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Dynamic Analysis of Outsourcing

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Abstract

This paper constructs a North-South endogenous growth model where final good producers in the North determine whether they outsource the production of intermediate goods to the South or not. When the final good producers outsource the production of intermediate goods to the Northern firms, the price of intermediate goods is high, whereas the cost of outsourcing is low. On the other hand, when they outsource to the Southern firms, the price of intermediate goods is low, whereas the cost of outsourcing is high. Using this model, this paper shows that, as the economy develops, the wage inequality between the North and the South widens and that the outsourcing location for the Northern final good producers switches from the North to the South.

Keywords: Outsourcing, North-South, R&D, Economic Growth
JEL classification: F43, O31

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1 Introduction

When firms produce final goods, they use many varieties of intermediate goods. The firms can buy the intermediate goods either from their home country or from foreign countries. Recently, many firms have outsourced the production of needed intermediate goods: they buy them from all over the world and use them to produce the final goods. For example, according to the 1998 World Trade Organization annual report, in the production of an "American" car, 30 percent of the car's value originates in Korea, 17.5 percent in Japan, 7.5 percent in Germany, 4 percent in Taiwan and Singapore, 2.5 percent in the United Kingdom, and 1.5 percent in Ireland and Barbados. Thus, only 37 percent of the production value is generated in the United States. Thus, there are rarely all-U.S. products in the U.S..

Feenstra and Hanson (1996) found that the import of intermediate goods rapidly increased during the period 1972 to 1990. Since there are no figures directly representing outsourcing, they measured it by calculating the share of the imported intermediate goods within the total purchase of non-energy materials.[1] The increase in international trade of intermediate goods could be because of the decision of final good firms in the developed countries to switch from intermediate goods suppliers in the home country to those in a country where the wage rate is lower, in order to reduce costs.[2]

To investigate the outsourcing issues, researchers have used static models. One of these models is developed by Antràs and Helpman (2004). It is a two-country Ricardian model of international trade. In their model, final good producers with different productivity levels decide to whether to integrate the suppliers or to outsource the production of intermediate goods. Furthermore, they have to choose a country to which to outsource the production of intermediate goods. Antràs and Helpman (2004) show that the final good producers' strategy crucially depends on

[1]Many researchers also analyze outsourcing empirically, for example, Campa and Goldberg (1997), Hummels, Ishii, and Yi (2001), Yeats (2001), and Hanson, Mataloni and Slaughter (2001, 2005).

[2]When Canon Inc-. produced scanners in 1997, they outsourced the production of intermediate goods to domestic firms. As Taiwanese companies entered the scanner market and the price competition became severe, Canon was concerned about the high wage rate in Japan. As a result, Canon outsourced the production of intermediate goods to Taiwanese firms whose wage rates were lower. Recently, because the wage rate in Taiwan became higher, Canon outsourced the production of intermediate goods to China, where the wage rate is much lower.

their productivity. Although this study has suggested some important results about the final good firms' strategies, it only analyzes the static equilibria.

In contrast to this static analysis, Naghavi and Ottaviano (2008) present a North-South endogenous growth model with offshoring in order to investigate how many final good producers outsource the production of intermediate goods to the South. They analyze the two types of equilibrium: first, all of the producers outsource the production of intermediate goods; second, only some of the producers outsource the production of intermediate goods. Gao (2007) also proposes an endogenous growth model and analyzes the relationship between trade cost and the place where the intermediate goods are produced. He shows that, as the trade cost falls, the number of intermediate goods produced in the South increases. Although these studies are based on dynamic models, they focus only on the steady state.

In this paper, we construct an endogenous growth model of the variety-expansion type to investigate the dynamic choice problem of final good producers: when should they outsource the production of their inputs to the South? We explore transition paths and the point at which final good producers outsource the production of intermediate goods from the domestic suppliers to the foreign suppliers. On the one hand, when the final good producers outsource the production of intermediate goods to Northern firms, the price of intermediate goods is high, whereas the management cost paid by the final good producers to obtain the intermediate goods is low. On the other hand, when the final good producers outsource the production of intermediate goods to Southern firms, the price of intermediate goods is low, whereas the management cost is high. Therefore, the final good producers face the problem of where to outsource the production of intermediate goods.

