, $\bar{b} > \frac{\tau}{(1-\tau)\alpha}$, and two types of diagrams emerge. Figure 2-(c)-(i) is the case where both of the steady-state values k_g

[8] Transversality condition (7) excludes all paths if there exist no steady states. We see this in the next subsection.

[9] Please see Appendix B regarding the shapes and positions of the $\dot{c} = 0$ and $\dot{k}_{g,t} = 0$ loci in the phase diagrams.

become smaller than $\bar{b}$, whereas Figure 2-(c)-(ii) is the case where both of them become larger than $\bar{b}$. In any case, we have the following common features regarding the dynamic behaviors. The low-growth steady state is unstable, whereas there exists a saddle path converging to the high-growth steady state. The stable arm is always upward sloping. If (21) is violated, there are no steady states. In this case, there are no paths that satisfy transversality condition (7). The paths that go to $k_g = 0$ or $c = 0$ do not satisfy (7) obviously. The paths in which both k_g and c continue to increase are excluded because private capital, K_t, keeps decreasing and eventually becomes zero on those paths.[10]

[Figure 2]

Next, we check the local stability of the system including the case of $b_t \neq \bar{b}$. We obtain the following proposition on the stability of the dynamic system (10), (17) and (18).

Proposition 2

The high-growth steady state is locally saddle-point stable, whereas the low-growth steady state is unstable.

Poof. See Appendix C.

Proposition 2 and the phase diagrams state the following three facts. First, although there are two steady states, only the high-growth steady state is meaningful economically. Second, it is determinate. Third, given $b_0 \simeq \bar{b}$, the economy falls into the development trap if the initial ratio of public capital to private capital, $k_{g,0}$, is smaller than $k^*_{g,L}$. In the trap, there exists no path that the representative household with perfect foresight and rational expectations can follow, and hence the economy cannot develop.[11]

The intuition for the development trap is as follows: when k_g is small, production relative to private capital is small.[12] The tax revenue of the government becomes small relative to private capital, which tightens the budget constraint of the government. Then, public investment becomes small relative to private capital. This can be verified using $b_t = \bar{b}$, (15) and (22), so that we can derive:

$$g = \tau A {k_g}^{1-\alpha} - \rho \bar{b}. \tag{25}$$

This equation shows that in the steady state, g is small when k_g is small. A small g implies that the government cannot accumulate enough public capital. Then, the economy falls into the development trap when $k_{g,0}$ is sufficiently small.

Our results are totally different from those of Futagami et al. (2008), where a productive input of the government is a flow variable. The results in Futagami et al. (2008) can

[10]The reason for this is as follows. From (12), if private capital keeps decreasing, $Ak_{g,t}^{1-\alpha} - c_t - g_t < 0$ holds. By using (16), this condition can be rewritten as $c_t > (1-\tau)(1+\alpha\bar{b})Ak_{g,t}^{1-\alpha}$. Next, from (24), the $\dot{k}_{g,t} = 0$ locus is rewritten as $c = \left\{(1-\tau)(1+\alpha\bar{b}) + \frac{(1-\tau)\alpha\bar{b}-\tau}{k_g-\bar{b}}(1+\bar{b})\right\} Ak_g^{1-\alpha}$. Because the paths in which both k_g and c continue to increase pass through the $\dot{k}_{g,t} = 0$ locus, they satisfy $c_t > \left\{(1-\tau)(1+\alpha\bar{b}) + \frac{(1-\tau)\alpha\bar{b}-\tau}{k_{g,t}-\bar{b}}(1+\bar{b})\right\} Ak_{g,t}^{1-\alpha}$. Because $\bar{b} > \frac{\tau}{(1-\tau)\alpha}$ when (21) is violated, these paths satisfy $c_t > (1-\tau)(1+\alpha\bar{b})Ak_{g,t}^{1-\alpha}$ and then K_t keeps decreasing on the paths.

[11]The transversality condition (7) excludes any paths that go to the states in which k_g becomes zero.

[12]Production relative to private capital is given by $Y_t/K_t = A{k_{g,t}}^{1-\alpha}$.

be summarized as follows. First, the two steady states are both stable and meaningful economically. Second, the high-growth steady state can become locally indeterminate. Finally, because the low-growth steady state is stable and meaningful economically, the economy can be stuck in a low-growth trap.[13] The low-growth trap in Futagami et al. (2008) occurs when the household has pessimistic expectations that the low-growth steady state will be realized. The development trap in this paper occurs when $k_{g,0}$ is smaller than $k^*_{g,L}$. Therefore, the low-growth trap in Futagami et al. (2008) has a feature different from the development trap in this paper.

In Futagami et al. (2008) where public services enter the production function, public productive expenditure comes into operation in final goods production immediately. If the government increases public productive expenditure, final goods production and also the tax revenue of the government increase immediately, which allows the government to spend more on productive public expenditure. There is a complementarity between productive public expenditure and final goods production. As a result, the development trap does not occur and equilibrium indeterminacy can arise in Futagami et al. (2008).

4 Characteristics of the Steady States

This section analyzes the effects of changes in the income tax rate and in the target level of government bonds in the steady states. Because the economically meaningful steady state is unique (the high-growth steady state), we should examine the policy effect in the high-growth steady state. Because the effects of policy changes on $k^*_{g,L}$ are crucial for the development trap, we also pay attention to the policy effects on $k^*_{g,L}$.

We first examine the effects of changes in $\bar{b}$. As shown in Figure 3-(a), when $\bar{b}$ rises, only the straight line $\Pi(k_g)$, which represents the RHS of (20), shifts downward. As a result, $k^*_{g,L}$ rises whereas $k^*_{g,H}$ falls. We next examine the effects of changes in τ. As shown in Figure 3-(b), when τ rises, only the U-shaped curve $\Lambda(k_g)$, which represents the LHS of (20), shifts downward. Then, $k^*_{g,L}$ falls whereas $k^*_{g,H}$ rises. We obtain the following lemma.

[Figure 3]

Lemma

The effects of changes in policy variables $\bar{b}$ and τ are as follows:

$$\text{(i)}\quad \frac{\partial k^*_{g,L}}{\partial \bar{b}} > 0,\ \ \frac{\partial k^*_{g,H}}{\partial \bar{b}} < 0. \quad \text{(ii)}\quad \frac{\partial k^*_{g,L}}{\partial \tau} < 0,\ \ \frac{\partial k^*_{g,H}}{\partial \tau} > 0.$$

This lemma reveals the following. In the stable high-growth steady state, increases in $\bar{b}$ (τ) have negative (positive) effects on the ratio of public capital to private capital, $k^*_{g,H}$. In the unstable low-growth steady state, the ratio of public capital to private capital, $k^*_{g,L}$, increases (decreases) when $\bar{b}$ (τ) increases. Hence, when public capital is scarce relative to private capital, the economy is more (less) likely to fall into the development trap.

[13] In Futagami et al. (2008), the welfare level in the low-growth steady state is lower than that in the high-growth steady state.

When $\bar{b}$ increases, the interest payments on government debt increase in the steady state, as (25) shows. As a result, public investment becomes small and the accumulation of public capital is depressed. Consequently, when public capital is scarce relative to private capital, the economy is more likely to fall into the development trap. Besides, the depressed accumulation of public capital decreases the ratio of public capital to private capital, $k^*_{g,H}$, in the high-growth steady state. When τ increases, tax revenue increases. Public investment relative to private capital, g, increases, as (25) shows. Then, an increase in τ has effects opposite to those for an increase in $\bar{b}$.

In Futagami et al. (2008), where public services enter the production function, the economy falls into a low-growth trap when the household has pessimistic expectations. Because it is not easy for the government to control households' expectations, the government has difficulty in helping the economy escape from the low-growth trap. In contrast, in our case, the stock of public capital enters the production function, the government can relatively easily help the economy to escape from the development trap by decreasing $\bar{b}$, or increasing τ.

We next investigate the relationship between the long-run growth rate and the main policy variable, $\bar{b}$.[14] From now on, we focus only on the high-growth steady state that is stable. As the growth rate in the high-growth steady state is given by $\gamma_H = (1 - \tau)\alpha A(k^*_{g,H})^{1-\alpha} - \rho$, we can prove the next proposition by using Lemma-(i).

Proposition 3

A decrease in the target level, $\bar{b}$, enhances the growth rate in the high-growth steady state.

In the rest of this section, we investigate the effect of $\bar{b}$ on the ratio of public investment to private capital in the high growth steady state, g^*_H. From (25) and Lemma-(i), we obtain immediately:

$$\frac{\partial g^*_H}{\partial \bar{b}} = \tau(1-\alpha)A(k^*_{g,H})^{-\alpha}\frac{\partial k^*_{g,H}}{\partial \bar{b}} - \rho < 0. \tag{26}$$

This states the following proposition.

Proposition 4

A decrease in the target level, $\bar{b}$, boosts the ratio of public investment to private capital in the high-growth steady state.

Proposition 4 states that if the government reduces $\bar{b}$, g^*_H always increases in the long run. The reason is simple. As $\bar{b}$ decreases, outstanding public debt is reduced in the long run. Therefore, the interest payments are reduced and then the budget constraint of the government becomes loose. Please remember that in the short run, a decrease in $\bar{b}$ reduces public investment. However, this steadily decreases outstanding debt and the interest payments and can increase public investment gradually. In the long run, Proposition 4 states that the volume of public investment can become larger than that in the original

[14] How the tax rate influences the growth rate in the long run may be an important issue. We obtain a tax rate that maximizes the growth rate in the high-growth steady state in this model. Furthermore, we find this tax is increasing in $\bar{b}$.

state.

5 Transitional Dynamics

This section examines the transitional dynamics numerically. Our scenario is as follows: the economy is in a high-growth steady state initially with $\bar{b} = \bar{b}_{init}$, where $\bar{b}_{init}$ denotes the initial level of $\bar{b}$. At time 0, the government reduces $\bar{b}$ from $\bar{b}_{init}$ to $\bar{b}_{new}$ unexpectedly, where $\bar{b}_{new}$ is the level of $\bar{b}$ after the policy change. Then, the economy begins to move toward the new high-growth steady state along the saddle path and the transitional dynamics are generated.

In Section 4, we examined the long-run effects of changes in $\bar{b}$: decreases in $\bar{b}$ have positive effects on both the growth rate and on public investment in the long run. This section shows the policy effects of $\bar{b}$ in the short run and on the transition path. We also show that the transitional dynamics and the strength of policy effects in the short run and on the transition path depend heavily on the value of ϕ, which has no effect in the long run. Remember that ϕ determines the adjustment speed of b_t (see (10)).

