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Saving and retirement behavior under quasi-hyperbolic discounting

Lin Zhang

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GCOE Secretariat
Graduate School of Economics
OSAKA UNIVERSITY
1-7 Machikaneyama, Toyonaka, Osaka, 560-0043, Japan

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Abstract This paper investigates the saving and retirement behavior of consumers using a quasi-hyperbolic discounting model with endogenous labor supply. The optimal behavior, which is obtained under exponential discounting, is compared with the behavior of naive and sophisticated consumers. If and only if the present-biased marginal utility of future consumption decreases with stronger present bias, the quasi-hyperbolic discounter, whether naive or sophisticated, under-saves and retires early compared with an exponential discounter. In other words, quasi-hyperbolic discounting explains why, under-savers might also be early retirees. This is consistent with previous empirical studies. Further, two functional forms are employed as examples of this necessary and sufficient condition. And I show that under logarithmic utility, a wage tax and an interest subsidy can improve consumer welfare.

Keywords: Saving · Retirement · Quasi-Hyperbolic discounting · Sophisticated · Naive

JEL Classification: D91 · E21 · J26

1 Introduction

The quasi-hyperbolic discounting model has been widely applied to study how people save for retirement, because of its good approximation to hyperbolic discounting's accurate description of time-inconsistent impatience, namely discounting the near future much more heavily than the distant future for the same length of time. This research approach has been inspired by experimental research as well as by common intuitions. By contrast, time-consistent preference can be described by exponential discounting under which the marginal rate of the substitution of consumption between any two points of time depends only on the distance between these. Intuitively, the quasi-hyperbolic discounter does not save as much as the exponential discounter does, owing

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L. Zhang ✉
Graduate School of Economics, Osaka University, 1-7 Machikaneyama, Toyonaka, Osaka 560-0043 Japan
Email: kge017zl@mail2.econ.osaka-u.ac.jp

to the stronger impatience to consume immediately. Consequently, he or she cannot accumulate enough wealth to support him-or herself after stopping working, has to delay retirement, and thus works for a longer time compared with the exponential discounter. This trade-off relation between saving level and retirement age under quasi-hyperbolic discounting has been discussed by Laibson et al. (1998). Retiring early is also desirable for the quasi-hyperbolic discounter; however, in order to realize this, he or she has to forego a proportion of current consumption and saves at a higher level (Diamond and Koszegi, 2003). Given that either under-saving or early retirement has only been predicted separately by previous studies, the aim of this paper is to investigate whether the consumers can under-save and still retire early under quasi-hyperbolic discounting.

Although the saving and retirement behavior of quasi-hyperbolic discounter has been empirically investigated, these behavioral traits have not been jointly examined thus far. Eisenhauer and Ventura (2006) find that wealth accumulation is negatively related to hyperbolic discounting in Italy and that hyperbolic discounters are less likely to utilize commitment devices to control their choices. Fang and Silverman (2007) show that unmarried mothers in the US fail to maintain sufficient labor supply under quasi-hyperbolic discounting. By relating wealth level and labor supply to discounting styles, under-saving jointly determined with early retirement can be indirectly shown. In this context, the present analysis helps explain this as-yet underexplored phenomenon.

A number of empirical studies discuss resent trends in under-saving or early retirement. For example, Attanasio (1993) uses a CEX data set provided the Bureau of Labor Statistic tapes of the Consumer Expenditure Survey to show that aggregate personal saving has declined in the US since the 1980s. Bosworth et al. (1991) show that the private saving rate in the US has declined steadily over past 20 years. Gendell (2001) shows that the average age at retirement declined in the 1990s, while Gruber and Wise (2002) demonstrate that workers have been leaving the labor force at younger and younger ages in recent years. These empirical studies imply that over the past 20 years American household saving has declined, even though people have been retiring earlier. The findings in this paper could help explain these seemingly paradoxical empirical phenomena.

This paper employs a three-period quasi-hyperbolic discounting model with endogenous labor supply. Saving level is determined in the first period and retirement age in the second period. This setup is different from that presented by Diamond and Koszegi (2003) in that the labor supply chosen in the second period is continuous. Following O'Donoghue and Robin (1999), two types of consumer can be distinguished: (i) those who can foresee self-control problems (defined as "sophisticated consumers") and (ii) those unaware of such problems (defined as "naive consumers").

