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Dynamic analysis of an endogenous growth model with investment-specific technological change*

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Abstract

This paper examines some dynamic properties of the endogenous growth model developed by Krusell[Krusell, P., 1998. Investment-specific R&D and the decline in the relative price of capital. Journal of Economic Growth 3, 131-141], in which investment-specific technological progress occurs endogenously due to firm-specific R&D performed by monopolistic firms. I show that when the function of R&D technology is assumed to be linear to labor inputs, the steady state is saddle-point stable under a continuous time setting. Furthermore, I also show that a combination of the time-invariant subsidy for investment and the time-variant subsidy for R&D can replicate the market equilibrium to the socially optimal allocation in this setting.

Keywords: Investment, R&D, Transitional dynamics

JEL classification: D92, O32, O41

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1 Introduction

Investment-specific technological change means technological progress in the production of capital goods. This technological progress lowers the relative price of capital goods, thereby encouraging capital accumulation and promoting economic growth. In fact, in the postwar period, the relative price of capital goods has fallen and the ratio of capital investment to GDP has risen substantially in the US. For instance, information and computer technologies have advanced dramatically in this period. Greenwood, Hercowitz, and Krusell(1997) quantitatively show that extending Gordon(1990)'s price index, the presence of this technological progress accounts for about 60% of the US economic growth after World War Ⅱ.

Although recently this technological progress has been recognized as a major source of economic growth, only a few papers have attempted to develop an endogenous growth model with investment-specific technological change [1]. As the seminal paper, Krusell(1998) presents a model in which investment-specific technological change occurs endogenously due to firm-specific R&D performed by monopolistic firms. However, Krusell(1998) analyzes only the steady state and consequently does not explore the dynamic properties of his model. Further, he does not analyze the policy implication, which is an important goal of this paper.

In this paper, I examine the dynamic properties of the model developed by Krusell(1998). To get clear results, I use a continuous time model for simplifying the analysis, although Krusell(1998) uses a discrete time model. I assume that R&D technologies are specified by a linear function of labor inputs. In this setting, I re-construct the model and analyze the transitional dynamics, further enhancing the analysis of Krusell(1998) in two respects.

First, I show that the steady state is saddle-point stable. Hence, indeterminacy and cyclical movements can be ruled out. If the steady state is unstable, a policy analysis of the equilibrium is not meaningful because no equilibrium path converges to the steady state. Second, I also analyze an optimal subsidy policy that replicates the market equilibrium to the socially optimal allocation.

In the same manner as Krusell(1998), the economy has two distortions: monopoly power and social knowledge spillovers in R&D. Monopoly power makes firms produce and innovate at less than an optimal level. Technological advances in the production of capital goods decrease the rental price of capital goods, which lowers revenues generated from capital stocks possessed by producers of capital goods. This means that capital goods produced in the past become obsolete. Each firm internalizes this obsolescence effect of the technological advance, which discourages the firm's inventive to innovate. This is a salient feature, as compared to the familiar endogenous growth models based on R&D. Another distortion is the presence of social knowledge spillovers in R&D, which too distort a firm's incentive to innovate.

As the main contribution, I show that a combination of the time-invariant subsidy for investment and the time-variant subsidy for R&D can replicate the market equilibrium to the socially optimal allocation. Because the speed of convergence may be slow[2], it is important to note that a simple subsidy policy cannot replicate the market economy to socially optimal

[1] Boucekkine(2003) develops an endogenous growth model where learning-by-doing is the engine of investment-specific technological change. Boucekkine(2005) and Huffman(2007, 2008) develop an endogenous growth model where investment-specific technological change is determined through R&D. However, their models examine only the steady state.

[2] For example, Steger(2003) shows that the speed of convergence is slow in a calibrated version of the model constructed using a quality-ladder model developed by Segerstrom(1998).

allocation during the transition to the steady state[3].

The rest of the paper is structured as follows: Section 2 describes the model, following Krusell(1998). Section 3 analyzes the dynamic system of the model. Section 4 analyzes the socially optimum allocation and compares the market economy with socially optimal allocation. Finally, section 5 analyzes an optimal subsidy policy.

2 Model

In this section, I set up a model, following Krusell(1998). In contrast to Krusell(1998), I use a continuous time model to simplify the analysis of transition paths. Except for the specification of R&D technology, the environment of the model is the same as in Krusell(1998). An economy is composed of a final goods sector, a capital goods sector, and households. The production technology of final goods and capital goods are different. First, I consider the final goods sector.

2.1 Final goods sector

The final good, Y_t, is produced by the following production function:

$$Y_t = \int_0^1 K_{jt}^{\alpha}\, dj\, L_{Yt}^{1-\alpha}, \ \ \alpha \in (0,1), \tag{1}$$

where L_{Yt} and K_{jt}, respectively, represent labor inputs and inputs of the jth capital good at time t. Perfect competition prevails in the final goods market. The price of final goods is normalized to one. Therefore, I obtain the following profit-maximization conditions:

$$w_t = (1-\alpha) \int_0^1 K_{jt}^{\alpha}\, dj\, L_{Yt}^{-\alpha}, \tag{2}$$

$$p_{jt} = \alpha K_{jt}^{\alpha-1} L_{Yt}^{1-\alpha}, \tag{3}$$

where w_t and p_{jt}, respectively, represent the wage rate and the rental price of jth capital good at time t.