We analyze the transition paths numerically using the relaxation algorithm.[15] As a benchmark, we choose the following parameter values. The subjective time discount rate, ρ, is set to 0.05, which is an often-used value in the growth literature.[16] The elasticity of production with respect to public capital, $1 - \alpha$, is set to 0.25. This value is often used in studies such as Barro (1990), Greiner (2007) and Gómez (2004). We use $\tau = 0.1$ following Greiner (2007), who studies how public investment financed by public debts affects the transitional dynamics of the economy. The value of A is chosen so that the long-run growth rate of the new steady state is equal to 0.02, which results in $A = 0.1313$. The values of $\bar{b}_{init}$ and $\bar{b}_{new}$ are chosen so that the debt–GDP ratio in the initial and new steady states is equal to 80 percent and 50 percent, respectively. This yields $\bar{b}_{init} = 0.0802$ and $\bar{b}_{new} = 0.0519$. The reason for setting the initial debt–GDP ratio to 80 percent is that the average debt–GDP ratio in the EU27 was almost 80 percent in 2010, as we mentioned in Section 1. Because of the recent public finance crises in the EU, using the EU in the determination of the parameter values is appropriate. Because the member countries of the EU must maintain a debt–GDP ratio of less than 60 percent under the Maastricht criterion, we assume that the debt–GDP ratio in the new steady state is less than 60 percent. It is also noted that the debt–GDP ratio in the high-growth steady state is given by $B/Y = \bar{b}/(A(k^*_{g,H})^{1-\alpha})$ and hence it increases with $\bar{b}$ in the high-growth steady state (see Lemma-(i)). Under these parameter values, $k^*_{g,H}(\equiv K_g/K)$ is around 0.34 and 0.39 in the initial and new steady states, respectively.

The value of ϕ has no effect on the steady state. However, as for transitional dynamics, ϕ has an important roles because it governs the dynamics of b_t, and hence the time paths of other endogenous variables are affected strongly by the value of ϕ. We use five values: $\phi = 0.05$, 0.1, 0.2, 0.3 and 0.4. In the case of $\phi = 0.05$, it takes about 25.5 years until the debt–GDP ratio reduces to around 60 percent. As ϕ increases, it takes fewer years. When

[15] Trimborn et al. (2008) detail the relaxation algorithm. They also provide MATLAB programs for the relaxation algorithm, freely downloadable at http://www.uni-siegen.de/fb5/vwli/forschung/relaxation/the_relaxation_method.html?lang=de.

[16] Greiner (2008) also sets $\rho = 0.05$.

$\phi = 0.1$, 0.2, 0.3 and 0.4, the debt–GDP ratio reduces to around 60 percent in around 13, 6.5, 4.5 and 3.5 years.[17]

[Figure 4]

Under these parameter values, we examine numerically the transitional dynamics generated by the reduction of $\bar{b}$ from $\bar{b}_{init}$ to $\bar{b}_{new}$, using the relaxation algorithm. Figure 4 presents the results. Panels (a) and (b) show that b_t and the debt–GDP ratio decrease monotonically and converge to their new-steady state values. As ϕ increases, the speeds of convergence of b_t and the debt–GDP ratio increase.

Panel (c) shows that just after the policy change, $g_t(\equiv G/K)$ drops sharply below the initial level and before rising to the new steady-state level, which is higher than the initial level. To reduce b_t to $\bar{b}_{new}$, the government must reduce its expenditure initially, which results in a short-run decrease in g_t, as is shown by the third term in parentheses in (16). However, as b_t declines steadily, the interest payments on the government debt decrease gradually and its budget constraint becomes gradually loose. Then, g_t increases gradually and eventually exceeds the initial level (see (25) and Proposition 4). When the values of ϕ are larger, the initial declines in g_t also become larger. A large ϕ means that the government must reduce its debt at a higher speed and hence its budget becomes tight immediately after the policy change. As a result, the government must reduce public investment by a large amount. When ϕ is as large as 0.3 or 0.4, g_t becomes negative immediately after the policy change, which implies that the government must sell its capital in order to meet its budget. Although the initial declines in g_t increase with ϕ, it takes shorter periods of time until g_t recovers its initial level when the value of ϕ is large: when ϕ is equal to 0.05, 0.1, 0.2, 0.3 and 0.4, it takes about 14.5, 11.5, 8.5, 6.5 and 6 years, respectively, until g_t returns back to its initial level. Because public investment is the source of the accumulation of public capital $K_{g,t}$, the growth rates of public capital follow dynamical paths similar to those of g_t, as shown in Panel (d).

In contrast to g_t, the growth rate of private capital K_t increases sharply above its initial level immediately after the policy change, and then decreases (see Panel (e)). After the policy change, public investment decreases as we have just discussed, which has crowding-in effects on private investment. Then, the growth rate of K_t increases initially. However, as g_t increases gradually, the crowding-out effects on private investment begin to prevail, which leads to gradual declines in the growth rate of K_t. When ϕ is as large as 0.3 or 0.4, the crowding-out effects of public investment become so strong that the growth rate of K_t follows a nonmonotonic transitional path.

The dynamics of the growth rates of $K_{g,t}$ and K_t give rise to the nonmonotonic transitional paths of $k_{g,t} \equiv K_{g,t}/K_t$, as shown in Panel (f). In the early stages of transition, $k_{g,t}$ decreases gradually because of decreased public investment and increased private investment. The larger ϕ is, the larger the declines in k_g are. However, because of gradual increases in public investment and gradual decreases in private investment, $k_{g,t}$ starts to increase to its new steady-state level several years after the policy change. When ϕ becomes larger, it takes less time until $k_{g,t}$ begins to increase: when ϕ is equal to 0.05, 0.1, 0.2, 0.3 and 0.4, $k_{g,t}$ begins to increase in about 13, 10, 7, 6 and 5 years, respectively.

[17] When $\phi = 0.05$, 0.1, 0.2, 0.3 and 0.4, $b_t - \bar{b}$ reduces to 50 percent of its initial size in around 14, 7, 3.5, 2.5 and 1.5 years, respectively.

From (2) and (6), the growth rate of private consumption becomes a function of k_g. Then, as shown in Panel (g), the growth rates of private consumption follow dynamical paths similar to those of $k_{g,t}$.

Panel (h) provides the transitional paths of c_t. From the panel, we know the effects of the policy change on the initial consumption levels, C_0, because $C_0 \equiv c_0 K_0$ holds. The effects on C_0 depend apparently on the value of ϕ. When ϕ is small ($\phi = 0.05$), C_0 decreases in reaction to the policy change. As ϕ becomes larger, C_0 increases to a higher level. For sufficiently large values of $\phi(= 0.4)$, C_0 increases to a higher level than the initial level as a result of the policy change. As the value of ϕ increases, the initial declines in public investment also become large, and hence more resources are released to the private sector (see (12)). Consequently, the household can consume more as ϕ increases. To see why, let us integrate (5) and use (6).

$$C_0 = \rho(W_0 + \tilde{w}_0). \tag{27}$$

C_0 increases with the initial asset, W_0, and the present value of wage income, $\tilde{w}_0 = \int_0^\infty w_t \exp\left\{\left(-\int_0^t r_\omega d\omega\right) t\right\} dt$. w_t depends on $k_{g,t}$ and K_t (see (3)). After the policy change, the growth rate of private capital K_t increases beyond the initial level (see Panel (e)), which enhances w_t and exerts positive effects on $\tilde{w}_0$. In contrast, k_g decreases in the early stages of transition as shown in Panel (f), which lowers w_t and has negative effects on $\tilde{w}_0$. As ϕ increases, the positive effects tend to dominate the negative effects. Therefore, C_0 tends to be larger (smaller) than the initial level when ϕ is large (small) and C_0 increases with ϕ.

6 Welfare

As in Section 5, we consider the following scenario. The economy is initially in a high-growth steady state with $\bar{b} = \bar{b}_{init}$. At time 0, the government reduces $\bar{b}$ from $\bar{b}_{init}$ to $\bar{b}_{new}$ unexpectedly. Then, the transitional dynamics are generated. The purpose of this section is to investigate the welfare effects of this policy change.

Our welfare measure is (4). Because the growth rate of consumption is given by $\gamma_{C,t} \equiv (1-\tau)\alpha A {k_{g,t}}^{1-\alpha} - \rho$ and we have $C_t = C_0 e^{\int_0^t \gamma_{C,\nu} d\nu}$, we can rewrite (4) as:

$$U_0 = \frac{\ln C_0}{\rho} + \int_0^\infty \left(\int_0^t \gamma_{C,\nu} d\nu\right) e^{-\rho t} dt.$$

The above equation shows that decreases in $\bar{b}$ affect the welfare level through its effects on C_0 and the paths of $\gamma_{C,t}$. As we observed in Section 5, the effects of the policy change on C_0 and the transitional paths of $\gamma_{C,t}$ depend heavily on the value of ϕ. Consequently, we will observe that the welfare effects of the policy change are influenced by the value of ϕ.

We examine the welfare effects numerically by considering the case where the government reduces its debt-GDP ratio from B_0/Y_0 to 0.5, where B_0/Y_0 =0.6, 0.7, 0.8, 0.9 and 1. As in Section 5, we assume that the debt–GDP ratio in the new steady state is less than 60 percent because the member countries of the EU must maintain a debt–GDP ratio of less than 60 percent under the Maastricht criterion. In the benchmark case, the values of A, ρ and α are the same as those employed in Section 5. The values of ϕ we consider are

$\phi = 0.01$, 0.025, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.75 and 1. When ϕ is smaller (larger) than 0.05 (0.4), it takes considerably more (fewer) years until the debt–GDP ratio reduces to 60 percent. However, to clarify the welfare effects of the policy change, we include extremely small and large values of ϕ. We consider three values of income tax rates: τ =0.1, 0.2 and 0.3. The values of $\bar{b}_{init}$ and $\bar{b}_{SS}$ are chosen for each τ and B_0/Y_0 so that the initial debt–GDP ratio equals B_0/Y_0 and the debt–GDP ratio in the new steady state equals 0.5.

Let us denote the welfare level without the policy change as $U^*_{0,N}$. We have $U^*_{0,N} = \ln(c^*_{H,N}K_0)/\rho + \gamma^*_{C,N}/\rho^2$, where $c^*_{H,N}$ and $\gamma^*_{C,N}$ are the initial steady-state values of c and γ_C. To calculate $U^*_{0,N}$, we need the initial value of private capital, K_0. Here, we set $K_0 = 1$. The welfare level immediately after the policy change is denoted as U^{**}_0. We calculate U^{**}_0 by setting $K_0 = 1$ and using the relaxation algorithm (see Appendix D for more detail).

