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Inequality, Growth and the Politics of Education and Redistribution

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Abstract

This paper analyzes the political economy of public education and redistribution in an overlapping-generation model of a two-class society in which growth is driven by the accumulation of human capital. The levels of public education and lump-sum financial transfers are determined by voting, while private education which supplements public education is purchased individually. The model, which includes two-dimensional voting, demonstrates multiple steady-state political equilibria. One is an equilibrium with a high share of public education in government expenditure; the other is an equilibrium with a high share of lump-sum transfers. Numerical analysis shows empirically plausible result of growth, inequality and the composition of redistributive expenditures.

- JEL Classification Number: D72, D91, I24

  Key words: Education, political economy, inequality, growth

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1 Introduction

In most countries redistribution is carried out through in-kind transfer programs such as public education for successive generations, as well as through in-cash transfer programs such as social security and welfare budgets within the current generation. The size and the composition of redistributive policies in democratic countries are determined via voting, and redistributive expenditures affect growth and income distribution, which in turn have an effect on voting over in-cash and in-kind transfer programs. Therefore, it is natural to expect some correlation among inequality, growth and size with the composition of redistributive expenditures.

For the past decades, there has been an increasing amount of literature on the political economy of growth, inequality and redistribution (see, for example, Persson and Tabellini (2000) for a survey). However, most of these focus on either redistribution in cash (Saint-Paul and Verdier, 1996; Benabou, 1996; Bertola, 1993; Alesina and Rodrik, 1994; Persson and Tabellini, 1994; Perotti, 1993) or redistribution in kind through public education (Glomm and Ravikumar, 1992, 1996, 2001; Glomm, 2004; Gradstein and Justman, 1996, 1997). There are few studies that focus on both types of redistribution in the context of growth and inequality.

Exceptions are Creedy, Li and Moslehi (2011) and Bernasconi and Profeta (2012). Creedy, Li and Moslehi (2011) focus on financial transfer and pure public goods provision, and consider voting over the composition of government expenditure in an overlapping-generation model. They successfully demonstrate the endogenous determination of the composition of redistributive spending. However, they show nothing about how the size of government spending is determined via voting because the tax rate is taken as exogenous to ensure voting over one dimension. In addition, there is no link between growth and redistribution because income is assumed to grow exogenously in their framework.

Bernasconi and Profeta (2012) overcome the two limitations of Creedy, Li and Moslehi (2011). They develop a politico-economic model of a two-class society with human capital accumulation and consider probabilistic voting over public education (i.e., redistribution in kind) and lump-sum transfer (i.e., redistribution in-cash). However, private education as an alternative to public education is abstracted away from their analysis because their focus is on the role of public education as a device to provide for poor-born children to be recognized for their talent. Therefore, the following questions still remain unresolved: how the politics of public education and lump-sum transfer affect economic decisions over private education of agents; how the decisions in turn affect the inequality among agents and their preferences over the size and the composition of redistributive expenditures; what is the long-run consequence of this interaction for growth and inequality.

In order to answer to these questions, this paper utilizes the framework in which

Gradstein and Justman (1996) investigate the role of private education as an alternative to public education. This model differs from that of Gradstein and Justman (1996) in that Gradstein and Justman (1996) consider public education as an only means of redistribution, while this model allows for lump-sum transfer as an alternative to public education. The presence of lump-sum transfer might give some agents an incentive to prefer lump-sum transfer to public education, and to make use of the transfer benefits for private education for their children. A model which includes this incentive enables us to answer the abovementioned questions.

For the purpose of analysis, this paper develops a two-period overlapping-generation model based on that by Gradstein and Justman (1996). There are two types of family dynasties classified according to their endowed levels of human capital: low and high. An agent in each type of family enters adulthood with a stock of human capital invested by his/her parents, earns after-tax labor income, and receives lump-sum transfer benefits from the government. He/she decides the allocation of disposable income between current consumption and private investment in his/her child's further education. The private educational investment combined with public education determines his/her child's human capital level.

Every adult agent votes over the tax rate as well as the allocation of tax revenue between public education and lump-sum transfer. Given the bidimensional issue space, the Nash equilibrium of a majority voting game may fail to exist. To deal with this feature, we use the concept of issue-by-issue voting; that is, notion of structure-induced Nash equilibrium voting game formalized by Shepsle (1979) and applied by Conde-Ruiz and Galasso (2003, 2005) for the framework of overlapping generations.

Voting may result in multiple political equilibria. One is the equilibrium where the tax revenue is fully utilized for lump-sum transfer payments; no public education is provided. Both high and low types of families privately invest in education to complement the lack of public education. The other is the equilibrium where all the tax revenue is spent on public education. Low-type families leave their children in the hands of public education and make no private investments in their children. However, high-type families spend a part of their income on private educational investment to further improve their children's human capital level. The result suggests that the interaction between economic and political decisions differ between the two equilibria.

The characterization of the political equilibria has the following two features. First, the tax revenues are exclusively used for either public education or lump-sum transfer. These extreme usage patterns come from the simplified assumption of the utility function and the human capital formation, both of which enable us to solve the model analytically. Second, multiple political equilibria arise due to bidimensional voting. If we instead consider one dimensional voting as in Gradstein and Justman (1996), we find that multiple equilibria do not arise.

Based on the characterization of the political equilibria, we undertake numerical analysis to demonstrate the long-term consequences of political equilibria in terms of growth and inequality. In particular, we focus on the case in which public education is less efficient than private education because the former provides standardized, rather than individualized, education to each child. In this situation, we find that an equilibrium which exclusively uses tax revenue for public education attains a higher inequality level than an equilibrium which exclusively uses tax revenue for lump-sum transfer. This result is consistent with the empirical evidence in OECD countries: higher inequality is associated with a larger share of public education in redistributive spending (see Panel (a) of Figure 1). The result also applies to the evidence demonstrated in Glomm (2004) where the set of countries is expanded to low and middle-income countries.

The evidence in OECD countries also suggests that higher inequality is associated with a lower growth rate (see Panel (b) of Figure 1). However, the result is not robust as indicated by Persson and Tabellini (1994) and Alesina and Rodrik (1994). In fact, the panel (b) of Figure 1 indicates a positive correlation between growth and inequality in some groups of countries. The numerical analysis can demonstrate this mixed evidence in terms of growth and inequality by controlling the degree of efficiency of public education. In other words, the degree of efficiency of public education is a key to explain the relationship between growth and inequality.

The organization of this paper is as follows. In section 2, we set up the model and characterize economic equilibrium. Section 3 considers voting behavior of agents and characterizes political equilibria in each period. Section 4 demonstrates an equilibrium path of the measure of inequality over time, and shows the existence and multiplicity of steady-state equilibria. Section 5 investigates how the efficiency of public education affects the steady-state growth rates and inequality levels. Section 6 provides concluding remarks.

2 The Model and Economic Equilibrium

We consider a discrete-time overlapping-generation economy that starts at time 0. The economy is populated by two types of family dynasties, indexed by $i \in \{L, H\}$, of agents whose lives consist of two periods, youth and adult age. A type-i adult agent in period 0 is endowed with h_0^i units of human capital where $0 < h_0^L < h_0^H$. Thus, period-0 type-L and type-H agents are endowed with low and high human capital, respectively.

Each adult agent produces one offspring, hence the population remains constant in every generation. A fraction of type-i agents within each generation is given by $\phi^i \in (0, 1)$

where ϕ^i is constant across generations and satisfies $0 < \phi^H < 0.5 < \phi^L < 1$ with $\sum_i \phi^i = 1$. The assumption implies that in every period, type-L agents are the majority in the economy. This assumption reflects the right-skewed income distribution in the real world.

A type-i adult agent at time t is endowed at the time he/she enters the adulthood with the stock of human capital h_t^i which also defines his/her effective labor capacity. He/she receives lump-sum transfer from the government, b_t. Given the income tax τ_t and the transfer b_t, a type-i adult decides the allocation of disposable income between current consumption, c_t^i, and private investment in his/her child's further education, z_t^i, subject to the budget constraint:

$$c_t^i + z_t^i \leq (1 - \tau_t)h_t^i + b_t.$$

A type-i adult of generation t derives utility from current consumption, c_t^i, and from his/her child's anticipated future income, h_{t+1}^i. A type$-i$'s preferences are specified by the following utility function:

$$u_t^i = (1 - \delta) \ln c_t^i + \delta \ln h_{t+1}^i,$$

where $\delta \in (0, 1)$ is a common parameter reflecting the bequest motive. A higher δ implies a greater incentive for educational investment. We employ logarithmic utility function for the tractability of analysis.