Saving level and retirement age under quasi-hyperbolic discounting are compared with those under exponential discounting. Following O'Donoghue and Robin (1999), the outcome of time-consistent preference is considered to be optimal from the long-run perspective, and the definition of under-saving and early retirement is derived accordingly.

Furthermore, how a wage tax and an interest subsidy affect consumer welfare is investigated. This paper thus extends the model by including government policy, which aims to optimize a consumer's long-run well-being.

This paper makes four principal findings. First, if and only if the present-biased marginal utility decreases as present bias strengthens, consumers under quasi-hyperbolic discounting, whether sophisticated or naive, save less and retire earlier than those under exponential discounting. This result shows the co-existence of under-saving and early retirement. Second, two examples of utility function form (i.e., constant absolute risk aversion (CARA) and logarithmic) are found to show the establishment of this necessary and sufficient condition. With these two functional forms, the consumer definitely under-saves; however, consistent with the finding of Diamond and Koszegi (2003), consumers under quasi-hyperbolic discounting may either retire either earlier or later.

Third, a necessary and sufficient condition is provided under which the government improves consumer welfare by imposing a proportional wage tax. Interest subsidy definitely improves consumer welfare. Fourth, because the discounting effect of the present bias parameter can increase or decrease labor supply, the long-run Phillips curve relation may display a trade-off or stagflation between unemployment and inflation.

This paper makes the following original contributions to the body of knowledge in this area. A necessary and sufficient condition of the general form for under-saving and early retirement is derived, while two examples of utility function form are used to obtain closed-form solutions to the utility maximization problem with present bias, thereby conducting comparative dynamics with respect to the present bias parameter. Furthermore, a proportional wage tax and an interest subsidy are included to extend the model by discussing how they affect consumer welfare when the government's budget constraint is satisfied.

The remainder of this paper is organized as follows. Section 2 introduces the theoretical model. Section 3 presents the saving and retirement behavior of naïve consumers. Section 4 discusses the solutions to the problem of sophisticated consumers. Section 5 analyzes how representative consumer welfare is affected by a wage tax and an interest subsidy. Section 6 discusses on the results of this analysis and explores how these results work upon the long-run Phillips curve relation. Section 7 concludes.

2 The Model

To describe the saving and retirement behavior of quasi-hyperbolic discounters, the quasi-hyperbolic discounting model of Laibson (1997) is extended by incorporating endogenous labor supply. This model enables us to approximate present-biased preferences in a tractable form.

Consider a representative consumer who lives for three periods. In the first period, he or she has to supply labor in-elastically. He or she retires in the second period. Retirement is decided by choosing period-2 working hours *l*. Before retirement, he or she receives wage at a rate of w. Following Frogneux (2009), she has to suffer the disutility of working *e(l)*. After he or she retires in period 2, there is no further income. In period 3, he or she only consumes the amount saved before retirement. Variable c_t denotes the consumption level in each period and s_1 is the saving level in the first period ($s_1 = w - c_1$).

Following Laibson (1997), the representative consumer's lifetime utility function with present bias is

$$U_t = u(c_t) + \beta \sum_{\tau=1}^{3-t} \delta^{\tau} u(c_{t+\tau}) - (\beta\delta)^{2-t} e(l), t = 1,2 \quad (1)$$

where 0< β <1 and 0< δ <1. β captures present bias or diminishing impatience, whereas δ represents the long-run discount factor. A smaller β implies a stronger present bias. Therefore, β could be referred to as the present bias parameter. When β equals 1, (1) reduces to the case of exponential discounting.

In the last period, the consumer exhausts all the remaining savings and faces the inter-temporal budget constraint:

$$c_3 = R^2 w - R^2 c_1 + Rwl - Rc_2 \quad (2)$$

where R is the gross interest rate.

I assume the follow:

ASSUMPTION 1 The instantaneous utility function $u(\cdot)$ is concave in consumption c_t and the working disutility function $e(\cdot)$ is convex in labor supply l.

In the following sections, functional-form assumptions are made to obtain analytical results and to show that the propositions are robust to change in functional form. Two specific functional forms are employed as examples and they both meet Assumption 1.