[Table 1]

In Table 1, we present the difference between U^{**}_0 and $U^*_{0,N}$. The table shows that when $\tau = 0.1$, $B_0/Y_0 = 1$ and $\phi = 0.05$ hold, and social welfare can be improved by $0.04048 (= U^{**}_0 - U^*_{0,N})$ if the government reduces its debt–GDP ratio. In addition, the table shows that it takes about 37 years until the debt–GDP ratio reduces to about 60 percent.[18]

The first notable result in Table 1 is that in all the cases we consider, reductions in $\bar{b}$ have positive welfare effects. As shown in Section 5, when the government reduces $\bar{b}$, whether C_0 decreases or increases in the short run depends on the values of ϕ. Furthermore, the growth rate of consumption,γ_C, decreases in the early stages of transition, which has negative effects on welfare. However, in the long run, the growth rate becomes higher than the initial level, which has positive welfare effects. Our results show that in all cases, the positive effects dominate the negative effects, and hence social welfare improves if the government reduces $\bar{b}$ and decreases its debt–GDP ratio.

Table 1 also shows that given τ and ϕ, when the initial debt–GDP ratio is larger, the welfare gains are also larger. For example, when $\tau = 0.1$ and $\phi = 0.05$, the welfare gains decrease from 0.03379 to 0.02627 as B_0/Y_0 decreases from 0.9 to 0.8. A large B_0/Y_0 means that there are large gaps between the initial and target debt–GDP ratios. To fill this large gap, the government must reduce g_t initially by a large amount, as shown in Table 2.[19] As discussed in Section 5, the initial declines in g_t, leading to decreases in $k_{g,t}$ at the early stages of transition, are the source of the negative welfare effect (the declines in $\gamma_{C,t}$). Then, the negative welfare effect is large when B_0/Y_0 is large. However, the large initial declines in g_t suggest that more resources are released to the private sector and hence the household can consume more compared with the case where B_0/Y_0 is small. Then, the short-run effect (changes in C_0) tends to be weakly negative or even positive when B_0/Y_0 is large. In addition, a large B_0/Y_0 indicates a large difference between the values of k_g in the initial and new steady states. Then, the growth rate of consumption in the new steady state is much higher than that in the initial steady state, which suggests that the positive long-run effect is large when B_0/Y_0 is large. Because the dominance of the positive effect over the negative effect increases as B_0/Y_0 increases, the welfare gains from the policy change are large when B_0/Y_0 is large.

[18]When B_0/Y_0 is equal to 0.6, we do not calculate the years that it takes until the debt–GDP ratio reduces to about 60 percent.

[19]The initial decline of g_t is calculated by $(g_0 - g_{int})/g_{int}$ where g_{int} is the initial steady-state value and g_0 is the value immediately after the policy change.

[Table 2]

As expected, the values of ϕ affect the welfare effects. Table 1 shows that, as for the relationship between the welfare gains and ϕ, the following two patterns are observed. In many cases, (i) the welfare gains decrease in ϕ when ϕ is small, but increase in ϕ as ϕ becomes relatively large. Thus, the relationships between the welfare gains and ϕ are U-shaped in many cases. However, (ii) the welfare gains decrease monotonically in ϕ when τ is low and B_0/Y_0 is large: $\tau = 0.1$ and $B_0/Y_0 \geq 0.9$.

To understand the intuition for these results, the following points should be noted. Table 2 illustrates that the size of the initial declines of g_t under $\phi = 0.05$ depend crucially on τ and B_0/Y_0. When τ is lower (higher), and B_0/Y_0 is larger (smaller), the initial declines in g_t become larger (smaller). This is because when the tax rate is lower (higher), the government can rely less (more) on tax revenue to reduce its debt–GDP ratio. Furthermore, when B_0/Y_0 is larger (smaller), the initial declines of g_t also become larger (smaller) to fill larger gaps between the initial and target debt–GDP ratios. Next, the increase in ϕ has the following two opposing effects on welfare. It accelerates the decline of g_t, as we see in Figure 4-(c). However, it leads to a recovery in the growth rate of consumption sooner, as we see in Figure 4-(g). Which effect is dominant depends on the size of the initial decline of g_t. When the tax rate is low and the initial debt–GDP ratio is large, such as $\tau = 0.1$ and $B_0/Y_0 \geq 0.9$, the initial decline of g_t is too large when the government increases ϕ. Then, the welfare gains decrease monotonically in ϕ. In contrast, when the tax rate is high or B_0/Y_0 is small, the initial decline of g_t becomes small. Then, when ϕ is somewhat large, the positive effect of the earlier recovery of the growth of consumption dominates the negative effect of the initial decline of g_t. Thus, the relationships between the welfare gains and ϕ are U-shaped.

The results we have obtained may be crucially dependent on the subjective discount rate. When ρ is small (large), it is likely that the welfare effects in the long run (in the short run and on the transition path) dominate those in the short run and on the transition path (in the long run). Hence, the total welfare effects may change if we choose different values of ρ. We conduct the same exercise under different values of the discount rates (see Table 3). The values of A, α and ϕ are the same as those employed in Table 1. The values of $\bar{b}_{init}$ and $\bar{b}_{new}$ are chosen for each ρ, τ and B_0/Y_0 so that the initial debt–GDP ratio equals B_0/Y_0 and the debt–GDP ratio in the new steady state equals 0.5. Table 2 shows again that: (i) in all cases we consider, reductions in $\bar{b}$ have positive welfare effects; and (ii) as the initial debt–GDP ratio is larger, the welfare gains are also larger. With respect to the influences of ϕ, the findings when ρ is 0.03 are identical to those in Table 1. When τ is low and B_0/Y_0 is large ($\tau = 0.1$ and $B_0/Y_0 \geq 0.9$), they are decreasing monotonically. The relationships between the welfare gains and ϕ are U-shaped in many cases. In the case of $\rho = 0.07$, the results are also the same when B_0/Y_0 is small. Therefore, the results are robust.

However, in the case of $\rho = 0.07$, some results that are different from those in Table 1 are obtained when B_0/Y_0 is large. When $\tau \geq 0.2$ holds, the welfare gains increase monotonically with ϕ if B_0/Y_0 is large enough. As discussed above, when B_0/Y_0 is large, the short-run effect (changes in C_0) tends to be weakly negative or even positive. In addition, C_0 increases with ϕ as shown in Panel (h) of Figure 4. Because a large ρ indicates that the household cares more about the short-run effect, the welfare gains increase monotonically with ϕ when B_0/Y_0 is large and $\tau \geq 0.2$ holds. In the case of $\tau = 0.1$ and $B_0/Y_0 = 0.9$, the welfare gains increase with ϕ for large values of ϕ, in contrast to the results obtained in Table 1. These increases in the welfare gains also reflect the short-run effect. Furthermore, when $\tau = 0.1$ and $B_0/Y_0 = 1$ hold, the short-run effect (changes in C_0) tends to be weakly negative or even positive and C_0 increases with ϕ. However, the negative effect coming from the decline of g_t is so strong that the welfare gains decrease with ϕ even when the discount rate is large ($\rho = 0.07$).

[Table 3]

In the benchmark case, when τ is equal to 0.2 or 0.3, the long-run growth rates are much larger than 0.02. When τ is equal to 0.2 and 0.3, the long-run growth rate is about 0.026 and 0.027, respectively, in the benchmark case. A small difference by 0.006 or 0.007 in the long-run growth rate may lower the reliability of the result. Therefore, in the case that τ is 0.2 and 0.3, we conduct the same numerical exercise as when we change the value of total factor productivity, A, so that the long-run growth rate equals 0.02. However, Table 4 shows that the results are qualitatively the same as those obtained in Table 1. When $\tau = 0.2$, the welfare gains decrease at first and then begin to increase as the value of ϕ increases. On the other hand, the welfare gains increase with ϕ when τ is as large as 0.3.

[Table 4]

7 Conclusion

We investigated the policy effects of a reduction in the government debt–GDP ratio to below the 60 percent criterion. For an expenditure-based reduction of public debt, the government decreases public investment initially to reduce its debt. This policy change causes the following response during the transition and in the steady state, and causes opposite effects regarding welfare changes between the transition path and in the long run. In the short run, the investment in private capital rises initially because of the fall in public investment, and subsequently the return from private capital decreases. Next, the growth rate of consumption begins to decline after the policy change. This exerts negative effects on welfare on the transition path. However, as the size of public debt decreases, the interest payments on the debt decline and the government can spend more of its tax revenue on public investment. The recovery of public investment, in turn, hinges on private investment, and subsequently the return from private capital decreases. As a result, in the long run, the growth rates of consumption, private capital and public capital exceed those in the initial state. This exerts positive effects on welfare in the long run. We find the positive effects dominate the negative effects, meaning that the policy change improves welfare.

The adjustment speed of debt reduction has important effects on welfare. How the welfare gains are influenced as the government changes the speed of debt reduction depends on the tax rate and on how much the debt–GDP ratio initially exceeds the target level. The relationships between the welfare gains and the adjustment speed are U-shaped in many cases. However, they decrease monotonically when the tax rate is low and initial debt-GDP ratio is sufficiently large. When the tax rate is low, the government must reduce the gap between the initial debt–GDP ratio and the target without adequate tax revenue. Then the initial decline of public investment becomes large. When the initial debt–GDP ratio is large, the government must reduce the large gap between the initial and the target level of debt. Then, the initial decline of public investment also becomes large. The increase in the speed of reduction of public debt leads to the following trade off. On the one hand, the initial decline of public investment increases. On the other hand, the growth of consumption recovers sooner. When the tax rate is low and the initial debt–GDP ratio is large, the initial decline of public investment is too large when the government increases the adjustment speed. Then, the welfare gains decrease monotonically in the adjustment speed. In contrast, when the tax rate is high or the initial debt–GDP ratio is small, the initial decline of public investment becomes small. Next, when the adjustment speed is somewhat high, the positive effect of the earlier recovery of the growth of consumption dominates the negative effect of the initial decline of public investment. Then, the relationship between the welfare gains and the adjustment speed are U-shaped.

Besides these main results, this paper shows the possibility that the economy fails to develop sustainably when the initial level of public capital is sufficiently small relative to private capital. In the development process, the government should rely more on tax finance rather than debt to avoid falling into the development trap.