The level of offspring's education, h_{t+1}^i, is determined by public schooling, e_t, as well as by privately purchased supplementary education, z_t^i. Following Gradstein and Justman (1996), we assume that the individual level of education is determined by the following linear equation:

$$h_{t+1}^i = A^i \cdot \left\{(1 - \gamma) \cdot e_t + z_t^i\right\}.$$

The parameter $A^i (> 0)$ represents a durable productive asset handed from generation to generation such as genetic ability or cultural capital. The distribution of A^i is assumed to be stationary over time and to be positively correlated to human capital, h_t^i:

$$A^H = A > 0 \text{ and } A^L = \alpha A \text{ where } \alpha \in (0, 1).$$

This assumption implies that in average, children born in higher-income families are endowed with higher genetic ability or a higher level of cultural capital.[1] The parameter $\gamma \in [0, 1)$ implies that public education may be less efficient than private education

[1] A possible extension is to assume that children have the same genetic ability with a probability q. For example, children born in higher-income families have high genetic ability, A^H, with a probability q, while they have low genetic ability, A^L, with a probability $1 - q$. Bernasconi and Profeta (2012) assume that this genetic probability of talent transmission, q, is not generally known in public, thereby resulting in the talent mismatch. The current paper abstracts away the talent transmission and mismatch; instead, it focuses on the interaction between public and private education choice.

because the former provides standardized, rather than individualized, education to each child. The role of γ will be further investigated in Section 5.

In each period, the government raises tax revenue to finance the provision of uniform public schooling for all children, e_t, as well as lump-sum transfer, b_t. The fraction $\lambda_t \in [0, 1]$ of the tax revenue is devoted to lump-sum transfer; and the rest is devoted to public schooling. Thus, the government budget constraint is given by:

$$
\begin{aligned}
b_t &= \lambda_t (1 - \tau_t) \tau_t \bar{h}_t; \\
e_t &= (1 - \lambda_t)(1 - \tau_t) \tau_t \bar{h}_t;
\end{aligned}
$$

where the term $\bar{h}_t$ is the average human capital in period t, which is equivalent to the aggregate income in that period. The term $(1 - \tau_t)$ denotes the distortionary factor that represents efficiency loss of taxation. This assumption is solely to ensure an interior solution to preferred tax rates and otherwise plays no role.

The timing of events in period t is as follows. First, adult agents vote on the tax rate τ_t as well as the fraction of tax revenue devoted to lump-sum transfer λ_t by majority vote. Second, each agent decides on the allocation of disposable income between consumption and private education subject to the budget constraint. We solve the model by backward induction.

2.1 Economic Equilibrium

Given a sequence of tax rates and the sizes of redistribution and public education, $\{\tau_t, b_t, e_t\}_{t=0}^{\infty}$, an *economic equilibrium* is a sequence of allocations, $\{z_t^i, c_t^i, h_t^i\}_{i=L,H}^{t=0,\ldots,\infty}$ with the initial condition h_0^i $(i = L, H)$, such that (i) in every period, a type-i agent maximizes his/her utility subject to the budget constraint and the non-negativity constraint of investment in private education and (ii) the government budget is balanced in every period.

Solving the utility maximization problem of a type-i agent leads to the following private education decision:

$$
z_t^i = \max \left\{ 0, \delta \cdot \left[(1 - \tau_t) h_t^i + b_t - \frac{1 - \delta}{\delta} (1 - \gamma) e_t \right] \right\}. \tag{1}
$$

Eq. (1) indicates that the investment decision depends on an adult's human capital h_t^i as well as government policy variables, τ_t, b_t and e_t. In particular, an agent chooses to invest privately in education if his/her human capital is high, the tax rate is low, the size of redistribution is large and/or the level of public education is low; otherwise, he/she chooses no private investment in education and consumes all of his/her disposable income. Therefore, the consumption function is given by:

$$c_t^i = \min\left\{(1-\tau_t)h_t^i + b_t, (1-\delta)\cdot\left[(1-\tau_t)h_t^i + b_t + (1-\gamma)e_t\right]\right\}.$$

The utility obtained by agents in economic equilibrium is represented by their indirect utility functions. We use the abovementioned investment and consumption functions to obtain an indirect utility function of a type-i agent:

$$V_t^i = \begin{cases} V_{t,z>0}^i \equiv \ln\left[(1-\tau_t)h_t^i + b_t + (1-\gamma)e_t\right] + (1-\delta)\ln(1-\delta) + \delta\ln\delta \\ \qquad\qquad \text{if } (1-\tau_t)h_t^i + b_t > \frac{1-\delta}{\delta}(1-\gamma)e_t; \\ V_{t,z=0}^i \equiv (1-\delta)\ln\left[(1-\tau_t)h_t^i + b_t\right] + \delta\ln e_t + \delta\ln(1-\gamma) \\ \qquad\qquad \text{if } (1-\tau_t)h_t^i + b_t \le \frac{1-\delta}{\delta}(1-\gamma)e_t. \end{cases}$$

$V_{t,z>0}^i$ denotes the indirect utility of a type-i agent when he/she invests some portion of his/her income in private education, and $V_{t,z=0}^i$ denotes the indirect utility when he/she invests nothing in education.

With the use of the government budget constraints $b_t = \lambda_t(1-\tau_t)\tau_t\bar{h}_t$ and $e_t = (1-\lambda_t)(1-\tau_t)\tau_t\bar{h}_t$, the condition that determines investment decisions is rewritten as follows:

$$z_t^i > 0 \Leftrightarrow \lambda_t > \frac{\frac{1-\delta}{\delta}(1-\gamma)}{1+\frac{1-\delta}{\delta}(1-\gamma)}\cdot\left(1 - \frac{1}{\tau_t}\cdot\frac{h_t^i}{\bar{h}_t}\cdot\frac{1}{\frac{1-\delta}{\delta}(1-\gamma)}\right). \tag{2}$$

This condition states that a lower tax rate and a higher share of redistribution in government expenditure produce a larger income effect, thereby giving an agent an incentive to invest in education.

With the condition (2) in mind, we can write the indirect utility function in terms of the tax rate τ_t and the fraction λ_t as follows:

$$V_t^i = \begin{cases} V_{t,z>0}^i \equiv \ln(1-\tau_t) + \ln\left[h_t^i/\bar{h}_t + \{(1-\gamma)+\gamma\lambda_t\}\cdot\tau_t\right] \\ \qquad + \ln\bar{h}_t + (1-\delta)\ln(1-\delta) + \delta\ln\delta \\ \qquad\qquad \text{if } \lambda_t > \frac{\frac{1-\delta}{\delta}(1-\gamma)}{1+\frac{1-\delta}{\delta}(1-\gamma)}\cdot\left(1-\frac{h_t^i}{\tau_t\bar{h}_t}\cdot\frac{1}{\frac{1-\delta}{\delta}(1-\gamma)}\right); \\ V_{t,z=0}^i \equiv \ln(1-\tau_t) + (1-\delta)\ln\left[h_t^i/\bar{h}_t + \lambda_t\tau_t\right] + \delta\ln(1-\lambda_t) + \delta\ln\tau_t \\ \qquad + \ln\bar{h}_t + \delta\ln(1-\gamma) \\ \qquad\qquad \text{if } \lambda_t \le \frac{\frac{1-\delta}{\delta}(1-\gamma)}{1+\frac{1-\delta}{\delta}(1-\gamma)}\cdot\left(1-\frac{h_t^i}{\tau_t\bar{h}_t}\cdot\frac{1}{\frac{1-\delta}{\delta}(1-\gamma)}\right). \end{cases} \tag{3}$$

3 Period$-t$ Political Equilibrium

In each period t, the tax rate τ_t and the proportion λ_t are determined by period-t adult agents through a political process of majority voting. Type-L and type-H adult agents cast a ballot over τ_t, the income tax rate, and λ_t, the share of lump-sum transfer in government expenditure. Individual preferences over the two issues are represented by the indirect utility function in (3) for $i = L, H$. Every agent has zero mass and thus no individual vote can change the outcome of the election. Thus, we assume agents vote sincerely.

The current majority voting game is characterized by a bidimensional issue space, τ and λ. Thus, a Nash equilibrium may not exist within the majority voting game. To deal with this characteristic, we use the concept of issue-by-issue voting, or the structure-induced Nash equilibrium, as formalized by Shepsle (1979) and applied by Conde-Ruiz and Galasso (2003, 2005) to the framework of overlapping generations. Under the concept of the structure-induced Nash equilibrium, a sufficient condition for (τ_t^*, λ_t^*) to be a *period-t political equilibrium* of the voting game is that τ_t^* represents the outcome of majority voting over τ_t when the other dimension is fixed at its level λ_t^*, and vice versa, provided that preferences are single peaked along every dimension of the issue space.