2.2 Capital goods sector

Monopolistic competition prevails in the capital goods market and there is a continuum of goods, indexed by type $j \in [0,1]$. Each firm accumulates its specific capital goods and rents them to producers of final goods exclusively. Initially, this monopoly is assumed to be protected by perfect patent protection, and the entry of new firms is not considered. Investing one unit of final goods produces T_{jt} units of capital goods. That is, T_{jt} represents the level of the jth firm's production technology of producing capital goods at time t. Further, each

[3] As related studies, Arnold(2000a,b) examine the stability of an equilibrium in Romer(1990)'s model and show that a combination of the time-invariant subsidy for production and the time-variant subsidy for R&D can replicate the market equilibrium to the socially optimal allocation.

firm can conduct cost-reducing innovations with labor inputs. The profit of the jth firm at time t is

$$\pi_{jt} = p_{jt}K_{jt} - I_{jt} - w_t L_{Ajt},$$

where K_{jt}, I_{jt}, and L_{Ajt}, respectively, represent capital stocks, inputs of final goods (hereafter, referred to as investment), and labor inputs to R&D of the jth firm at time t. The law of motion of the capital stocks of the jth firm is

$$\dot{K}_{jt} = T_{jt}I_{jt} - \delta K_{jt}, \tag{4}$$

where δ represents the physical depreciation rate of capital stocks. As the production technology improves, more capital goods can be produced with one unit of final goods. This form of technological progress is characterized as "investment-specific technological change," which is the only engine of economic growth in this model. Moreover, the law of motion of the production technology of the jth firm is

$$\dot{T}_{jt} = \psi T_{jt}^{\gamma} \bar{T}_t^{1-\gamma} L_{Ajt}, \;\; \gamma \in [0, 1], \tag{5}$$

where $\psi > 0$ represents the productivity in the R&D technology and $\bar{T}_t \equiv \int_0^1 T_{jt} dj$ represents the average level of the production technology across firms. In contrast to Krusell(1998)[4], I assume that the R&D technology is specified to be linear in labor inputs, following the specification of standard endogenous growth models based on R&D; for instance, Grossman and Helpman(1991, ch3) and Romer(1990). T_{jt} captures in-house knowledge spillovers and $\bar{T}_t$ captures social knowledge spillovers of R&D. γ measures the relative importance of dynamic returns to R&D. If $\gamma > 0$, each firm internalizes such in-house knowledge spillovers. If $\gamma = 1$, each firm internalizes these spillovers completely. Higher values of γ make each firm internalize the effects more aggressively.

The present value of the sum of the jth firm's operating profit at time 0 is

$$V_{j0} = \int_0^{\infty} (\pi_{jt}) \exp\left(-\int_0^t r_s ds\right) dt, \tag{6}$$

where r_t represents the return on safe assets at time t. Symmetry across firms is assumed, and as such I can drop the subscript j. Hence, in equilibrium,

$$T_t = \bar{T}_t.$$

Each firm maximizes (6), subject to (3), (4), and (5), given $\bar{T}_t$[5]. To solve the inter-temporal maximization problem, I define the following Current-Value Hamiltonian as

$$H = \alpha K_t^{\alpha} L_{Yt}^{1-\alpha} - I_t - w_t L_{At} + \mu_t[\, T_t I_t - \delta K_t \,] \; + \; q_t[\, \psi T_t^{\gamma} \bar{T}_t^{1-\gamma} L_{At} \,],$$

where the co-state variables, μ_t and q_t, respectively, represent the shadow value of investment and R&D at time t. In this paper, I restrict the analysis to an interior solution. I obtain the

[4]Krusell(1998) assumes that the R&D technology is $T_{t+1} = T_{jt}^{\gamma} \bar{T}_{jt}^{1-\gamma} H(L_{Ajt})$, where $H'() > 0$, $H''() < 0$.

[5]In this model, incumbent firms innovate firm-specific technology repeatedly. This formulation is similar to Peretto(1998) and Smulderts and van de Klundert(1995).

following first-order conditions:

$$\mu_t = \frac{1}{T_t}, \tag{7}$$

$$r_t\mu_t - \dot{\mu}_t = \alpha^2 K_t^{\alpha-1} L_{Yt}^{1-\alpha} - \delta\mu_t, \tag{8}$$

$$q_t = \frac{w_t}{\psi T_t^{\gamma} \bar{T}_t^{1-\gamma}}, \tag{9}$$

$$r_t q_t - \dot{q}_t = \mu_t I_t + q_t \psi\gamma T_t^{\gamma-1} \bar{T}_t^{1-\gamma} L_{At}. \tag{10}$$

(7) means that the shadow value of investment must be equal to installed costs. (9) means that the shadow value of R&D must be equal to innovation costs. Further, (8) and (10), respectively, represent the no-arbitrage condition of investment and R&D. Furthermore, the following transversality conditions must be satisfied: $\lim_{t\to\infty} \mu_t K_t \exp\left(-\int_0^t r_s ds\right) = 0$ and $\lim_{t\to\infty} q_t T_t \exp\left(-\int_0^t r_s ds\right) = 0$.