Finally, we provided directions for future research. For countries facing government financial crises, there may be two ways to improve government financial conditions. The first is to reduce government spending. The second is to increase the tax rate. We investigate the first way in this paper. However, countries such as France, Italy, Ireland, Greece, Portugal, Spain and the USA have attempted to implement both spending cuts and tax increases. Investigating debt reduction by including tax increases is very important. An additional possibility is that under some conditions, reductions in tax rates rather than increases in tax rates may be a better policy for reducing debt. Bruce and Turnovsky (1999) show that both expenditure cuts and cuts in the tax rate generate higher economic growth, which leads to higher tax revenue in the future. In turn, these increase the present discounted value of all future tax revenues. However, they do not investigate the policy effects on the transition dynamics but rather those in the steady state. Investigating which is better, increasing or decreasing tax rates, for welfare including transition paths is also important. Furthermore, we assumed public infrastructure only influences the production of goods as in Futagami et al. (1993) and Turnovsky (1997) and so on. However, it may also affect households' utility functions if public capital includes public health capital, as in Agénor (2008). How this extension changes the policy effects on the economy may be worth further research. These are left for future research.

Acknowledgments

We would like to thank Akira Momota, Akira Yakita and Kazuki Hiraga for helpful comments and suggestions. We are responsible for any remaining errors.

Appendix

A Properties of $\Lambda(k_g)$

We can show that $\Lambda(k_g)$ has the following properties:

$$\Lambda'(k_g) = Ak_g^{-\alpha}\{(1-\tau)\alpha(2-\alpha)k_g - \tau(1-\alpha)\} < (=)(>)0 \text{ if and only if } k_g < (=)(>)\breve{k}_g,$$
$$\Lambda''(k_g) = (1-\tau)\alpha(1-\alpha)(2-\alpha)Ak_g^{-\alpha} + \tau\alpha(1-\alpha)Ak_g^{-\alpha-1} > 0,$$
$$\lim_{k_g\to 0}\Lambda'(k_g) = -\infty, \quad \lim_{k_g\to\infty}\Lambda'(k_g) = \infty, \quad \text{and} \quad \Lambda(0) = \Lambda\left(\frac{\tau}{(1-\tau)\alpha}\right) = 0,$$

where $\breve{k}_g \equiv \frac{(1-\alpha)\tau}{(1-\tau)(2-\alpha)\alpha}$. Apparently, $\Lambda(k_g)$ is a convex function of k_g as shown in Figure 1. The above properties ensure the existence and the uniqueness of $\tilde{k}_g(>0)$ that satisfies $\Lambda'(\tilde{k}_g) = \rho$.

B Phase diagram of (k_g, c)

From (23), the $\dot{c} = 0$ locus is given by $c = \zeta(k_g, \tau, \bar{b})$. It is easy to show $\zeta(0, \tau, \bar{b}) > 0$, $\partial\zeta(k_g, \tau, \bar{b})/\partial k_g > 0$ and $\partial^2\zeta(k_g, \tau, \bar{b})/\partial k_g^2 < 0$. Eq. (23) shows that $\dot{c}_t \geq (<)0$ holds when $c_t \geq (<)\zeta(k_{g,t}, \tau, \bar{b})$. Summarizing these results, Figures 2 (a)–2(c) show the shape of the $\dot{c} = 0$ locus and the motion of c_t.

Next, we move on to the $\dot{k}_g = 0$ locus. Depending on the value of $\bar{b}$, the $\dot{k}_g = 0$ locus takes different shapes. We consider the following three cases: (i) $\bar{b} = \tau/\{(1-\tau)\alpha\}$, (ii) $\bar{b} < \tau/\{(1-\tau)\alpha\}$ and (iii) $\bar{b} > \tau/\{(1-\tau)\alpha\}$.

(i) We consider the case of $\bar{b} = \tau/\{(1-\tau)\alpha\}$. Because $\eta(k_{g,t}, \tau, \bar{b}) = (k_{g,t} - \bar{b})Ak_g^{1-\alpha}$ holds, we know from (24) that the $\dot{k}_g = 0$ locus is given by:

$$k_g = \bar{b} = \frac{\tau}{(1-\tau)\alpha} \quad \text{and} \quad c = Ak_g^{1-\alpha}. \tag{B.1}$$

The first equation of (B.1) indicates that one of the steady-state values of k_g is given by $k_g^* = \bar{b}$. Solving the $\dot{c} = 0$ locus and the second equation of (B.1) for k_g, we know that the other steady-state value of k_g is given by:

$$x \equiv \left\{\frac{\rho(1 + \frac{\tau}{(1-\tau)\alpha})}{A[1-(1-\tau)(1-\alpha)]}\right\}^{\frac{1}{1-\alpha}}.$$

The upper panel of Figure 2-(a) presents the $\dot{k}_g = 0$ locus in the case when ρ is small enough to satisfy $\bar{b} > x$. When ρ is large enough to satisfy $\bar{b} < x$, the $\dot{k}_g = 0$ locus is shown in the lower panel of Figure 2-(a).

We know from (24) that:

$$\dot{k}_{g,t} \geq 0, \ \text{(a) if } c_t \leq Ak_g^{1-\alpha} \text{ and } k_g \leq \bar{b}, \text{ or (b) if } c_t \geq Ak_g^{1-\alpha} \text{ and } k_g \geq \bar{b},$$
$$\dot{k}_{g,t} < 0, \ \text{(c) if } c_t < Ak_g^{1-\alpha} \text{ and } k_g > \bar{b}, \text{ or (d) if } c_t > Ak_g^{1-\alpha} \text{ and } k_g < \bar{b}.$$

Based on the discussion so far, the phase diagrams are represented in Figure 2-(a) when $\bar{b} = \frac{\tau}{(1-\tau)\alpha}$ holds.

We next move on to cases (ii) and (iii) where $\bar{b} \neq \frac{\tau}{(1-\tau)\alpha}$. From (24) and the definition of $\eta(k_{g,t}, \tau, \bar{b})$, the $\dot{k}_g = 0$ locus is given by:

$$c = \Gamma(k_g, \tau, \bar{b}) \equiv \frac{[(1-\tau)(1+\alpha\bar{b})k_g - \{[1-(1-\tau)\alpha]\bar{b} + \tau\}]Ak_g^{1-\alpha}}{k_g - \bar{b}}. \qquad \text{(B.2)}$$

Apparently, $\Gamma(k_g, \tau, \bar{b})$ equals zero when $k_g = 0$ and $k_g = \hat{k}_g \equiv \frac{[1-(1-\tau)\alpha]\bar{b}+\tau}{(1-\tau)(1+\alpha\bar{b})}$. Then, the $\dot{k}_g = 0$ locus passes through $(0, 0)$ and $(\hat{k}_g, 0)$ as shown in Figures 2-(b) and (c). $\Gamma(k_g, \tau, \bar{b})$ is continuous with respect to k_g except at $k_g = \bar{b}$.

(ii) We consider the case where $\bar{b} < \frac{\tau}{(1-\tau)\alpha}$ holds. In this case, $\bar{b} < \hat{k}_g$ holds because we have $\hat{k}_g - \bar{b} = \frac{(1+\bar{b})[\tau-(1-\tau)\alpha\bar{b}]}{(1-\tau)(1+\alpha\bar{b})}$. We rewrite (B.2) as:

$$\Gamma(k_g, \tau, \bar{b}) = \left\{(1-\tau)(1+\alpha\bar{b}) + \frac{(1-\tau)\alpha\bar{b} - \tau}{k_g - \bar{b}}(1+\bar{b})\right\} Ak_g^{1-\alpha}. \qquad \text{(B.3)}$$

Because $\bar{b} < \frac{\tau}{(1-\tau)\alpha}$ holds, $\partial\Gamma(k_g, \tau, \bar{b})/\partial k_g$ has a positive sign if $k_g \neq \bar{b}$. We also have $\lim_{k_g \to \bar{b}-0} \Gamma(k_g, \tau, \bar{b}) = +\infty$. Because we have $\Gamma(0, \tau, \bar{b}) = 0 < \zeta(0, \tau, \bar{b})$, the continuity of $\Gamma(k_g, \tau, \bar{b})$ and $\zeta(k_g, \tau, \bar{b})$ indicates that the $\dot{k}_g = 0$ locus has at least one intersection with the $\dot{c} = 0$ locus in the region of $k_g < \bar{b}$ as shown in Figure 2-(b). We can show $\lim_{k_g \to \infty} \Gamma(k_g, \tau, \bar{b})/\zeta(k_g, \tau, \bar{b}) = (1+\alpha\bar{b})/(1-\alpha) > 1$, which means that for large k_g, the $\dot{k}_g = 0$ locus is located above the $\dot{c}_t = 0$ locus. Because we have $\Gamma(\hat{k}_g, \tau, \bar{b}) = 0 < \zeta(\hat{k}_g, \tau, \bar{b})$, the continuity indicates that the $\dot{k}_g = 0$ locus intersects with the $\dot{c} = 0$ locus at least once in the region of $k_g > \bar{b}$. Because there exist two steady states, we now know that one of the two intersection of the $\dot{c} = 0$ and $\dot{k}_g = 0$ loci is in the region of $k_g < \bar{b}$ and the other intersection is in the region of $k_g > \bar{b}$. From the discussion so far, we depict the $\dot{k}_g = 0$ locus as shown in Figure 2-(b). From (24), we obtain:

$$\dot{k}_{g,t} \geq 0, \ \text{(a) if } c_t \leq \Gamma(k_{g,t}, \tau, \bar{b}) \text{ and } k_g \leq \bar{b}, \text{ or (b) if } c_t \geq \Gamma(k_{g,t}, \tau, \bar{b}) \text{ and } k_g \geq \bar{b}, \quad \text{(B.4)}$$

$$\dot{k}_{g,t} < 0, \ \text{(c) if } c_t < \Gamma(k_{g,t}, \tau, \bar{b}) \text{ and } k_g > \bar{b}, \text{ or (d) if } c_t > \Gamma(k_{g,t}, \tau, \bar{b}) \text{ and } k_g < \bar{b}. \quad \text{(B.5)}$$

Thus, we can draw the phase diagram as shown in Figure 2-(b).

(iii) We finally turn to the case of $\bar{b} > \frac{\tau}{(1-\tau)\alpha}$. The second term in parentheses of the RHS of (B.3) is negative (positive) when $k_g < (>)\bar{b}$ holds. In the region where $k_g < (>)\bar{b}$ holds, the graph of $c = \Gamma(k_g, \tau, \bar{b})$ is located below (above) the graph of $c = (1-\tau)(1+\alpha\bar{b})Ak_g^{1-\alpha}$, as is shown in Figure 2-(c). Eq (B.3) also reveals $\lim_{k_g \to \bar{b}+0} \Gamma(k_g, \tau, \bar{b}) = +\infty$. In addition, the graph of $c = \Gamma(k_g, \tau, \bar{b})$ asymptotically becomes close to the graph of $c = (1-\tau)(1+\alpha\bar{b})Ak_g^{1-\alpha}$ as k_g increases to $+\infty$. Therefore, in the region of $k_g > \bar{b}$, the $\dot{k}_g = 0$ locus takes the shape shown in Figure 2-(c).