Under the current framework, type-L agents are the majority for each issue; and the preferences of type-L agents, specified in Eq. (3), are singled-peaked for each issue. We can apply the concept of the structure-induced Nash equilibrium to the current framework. Let $\lambda_t^L(\tau)$ denote type-L's most preferred share as a function of the tax rate τ_t, and let $\tau_t^L(\lambda)$ denote type-L's most preferred tax rate as a function of λ_t. The point where these two reaction functions cross corresponds to the structure-induced Nash equilibrium outcome of the voting game.

In the following analysis, we focus on the low-to-mean ratio of income as the measure of inequality:

$$\rho_t \equiv h_t^L/\bar{h}_t \in (0, 1).$$

This captures the extent of income inequality in the economy; a higher ρ implies less inequality. Given the assumption that type-L agents are the majority for each issue, we investigate two sorts of equilibria, an equilibrium with $z_t^L > 0$ and an equilibrium with $z_t^L = 0$, respectively.

3.1 $z_t^L > 0$ equilibrium

Suppose that the type-L agent invests a part of his/her income in education. The condition of $z_t^L > 0$ in (2) is rewritten in terms of ρ_t:

$$z_t^L > 0 \Leftrightarrow \lambda_t > \frac{\frac{1-\delta}{\delta}(1-\gamma)}{1+\frac{1-\delta}{\delta}(1-\gamma)} \cdot \left(1 - \frac{\rho_t}{\tau_t} \cdot \frac{1}{\frac{1-\delta}{\delta}(1-\gamma)}\right). \tag{4}$$

Under the condition (4), the type-L agent, as a decisive voter, chooses τ_t to maximize his/her indirect utility $V_{t,z>0}^L$.

The first derivative of $V_{t,z>0}^L$ with respect to τ_t is:

$$\frac{\partial V_{t,z>0}^L}{\partial \tau_t} = \frac{-1}{1-\tau_t} + \frac{(1-\gamma)+\gamma\lambda_t}{\rho_t + \{(1-\gamma)+\gamma\lambda_t\} \cdot \tau_t}.$$

The first term on the right-hand side shows the marginal cost of taxation; the second term shows the marginal benefit of taxation. The above equation indicates that the marginal cost is independent of the share of redistribution in government expenditure, λ_t, while the marginal benefit is increasing in λ_t. Therefore, there is a critical value of λ_t, and type$-L$ agents will find it optimal to owe no tax burden when λ_t is below this critical value, that is, $\lambda_t \leq (\rho_t - (1-\gamma))/\gamma$, while they find it optimal to owe some tax burden when $\lambda_t > (\rho_t - (1-\gamma))/\gamma$. The optimal choice of τ_t by type-L agents is summarized as:

$$\tau_t = \tau^L_{z>0}(\lambda_t) \Leftrightarrow \begin{cases} \tau_t = 0 & \text{if } \lambda_t \leq \frac{1}{\gamma} \cdot (\rho_t - (1-\gamma)); \\ \lambda_t = \frac{1}{\gamma} \cdot \left[\frac{\rho_t}{1-2\tau_t} - (1-\gamma)\right] & \text{if } \lambda_t > \frac{1}{\gamma} \cdot (\rho_t - (1-\gamma)). \end{cases} \tag{5}$$

Next, consider the choice of λ_t by type$-L$ agents with $z^L_t > 0$. A marginal increase in λ_t results in an increase of redistribution by one unit whereas it results in a decrease in public education by $1-\gamma$ units. The net benefits are positive: $1-(1-\gamma) > 0$. Therefore, they always choose $\lambda_t = 1$:

$$\lambda_t = \lambda^L_{z>0}(\tau_t) = 1 \forall \tau_t \in [0,1].$$

The period$-t$ political equilibrium when $z^L_t > 0$ is the point where the two reaction functions, $\tau_t = \tau^L_{z>0}(\lambda_t)$ and $\lambda_t = \lambda^L_{z>0}(\tau_t)$, cross (see Figure 2). The period-t equilibrium policy when $z^L_t > 0$ is calculated as:

$$(\tau_t, \lambda_t) = \left(\frac{1}{2}(1-\rho_t), 1\right).$$

We substitute this solution into the condition of $z^L_t > 0$ in (4) and find that the condition folds for any $\rho_t \in [0,1]$. We also find that $z^H_t > 0$ in the current equilibrium because $z^H_t > z^L_t$ holds.

[Figure 2 about here.]

- **Proposition 1.** *There exists a period-t political equilibrium of the voting game with $z^L_t > 0$ and $z^H_t > 0$ such that $(\tau_t, \lambda_t) = \left(\frac{1}{2}(1-\rho_t), 1\right) \forall \rho_t \in [0,1]$.*

Two remarks are in order. First, type$-L$ agents prefer a higher tax rate as ρ_t becomes lower, that is, as their income becomes lower compared to the average income. Second, for $\rho_t \in [0,1]$, there always exists a voting equilibrium where type$-L$ agents invest privately in education and prefer no public education provision. In other words, they prefer redistribution to public education regardless of whether they are endowed with high income or not. However, there is also the equilibrium where type$-L$ agents do not invest privately in education and prefer public education to redistribution when their income is below the critical value, which will be demonstrated in the next subsection.

3.2 $z_t^L = 0$ equilibrium

Suppose that the type-L agent invests nothing in education: $z_t^L = 0$. The condition of $z_t^L = 0$ in terms of ρ_t is given by:

$$z_t^L = 0 \Leftrightarrow \lambda_t \leq \frac{\frac{1-\delta}{\delta}(1-\gamma)}{1+\frac{1-\delta}{\delta}(1-\gamma)} \cdot \left(1 - \frac{\rho_t}{\tau_t} \cdot \frac{1}{\frac{1-\delta}{\delta}(1-\gamma)}\right). \tag{6}$$

Under the condition (6), the type-L agent, as a decisive voter, chooses τ_t to maximize his/her indirect utility $V_{t,z=0}^L$.

The first derivative of $V_{t,z=0}^L$ with respect to τ_t is:

$$\frac{\partial V_{t,z=0}^L}{\partial \tau_t} = \frac{-1}{1-\tau_t} + (1-\delta) \cdot \frac{\lambda_t}{\rho_t + \lambda_t} + \frac{\delta}{\tau_t}.$$

The first term on the right-hand side shows the marginal cost of taxation; the second term shows the marginal benefit of taxation via redistribution; and the third terms shows the marginal benefit of taxation via public education.

Corner solutions, $\tau_t = 0$ and 1, are not optimal for type$-L$ agents because $\partial V_{t,z=0}^L/\partial \tau_t\big|_{\tau_t=0} = +\infty > 0$ and $\partial V_{t,z=0}^L/\partial \tau_t\big|_{\tau_t=1} = -\infty < 0$ hold. Thus, the optimal solution satisfies $\partial V_{t,z=0}^L/\partial \tau_t = 0$, which results in:

$$\tau_t = \tau_{z=0}^L(\lambda_t) \Leftrightarrow \lambda_t = \frac{(1+\delta)\rho_t\left(1 - \frac{1}{\tau_t}\cdot\frac{\delta}{1+\delta}\right)}{1-2\tau_t}.$$

Next, consider the choice of λ_t by the type$-L$ agents when they invest nothing in education, $z_t^L = 0$. The first derivative of $V_{t,z=0}^L$ with respect to λ_t is:

$$\frac{\partial V_{t,z=0}^L}{\partial \lambda_t} = (1-\delta) \cdot \frac{\tau_t}{\rho_t + \lambda_t \tau_t} - \frac{\delta}{1-\lambda_t}. \tag{7}$$

The first term on the right-hand side shows the marginal benefit from an increase in redistribution; and the second term shows the marginal cost from a decrease in spending on public education.

When $\rho_t > (1-\delta)(1-\gamma)/(1+\delta)$, the latter effect overcomes the former one; type$-L$ agents prefer no redistribution, $\lambda_t = 0$. However, when $\rho_t \leq (1-\delta)(1-\gamma)/(1+\delta)$, the two opposing effects are offset at some level of $\lambda_t \in (0,1)$. In this case, the optimal share satisfies $\partial V_{t,z=0}^L/\partial \lambda_t = 0$, i.e., $\lambda_t = (1-\delta) - \delta\rho_t/\tau_t$. Therefore, the preferred share λ_t for type$-L$ agents is summarized as

$$\lambda_t = \lambda_{z=0}^L(\tau_t) \equiv \max\left\{0, (1-\delta) - \frac{\delta\rho_t}{\tau_t}\right\}.$$

Figure 3 illustrates the reaction functions $\tau_t = \tau_{z=0}^L(\lambda_t)$ and $\lambda_t = \lambda_{z=0}^L(\tau_t)$.

[Figure 3 about here.]