From (7) and (8), the amount of capital stocks at time t is as follows:

$$K_t = \alpha^{\frac{2}{1-\alpha}} L_{Yt} T_t^{\frac{1}{1-\alpha}} u_t^{\frac{1}{\alpha-1}}, \tag{11}$$

where u_t is defined as:

$$u_t \equiv r_t + \delta + \frac{\dot{T}_t}{T_t}. \tag{12}$$

From (3) and (11), the rental price of capital goods at time t is given by

$$p_t = \frac{1}{\alpha} u_t \frac{1}{T_t},$$

where $u_t \frac{1}{T_t}$ represents the user-cost of capital at time t, which is the marginal cost to produce one unit of capital goods. Each firm charges a monopoly markup price. Technological progress decreases the rental price of capital goods, which lowers the revenue generated from capital invested as a preliminary. On the other hand, technological progress enhances the productivity of R&D through in-house knowledge spillovers. Therefore, each firm creates a schedule of R&D internalizing the above effects and costs.

2.3 Households

Here, I consider a representative household's problem. Population size, L, is constant over time. Each individual supplies one unit of labor inelastically. Each household maximizes the following lifetime utility function[6]:

$$U_0 = \int_0^\infty (\log c_t) \exp(-\rho t)\, dt, \quad \rho > 0,$$

where c_t and ρ, respectively, represent consumption of final goods per-capita at time t and the individual discount rate. Solving the inter-temporal optimization problem, I obtain the following Euler equation:

$$\frac{\dot{c}_t}{c_t} = r_t - \rho. \tag{13}$$

[6]For simplicity, the utility function is specified to be of log-utility type although it is specified to be of CRRA type in Krusell(1998).

Furthermore, the following transversality condition must be satisfied: $\lim_{t\to\infty} \frac{1}{c_t} a_t \exp(-\rho t) = 0$.

3 Market equilibrium and dynamics

In this section, I derive the market equilibrium and construct a dynamic system based on the model in the preceding section. The market equilibrium condition of final goods is

$$Y_t = c_t L + I_t, \tag{14}$$

where $c_t L$ and I_t, respectively, represent aggregate consumption and investment at time t. Further, the market equilibrium condition of labor is

$$L = L_{Yt} + L_{At}. \tag{15}$$

To derive the dynamic system of the economy, I define

$$\begin{aligned} Z_t &\equiv T_t^{\frac{1}{\alpha-1}} K_t, \\ S_t &\equiv T_t^{\frac{\alpha}{\alpha-1}} c_t, \\ Q_t &\equiv q_t^{\frac{\alpha-1}{\alpha}} T_t^{\frac{2\alpha-1}{\alpha}}. \end{aligned}$$

In Appendix A, I show that the following three equations constitute the dynamic system of the economy:

$$\frac{\dot{Z}_t}{Z_t} = \kappa Q_t + \kappa Q_t^{\frac{1}{1-\alpha}} Z_t - S_t Z_t^{-1} L - \frac{\psi L}{1-\alpha} - \delta, \tag{16}$$

$$\frac{\dot{S}_t}{S_t} = \alpha^2 \kappa Q_t + \kappa Q_t^{\frac{1}{1-\alpha}} Z_t - \frac{\psi L}{1-\alpha} - (\delta + \rho), \tag{17}$$

$$\begin{aligned} \frac{\dot{Q}_t}{Q_t} = &\frac{(1-\alpha)^2}{\alpha}(1-\gamma)\kappa Q_t^{\frac{1}{1-\alpha}} Z_t - \alpha(1-\alpha)\kappa Q_t - \frac{1-\alpha}{\alpha} Q_t^{\frac{\alpha}{1-\alpha}} S_t L \\ &+ \left[\frac{\alpha + (1-\alpha)\gamma}{\alpha}\right] \psi L + \frac{1-\alpha}{\alpha}\delta, \end{aligned} \tag{18}$$

where $\kappa \equiv (1-\alpha)^{\frac{1-\alpha}{\alpha}} \psi^{\frac{\alpha-1}{\alpha}}$. In Appendix B, I show that the steady state $\{Z^*, S^*, Q^*\}$ is determined by

$$\begin{aligned} Z^* &= F(Q^*) \equiv \left[\frac{\psi L}{(1-\alpha)\kappa} + \frac{(\delta+\rho)}{\kappa}\right] Q^{*\frac{1}{\alpha-1}} - \alpha^2 Q^{*\frac{\alpha}{\alpha-1}}, \\ S^* &= G(Q^*) \equiv \left[\frac{\psi}{1-\alpha} + \frac{\{(1-\alpha)(1-\gamma)+1\}\delta}{L} + \frac{(1-\alpha)(1-\gamma)\rho}{L}\right] Q^{*\frac{\alpha}{\alpha-1}} \\ &\quad - \left[\frac{(1-\alpha)(1-\gamma)+1}{L}\right] \alpha^2 \kappa Q^{*\frac{2\alpha-1}{\alpha-1}}, \\ Q^* &= \frac{-N + \sqrt{N^2 - 4MP}}{2\kappa M}, \end{aligned}$$

where M, N, and P, respectively, are defined as

$$\begin{aligned}
M &\equiv \alpha^4 + \alpha^2(1-\alpha)(1-\gamma), \\
N &\equiv \frac{\psi L}{1-\alpha}(-\alpha^2) + \delta[-\alpha^2 - (1-\alpha)(1-\gamma)] \\
&\quad + \rho[1 - 2\alpha^2 - (1-\alpha)(1-\gamma)], \\
P &\equiv \left[\frac{\psi L}{1-\alpha} + (\delta+\rho)\right]\rho.
\end{aligned}$$

In the steady state, $\{L_{Yt}, L_{At}, r_t, u_t\}$ are constant.