By using (B.2), we next show that $\Gamma(k_g, \tau, \bar{b}) > 0$ holds in the region of $k_g < \hat{k}_g$. From (B.2), we know that the denominator of $\Gamma(k_g, \tau, \bar{b})$ becomes negative when $k_g < \bar{b}$ and the numerator becomes negative when $k_g < \hat{k}_g$. Because we have $\hat{k}_g < \bar{b}$ when $\bar{b} > \frac{\tau}{(1-\tau)\alpha}$ holds, both the denominator and numerator of $\Gamma(k_g, \tau, \bar{b})$ are negative in the region where $k_g < \hat{k}_g$ holds. Therefore, $\Gamma(k_g, \tau, \bar{b}) > 0$ holds in the region of $k_g < \hat{k}_g$.

From the discussion so far, together with (B.4) and (B.5), we can draw the phase diagram as shown in Figure 2-(c). The upper (lower) panel shows the case where both $k_{g,L}^*$ and $k_{g,H}^*$ are smaller (larger) than $\bar{b}$.

C Proof of Proposition 2

Approximating (10), (17) and (18) linearly in the neighborhood of the steady states, we obtain:

$$\begin{pmatrix} \dot{b} \\ \dot{c} \\ \dot{k}_g \end{pmatrix} = \begin{pmatrix} -\phi & 0 & 0 \\ J_{cb} & J_{cc} & J_{ck_g} \\ J_{k_g b} & J_{k_g c} & J_{k_g k_g} \end{pmatrix} \begin{pmatrix} b_t - \bar{b} \\ c_t - c^* \\ k_{g_t} - k_g^* \end{pmatrix}. \tag{C.1}$$

$J = (J_{ij})$ denotes the coefficient matrix of the former system:

$$\begin{aligned} J_{cb} &= -\frac{\rho+\phi}{1+\bar{b}}c^*, \quad J_{cc} = \frac{c^*}{1+\bar{b}}, \quad J_{ck_g} = -\frac{(1-\alpha)^2(1-\tau)c^*}{1+\bar{b}}A(k_g^*)^{-\alpha}, \\ J_{k_g b} &= -(1+k_g^*)\frac{\rho+\phi}{1+\bar{b}}, \quad J_{k_g c} = \frac{k_g^* - \bar{b}}{1+\bar{b}}, \\ J_{k_g k_g} &= -\frac{(1-\alpha)\eta(k_g^*, \tau, \bar{b})}{(1+\bar{b})k_g^*} - [(1-\tau)\alpha A(k_g^*)^{1-\alpha} - \rho]. \end{aligned} \tag{C.2}$$

where $k_g^* = k_{g,L}^*$ or $k_{g,H}^*$, and $c^* = \zeta(k_g^*, \tau, \bar{b})$.

Let us denote the eigenvalues of the coefficient matrix J as ν_i ($i = 1$, 2 and 3). The structure of the first column of J entails that $-\phi$ is an eigenvalue ν_1. The remaining eigenvalues, ν_2 and ν_3, of J are those of the matrix, $\bar{J}$, derived by deleting the first row and column from J. The eigenvalues, ν_2 and ν_3, are the solution of the characteristic equation, $\nu^2 - (J_{cc} + J_{k_g k_g})\nu + J_{cc}J_{k_g k_g} - J_{k_g c}J_{ck_g} = 0$. Because we will use ν_2 and ν_3 in Appendix E, we derive these by solving this equation:

$$\nu_2 = \frac{J_{cc} + J_{k_g k_g} + \sqrt{(J_{cc} + J_{k_g k_g})^2 - 4(J_{cc}J_{k_g k_g} - J_{k_g c}J_{ck_g})}}{2}, \tag{C.3}$$

$$\nu_3 = \frac{J_{cc} + J_{k_g k_g} - \sqrt{(J_{cc} + J_{k_g k_g})^2 - 4(J_{cc}J_{k_g k_g} - J_{k_g c}J_{ck_g})}}{2}. \tag{C.4}$$

To check the stability, we examine the sign of $det\bar{J} = J_{cc}J_{k_g k_g} - J_{k_g c}J_{ck_g}$. Using (C.2), we obtain:

$$det\bar{J} = -\frac{c^*}{(1+\bar{b})^2}\left\{(2-\alpha)(1-\tau)\alpha A(k_g^*)^{1-\alpha} - (1-\alpha)\tau A(k_g^*)^{-\alpha} - \rho\right\}. \tag{C.5}$$

To simplify the above equation, we subtract $\Pi'(k_g^*)$ from $\Lambda'(k_g^*)$.

$$\Lambda'(k_g^*) - \Pi'(k_g^*) = (2-\alpha)(1-\tau)\alpha A(k_g^*)^{1-\alpha} - (1-\alpha)\tau A(k_g^*)^{-\alpha} - \rho.$$

Thus, (C.5) can be rewritten as:

$$det\bar{J} = -\frac{c^*}{(1+\bar{b})^2}\left\{\Lambda'(k_g^*) - \Pi'(k_g^*)\right\}.$$

Figure 1 shows that in the high-growth steady state, $\Lambda'(k_{g,H}^*) > \Pi'(k_{g,H}^*)$ holds and hence $det\bar{J} < 0$. One of the eigenvalues of $\bar{J}$ has a positive real part and the other has a negative real part. Moreover, because the inequality $det\bar{J} < 0$ implies $(J_{cc} + J_{k_g k_g})^2 - 4(J_{cc}J_{k_g k_g} - J_{k_g c}J_{ck_g}) > 0$, both ν_2 and ν_3 are real numbers. We then have $\nu_2 > 0$ and $\nu_3 < 0$. The high-growth steady state is saddle-point stable.

Figure 1 shows that in the low-growth steady state, $\Lambda'(k^*_{g,L}) < \Pi'(k^*_{g,L})$ holds and then $det\bar{J} > 0$ holds. This implies that the real parts of ν_2 and ν_3 have the same signs. To determine the sign of the real parts, we check the sign of $Tr\bar{J} = J_{cc} + J_{k_g k_g}$. As we have just shown, we have $det\bar{J} = J_{cc}J_{k_g k_g} - J_{k_g c}J_{c k_g} > 0$ in the low-growth steady state. This inequality can be written as:

$$J_{cc}J_{k_g k_g} > J_{k_g c}J_{c k_g} = -\frac{(1-\alpha)^2(1-\tau)(k_g^* - \bar{b})}{1+\bar{b}}\frac{c^*}{1+\bar{b}}A(k_g^*)^{-\alpha}.$$

We divide both sides of the above inequality by $J_{cc} = c^*/(1+\bar{b})(>0)$.

$$J_{k_g k_g} > -\frac{(1-\alpha)^2(1-\tau)(k_g^* - \bar{b})}{1+\bar{b}}A(k_g^*)^{-\alpha}.$$

Adding J_{cc} to both sides of the above inequality and using $c^* = \zeta(k_g^*, \tau, \bar{b})$, we obtain:

$$\begin{aligned} J_{cc} + J_{k_g k_g} &> \frac{1}{1+\bar{b}}\left\{c^* - (1-\alpha)^2(1-\tau)(k_g^* - \bar{b})A(k_g^*)^{-\alpha}\right\}, \\ &= \frac{1}{1+\bar{b}}\left\{(1-\tau)(1-\alpha)[\alpha + (1-\alpha)\bar{b}]A(k_g^*)^{1-\alpha} + \rho(1+\bar{b})\right\}, \\ &> 0. \end{aligned}$$

In the low-growth steady state, $Tr\bar{J} = J_{cc} + J_{k_g k_g} > 0$ holds. The real parts of ν_2 and ν_3 are positive. The low-growth steady state is unstable.

D Welfare effects of $\bar{b}$

To calculate the value of U_0^{**}, we calculate the dynamic path and the initial value of $U_t \equiv \int_t^\infty (\ln C_v)e^{-\rho(v-t)}dv$ by using the relaxation algorithm. However, we cannot calculate the dynamic path of and the initial value of U_t directly because U_t does not remain constant in the high-growth steady state. Let us define $X_t \equiv U_t - \ln K_t/\rho$. Because $C_t \equiv c_t K_t$, we have:

$$\dot{X}_t = \rho X_t - \ln c_t - \frac{1}{\rho}(A{k_{g,t}}^{1-\alpha} - c_t - \rho).$$

X_t becomes constant over time in the high-growth steady state. Then, we calculate the dynamic path of and the initial value of X_t using the relaxation algorithm. As K_0 is normalized to one, we have $U_0^{**} = X_0$.

References

[1] Agénor, P.R., 2008. Health and infrastructure in a model of endogenous growth. Journal of Macroeconomics 30, 1407–1422.

[2] Barro, R.J., 1990. Government spending in a simple model of endogenous growth. Journal of Political Economy 98, S103–S125.

[3] Bruce, N., Turnovsky, S.J., 1999. Budget balance, welfare, and the growth rate: "Dynamic scoring" of the long-run government budget. Journal of Money, Credit, and Banking 31, 162–186.

[4] European Commission, 2011. Public finances in EMU – 2011, *European Economy* No. 3, 2011.

[5] Futagami, K., Iwaisako, T., Ohdoi, R., 2008. Debt policy rule, productive government spending, and multiple growth paths. Macroeconomic Dynamics 12, 445–462.

[6] Futagami, K., Morita, Y., Shibata, A., 1993. Dynamic analysis of an endogenous growth model with public capital. Scandinavian Journal of Economics 95, 607–625.

[7] Ghosh, S., Mourmouras, I.A., 2004. Endogenous growth, welfare and budgetary regimes. Journal of Macroeconomics 26, 623–635.

[8] Gómez, M.A., 2004. Optimal fiscal policy in a growing economy with public capital. Macroeconomic Dynamics 8, 419–435.

[9] Greiner, A., Semmler, W., 2000. Endogenous growth, government debt and budgetary regimes. Journal of Macroeconomics 22, 363–384.

[10] Greiner, A., 2007. An endogenous growth model with public capital and sustainable government debt. Japanese Economic Review 58, 345–361.

[11] Greiner, A., 2008. Human capital formation, public debt and economic growth. Journal of Macroeconomics 30, 415–427.