***Example 1**: Constant Absolute Risk Aversion Utility Function*

CARA forms the utility function into

$$u(c_t) = -e^{-Ac_t} \qquad \text{(A>0),} \quad (3)$$

and assume that the disutility function of working is

$$e(l) = e^{Bl} - 1 \qquad \text{(B>0).} \quad (4)$$

***Example 2**: Logarithmic Utility Function*

The utility and disutility functions are in the forms:

$$u(c_t) = \ln c_t \quad , \quad (5)$$

$$e(l) = -\ln(-l) \quad . \quad (6)$$

In the following sections, consumption level and retirement age are compared between the cases under quasi-hyperbolic discounting (β<1) and under exponential discounting (β=1). As pointed out by O’Donoghue and Robin (1999), the behavior of exponential discounter is considered to be the optimal outcome from the long-run perspective, because their preference is time-consistent. This comparison is necessary to evaluate normatively the effect of present bias. Specifically, I use the following terminology to characterize saving and retirement behavior normatively:

DEFINITION 1 A consumer is said to ***under-save*** when his or her saving is lower than that under exponential discounting. He or she is also said to ***retire too early*** when his or her retirement is earlier than that under exponential discounting.

When the present bias parameter β is less than 1, the marginal rates of substitution of c_2 for c_3 (and l for c_3) in period 1 differ from those in period 2. Time-inconsistency arises if this change in the marginal rates of substitution is not incorporated into the agent's consumption plan. A representative consumer who is unaware of this problem and optimistically believes that he or she will carry out what he or she plans now is defined as being *naive* (denoted with a superscript N). By contrast, a representative consumer who regards the life time utility maximizing problem as a game between different periods and behaves following the sub-game perfect Nash equilibrium solution by solving the problem backwardly is defined as being *sophisticated* (denoted with a superscript S). Under logarithmic utility, the solution for naive consumers coincides with that for sophisticated consumers.

3 Naïve Consumers

Consider that a consumer is naïve, that is to say, the consumer cannot foresee his or her present bias and the corresponding time inconsistency and believes that he or she will carry out whatever he or she plans today. His or her optimizing consumption levels and labor supply are derived by solving the utility maximization problems in period 1 and 2 consecutively.

In period 1, the naive consumer consumes c_1^N and optimistically plans the consumption levels $\tilde{c}_2^N$ and $\tilde{c}_3^N$ as well as labor supply $\tilde{l}$ in terms of:

$$u'(c_1^N) = \beta\delta^2 R^2 u'(\tilde{c}_3^N), \tag{7}$$

$$u'(\tilde{c}_2^N) = \delta R u'(\tilde{c}_3^N), \tag{8}$$

$$e'(\tilde{l}^N) = \delta R w u'(\tilde{c}_3^N). \tag{9}$$

The inter-temporal budget constraint (2), and (7), (8), and (9) jointly determine the consumption levels and labor supply for the lifetime utility maximization problem in period 1. However, time inconsistency will arise by neglecting the change in the marginal rates of substitution of c_2 for c_3 (and l for c_3) in period 2. In order to separate the actual consumption level and labor supply from those planned in period 1, those are optimistically planned are denoted as $\tilde{c}_2^N$, $\tilde{c}_3^N$, and $\tilde{l}$. As a result, only c_1^N is realized in period 1.

When period 2 comes and given c_1^N, the consumer re-solves the lifetime utility maximization problem in terms of:

$$u'(c_2^N) = \beta\delta R u'(c_3^N), \tag{10}$$

$$e'(l^N) = \beta\delta R\, w\, u'(c_3^N). \tag{11}$$

Consumption levels $c_2{}^N$ and $c_3{}^N$, and labor supply l^N are determined by (2), (10), and (11). In the presence of present bias (β<1), $c_2{}^N$, $c_3{}^N$, and l^N deviate from the plan made in period 1. By differentiating consumption level and labor supply with respect to β, the effect of present bias on the saving and labor supply of quasi-hyperbolic discounters is investigated.

LEMMA 1 Suppose that the representative consumer is naïve. Then, in the setting specified in the previous section, stronger present bias (i.e., a smaller β) leads to less saving if and only if $\dfrac{d[\beta u'(\tilde{c}_3^N(\beta))]}{d\beta} > 0$.

The proof of Lemma 1 is shown in the Appendix. Lemma 1 demonstrates the necessary and sufficient condition for under-saving by naive consumers. Notice that $u'(c_t^N(\beta))$ is the marginal utility of consumption in period t, which can ultimately be expressed as a function of the present bias parameter β, while $\beta u'(c_t^N(\beta))$ represents marginal utility discounted by the short-run discounting factor or marginal utility in the next period. This paper follows the expression "weighted marginal utility" in line with Pareto (1909), while $\beta u'(c_t)$ is referred to as the "present-biased marginal utility of consumption".