Proposition 1. *The steady state is saddle-point stable.*

Proof. See Appendix C. □

4 Socially optimal growth path

In this section, I consider the first-best resource allocation in which a social planner maximizes the lifetime utility function of a representative household, subject to the market constraints of final goods and labor, and the law of motion of capital stocks and the production technology of capital goods. Therefore, the inter-temporal optimization problem of the social planner can be described as follows:

$$\begin{aligned}
\max_{I_t, K_t, L_{At}, T_t} \quad & \int_0^\infty (\log c_t) \exp(-\rho t) dt, \\
\text{s.t} \quad & Y_t = c_t L + I_t, \\
& L = L_{Yt} + L_{At}, \\
& \dot{K}_t = T_t I_t - \delta K_t, \\
& \dot{T}_t = \psi T_t L_{At}.
\end{aligned}$$

In Appendix D, I show that the following three equations constitute the dynamic system of the socially optimal allocation:

$$\frac{\dot{Z}_t}{Z_t} = \kappa Q_t + \kappa {Q_t}^{\frac{1}{1-\alpha}} Z_t - S_t {Z_t}^{-1} L - \frac{\psi L}{1-\alpha} - \delta, \tag{19}$$

$$\frac{\dot{S}_t}{S_t} = \alpha\kappa Q_t + \kappa {Q_t}^{\frac{1}{1-\alpha}} Z_t - \frac{\psi L}{1-\alpha} - (\delta+\rho), \tag{20}$$

$$\frac{\dot{Q}_t}{Q_t} = -(1-\alpha)\kappa Q_t - \frac{1-\alpha}{\alpha} {Q_t}^{\frac{\alpha}{1-\alpha}} S_t L + \frac{1}{\alpha}\psi L + \frac{1-\alpha}{\alpha}\delta. \tag{21}$$

Because the dynamic systems are too complicated, I resort to numerical simulations to see the differences between the market equilibrium and the socially optimal allocation. Using linearized approximation around the steady state, I numerically calculate the transition path to the steady state.

As a benchmark, I choose the parameter values as follows. The time preference rate, ρ, and the physical depreciation rate of capitals, δ, respectively, are set as 0.05 and 0.2, as is conventional in macroeconomic literature. α is set as 0.8, which implies that the mark-up rate of capital goods is 25%. The scale parameter, L, is normalized to one. ψ is set as 0.025 so that the economic growth rate of the economy is around 2%. To examine the transition path to the steady state, I choose $Z_0 = Z^* \times 0.5$ as the initial value of the state variable.

Figure 1 shows how the linearly approximated transition path of $\{L_{At}\}$ evolves over time under the benchmark parameter case. The solid line stands for the transition path in the market equilibrium, and the dotted line stands for that in the socially optimal allocation. Figure 1 shows that labor inputs to R&D in the market equilibrium are less than in the socially optimal allocation. Each firm internalizes the obsolescence effect of a decline in the rental price. Furthermore, social knowledge spillovers in R&D also discourage the firm's inventive to innovate.

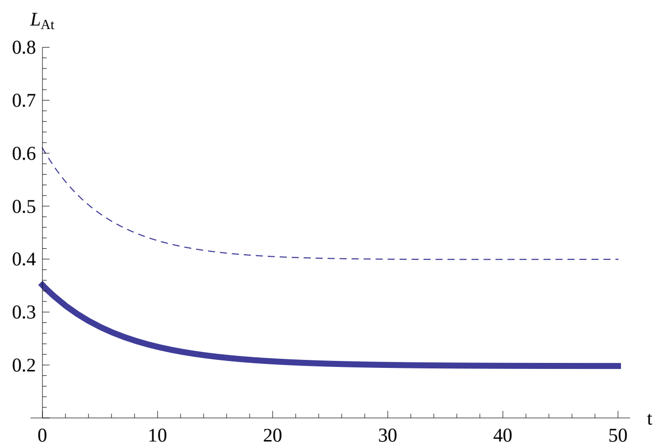

Figure 1: The transition path of labor inputs to R&D

5 Optimal subsidy policy

In this section, I analyze the subsidy policy that enables the market equilibrium to replicate the socially optimal allocation. The following two aspects must be considered. First, the market economy must attain the steady state of the socially optimal allocation. Second, the transition path to the steady state too must correspond with that of the socially optimal allocation.