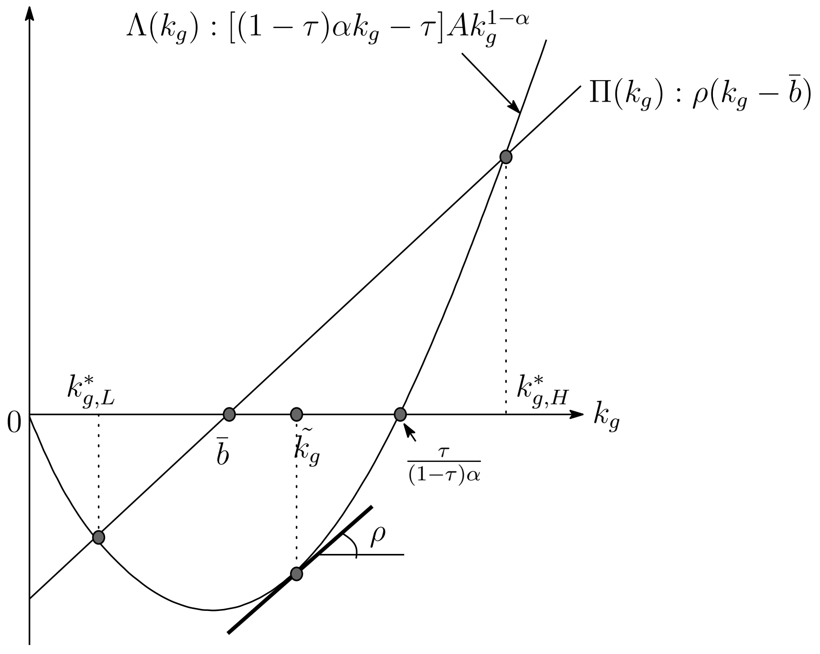
[12] Minea, A., Villieu, P., 2012. Debt policy rule, productive government spending, and multiple growth paths: A note. Macroeconomic Dynamics, doi:10.1017/S1365100511000642.

[13] Rubianes, Á.C., 2010. Patterns and trends of public investment in the new member states of the European Union. Working Paper No. 3, 2010.

[14] Trimborn, T., Koch, K.J., Steger, T.M., 2008. Multidimensional transitional dynamics: a simple numerical procedure. Macroeconomic Dynamics 12, 301–319.

[15] Turnovsky, S.J., 1997. Fiscal policy in a growing economy with public capital. Macroeconomic Dynamics 1,615–639.

[16] Yakita, A., 2008. Sustainability of public debt, public capital formation, and endogenous growth in an overlapping generations setting. Journal of Public Economics 92, 897–914.



$\Lambda(k_g) : [(1-\tau)\alpha k_g - \tau]Ak_g^{1-\alpha}$
$\Pi(k_g) : \rho(k_g - \bar{b})$
$k^*_{g,L}$
$k^*_{g,H}$
0
k_g
$\bar{b}$
$\tilde{k}_g$
$\frac{\tau}{(1-\tau)\alpha}$
ρ

Figure 1

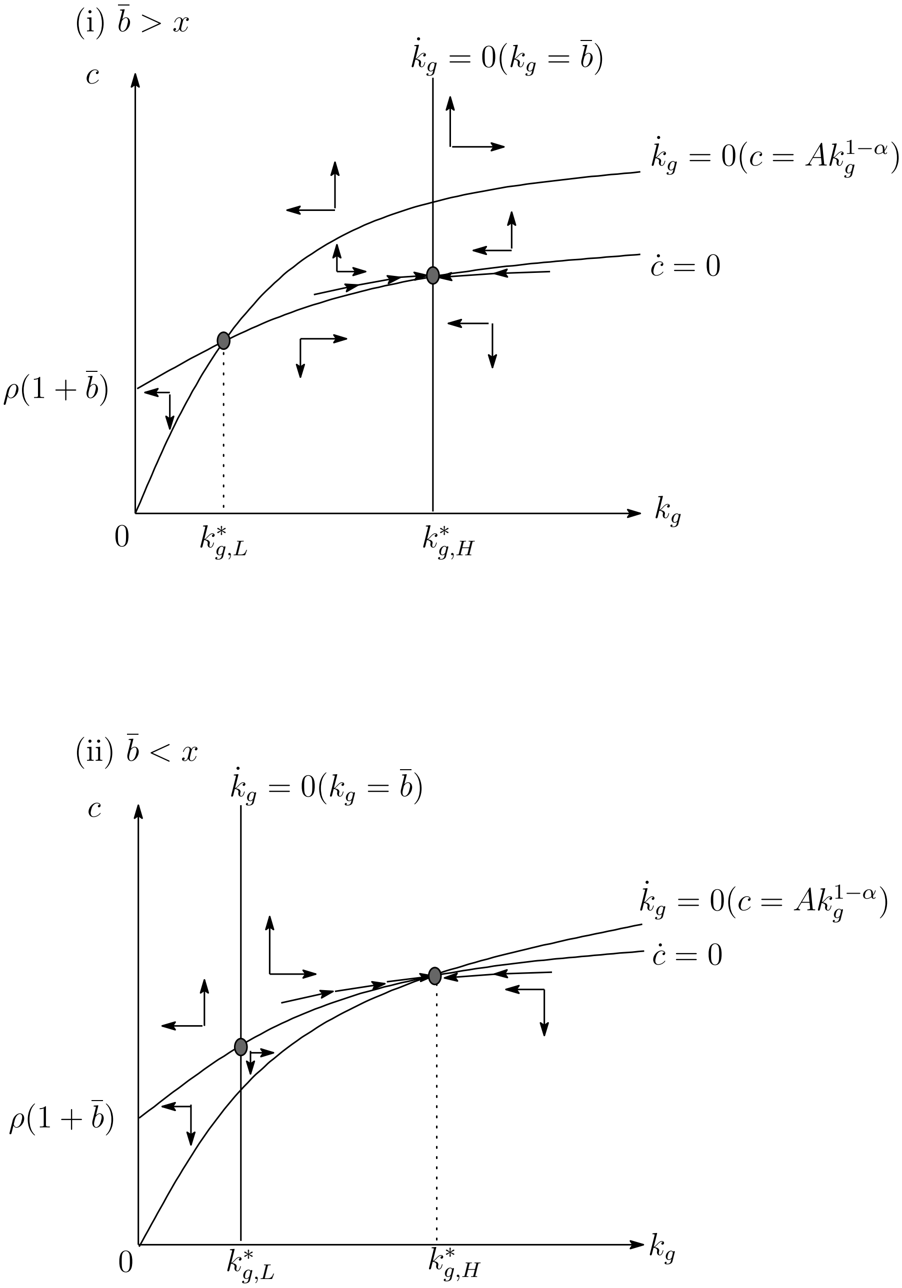


Figure 2-(a) $\bar{b} = \frac{\tau}{(1-\tau)\alpha}$

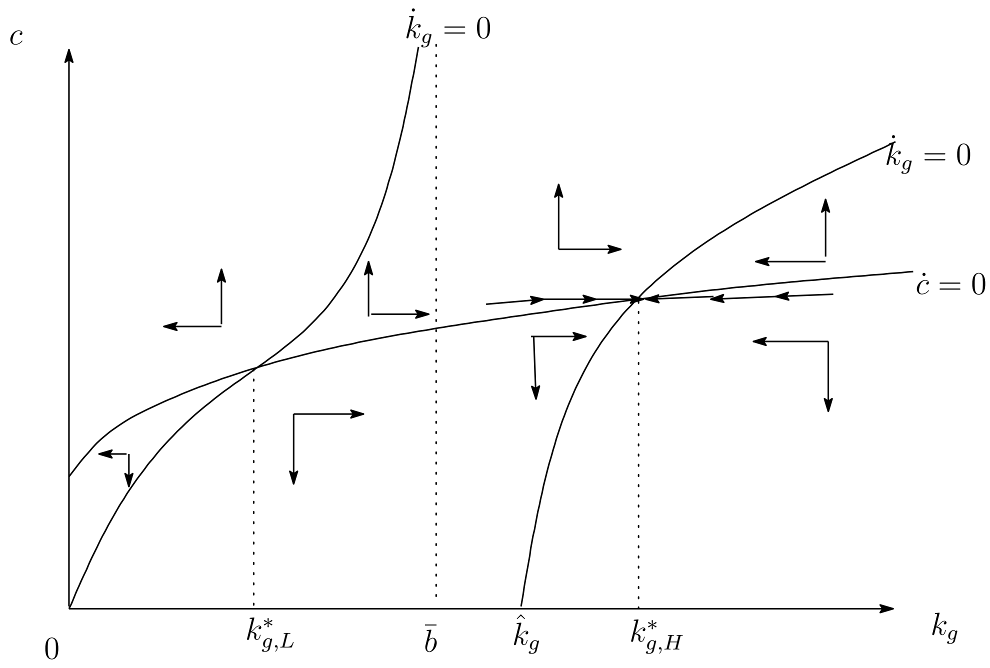
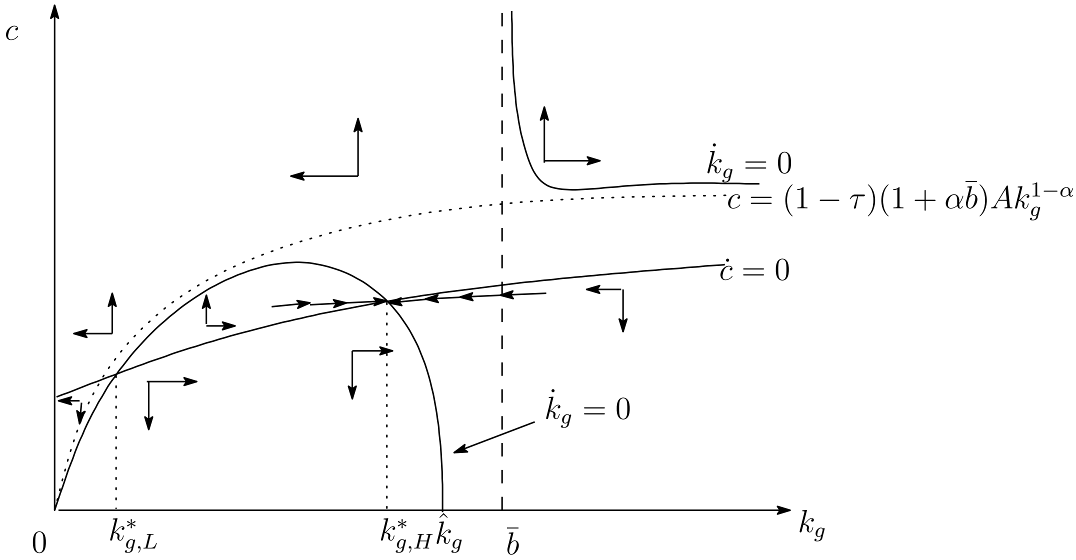