In the three-period model, $\beta u'(c_t)$ captures the degree to which a consumer is concerned about his or her consumption in the future. Therefore, in Lemma 1 $\beta u'(\tilde{c}_3^N(\beta))$ represents the present-biased marginal utility of period-3 consumption in the naive consumer's plan, which affects how much c_1^N is consumed. For the naive consumer making plans in period 1, a smaller β means stronger present bias for the utility in the future. If in the naive consumer's plan, a smaller present-biased marginal utility of period-3 consumption follows a smaller β, this implies that he or she is less concerned about future consumption than might an exponential discounter, because present bias makes he or her so desirous of the current consumption. Then she does not save much for future in period 1.

Saving level is determined in period 1, and thus it is based on the naive consumer's plans. However, these plans are too optimistic for a quasi-hyperbolic discounter and thus they will be modified in period 2.

LEMMA 2 Suppose that the representative consumer is naïve. Then, in the setting specified in the previous section, stronger present bias (i.e., a smaller β) leads to an earlier retirement if and only if $\dfrac{d[\beta u'(c_3^N(\beta))]}{d\beta} > 0$.

The proof of Lemma 2 is similar to that of Lemma 1. Notice that Lemma 2 concerns the present-biased marginal utility of actual period-3 consumption c_3^N, which is determined in period 2. In period 2, the naive consumer modifies his or her consumption level and labor supply because he or she aims to maximize the lifetime utility from the perspective of the present period. The consumer in period 2 is the victim of his or her under-saving in period 1 and has to work longer for period-3. However, he or she is reluctant to work and faces a trade-off between enjoying the current leisure or working longer for future. When a smaller present-biased marginal utility of actual period-3 consumption c_3^N follows stronger present bias, the unwillingness to work dominates and the consumer chooses to retire earlier, because he or she is still concerned less about his or her future (period-3) consumption.

PROPOSITION 1 Suppose that the representative consumer is naïve. Then, if and only if $\frac{d[\beta u'(\tilde{c}_3^N(\beta))]}{d\beta} > 0$ and $\frac{d[\beta u'(c_3^N(\beta))]}{d\beta} > 0$, the quasi-hyperbolic discounter under-saves and retires too early.

Proposition 1 implies a condition that makes saving and working less attractive to the consumer. When the naive consumer is less concerned about planned and actual period-3 consumption less, he or she chooses to save less and retire earlier than might an exponential discounter. In other words, the consumer can raise period-3 consumption by either increasing saving in period 1 or labor supply in period 2, and when he or she is less concerned about period-3 consumption in both periods, saving and labor supply are at a lower level. Therefore, under-saving can co-exist with early retirement, which is consistent with empirical studies.

However, there remains the question of whether this necessary and sufficient condition for under-saving and early retirement exists and whether is robust to a specific form of utility function. Two examples of utility function are included to show the co-existence of under-saving and early retirement.

COROLLARY 1 Under CARA utility, the naive consumer definitely under-saves; if and only if $w < \frac{B}{A} \cdot [\frac{1}{R(R+1)(R-1)} - 1]$, he or she retires too early.

Therefore, for a naive consumer with CARA utility under-saving can co-exist with earlier-retirement.

COROLLARY 2 Under logarithmic utility, the naive consumer definitely under-saves; if and only if $\beta^2 \delta^2 (\delta + 2) > 2$ she retires too early.

The logarithmic utility function form thus allows the co-existence of under-saving and early retirement.

4. Sophisticated Consumers

When the representative consumer is sophisticated, his or her optimal behavior is obtained by solving the sub-game perfect equilibrium backwardly, which is carried out by different inter-temporal selves.

First the sophisticated consumer anticipates his or her optimal behavior in period 2 and period 3 with given c_1^S :

$$u'(c_2^S) = \beta\delta R u'(c_3^S), \tag{12}$$

$$e'(l^S) = \beta w \delta R u'(c_3^S). \tag{13}$$

In combination with (2), he or she foresees how much will be consumed in periods 2 and 3 as well as how long to work for a given saving level. This representative consumer is sophisticated in that he or she integrates this anticipation into the lifetime utility maximization problem in period 1

$$u'(c_1^S) = \beta\delta^2 u'(c_3^S)[\beta R^2 - (1-\beta)\frac{\partial c_3^S}{\partial c_1^S}], \tag{14}$$

and determines the consumption level c_1^S. In this case, time inconsistency is absent.