To see this, I introduce two policy tools in the market economy: the proportional subsidy for investment and R&D. Let τ_t^I and τ_t^R be the rate of subsidy for investment and for R&D at time t, respectively. I assume that the government can impose a lump-sum tax on households to cover these subsidies. The net profit of the jth firm at time t is rewritten as

$$\pi_{jt} = p_{jt}K_{jt} - I_{jt}(1 - \tau_t^I) - w_t L_{Ajt}(1 - \tau_t^R).$$

Each firm maximizes the present value of the sum of profit, subject to (4) and (5), given the rate of these subsidies. The dynamic system of the economy is given by the following three equations:

$$\frac{\dot{Z}_t}{Z_t} = \kappa Q_t + \kappa Q_t^{\frac{1}{1-\alpha}} Z_t - S_t Z_t^{-1} L - \frac{\psi L}{1-\alpha} - \delta, \tag{22}$$

$$\frac{\dot{S}_t}{S_t} = \alpha^2 \kappa Q_t \left[\frac{1}{1-\tau_t^I}\right] + \kappa Q_t^{\frac{1}{1-\alpha}} Z_t - \frac{\psi L}{1-\alpha} - (\delta+\rho) - \frac{\dot{\tau}_t^I}{1-\tau_t^I}, \tag{23}$$

$$\begin{aligned}\frac{\dot{Q}_t}{Q_t} =& \frac{1-\alpha}{\alpha}\kappa Q_t^{\frac{1}{1-\alpha}} Z_t \left[-2\alpha + 1 - (1+\gamma)(1-\alpha) + \frac{1-\tau_t^I}{1-\tau_t^R}\right] \\ &- \alpha(1-\alpha)\kappa Q_t \left[\frac{1}{1-\tau_t^I}\right] - \frac{1-\alpha}{\alpha} Q_t^{\frac{\alpha}{1-\alpha}} S_t L \left[\frac{1-\tau_t^I}{1-\tau_t^R}\right] \\ &+ \left[\frac{\alpha + (1-\alpha)\gamma}{\alpha}\right]\psi L + \frac{1-\alpha}{\alpha}\delta + \frac{1-\alpha}{\alpha}\left[\frac{\dot{\tau}_t^I}{1-\tau_t^I}\right] \\ &- \frac{1-\alpha}{\alpha}\left[\frac{\dot{\tau}_t^R}{1-\tau_t^R}\right]. \end{aligned} \tag{24}$$

Now, I examine the optimal rate of two subsidies at which (22), (23), and (24) correspond with (19), (20), and (21). With or without the subsidy policy, (22) corresponds with (19). When $\gamma = 1$, $\tau_t^I = 1 - \alpha$ replicates (23) to (20), and $\tau_t^R = 1 - \alpha$ replicates (24) to (21). However, when $\gamma \neq 1$, the R&D subsidy must be time-variant to replicate (24) to (21). For (24) to correspond with (21), the following equation must be satisfied:

$$\begin{aligned}&\kappa Q_t^{op\frac{1}{1-\alpha}} Z_t^{op} \left[-\alpha - \gamma(1-\alpha) + \frac{\alpha}{1-\tau_t^R}\right] + Q_t^{op\frac{\alpha}{1-\alpha}} S_t^{op} L \left[1 - \frac{\alpha}{1-\tau_t^R}\right] \\ &+ (\gamma - 1)\,\psi L = \frac{\dot{\tau}_t^R}{1-\tau_t^R},\end{aligned}$$

where $\{Z_t^{op}, S_t^{op}, Q_t^{op}\}$ represent the equilibrium values during the transition to the steady state in the socially optimal allocation. Rewriting the above equation, I obtain the following differential equation:

$$\dot{\tau}_t^R + A(t)\tau_t^R = B(t), \tag{25}$$

where $A(t)$ and $B(t)$ are defined as follows:

$$\begin{aligned}A(t) &\equiv \kappa Q_t^{op\frac{1}{1-\alpha}} Z_t^{op}\left[-\alpha - \gamma(1-\alpha)\right] + Q_t^{op\frac{\alpha}{1-\alpha}} S_t^{op} L + (\gamma-1)\psi L, \\ B(t) &\equiv \kappa Q_t^{op\frac{1}{1-\alpha}} Z_t^{op}\left[-\alpha - \gamma(1-\alpha)\right] + Q_t^{op\frac{\alpha}{1-\alpha}} S_t^{op} L \\ &\quad + (\gamma-1)\psi L + \alpha\left[\kappa Q_t^{op\frac{1}{1-\alpha}} Z_t^{op} - Q_t^{op\frac{\alpha}{1-\alpha}} S_t^{op} L\right].\end{aligned}$$

First, I examine the optimal rate of the R&D subsidy in the steady state, τ^{*R}. By setting $\dot{\tau}_t^R = 0$ in (25), I obtain

$$\tau^{*R} = 1 - \frac{\alpha\rho}{[\alpha\kappa Q^{**} - \delta] - [\alpha + \gamma(1-\alpha)][\alpha\kappa Q^{**} - (\delta+\rho)]},$$

where Q^{**} is the steady-state equilibrium value in the socially optimal allocation. τ_*^R is a decreasing function of γ. This is because each firm does not further internalize social knowledge spillovers as γ moves toward 0.

I can analytically obtain the optimal transition path of τ_t^R by solving the above differential equation, (25). However, because (25) is too complicated, I resort to numerical simulations. Calculating under the benchmark parameter sets as previously discussed, I obtain the numerical example of the linearly-approximated optimal transition path of τ_t^R as given in Figure 2[7]. It is shown that when the social knowledge spillover in R&D exists, a higher subsidy rate for R&D is required during the early stage of the transition. To sum up, I state the following proposition.