The crossing points of the two reaction functions may correspond to the period-t structure-induced Nash equilibrium of the voting game. Figure 3 demonstrates that there are two possible solutions: one characterized by no redistribution, and the other characterized by some redistribution. They are given by:

$$(\tau_t, \lambda_t) = \left(\frac{\delta}{1+\delta}, 0\right) \text{ and } \left(\hat{\tau}_t, \hat{\lambda}_t\right) \text{ where } \hat{\lambda}_t > 0.$$

The former solution $(\delta/(1+\delta), 0)$ satisfies the $z_t^L = 0$ condition if $\rho_t \leq (1-\delta)(1-\gamma)/(1+\delta)$; the latter solution $\left(\hat{\tau}_t, \hat{\lambda}_t\right)$ does not satisfy the $z_t^L = 0$ condition.[2] Therefore, the solution is limited to $(\tau_t, \lambda_t) = (\delta/(1+\delta), 0)$ when $z_t^L = 0$.

The solution $(\tau_t, \lambda_t) = (\delta/(1+\delta), 0)$ is feasible if the low-to-mean income ratio, ρ_t, is low such that $\rho_t \leq (1-\delta)(1-\gamma)/(1+\delta)$. Because of a low ratio, the marginal benefit of redistribution is always greater than the marginal cost of redistribution as demonstrated in the first term on the right-hand side in Eq. (7). Type$-L$ agents find it optimal to set $\lambda = 0$, that is, no redistribution, from the viewpoint of their utility maximization if $\rho_t \leq (1-\delta)(1-\gamma)/(1+\delta)$ holds.

The absence of income redistribution has a negative impact on private education. Due to this effect, type$-L$ agents cannot afford to undertake private investment in education. However, type$-H$ agents can still undertake private educational investment because they are endowed with high income.[3]

The results established so far are summarized as in the following proposition.

- **Proposition 2.** *There exists a period-t political equilibrium of the voting game with $z_t^L = 0$ and $z_t^H > 0$ such that $(\tau_t, \lambda_t) = \left(\frac{\delta}{1+\delta}, 0\right) \forall \rho_t \in [0, (1-\delta)(1-\gamma)/(1+\delta)]$.*

The results established in Propositions 1 and 2 imply that the tax revenues are exclusively used for either public education or lump-sum transfer in the political equilibria. The extreme usage patterns come from the simplified assumption of the utility function and the human capital formation. The results also imply that there are multiple period$-t$

[2]Suppose that the solution $\left(\hat{\tau}_t, \hat{\lambda}_t\right)$ satisfies the $z^L = 0$ condition in (6). We substitute $\hat{\lambda}_t = (1-\delta) - \delta\rho_t/\tau_t$ into (6) and obtain $\gamma \cdot (1 + \rho_t/\tau_t) \leq 0$, which is a contradiction.

[3]The proof of $z_t^H > 0$ is as follows. We substitute $(\tau_t, \lambda_t) = (\delta/(1+\delta), 0)$ into the condition $z_t^H > 0$ in (3) to obtain:

$$0 > \frac{\frac{1-\delta}{\delta}(1-\gamma)}{1 + \frac{1-\delta}{\delta}(1-\gamma)} \cdot \left(1 - \frac{1}{1} \cdot \frac{y_t^H}{\bar{y}_t} \cdot \frac{1}{\frac{1-\delta}{\delta}(1-\gamma)}\right),$$

that is,

$$\frac{1-\delta}{1+\delta} \cdot (1-\gamma) \cdot \bar{y}_t < y_t^H.$$

This condition holds for any $\bar{y}_t$ and $y_t^H (> \bar{y}_t)$.

political equilibria if $\rho_t \in [0, (1-\delta)(1-\gamma)/(1+\delta)]$. The multiplicity arises because of the bidimensional voting.

If instead we consider one dimensional voting, we find that the multiple period$-t$ political equilibria never arise. To confirm this, let us consider voting over the tax rate provided that the share of redistribution λ is exogenously given at a certain level. Eq. (5) indicates that the tax rate is uniquely determined via voting at $\tau_t = 0$ if $\rho_t \geq \gamma\lambda_t + (1-\gamma)$; it is uniquely determined at $\tau_t = (1/2) \cdot \{1 - \rho_t / (\gamma\lambda_t + (1-\gamma))\}$ if $\rho_t < \gamma\lambda_t + (1-\gamma)$. The two ranges of ρ_t do not overlap when λ_t is fixed. However, they overlap for the range of $\rho_t \leq (1-\delta)(1-\gamma)/(1+\delta)$ when λ_t is chosen endogenously via voting. Therefore, bidimensional voting is a key to the multiple equilibria.

4 Dynamic Equilibrium

Given the characterization of the period$-t$ political equilibrium in the previous section, we are now ready to consider the *dynamic equilibrium* which presents the motion of ρ_t over time. Following the results in Propositions 1 and 2, we first derive separately the dynamic paths when $z^L > 0$ and $z^L = 0$. After that, we combine two cases and derive the condition for the existence of the steady-state equilibrium where ρ_t is stationary over time.

First, suppose that the type$-L$ agents privately invest in education: $z^L > 0$. The period$-t$ equilibrium policy is $(\tau_t, \lambda_t) = ((1-\rho_t)/2, 1)$; and both types of agents invest privately in education, $z^L > 0$ and $z^H > 0$ (Proposition 1). Under this situation, the next-period income levels of the both types of agents are:

$$
\begin{aligned}
h_{t+1}^L &= \alpha A\delta \cdot (1-\tau_t) \cdot \left[h_t^L + \{\lambda_t\tau_t + (1-\gamma)(1-\lambda_t)\cdot\tau_t\} \cdot \bar{h}_t\right], \\
h_{t+1}^H &= A\delta \cdot (1-\tau_t) \cdot \left[h_t^H + \{\lambda_t\tau_t + (1-\gamma)(1-\lambda_t)\cdot\tau_t\} \cdot \bar{h}_t\right],
\end{aligned}
$$

respectively; and the mean income level is:

$$
\begin{aligned}
\bar{h}_{t+1} &= \phi^L h_{t+1}^L + \phi^H h_{t+1}^H \\
&= A\delta \cdot (1-\tau_t) \cdot \left[\bar{h}_t + \{\lambda_t\tau_t + (1-\gamma)(1-\lambda_t)\cdot\tau_t\} \cdot \bar{h}_t\right] \\
&\quad - (1-\alpha) \cdot \phi^L A\delta \cdot (1-\tau_t) \cdot \left[h_t^L + \{\lambda_t\tau_t + (1-\gamma)(1-\lambda_t)\cdot\tau_t\} \cdot \bar{h}_t\right].
\end{aligned}
$$

The low-to-mean ratio of income in period $t+1$, ρ_{t+1}, becomes:

$$
\rho_{t+1} = \Omega^1(\rho_t) \equiv \frac{\alpha}{\frac{3-\rho_t}{1+\rho_t} - (1-\alpha)\cdot\phi^L} \text{ for } \rho_t \in [0,1].
$$

where the superscript "1" in $\Omega^1(\rho_t)$ implies the presence of private investment. The function $\Omega^1(\cdot)$ is strictly increasing and strictly convex in ρ_t with:

$$\Omega^1(0) = \frac{\alpha}{3 - (1-\alpha) \cdot \phi^L} \in (0,1) \text{ and } \Omega^1(1) = \frac{\alpha}{1 - (1-\alpha) \cdot \phi^L} \in (0,1).$$

Therefore, there always exists a steady-state equilibrium with $z^L > 0$ and $z^H > 0$; and it is unique.

Alternatively, suppose that the type$-L$ agents make no private investment in education: $z^L = 0$. The period$-t$ equilibrium policy is $(\tau_t, \lambda_t) = (\delta/(1+\delta), 0)$; and the type$-H$ agents still invest privately in education (Proposition 2). Under this situation, the next-period income levels of both types of agents are:

$$\begin{aligned} h^L_{t+1} &= \alpha A \cdot (1-\gamma) \cdot \tau_t \cdot (1-\tau_t) \cdot \bar{h}_t, \\ h^H_{t+1} &= A\delta \cdot (1-\tau_t) \cdot \left[h^H_t + (1-\gamma) \cdot \tau_t \cdot \bar{h}_t\right], \end{aligned}$$

respectively; and the mean income level is:

$$\begin{aligned} \bar{h}_{t+1} &= \phi^L h^L_{t+1} + \phi^H h^H_{t+1} \\ &= A \cdot (1-\tau_t) \cdot \bar{h}_t \cdot \left[\phi^L \alpha \cdot (1-\gamma) \cdot \tau_t + \delta + (1-\phi^L) \cdot (1-\gamma) \cdot \delta \tau_t\right] \\ &\quad - \phi^L A\delta \cdot (1-\tau_t) \cdot h^L_t. \end{aligned}$$