Similarly, I differentiate saving level and labor supply with respect to the present bias parameter β to compare the behavior under quasi-hyperbolic discounting with that under exponential discounting.

LEMMA 3 Suppose that the representative consumer is sophisticated. Then, in the setting specified in the previous section, stronger present bias (i.e., a smaller β) leads to less saving if and only if $\frac{d\{[\beta[\beta R^2 - (1-\beta)\frac{\partial c_3^S}{\partial c_1^S}]u'(c_3^S(\beta))\}}{d\beta} > 0$.

The proof of Lemma 3 is similar to that of Lemma 1. The only difference is that the necessary and sufficient condition in Lemma 3 for under-saving involves the anticipation of the next two periods. The present-biased marginal utility can thus be applied here, with an additional ***sophistication fraction***$[\beta R^2 - (1-\beta)\frac{\partial c_3^S(\beta)}{\partial c_1^S}]$. The sophisticated consumer foresees that increasing period-3 consumption c_3^S follows increasing saving level, and therefore he or she integrates this insight into consumption strategy in period 1 in order to induce an adverse effect.

Similarly to the naive consumer, if the present-biased marginal utility of period-3 consumption c_3^S with sophistication fraction decreases with stronger present bias, the representative consumer saves less than might the exponential discounter because he or she is too desires of current consumption and is less concerned about his or her future consumption.

LEMMA 4 Suppose that the representative consumer is sophisticated. Then, in the setting specified in the previous section, stronger present bias (i.e., a smaller β) leads to an earlier retirement if and only if $\frac{d[\beta u'(c_3^S(\beta))]}{d\beta} > 0$.

The necessary and sufficient condition for early retirement is also similar to that of the naive consumer. However, it should be noted that the labor supply levels of these two types of consumer are not necessarily identical.

The sophisticated consumer foresees his or her unwillingness to work in period 2 and incorporates it into peiod-1 plan. Meanwhile, he or she is also desires of consumption now. As a result, he or she has to sacrifices the consumption in period 3.

PROPOSITION 2 Suppose that the representative consumer is sophisticated. Then, if and only if $\frac{d\{[\beta[\beta R^2-(1-\beta)\frac{\partial c_3{}^S}{\partial c_1{}^S}]u'(c_3^S(\beta))\}}{d\beta}>0$ and $\frac{d[\beta u'(c_3^S(\beta))]}{d\beta}>0$, the quasi-hyperbolic discounter under-saves and retires too early.

Proposition 2 implies that for sophisticated consumers, if the present-biased marginal utility of period-3 consumption $c_3{}^S$ with (or without) the sophistication fraction decreases with stronger present bias, under-saving co-exists with early retirement. In this context, the consumer finds that more consumption now and less work in next period are attractive and eventually decides to save less and retire earlier than an exponential discounter because he or she cares his- or herself in period 3 less.

Two specific utility function forms are thus employed to show that the co-existence of under-saving and early retirement is allowed.

COROLLARY 3 Under CARA utility, the sophisticated consumer definitely under-saves; if and only if $w_2<\frac{B}{A}\cdot[\frac{1}{\beta R(R+1)(R-1)}-1]$, he or she retires too early.

COROLLARY 4 Under logarithmic utility, the sophisticated consumer definitely under-saves; if and only if $\beta^2\delta^2(\delta+2)>2$, he or she retires too early.

5. Government Policy

This section investigates the welfare effect of a proportional wage tax and an interest subsidy. In this partial equilibrium model, wage rate and gross interest rate are exogenously given, while interest makes saving more advantageous and a wage tax decreases income. The government's policy would thus considerably adjust consumption levels and labor supply.

Here, ex-ante welfare is evaluated at the point when the consumer is born. I evaluate the consumer's well-being from the long-run perspective, namely under exponential discounting:

$$W=u(c_1)+\delta u(c_2)+\delta^2u(c_3)-\delta e(l)\,. \tag{15}$$

Further, the utility function is specialized to logarithmic utility. This functional form assumption makes the model tractable and widely applicable.

The sophisticated consumer's behavior is different from that of the naive one in that incorporating foresight leads to time inconsistency disappearing. However, under logarithmic utility the naive consumer's consumption level and labor supply coincides with that of the sophisticated consumer. Hence, government policy has an identical effect on these two types of consumers, whether a proportional wage tax or an interest subsidy.