Proposition 2. *When the social knowledge spillover in R&D does not exist (that is, $\gamma = 1$), a combination of the time-invariant subsidy for investment and for R&D can replicate the market equilibrium to the socially optimal allocation. When social knowledge spillovers in R&D exist (that is, $\gamma \neq 1$), a combination of the time-invariant subsidy for investment and the time-variant subsidy for R&D can replicate the market equilibrium to the socially optimal allocation.*

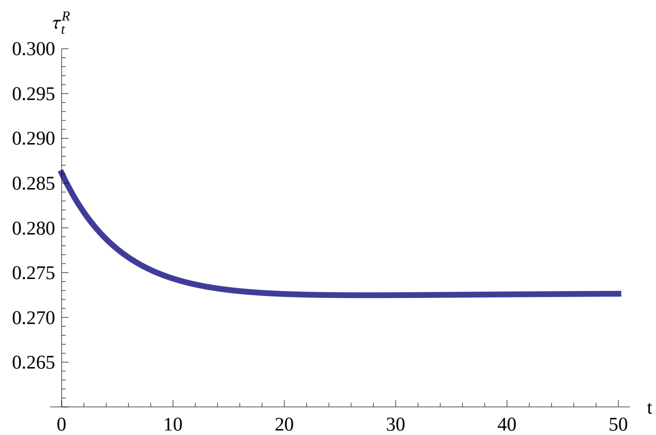

Figure 2: The optimal transition path of subsidy rate for R&D

6 Conclusion

This paper analyzes some dynamic properties of a modified version of Krusell(1998)'s model, in which investment-specific technological change occurs endogenously. I make and prove the two propositions. First, I show the steady state to be saddle-point stable. Second, I also show that a simple subsidy policy cannot replicate the market economy to the socially optimal allocation, and consequently, a complex subsidy policy is required in this model.

[7] An alternative choice of Z_0 does not change the basic result.

A Appendix A

In this appendix, I derive the dynamic system of the economy.

From (2) and (9), I can eliminate w_t and derive

$$(1-\alpha)K_t^{\alpha}L_{Yt}^{-\alpha} = \psi q_t T_t.$$

From the above equation and (15), I can eliminate L_{Yt} and derive

$$(1-\alpha)K_t^{\alpha}(L - L_{At})^{-\alpha} = \psi q_t T_t.$$

Then, L_{At} is derived as

$$\begin{aligned} L_{At} &= L - (1-\alpha)^{\frac{1}{\alpha}}\psi^{-\frac{1}{\alpha}}q_t^{-\frac{1}{\alpha}}T_t^{-\frac{1}{\alpha}}K_t \\ &= L - (1-\alpha)\psi^{-1}\kappa Q_t^{\frac{1}{1-\alpha}}Z_t. \end{aligned} \tag{A-1}$$

From (11), (15), and (A-1), u_t is derived as

$$u_t = \alpha^2\kappa Q_t, \tag{A-2}$$

From (12), (A-1), and (A-2), r_t is derived as

$$r_t = \alpha^2\kappa Q_t + (1-\alpha)\kappa Q_t^{\frac{1}{1-\alpha}}Z_t - \psi L - \delta. \tag{A-3}$$

Substituting (A-3) into (13), I obtain

$$\frac{\dot{c}_t}{c_t} = \alpha^2\kappa Q_t + (1-\alpha)\kappa Q_t^{\frac{1}{1-\alpha}}Z_t - \psi L - (\delta+\rho).$$

From (1), (4), (14), (15), and (A-1), I obtain

$$\frac{\dot{K}_t}{K_t} = \kappa Q_t - S_t Z_t^{-1} L - \delta.$$

Substituting (A-1) into (5), I obtain

$$\frac{\dot{T}_t}{T_t} = \psi L - (1-\alpha)\kappa Q_t^{\frac{1}{1-\alpha}}Z_t.$$

From (1), (7), (9), (10), (14), (15), (A-1), and (A-3), I obtain

$$\begin{aligned} \frac{\dot{q}_t}{q_t} = {} & \alpha^2\kappa Q_t + (1-\alpha)(1+\gamma)\kappa Q_t^{\frac{1}{1-\alpha}}Z_t - \kappa Q_t^{\frac{1}{1-\alpha}}Z_t \\ & + Q_t^{\frac{\alpha}{1-\alpha}}S_t L - (1+\gamma)\psi L - \delta. \end{aligned}$$

From the definitions of $\{Z_t, S_t, Q_t\}$, I can derive (16), (17), and (18).