The low-to-mean income ratio when $z^L_t = 0$ is:

$$\rho_{t+1} = \Omega^0(\rho_t) \equiv \frac{\alpha}{(\phi^L \alpha + (1-\phi^L)\delta) + \frac{1+\delta}{1-\gamma} - \frac{1+\delta}{1-\gamma}\phi^L \rho_t} \text{ for } \rho_t \in \left[0, \frac{(1-\delta)(1-\gamma)}{1+\delta}\right],$$

where the superscript 0 means the absence of private investment by type$-L$ agents. The function $\Omega^0(\cdot)$ is strictly increasing and strictly convex in ρ_t with:

$$\begin{aligned} \Omega^0(0) &= \frac{\alpha}{(\phi^L \alpha + (1-\phi^L)\delta) + \frac{1+\delta}{1-\gamma}} \in (0,1); \\ \Omega^0\left(\tfrac{(1-\delta)(1-\gamma)}{1+\delta}\right) &= \frac{\alpha}{\delta + \frac{1+\delta}{1-\gamma} - \phi^L(1-\alpha)} \in (0,1). \end{aligned}$$

Therefore, there exists a steady-state equilibrium with $z^L = 0$ and $z^H > 0$ if:

$$\Omega^0\left(\tfrac{(1-\delta)(1-\gamma)}{1+\delta}\right) < \tfrac{(1-\delta)(1-\gamma)}{1+\delta};$$

that is, if:

$$\phi^L < \frac{\delta}{1-\alpha} - \frac{(1+\delta)(1-\delta-\alpha)}{(1-\delta)(1-\gamma)(1-\alpha)}. \tag{8}$$

The results established so far are summarized in the following proposition.

- **Proposition 3.** *The dynamic equilibrium of* $\{\rho_t\}$ *satisfies:*

$$\rho_{t+1} = \begin{cases} \{\Omega^1(\rho_t), \Omega^0(\rho_t)\} & \text{for} \quad \rho_t \in \left[0, \frac{(1-\delta)(1-\gamma)}{1+\delta}\right], \\ \Omega^1(\rho_t) & \text{for} \quad \rho_t \in \left(\frac{(1-\delta)(1-\gamma)}{1+\delta}, 1\right]. \end{cases}$$

(i) *If (8) holds, there exist multiple steady-state equilibria: one is featured by* $z^L > 0$*; and the other is featured by* $z^L = 0$. (ii) *If (8) fails to hold, there exists a unique, locally stable steady-state equilibrium with* $z^L > 0$.

The dynamic equilibrium is unaffected by the parameter γ representing the inefficiency of public education as long as $z^L > 0$, while it is affected by γ if $z^L = 0$. The difference comes from the difference in the choice of λ in voting. When $z^L > 0$, the equilibrium value of λ becomes one (Proposition 1): all the tax revenue is spent on redistribution, and thus there is no provision of public education. The effect of γ via public education disappears in the equilibrium with $z^L > 0$. However, when $z^L = 0$, the effect of γ remains valid because all the tax revenue is spent on public education.

In order to observe the effect of γ on the dynamic equilibrium, we illustrate three numerical examples of the law of motion of ρ_t in Figure 4 : the cases of $\gamma = 0$ (panel (a)); $\gamma = 0.2$ (panel (b)); and $\gamma = 0.4$ (panel(c)). In producing the figure, the values of α, δ, A and ϕ^L are set to be 0.7, 0.3, 4.5 and 0.7, respectively. From the figure, we can find that a higher γ, which implies a lower efficiency of public education, leads to a lower low-to-mean ratio and thus a higher inequality level in the economy with $z^L = 0$. This point will be further investigated in the next section.

[Figure 4 about here.]

In closing this section, we briefly consider the dynamic motion of ρ_t by utilizing Figure 4. All the three panels demonstrate two steady-state equilibria. However, the properties of their stability in panel (a) differ from those in panels (b) and (c). In panel (a), the economy has no locally stable steady-state equilibrium: the economy may move back and forth between the two states, $z^L > 0$ and $z^L = 0$. In panels (b) and (c), such movement still arises when the initial level of ρ is below the critical value. However, when the initial level is above the critical value, the economy displays a monotone convergence toward the steady-state equilibrium with $z^L > 0$. Therefore, the (in)efficiency of public education critically affects the dynamic equilibrium in the economy.

5 Growth and Inequality in Steady States

This section analyzes the effects of the inefficiency of public education, represented by the parameter γ on growth and inequality. In particular, we focus on the growth rate of average income, $\bar{h}_{t+1}/\bar{h}_t$ and the low-to-mean income ratio, ρ_t. Because the model may attain multiple values of ρ_{t+1} for a given ρ_t, it is generally unable to demonstrate the evolution of growth and inequality over time without imposing any additional assumptions that select an equilibrium. Given this limitation, we here focus our attention on the two steady-state equilibria and discuss the empirical plausibility of the multiple political equilibria.

Let $\bar{\rho}^0$ and $\bar{\rho}^1$ denote the steady-state level of ρ when $z^L = 0$ and $z^L > 0$, respectively. The growth rates of the economy at $\rho_t = \bar{\rho}^0$ and $\bar{\rho}^1$ are computed as:

$$\left.\frac{\bar{h}_{t+1}}{\bar{h}_t}\right|_{\rho=\bar{\rho}^0} = A\frac{\delta}{1+\delta}\cdot\left[\phi^L\alpha(1-\gamma)\frac{1}{1+\delta}+1+(1-\phi^L)(1-\gamma)\frac{\delta}{1+\delta}\right]-\phi^L A\frac{\delta}{1+\delta}\bar{\rho}^0,$$
$$\left.\frac{\bar{h}_{t+1}}{\bar{h}_t}\right|_{\rho=\bar{\rho}^1} = \frac{A\delta}{4}(1+\bar{\rho}^1)\cdot\left[\left(3-(1-\alpha)\phi^L\right)-\bar{\rho}^1\left\{1+(1-\alpha)\phi^L\right\}\right],$$

respectively. Similar to the dynamic equilibrium of ρ_t, the steady-state growth rate is affected by γ if and only if there is no private education, $z^L = 0$.

In order to keep the result shown below comparable to the result in the previous section, we set the values of α, δ, and A to be 0.7, 0.3 and 4.5, respectively; and we consider the three cases: $\gamma = 0, 0.2$ and 0.4. Because of the assumption of the low-majority, we take the value of ϕ^L from 0.5 to 0.99. Figure 5 demonstrates the low-to-mean ratios of income (panel (a)) and the growth rates (panel (b)) for each ϕ^L. The numerical result indicates that the two similar economies may attain different education and redistribution policies, and experience different growth and inequality.

[Figure 5 about here.]

First, consider the case of $\gamma = 0$: there is no efficiency difference between private and public education. The equilibrium with $z^L = 0$ attains a higher growth rate and a higher equality level than the equilibrium with $z^L > 0$. The equilibrium with the absence of private education, $z^L = 0$, is superior to the equilibrium with the presence of private education, $z^L > 0$, in terms of growth and equality.

If the efficiency of public education is 20 percent points lower than the private education, that is, if $\gamma = 0.2$, then the $z^L = 0$ equilibrium attains a higher growth rate but a lower equality than the equilibrium with $z^L > 0$: there is a trade-off between growth and equality. In addition, the $z^L = 0$ equilibrium is characterized by a higher inequality and a higher share of public education in redistributive spending. The numerical result predicts a positive correlation between inequality and the share of public education in redistributive expenditure. This prediction is consistent with the empirical evidence in OECD countries as demonstrated in Panel (a) of Figure 1.

Finally, when the efficiency of public education is sufficiently low such that $\gamma = 0.4$, the $z^L = 0$ equilibrium realizes a lower growth rate and a higher inequality level than the equilibrium with $z^L > 0$. The result still predicts the positive correlation between inequality and public education expenditure. However, the $z^L = 0$ equilibrium attains a lower growth rate and a higher inequality level than the $z^L > 0$ equilibrium: this result is opposite to that in the case of $\gamma = 0.2$. The growth-inequality relationship is critically affected by the efficiency of public education; and the different results as regards growth and inequality might give one possible explanation for the mixed evidence of growth and inequality in OECD countries as illustrated in Introduction.

6 Conclusion

How the politics of public education and lump-sum transfer affect economic decisions over private education of agents; how the decisions in turn affect the inequality among agents and their preferences over the size and the composition of redistributive policies; what is the long-run consequence of this interaction for growth and inequality. In order to answer these questions, this paper developed an overlapping-generation model with private educational investments which are supplemented by public education and financed by lump-sum transfer. The tax on income as well as the allocation of tax revenue between public education and lump-sum transfer are determined by majority voting in every period.

Under this framework, we obtain the following results. First, there might be multiple political equilibria: one is the equilibrium featured by the exclusive use of tax revenue for public education; the other is the equilibrium featured by the exclusive use of tax revenue for lump-sum transfer. The multiplicity of equilibria comes from multidimensional voting.