Figure 2-(b) $\bar{b} < \frac{\tau}{(1-\tau)\alpha}$

(i) $k_{g,L}^* < k_{g,H}^* < \bar{b}$



(ii) $\bar{b} < k_{g,L}^* < k_{g,H}^*$

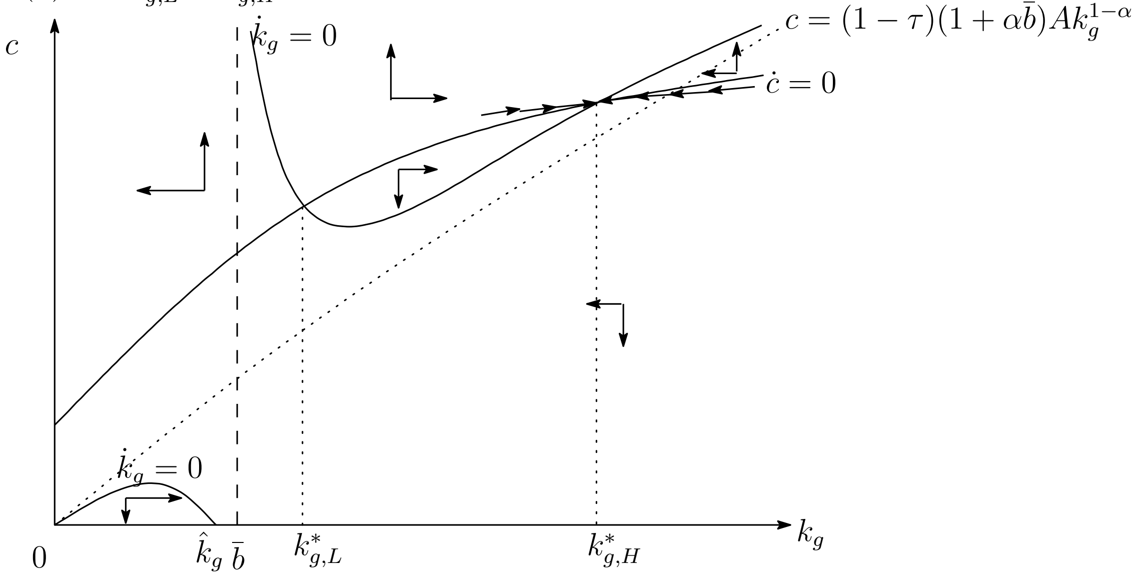
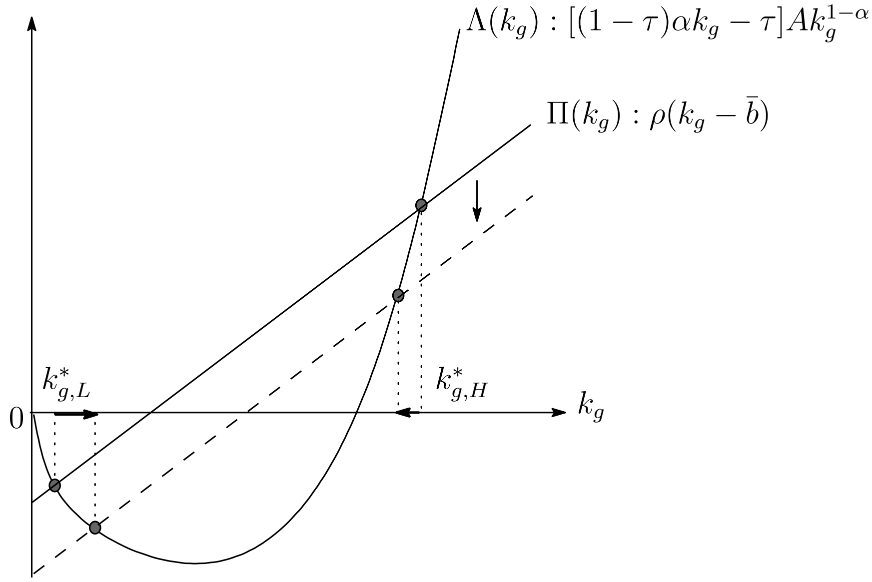