B Appendix B

In this appendix, I provide a detailed derivation of the steady state in the market economy. From (16), (17), and (18), the steady state, $\{Z^*, S^*, Q^*\}$, must satisfy the following equations:

$$\kappa Q^* + \kappa Q^{*\frac{1}{1-\alpha}} Z^* - S^* Z^{*-1} L - \frac{\psi L}{1-\alpha} - \delta = 0, \tag{B-1}$$

$$\alpha^2 \kappa Q^* + \kappa Q^{*\frac{1}{1-\alpha}} Z^* - \frac{\psi L}{1-\alpha} - (\delta + \rho) = 0, \tag{B-2}$$

$$\frac{(1-\alpha)^2}{\alpha}(1-\gamma)\kappa Q^{*\frac{1}{1-\alpha}} Z^* - \alpha(1-\alpha)\kappa Q^* - \frac{1-\alpha}{\alpha} Q^{*\frac{\alpha}{1-\alpha}} S^* L + \left[\frac{\alpha + (1-\alpha)\gamma}{\alpha}\right] \psi L + \frac{1-\alpha}{\alpha}\delta = 0. \tag{B-3}$$

From (B-2) and (B-3), I obtain $Z^* = F(Q^*)$ and $S^* = G(Q^*)$, respectively. Therefore, from (B-1), Q^* is derived from the following equation:

$$H(Q^*) \equiv \kappa Q^* + \kappa Q^{*\frac{1}{1-\alpha}} F(Q^*) - G(Q^*) F(Q^*)^{-1} L - \frac{\psi L}{1-\alpha} - \delta = 0.$$

Calculating the above equation, I can rewrite this as the following quadratic equation:

$$\kappa^2 M Q^{*2} + \kappa N Q^* + P = 0.$$

Solving the above quadratic equation, I obtain

$$Q^* = \frac{-N + \sqrt{N^2 - 4MP}}{2\kappa M} \text{ } or \tag{B-4}$$

$$Q^* = \frac{-N - \sqrt{N^2 - 4MP}}{2\kappa M}. \tag{B-5}$$

Here, I assume that ψL is high enough, and δ and ρ are low enough to obtain an interior solution of the positive growth rate. Further, $N < 0$ and $\kappa^2 N^2 - 4\kappa^2 MP > 0$ is assumed to be satisfied. Under the above assumptions, (B-5) cannot be an interior solution of positive growth rate because labor inputs to R&D activity are negative, that is, $L_A^* < 0$. Therefore, I focus only on (B-4) as the solution of the steady state.

C Appendix C

In this appendix, I consider the stability of the steady state. Linearizing (16), (17), and (18), around the steady state, I obtain

$$\begin{pmatrix} \dot{Z}_t \\ \dot{S}_t \\ \dot{Q}_t \end{pmatrix} = J \begin{pmatrix} Z_t - Z^* \\ S_t - S^* \\ Q_t - Q^* \end{pmatrix},$$

where $J \equiv \begin{pmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{pmatrix}$, and each element in this coefficient matrix, respectively, is given by

$$\begin{aligned}
J_{11} &= \kappa Q^{*\frac{1}{1-\alpha}} Z^* + S^* Z^{*-1} L, \\
J_{12} &= -L, \\
J_{13} &= \kappa Z^* + \frac{\kappa}{1-\alpha} Q^{*\frac{\alpha}{1-\alpha}} Z^{*2}, \\
J_{21} &= \kappa Q^{*\frac{1}{1-\alpha}} S^*, \\
J_{22} &= 0, \\
J_{23} &= \alpha^2 \kappa S^* + \frac{\kappa}{1-\alpha} Q^{*\frac{\alpha}{1-\alpha}} Z^* S^*, \\
J_{31} &= \frac{(1-\alpha)^2(1-\gamma)\kappa}{\alpha} Q^{*\frac{2-\alpha}{1-\alpha}}, \\
J_{32} &= \frac{\alpha-1}{\alpha} Q^{*\frac{1}{1-\alpha}} L, \\
J_{33} &= \frac{(1-\alpha)(1-\gamma)\kappa}{\alpha} Q^{*\frac{1}{1-\alpha}} Z^* \\
&\quad - \alpha(1-\alpha)\kappa Q^* - Q^{*\frac{\alpha}{1-\alpha}} S^* L.
\end{aligned}$$

As the dynamic system has one state variable, Z_t, and two jump variables, S_t, and Q_t, to ensure that the steady state is saddle-point stable, the characteristic equation with this coefficient matrix must have one positive eigenvalue and two negative eigenvalues.

If $\mathrm{Tr}J > 0$ and $\det J < 0$, the following two cases arise. In the first case, three eigenvalues are real numbers, of which two are positive and the third is negative. In the second case, two eigenvalues are complex with positive real parts, and the third is a negative real number. In each case, the steady state is saddle-point stable. In what follows, I show that $\mathrm{Tr}J > 0$ and $\det J < 0$.

The determinant of J is given by

$$\det J = A \times \begin{vmatrix} \kappa Q^{*\frac{1}{1-\alpha}} Z^* + S^* Z^{*-1} L & -1 & \kappa Z^* + \frac{\kappa}{1-\alpha} Q^{*\frac{\alpha}{1-\alpha}} Z^{*2} \\ Q^{*\frac{1}{1-\alpha}} & 0 & \alpha^2 + \frac{1}{1-\alpha} Q^{*\frac{\alpha}{1-\alpha}} Z^* \\ (1-\alpha)(1-\gamma)\kappa Q^* & -1 & (1-\gamma)\kappa Z^* - \alpha^2 \kappa Q^{*\frac{\alpha}{\alpha-1}} - \frac{\alpha}{1-\alpha} Q^{*-1} S^* L \end{vmatrix},$$

where $A \equiv \frac{1-\alpha}{\alpha} \kappa Q^{*\frac{1}{1-\alpha}} S^* L$. Calculating this, I obtain

$$\det J = A \times [-2\kappa M Q^* - N].$$

Therefore, substituting (B-4) into this, I can derive

$$\det J = A \times [-\sqrt{N^2 - 4MP}] < 0.$$

The trace of J is given by

$$\begin{aligned}\mathrm{Tr}J = &\kappa Q^*(1-\alpha)[1-\alpha(1-\alpha)(1-\gamma)] \\ &+ \frac{\psi L}{1-\alpha}\left[(1-\alpha)(1-\gamma) + \frac{(1-\alpha)^2(1-\gamma)}{\alpha}\right] \\ &+ \delta\left[\frac{(1-\alpha)^2(1-\gamma)}{\alpha}\right] + \rho\left[2 + \frac{(1-\alpha)^2(1-\gamma)}{\alpha}\right] > 0.\end{aligned}$$