Second, the inequality level in the former equilibrium is higher than the latter equilibrium when the efficiency of public education is lower than the private education to some extent. The result predicts the positive correlation between inequality and the share of public education in government transfer expenditure, which is consistent with the empirical evidence in OECD countries.

Third, the relationship between growth and inequality depends on the efficiency of public education. When the degree of efficiency is above a certain level, the numerical analysis predicts a positive correlation between growth and inequality. However, it predicts a negative correlation when the degree of efficiency is below a certain level. The two opposing results might provide one possible explanation for the mixed evidence of growth and inequality in OECD countries.

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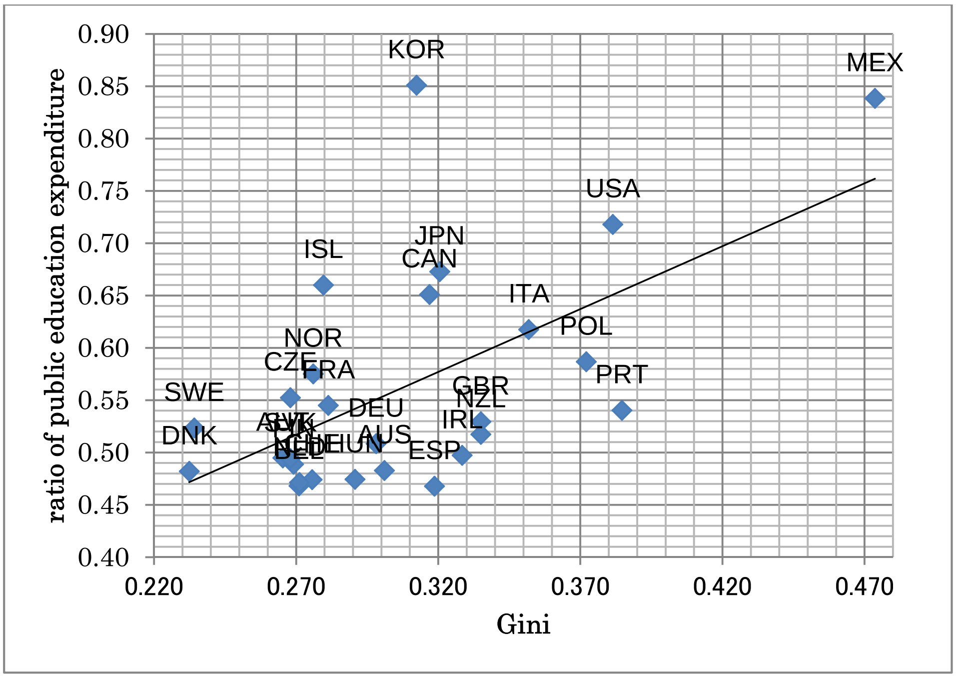
Panel (a) of Figure 1



Panel (b) of Figure 1

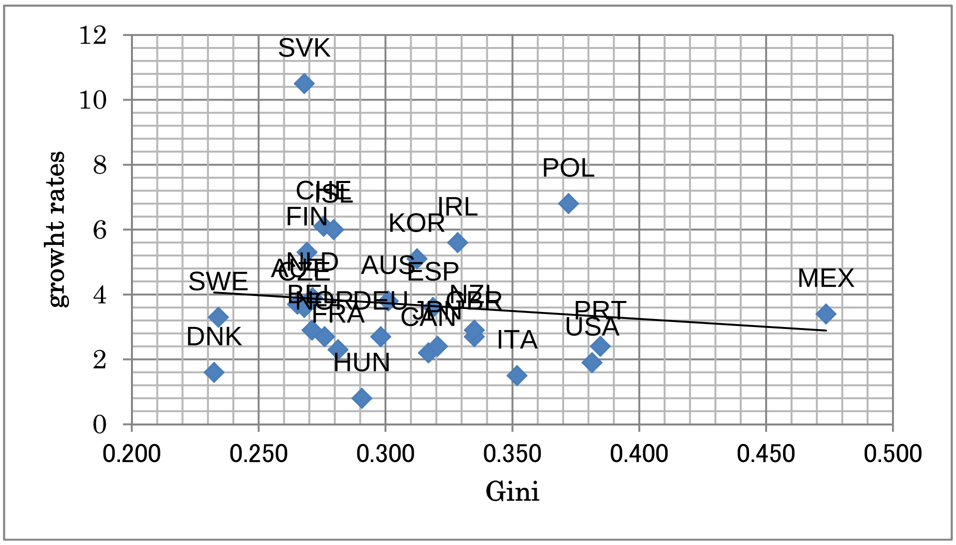
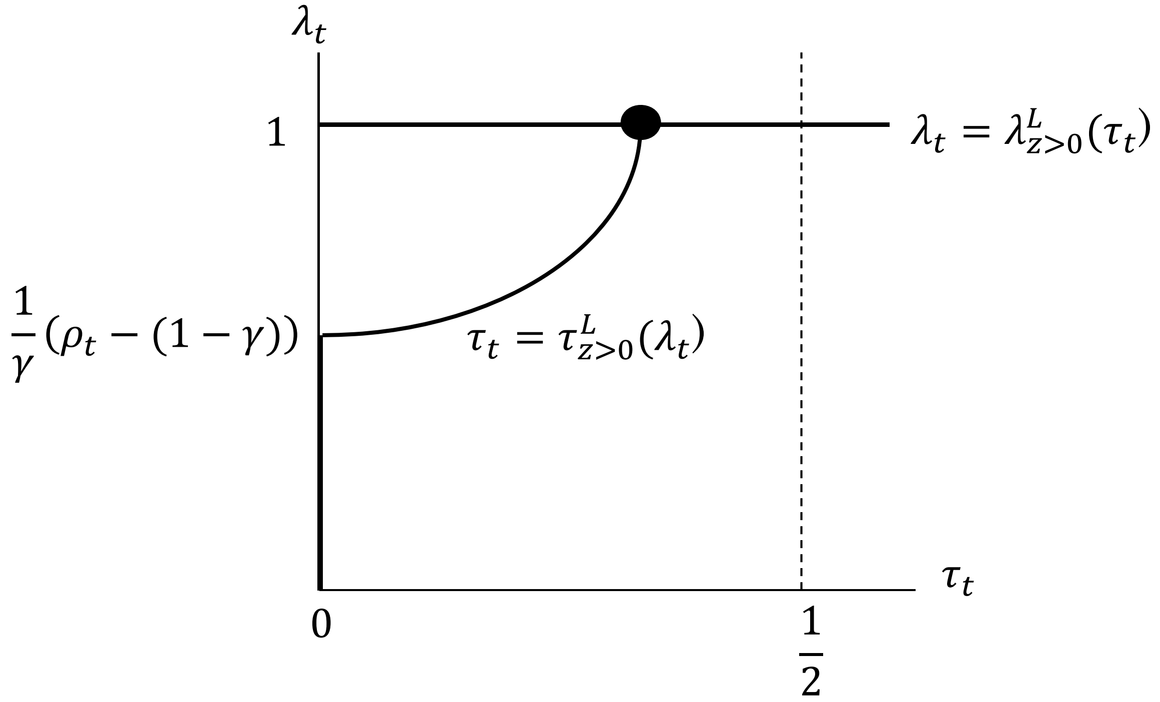


Figure 1. Panel (a) illustrates the cross-country data on the mid-2000 Gini coefficient and the ratio of public education expenditure per GDP in 2008 to the sum of the public education expenditure per GDP in 2008 and the income support to the working age population per GDP in 2007: Panel (b) illustrates the cross-country data on the mid-2000 Gini coefficient and growth rates in 2007. We compare data from different years because of the limited availability.

Source: OECD (2008) for the Gini coefficient, OECD (2011a) for public education expenditure per GDP and growth rates, and OECD (2011b) for income support to the working age population per GDP.