5.1 Proportional Wage Tax

Assume that the government imposes a proportional wage tax and gives a lump-sum transfer M in period 2. The wage tax rate is t ($0 < t < 1$). Therefore, the consumer's budget constrain is

$$c_3 = R^2 w - R^2 c_1 + Rwl(1-t) - Rc_2 + RM \,. \tag{16}$$

The budget constrain of the government is satisfied:

$$M = twl \,. \tag{17}$$

Accordingly, all consumers modify their first-order conditions regarding the proportional wage tax, whether naive or sophisticated. In period 2, labor is elastically supplied, while the proportional wage tax affects labor supply through two channels. First, it reduces income from working and hence decreases labor supply. Second, the representative consumer has to maintain consumption levels by increasing his or her labor supply. By contrast, labor is in-elastically supplied in period 1 and a lump-sum transfer would entirely counteract the effect caused by the wage tax in period 1, which has a negligible effect on welfare.

The government aims to optimize consumer welfare by selecting the tax rate properly. This effect is investigated by differentiating welfare with respect to the tax rate in the neighborhood of $t = 0$. Proposition 3 provides the necessary and sufficient condition for the government to improve consumer welfare by imposing wage tax.

PROPOSITION 3 In the setting of this model with logarithmic utility, for both naive and sophisticated consumer, the optimal proportional wage tax is positive if and only if $\beta < \dfrac{2}{\delta^2(2+\delta)}$.

The proof of Proposition 3 is provided in the Appendix.

Under logarithmic utility, the wage tax leads to a decrease in labor supply. Recalling from Corollary 2 and Corollary 4, we can conclude that when $\beta < \dfrac{2}{\delta^2(2+\delta)}$ the consumer retires later than the exponential discounter does. A positive wage tax mitigates later retirement and thus enhances consumer welfare.

5.2 Interest Subsidy

This sub-section considers the case in which the government subsidizes interest in period 2 at a rate r ($0 < r < 1$) and therefore a lump-sum transfer M occurs. The corresponding budget constraint of the consumer reads:

$$c_3 = (1+r)R^2 w - (1+r)R^2 c_1 + Rwl - Rc_2 - RM \,. \tag{18}$$

The government's budget constraint meets

$$M = rR(w - c_1)\,. \tag{19}$$

In this case, the government optimizes the consumer's long-run well-being by choosing the interest subsidy rate r. Proposition 4 states that the interest subsidy improves consumer welfare, whether consumers are naive or sophisticated.

PROPOSITION 4 In the setting of this model with logarithmic utility, for both naive and sophisticated consumers, the optimal interest subsidy is positive.

The proof of Proposition 4 is in the Appendix. Under logarithmic utility, interest subsidy increases saving level in period 1. Recall from Corollary 1 can Corollary 3, under logarithmic utility the consumer definitely under-saves. A positive interest subsidy mitigates under-saving and thus improves consumer welfare.

6. Discussion

This study investigates how the saving level and labor supply are affected by discounting style by differentiating with respect to the present bias parameter. It finds that under-saving can co-exist with early retirement and how the present-bias parameter β affects saving level and labor supply depends on whether a smaller present-biased marginal utility of period-3 consumption c_3 (or with the sophistication fraction) follows stronger present bias.

On one hand, the necessary and sufficient condition for under-saving has been shown. On the other hand, *over-saving* is also possible, in line with Diamond and Koszegi (2003) and Salanie and Treich (2006). As long as the present-biased marginal utility of period-3 consumption c_3 (or with sophistication fraction) increases with stronger present bias, over-saving by quasi-hyperbolic discounting consumer arises. In this circumstance, such consumers are more concerned about their future consumption more compared with the exponential discounters and they find that saving at a higher level can make up for the consumption in period 3.

Further, the possibility of both early retirement and late retirement coincides with the results presented by Diamond and Koszegi (2003). In Laibson et al. (1998) and Diamond and Koszegi (2003), consumers face a trade-off between consuming more or retiring early, although both options are desirable. In this paper, the representative consumer also faces this trade-off, but when the benefit of working is not sufficient, even the income received from retiring later cannot support his or her lifestyle.

Note that the concavity of the instantaneous utility function and the convexity of the disutility function matter when they determine the necessary and sufficient conditions for under-saving and early retirement. In particular, the convex disutility function of working describes a consumer who is exceedingly reluctant to work in period 2.