Therefore, it is proved that the steady-state equilibrium is saddle-point stable.

D Appendix D

In this appendix, I derive the dynamic system of the socially optimal allocation. To solve the inter-temporal optimization problem of the social planner, the current-value Hamiltonian is set up as follows:

$$H^p = \log\left[K_t^\alpha(L-L_{At})^{1-\alpha} - I_t\right] + \mu_t^p[T_t I_t - \delta K_t] + q_t^p[\psi T_t L_{At}],$$

where μ_t^p and q_t^p represent the co-state variables. Restricting the analysis to an interior solution, I obtain the first-order conditions as follows:

$$\mu_t^p T_t = \frac{1}{K_t^\alpha(L-L_{At})^{1-\alpha} - I_t}, \tag{D-1}$$

$$\rho\mu_t^p - \dot{\mu}_t^p = \frac{\alpha K_t^{\alpha-1}(L-L_{At})^{1-\alpha}}{K_t^\alpha(L-L_{At})^{1-\alpha} - I_t} - \delta\mu_t^p, \tag{D-2}$$

$$q_t^p \psi T_t = \frac{(1-\alpha)K_t^\alpha(L-L_{At})^{-\alpha}}{K_t^\alpha(L-L_{At})^{1-\alpha} - I_t}, \tag{D-3}$$

$$\rho q_t^p - \dot{q}_t^p = \mu_t^p I_t + q_t^p \psi L_{At}. \tag{D-4}$$

Furthermore, the following transversality conditions must be satisfied:
$\lim_{t\to\infty} \mu_t^p K_t \exp\left(-\int_0^\infty r_s ds\right) = 0$ and $\lim_{t\to\infty} q_t^p T_t \exp\left(-\int_0^\infty r_s ds\right) = 0$.

From (D-1) and the resource constraint of final goods, I obtain

$$\mu_t^p = \frac{1}{c_t L T_t}. \tag{D-5}$$

From (D-1), (D-2), and (D-5), I obtain

$$K_t = \alpha^{\frac{1}{1-\alpha}} T_t^{\frac{1}{1-\alpha}}(L-L_{At})\left(\frac{\dot{c}_t}{c_t} + \rho + \delta + \frac{\dot{T}_t}{T_t}\right)^{\frac{1}{\alpha-1}}. \tag{D-6}$$

From (D-3) and the resource constraint of final goods, I obtain

$$q_t^p = \frac{(1-\alpha)K_t^\alpha(L-L_{At})^{-\alpha}}{\psi T_t} \times \frac{1}{c_t L}. \tag{D-7}$$

In the market economy, $q_t = \frac{(1-\alpha)K_t^\alpha (L-L_{At})^{-\alpha}}{\psi T_t}$. Therefore, the relation between q_t and q_t^p is given by

$$q_t = q_t^p c_t L. \tag{D-8}$$

From (D-7), (D-8), and the resource constraint of labor, I obtain

$$L_{At} = L - (1-\alpha)\psi^{-1}\kappa {Q_t}^{\frac{1}{1-\alpha}} Z_t \tag{D-9}$$

From the law of motion of the production technology and (D-9), I obtain

$$\frac{\dot{T}_t}{T_t} = \psi L - (1-\alpha)\kappa Q_t^{\frac{1}{1-\alpha}} Z_t. \tag{D-10}$$

From (D-6), (D-9), and (D-10), I obtain

$$\frac{\dot{c}_t}{c_t} = \alpha\kappa Q_t + (1-\alpha)\kappa Q_t^{\frac{1}{1-\alpha}} Z_t - \psi L - (\delta + \rho). \tag{D-11}$$

From the law of motion of capital stocks, the resource constraints of final goods and labor, and (D-9), I obtain

$$\frac{\dot{K}_t}{K_t} = \kappa Q_t - S_t Z_t^{-1} L - \delta.$$

From the resource constraints of final goods and labor, (D-4), (D-5), (D-7), (D-9), (D-10), and (D-11), I obtain

$$\begin{aligned}\frac{\dot{q}_t}{q_t} &= \alpha\kappa Q_t + 2(1-\alpha)\kappa Q_t^{\frac{1}{1-\alpha}} Z_t - \kappa Q_t^{\frac{1}{1-\alpha}} Z_t \\ &\quad + Q_t^{\frac{\alpha}{1-\alpha}} S_t L - 2\psi L - \delta.\end{aligned}$$

Thus, from definitions of $\{Z_t, S_t, Q_t\}$, I obtain (19), (20), and (21).

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