Panel (a) of Figure 2



Panel (b) of Figure 2

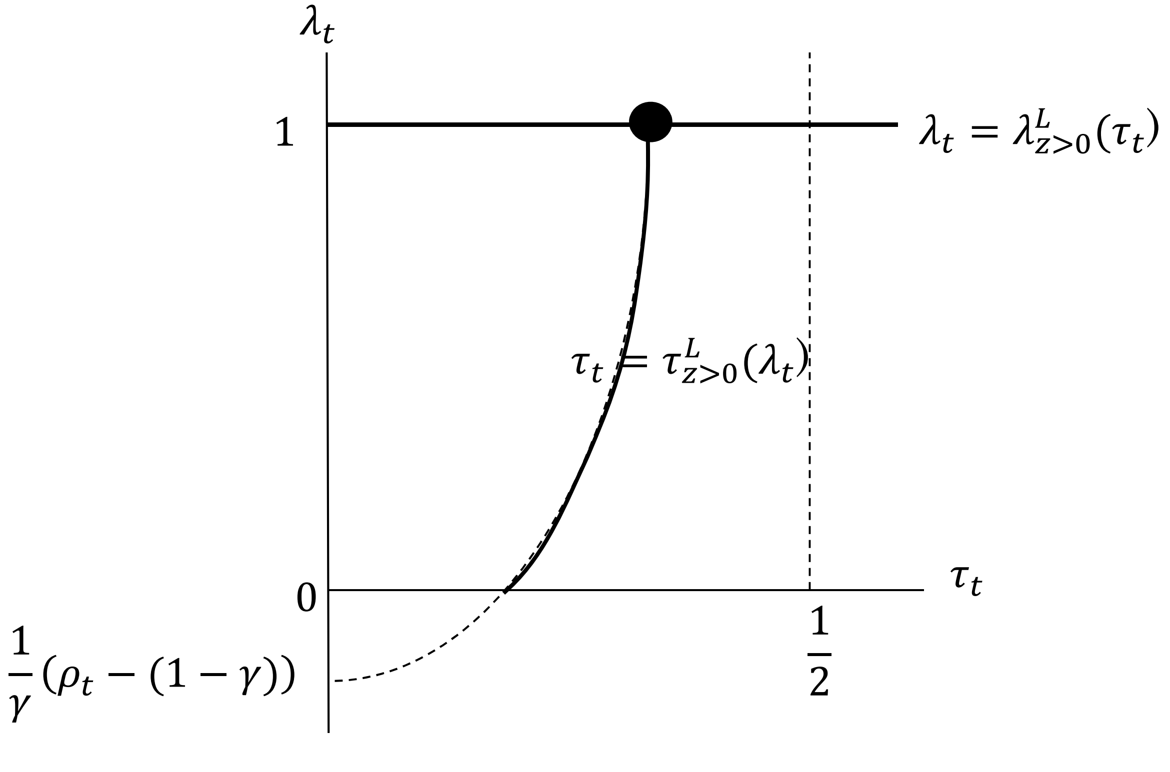


Figure 2. The reaction functions, $\tau_t = \tau^L_{z>0}(\lambda_t)$ and $\lambda_t = \lambda^L_{z>0}(\tau_t)$. Panel (a) illustrates the case of $(\rho_t - (1-\gamma))/\gamma > 0$; Panel (b) illustrates the case of $(\rho_t - (1-\gamma))/\gamma \leq 0$.

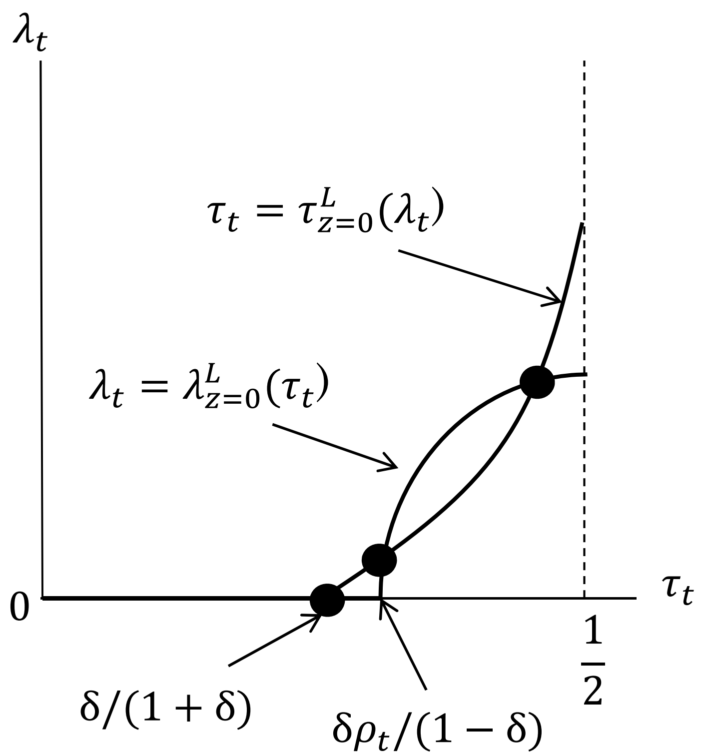
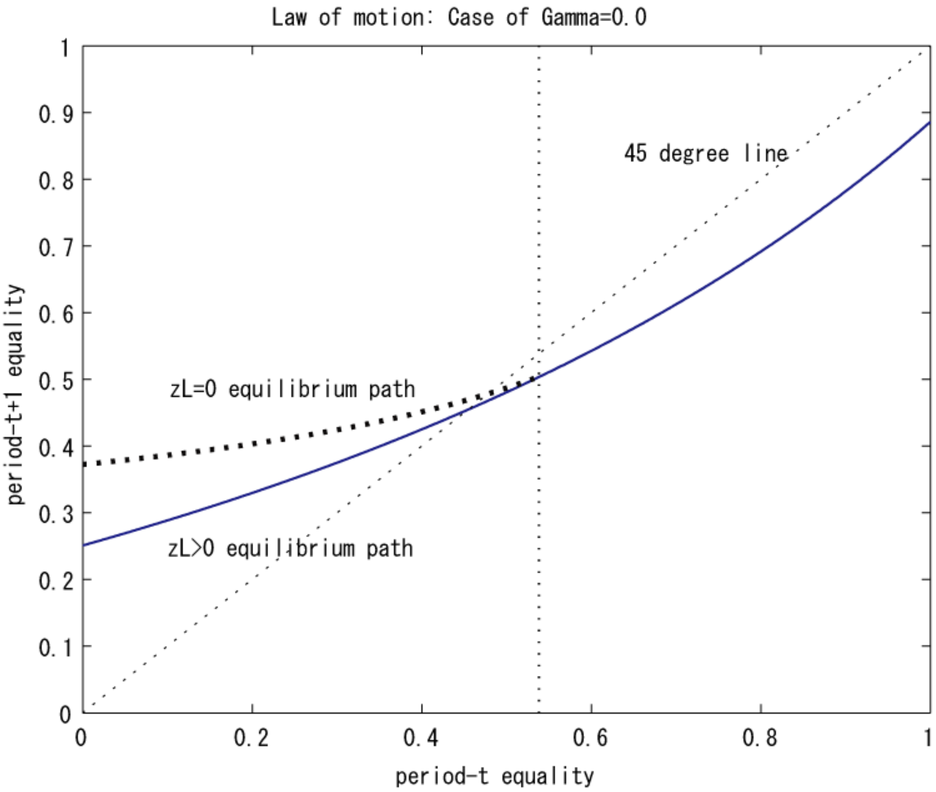
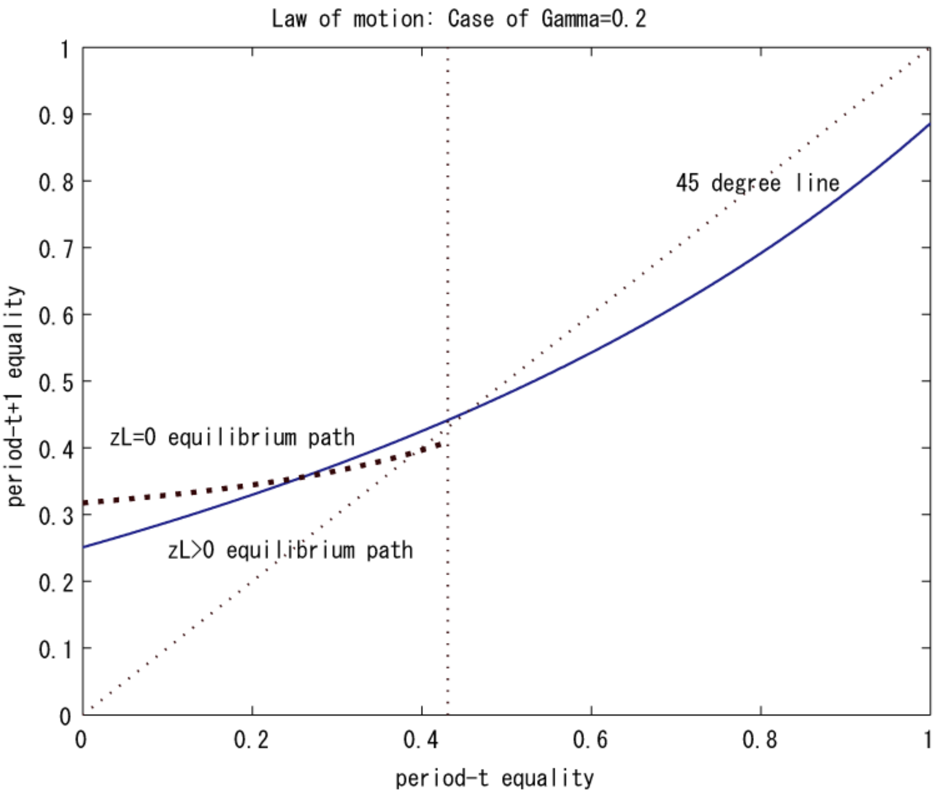


Figure 3. The reaction functions, $\tau_t = \tau^L_{z=0}(\lambda_t)$ and $\lambda_t = \lambda^L_{z=0}(\tau_t)$.