Figure 2-(c) $\bar{b} > \frac{\tau}{(1-\tau)\alpha}$

(i) Policy effect with respect to $\bar{b}(\bar{b}\uparrow)$



(ii) Policy effect with respect to $\tau(\tau\uparrow)$

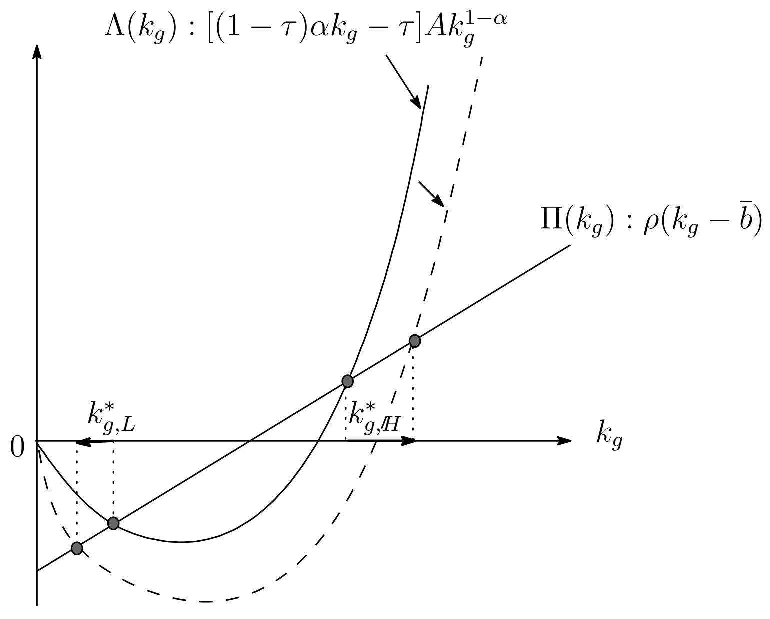


Figure 3

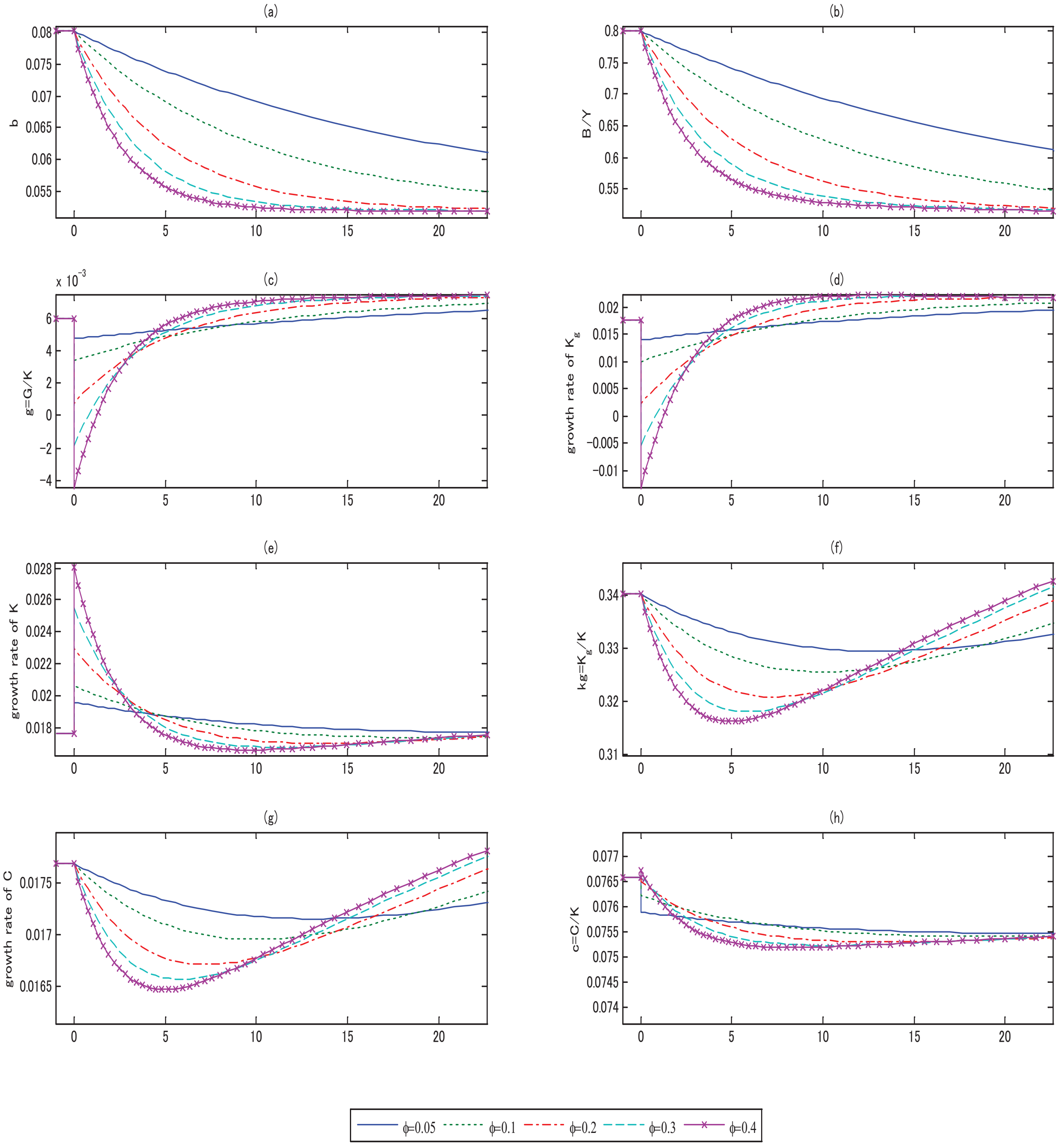


Figure 4. Transitional Dynamics

		ϕ									
τ	B_0/Y_0	0.01	0.025	0.05	0.1	0.2	0.3	0.4	0.5	0.75	1
0.1	1	0.06557	0.05236	0.04048	0.03064	0.02463	0.02261	0.02163	0.02104	0.02023	0.01976
		(165y)	(70y)	(37y)	(20y)	(11y)	(7y)	(5.5y)	(4.5y)	(3y)	(2y)
	0.9	0.05149	0.04190	0.03379	0.02776	0.02481	0.02412	0.02389	0.02380	0.02371	0.02364
		(140y)	(60.5y)	(32.5y)	(17y)	(9y)	(6y)	(4.5y)	(4y)	(2.5y)	(2y)
	0.8	0.03810	0.03148	0.02627	0.02293	0.02191	0.02199	0.02217	0.02234	0.02263	0.02278
		(113y)	(48y)	(25.5y)	(13y)	(6.5y)	(4.5y)	(3.5y)	(3y)	(2y)	(1.5y)
	0.7	0.02518	0.02105	0.01806	0.01652	0.01653	0.01692	0.01725	0.01750	0.01789	0.01811
		(74y)	(27.5y)	(15.5y)	(8y)	(4y)	(3y)	(2y)	(1.5y)	(1y)	(0.8y)
	0.6	0.01253	0.01057	0.00928	0.00880	0.00911	0.00946	0.00971	0.00989	0.01018	0.01034
0.2	1	0.06948	0.06231	0.05720	0.05570	0.05815	0.06041	0.06201	0.06314	0.06484	0.06570
		(165y)	(66.5y)	(35.5y)	(17.5y)	(9.5y)	(6.5y)	(5y)	(4y)	(2.5y)	(2y)
	0.9	0.05676	0.05056	0.04643	0.04549	0.04785	0.04988	0.05129	0.05229	0.05380	0.05459
		(140y)	(57.5y)	(30y)	(15.5y)	(8y)	(5y)	(4y)	(3y)	(2y)	(1.5y)
	0.8	0.04357	0.03853	0.03535	0.03483	0.03688	0.03855	0.03971	0.04053	0.04177	0.04243
		(112.5y)	(45.5y)	(23.5y)	(12y)	(6y)	(4y)	(3y)	(2.5y)	(1.6y)	(1.3y)
	0.7	0.02979	0.02614	0.02395	0.02370	0.02524	0.02646	0.02729	0.02788	0.02877	0.02926
		(74y)	(30y)	(15y)	(7.5y)	(4y)	(2.5y)	(2y)	(1.5y)	(1y)	(0.8y)
	0.6	0.01513	0.01314	0.01201	0.01192	0.01278	0.01343	0.01388	0.01419	0.01467	0.01494
0.3	1	0.06654	0.06262	0.06008	0.06099	0.06557	0.06888	0.07110	0.07264	0.07493	0.07606
		(165y)	(66.5y)	(34y)	(17.5y)	(9y)	(6y)	(4.5y)	(4y)	(2.5y)	(2y)
	0.9	0.05577	0.05161	0.04909	0.04975	0.05356	0.05630	0.05815	0.05943	0.06135	0.06232
		(140y)	(57.57)	(30y)	(15y)	(7.5y)	(5y)	(4y)	(3y)	(2y)	(1.5y)
	0.8	0.04373	0.03978	0.03752	0.03794	0.04090	0.04303	0.04446	0.04546	0.04697	0.04776
		(112.5y)	(45.5y)	(23.5y)	(12y)	(6y)	(4y)	(3y)	(2.5y)	(1.6y)	(1.3y)
	0.7	0.03067	0.02745	0.02567	0.02590	0.02793	0.02941	0.03040	0.03109	0.03213	0.03270
		(74y)	(29y)	(15y)	(7.5y)	(4y)	(2.5y)	(2y)	(1.5y)	(1y)	(0.8y)
	0.6	0.01609	0.01417	0.01314	0.01322	0.01427	0.01503	0.01554	0.01590	0.01645	0.01675

Table 1. Welfare effects when the debt–GDP ratio falls to 50 percent.
The numbers in parentheses denote the time required to reduce the debt–GDP ratio to approximately 60 percent.

	τ		
B_0/Y_0	0.1	0.2	0.3
1	41%	14%	8%
0.9	30%	11%	6.5%
0.8	21%	8%	5%
0.7	13%	5%	3%
0.6	6%	2.5%	1.5%

Table 2. Initial declines of g_t under $\phi = 0.05$.

τ	B_0/Y_0		ϕ									
			0.01	0.025	0.05	0.1	0.2	0.3	0.4	0.5	0.75	1
0.1	1	$\rho = 0.03$	0.20037	0.14441	0.1065	0.08091	0.06595	0.06034	0.05731	0.05537	0.05253	0.05091
		$\rho = 0.07$	0.01863	0.01729	0.01516	0.01270	0.01097	0.01043	0.01020	0.01001	0.00994	0.00981
	0.9	$\rho = 0.03$	0.15852	0.11767	0.09197	0.07697	0.07021	0.06826	0.06734	0.06679	0.06600	0.06552
		$\rho = 0.07$	0.01501	0.01390	0.01238	0.01084	0.00999	0.009847	0.009846	0.00988	0.00994	0.00996
	0.8	$\rho = 0.03$	0.1177	0.08969	0.07356	0.06600	0.06440	0.06461	0.06490	0.06514	0.06552	0.06569
		$\rho = 0.07$	0.01155	0.01061	0.00954	0.00864	0.00833	0.00841	0.00853	0.08627	0.00880	0.00888
	0.7	$\rho = 0.03$	0.07776	0.06074	0.05181	0.04899	0.05000	0.05112	0.05190	0.05244	0.05325	0.05368
		$\rho = 0.07$	0.00801	0.00726	0.00655	0.00607	0.00606	0.00622	0.00636	0.00648	0.00666	0.00677
	0.6	$\rho = 0.03$	0.03857	0.03074	0.02717	0.02677	0.02821	0.02921	0.02985	0.03029	0.03094	0.03129
		$\rho = 0.07$	0.00421	0.00376	0.00339	0.00319	0.00327	0.00339	0.00349	0.00367	0.00370	0.00377
0.2	1	$\rho = 0.03$	0.2335	0.1960	0.1811	0.1838	0.1961	0.2036	0.2082	0.2113	0.2158	0.02180
		$\rho = 0.07$	0.01576	0.01728	0.01779	0.01805	0.01887	0.01962	0.02019	0.02061	0.02123	0.02150
	0.9	$\rho = 0.03$	0.1876	0.1578	0.1466	0.1500	0.1611	0.1676	0.1717	0.1744	0.1783	0.1804
		$\rho = 0.07$	0.01431	0.01486	0.01487	0.01495	0.01568	0.01635	0.01686	0.01723	0.01780	0.01807
	0.8	$\rho = 0.03$	0.1413	0.1191	0.1113	0.1147	0.1238	0.1292	0.1325	0.1347	0.1379	0.1396
		$\rho = 0.07$	0.01212	0.01199	0.01168	0.01163	0.01221	0.01277	0.01319	0.01350	0.01397	0.01422
	0.7	$\rho = 0.03$	0.09472	0.07988	0.07502	0.07786	0.08453	0.08838	0.09075	0.09233	0.09464	0.09586
		$\rho = 0.07$	0.00909	0.00861	0.00817	0.00805	0.00846	0.00887	0.00917	0.00939	0.00974	0.00993
	0.6	$\rho = 0.03$	0.04753	0.04001	0.03783	0.03951	0.04311	0.04517	0.04643	0.04728	0.04851	0.04917
		$\rho = 0.07$	0.00505	0.00461	0.00426	0.00415	0.00436	0.00458	0.00474	0.00486	0.00506	0.00516
0.3	1	$\rho = 0.03$	0.2458	0.2141	0.2055	0.2159	0.2355	0.2465	0.2532	0.2577	0.2641	0.2673
		$\rho = 0.07$	0.00922	0.01324	0.01560	0.01719	0.01879	0.01983	0.02056	0.02107	0.0218	0.02209
	0.9	$\rho = 0.03$	0.1993	0.1731	0.1662	0.1749	0.1912	0.2003	0.2058	0.02095	0.2148	0.2175
		$\rho = 0.07$	0.01015	0.01231	0.01353	0.01446	0.01569	0.01656	0.01717	0.0176	0.01824	0.01853
	0.8	$\rho = 0.03$	0.1516	0.1312	0.1260	0.1329	0.1455	0.1525	0.1568	0.1596	0.1637	0.1659
		$\rho = 0.07$	0.00987	0.01066	0.01103	0.01147	0.01235	0.01302	0.01350	0.01384	0.01436	0.01462
	0.7	$\rho = 0.03$	0.1026	0.08844	0.08492	0.08974	0.09841	0.1032	0.1062	0.1081	0.1109	0.1124
		$\rho = 0.07$	0.00815	0.00809	0.00796	0.00806	0.00862	0.00908	0.00941	0.00966	0.01003	0.01023
	0.6	$\rho = 0.03$	0.05206	0.04473	0.04295	0.04546	0.04992	0.05240	0.05391	0.05491	0.05637	0.05716
		$\rho = 0.07$	0.00491	0.00456	0.00430	0.00425	0.00451	0.00475	0.00492	0.00505	0.00526	0.00537

Table 3. Welfare effects under $\rho = 0.03$ and 0.07.

τ	B_0/Y_0		ϕ 0.01	0.025	0.05	0.1	0.2	0.3	0.4	0.5	0.75	1
0.2	1	$A = 0.1161$	0.04687	0.04316	0.04046	0.04005	0.04220	0.04397	0.04518	0.04604	0.04731	0.04792
	0.9	$A = 0.1161$	0.03876	0.03528	0.03296	0.03274	0.03469	0.03623	0.03729	0.03803	0.03913	0.03969
	0.8	$A = 0.1161$	0.03011	0.02707	0.02519	0.02509	0.02671	0.02796	0.02881	0.02941	0.03031	0.03078
	0.7	$A = 0.1161$	0.02082	0.01849	0.01713	0.01709	0.01828	0.01917	0.01978	0.0202	0.02084	0.02119
	0.6	$A = 0.1161$	0.01081	0.00947	0.00873	0.00872	0.00936	0.00984	0.01016	0.01039	0.01073	0.01092
0.3	1	$A = 0.1146$	0.04210	0.04116	0.04045	0.04165	0.04502	0.04733	0.04886	0.04991	0.05145	0.05218
	0.9	$A = 0.1146$	0.03599	0.03428	0.03325	0.03409	0.03685	0.03877	0.04003	0.04091	0.04220	0.04283
	0.8	$A = 0.1146$	0.02881	0.02678	0.02564	0.02617	0.02830	0.02977	0.03076	0.03144	0.03245	0.03297
	0.7	$A = 0.1146$	0.02049	0.0186	0.01759	0.01787	0.01932	0.02033	0.02101	0.02148	0.02219	0.02256
	0.6	$A = 0.1146$	0.01091	0.00969	0.00905	0.00915	0.00989	0.01042	0.01077	0.01101	0.01138	0.01158

Table 4. Welfare effects when the long-run growth rate is equal to 0.02.