6.1 The Effect on the Long-run Phillips Curve

The foregoing sections investigate the saving level and labor supply of a representative quasi-hyperbolic discounter in comparison with an exponential discounter. This section investigates how these results affect the long-run Phillips curve relationship.

According to Graham and Snower (2008), inflation can affect the long-run Phillips curve by three channels:

(i) the employment cycling effect,
(ii) the labor smoothing effect,
(iii) the discounting effect.

As inflation increases, (i) labor demand reduces, (ii) labor supply reduces, and (iii) labor supply increases. The third effect dominates and thus there is a negative relation between inflation and unemployment in the long run.

However, in addition to this long-run trade-off relation between inflation and unemployment, a positive relation is found, namely stagflation. The results found in the foregoing sections support the conclusions of Graham and Snower (2008).

According to Lemma 2 and Lemma 4, whether the present bias of a consumer under quasi-hyperbolic discounting leads to more labor supply or less depends on whether the present-biased marginal utility of period-3 consumption c_3 increases with a stronger present bias parameter β. Therefore, the third channel that affects the long-run Phillips curve includes two aspects.

When later retirement arises, the discounting effect increases labor supply. Moreover, because this effect dominates other two effects, there is a long-run trade-off relation between inflation and unemployment. However, when stronger present bias leads to early retirement and hence the discounting effect decreases labor supply, as well as the employment cycling effect and labor smoothing effect, this generates a positive relation between long-run inflation and unemployment, which is stagflation.

7. Conclusion

This paper compares quasi-hyperbolic discounting with exponential discounting to explore saving and retirement behavior. In the quasi-hyperbolic discounting case, under-saving can co-exists with early retirement. However, this depends on whether the present-biased marginal utility of period-3 consumption decreases with stronger present bias. The consumer under quasi-hyperbolic discounting with the CARA (or logarithmic) utility function form definitely under-saves, but he or she can still earlier than might the exponential discounter.

Furthermore, By investigating the effects of a proportional wage tax and an interest subsidy, this paper shows that such policies can improve representative consumer welfare.

Because quasi-hyperbolic discounting can increase or decrease labor supply, the long-run Phillips curve relation can be expressed as a trade-off or stagflation between unemployment and inflation.

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Appendix

A.1 Proof of Lemma 1

LEMMA 1 Suppose that the representative consumer is naïve. Then, in the setting specified in the previous section, stronger present bias (i.e., a smaller β) leads to less saving if and only if $\dfrac{d[\beta u'(\tilde{c}_3^N(\beta))]}{d\beta} > 0$.

Proof: For a consumer under quasi-hyperbolic discounting, the consumption and labor supply plan he or she makes in period 1 is too optimistic. The actual consumption levels and labor supply in period 2 and 3 deviate from what he or she plans in period 1. How much is consumed in period 1 is based on

$$u'(c_1^N) = \beta\delta^2 R^2 u'(\tilde{c}_3^N). \tag{A.1}$$

Taking the total differential of (A.1) with respect to β leads to

$$\frac{du'(c_1^N)}{dc_1^N} \cdot \frac{dc_1^N}{d\beta} = \delta^2 R^2 \cdot \frac{d[\beta u'(\tilde{c}_3^N(\beta))]}{d\beta}. \tag{A.2}$$

According to Assumption 1, the concavity of the instantaneous utility function implies $\dfrac{du'(c_1^N)}{dc_1^N} < 0$. Because $s_1^N = w - c_1^N$, $\dfrac{ds_1^N}{d\beta}$ has the same sign as $\dfrac{d[\beta u'(\tilde{c}_3^N(\beta))]}{d\beta}$.

A.2 Proof of Proposition 3

PROPOSITION 3 In the setting of this model with logarithmic utility, for both naive and sophisticated consumers, the optimal proportional wage tax is positive if and only if $\beta < \dfrac{2}{\delta^2(2+\delta)}$.