Panel (a) of Figure 4



Panel (b) of Figure 4



Panel (c) of Figure 4

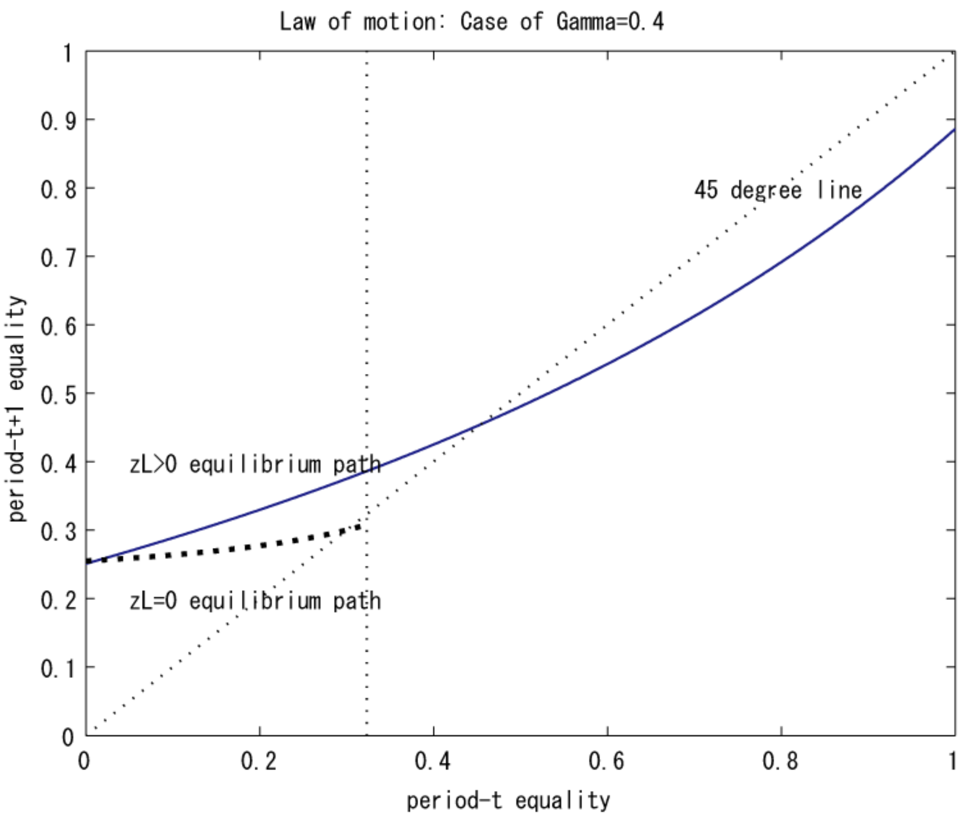
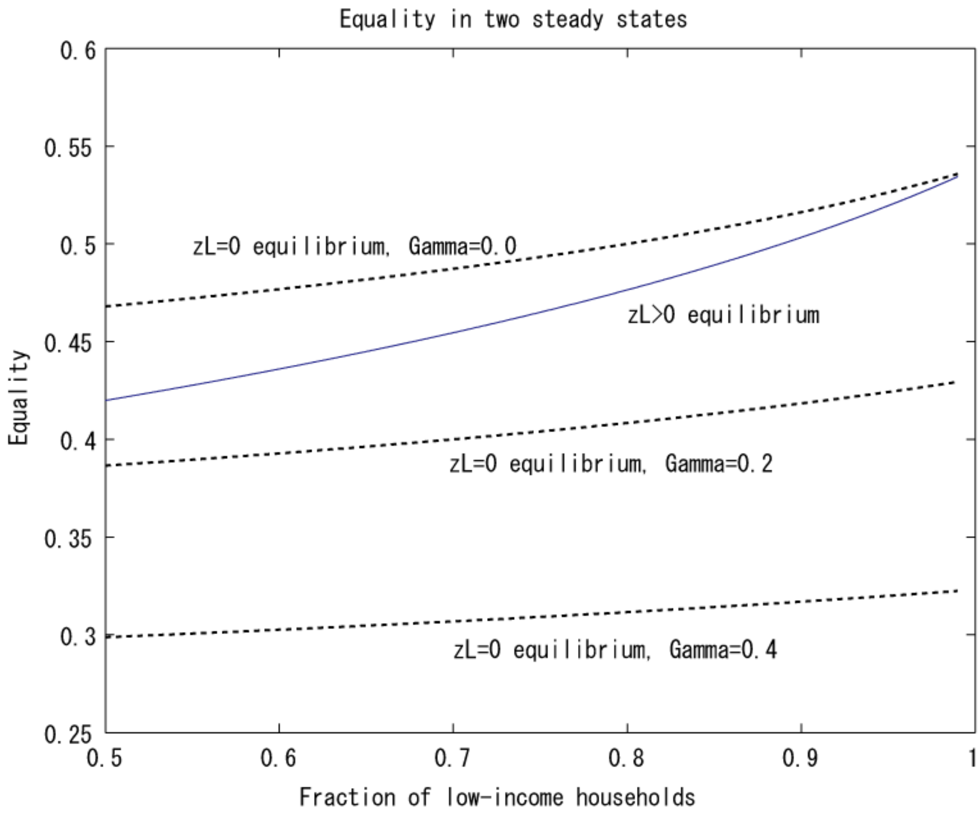


Figure 4. The figure illustrates the law of motions of the low-to-mean ratio of income for the case of $\gamma = 0.0$ (Panel (a)), $= 0.2$ (Panel (b)), and $\gamma = 0.4$ (Panel (c)), respectively. The solid curve indicates the law of motion in the equilibrium with the presence of private educational investment by low-income type agents; the dotted curve indicates the law of motion in the equilibrium with the absence of private educational investment by low-income type agents.

Panel (a) of Figure 5



Panel (b) of Figure 5

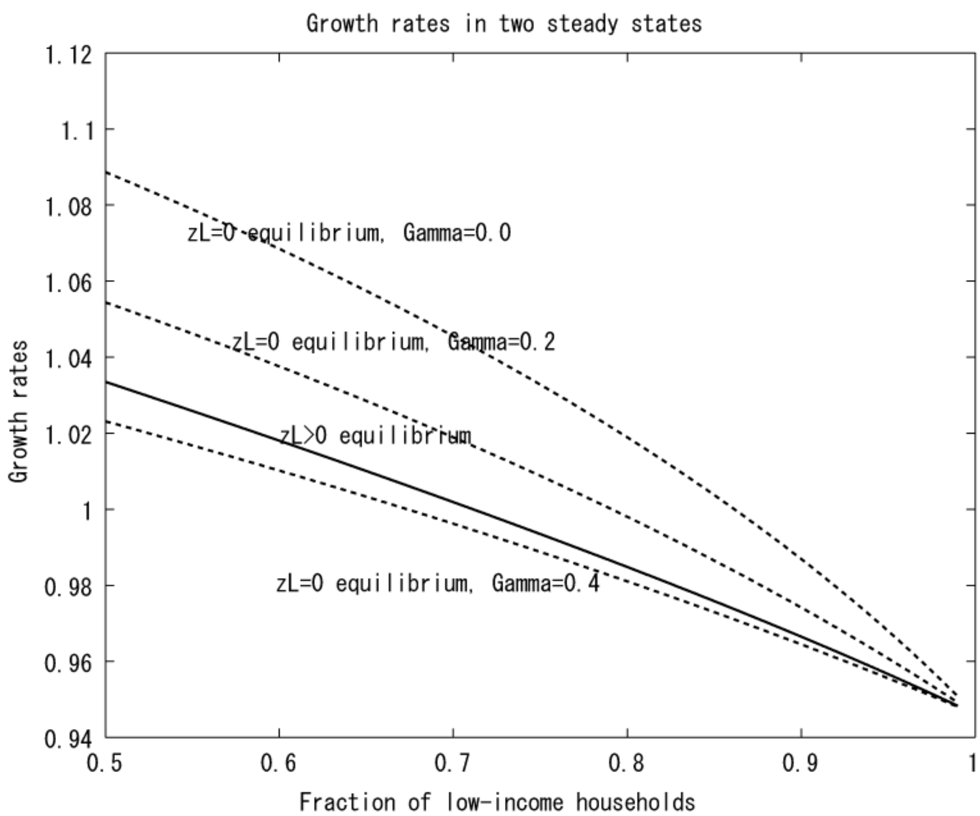


Figure 5. Equality levels (Panel (a)) and growth rates (Panel (b)) in steady states.