Proof: Because the naive consumer's consumption levels and labor supply coincides with that of the sophisticated one's, I only consider the naive consumer here. For the naive consumer under a proportional wage tax, the first-order conditions imply that in period 1:

$$u'(c_1{}^N) = \beta\delta^2 R^2 u'(\tilde{c}_3{}^N)\,, \tag{A.3}$$

$$u'(\tilde{c}_2{}^N) = \delta R u'(\tilde{c}_3{}^N)\,, \tag{A.4}$$

$$e'(\tilde{l}^N) = \delta R w(1-t) u'(\tilde{c}_3{}^N)\,; \tag{A.5}$$

Further in period 2:

$$u'(c_2{}^N) = \beta\delta R u'(c_3{}^N)\,, \tag{A.6}$$

$$e'(l^N) = \beta\delta R w(1-t) u'(c_3{}^N)\,. \tag{A.7}$$

The first order conditions (A.3), (A.4), (A.5), (A.6), and (A.7) and the consumer's budget constraint (16) jointly determine the consumption levels $c_1{}^N, c_2{}^N$, and $c_3{}^N$ as well as labor supply l^N.

I take the derivative of welfare with respect to the wage tax rate t in the neighborhood of $t = 0$ in order to find the optimal wage tax policy:

$$\left.\frac{dW}{dt}\right|_{t\to 0} = u'(c_1^N)\left.\frac{dc_1^N}{dt}\right|_{t\to 0} + \delta u'(c_2^N)\left.\frac{dc_2^N}{dt}\right|_{t\to 0} + \delta^2 u'(c_3^N)\left.\frac{dc_3^N}{dt}\right|_{t\to 0} - \delta e'(l^N)\left.\frac{dl^N}{dt}\right|_{t\to 0}\,. \tag{A.8}$$

Substituting these first-order conditions and taking the derivatives with respect to the wage tax rate t leads to:

$$\left.\frac{dW}{dt}\right|_{t\to 0} = \frac{\beta^2\delta^3(\delta+2)Rw(1-\beta)}{(2+\beta\delta)^2(1+2\beta\delta+\beta\delta^2)^2} u'(c_3^N)[2-\beta\delta^2(2+\delta)]\,. \tag{A.9}$$

$\left.\frac{dW}{dt}\right|_{t\to 0} > 0$ implies $\beta < \frac{2}{\delta^2(2+\delta)}$.

Therefore the optimal wage tax of the naive consumer is positive if and only if $\beta < \frac{2}{\delta^2(2+\delta)}$.

A.3 Proof of Proposition 4

PROPOSITION 4 In the setting of this model with logarithmic utility, for both naive and sophisticated consumer, the optimal interest subsidy is positive.

Proof: Because the naive consumer's consumption levels and labor supply coincides with that of the sophisticated one's, I only consider the naive consumer here. The first-order conditions of the naive consumer in period 1:

$$u'(c_1^N) = \beta\delta^2(1+r)R^2u'(\tilde{c}_3^N)\,, \tag{A.10}$$

$$u'(\tilde{c}_2^N) = \delta Ru'(\tilde{c}_3^N)\,, \tag{A.11}$$

$$e'(\tilde{l}^N) = \delta Rwu'(\tilde{c}_3^N)\,; \tag{A.12}$$

those in period 2:

$$u'(c_2^N) = \beta\delta Ru'(c_3^N)\,, \tag{A.13}$$

$$e'(l^N) = \beta\delta Rwu'(c_3^N)\,. \tag{A.14}$$

The first-order conditions (A.10), (A.11), (A.12), (A.13), and (A.14) and the consumer's budget constraint (18) jointly determine the consumption levels c_1^N, c_2^N, and c_3^N as well as labor supply l^N.

I take the derivative of welfare with respect to the interest subsidy rate r in the neighborhood of $r = 0$ in order to find the optimal interest subsidy policy:

$$\left.\frac{dW}{dr}\right|_{r\to 0} = u'(c_1^N)\left.\frac{dc_1^N}{dr}\right|_{r\to 0} + \delta u'(c_2^N)\left.\frac{dc_2^N}{dr}\right|_{r\to 0} + \delta^2 u'(c_3^N)\left.\frac{dc_3^N}{dr}\right|_{r\to 0} - \delta e'(l^N)\left.\frac{dl^N}{dr}\right|_{r\to 0}\,, \tag{A.15}$$

Substituting these first-order conditions and taking the derivatives with respect to the interest subsidy rate r leads to:

$$\left.\frac{dW}{dr}\right|_{r\to 0} = \frac{3-\beta\delta(2+\delta)}{1+2\beta\delta+\beta\delta^2}\,, \tag{A.16}$$

which is definitely positive because $0 < \beta < 1$ and $0 < \delta < 1$.

Therefore, the optimal interest subsidy is positive.

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