This paper shows that, as the economy develops, the wage inequality between the North and the South widens, and the place to which the final good producers outsource the production of their inputs is switched from the North to the South. Moreover, in the steady state, a decrease in the management cost paid by the final good producers to obtain the intermediate goods from Southern firms causes an increase in the North of both the number of firms and the wage rate in the North. Indeed, an increase in the productivity of R&D raises the wage rate in the North. An increase in the labor endowment in the North also increases the number of firms. An increase in the labor force in the South affects neither the number of firms nor the wage rate in the North.

The remainder of the paper is structured as follows. Section 2 introduces the model. In Section 3, we derive the equilibrium path of the model and prove that there exists a unique equilibrium path converging to the steady state. In Section 4, we conduct comparative statics with respect to the management cost, population size, and productivity of R&D. In Section 5, we deal with the case where the equilibrium path is indeterminate. Section 6 concludes.

2 The Model

We develop a dynamic general equilibrium model where final good producers outsource production of the intermediate goods to intermediate producers either in their home country or in foreign countries. Our model has a similar structure to the model of Grossman and Helpman (1991, Ch. 3).

The world economy consists of two countries, the North and the South indexed by $l \in \{N, S\}$. The population size in the world is 1. Each individual lives forever and is endowed with L^l units of labor services, which is inelastically supplied at each point of time. There exist three types of goods: a homogeneous good, intermediate goods, and final goods. The homogeneous good can be produced only in the South, and the final goods can be produced only in the North. The intermediate goods can be produced in either country. Individuals consume the homogeneous good and the final goods. Figure 1 shows the production structure of the world economy. In the North, there can exist at most three sectors: a R&D sector, a final goods sector, and an intermediate goods sector. In the South, there can exist at most two sectors: an intermediate goods sector, and a homogeneous good sector. R&D activities can be conducted only in the North. The final good firms outsource the production of intermediate goods either to the Northern firms or to the Southern firms to maximize their profits. When the final good producers outsource the production of intermediate goods to the Northern suppliers, there are three sectors in the North: the R&D sector, the final good sector, and the intermediate goods sector. In the South, there is only one sector: the homogeneous good sector. On the other hand, when the final good producers outsource the production of intermediate goods to the Southern firms, there exist only two sectors in the North: the R&D sector and the final good sector. In the South, there are now two sectors: the intermediate goods sector and the homogeneous good sector.

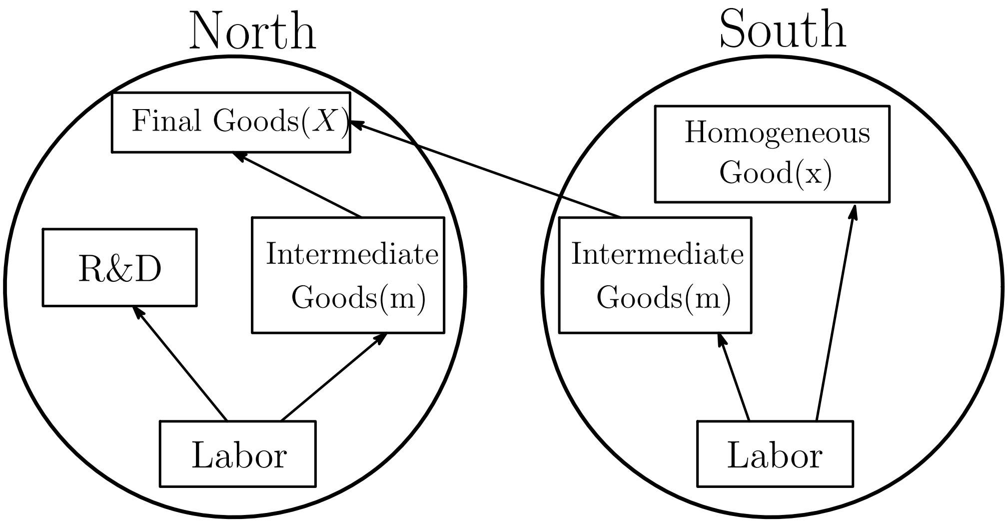

Figure 1: Production structure of the world economy

2.1 Consumers

Individuals in both countries have identical preferences:

$$\int_t^{\infty} e^{-\rho(\tau-t)} U(\tau) d\tau, \quad 0 < \rho < 1, \tag{1}$$

where ρ is the constant subjective discount rate. $U(\tau)$ is is the instantaneous utility per person at time τ. It is specified as follows:

$$U(\tau) = x(\tau) + \frac{1}{\mu} X(\tau)^{\mu}, \quad 0 < \mu < 1, \tag{2}$$

where μ is a parameter, $x(\tau)$ stands for consumption of the homogeneous good at time τ, and $X(\tau)$ is a composite good at time τ that is made up of differentiated final goods. For simplicity, we drop the time index τ from all the variables. A composite good X is given by:

$$X = \left[\int_0^n x(i)^{\alpha} di \right]^{\frac{1}{\alpha}}, \qquad 0 < \alpha < 1, \tag{3}$$

where $x(i)$ represents consumption of different final goods i and n stands for the number of final goods. $\frac{1}{1-\alpha}$ is the elasticity of substitution between any two varieties in a given sector. If α is close to one, the goods are nearly perfect substitutes and the sector is highly competitive. If α is close to zero, the goods are distinct products and the sector becomes monopolistic. We assume that $\alpha > \mu$ in the following analysis.

The utility maximization problem of the individual can be solved in two steps. The first step is to solve the following static optimization problem:

$$\max_{x,x(i)} \quad x + \frac{1}{\mu} X^{\mu},$$
$$\text{subject to } \; x + \int_0^n P(i)x(i)di = E,$$

where the homogeneous good is chosen to be the numeraire, $P(i)$ stands for the price of the product i, and E is the total expenditure.

From the first-order condition, we can obtain the following inverse demand function:

$$P(i) = X^{\mu-\alpha} x(i)^{\alpha-1}, \; i \in [0, n]\,. \tag{4}$$

The second step is to solve the intertemporal optimization problem. From (4), the indirect utility function is given by:

$$U = E - \left(1 - \frac{1}{\mu}\right) \left[\int_0^n P(i)^{\frac{\alpha}{\alpha-1}} di\right]^{\frac{\mu(\alpha-1)}{\alpha(\mu-1)}}. \tag{5}$$

As is clear from the indirect utility function (5), the marginal utility of expenditures is constant. The market interest rate at time t, $r(t)$, must be equal to the subjective discount rate, as follows:

$$r(t) = \rho \qquad \text{for all } t\,. \tag{6}$$

2.2 Production

We denote the wage rate in the North by w_N, and the wage rate in the South by w_S. This economy has no transportation costs and tariffs.

Production of the homogeneous good uses labor only. The homogeneous good is supposed to be produced only in the South. The production of one unit of the homogeneous good requires one unit of Southern labor. We assume that the homogeneous good market is perfectly competitive. Thus, the price of the homogenous good becomes equal to the wage rate in the South. Since the homogeneous good is chosen to be the numeraire, the wage rate in the South becomes unity, that is, $w_S = 1$.

Production of each final good requires a variety-specific intermediate good. We assume that all final goods are produced only in the North. The production function of firm i is given by:

$$x(i) = \theta m(i), \tag{7}$$

where $m(i)$ stands for an intermediate good used for the production of final good i, and θ is a productivity parameter. The intermediate input $m(i)$ can be produced in both countries. Production of one unit of each intermediate good requires one unit of labor. We assume that perfect competition prevails in the intermediate goods markets in both countries. Thus, when it is produced in the North, the price of each intermediate good is equal to the marginal cost, that is, the wage rate in the North. Similarly, the price of each intermediate good produced in the South becomes the wage rate in the South.

We assume that the final good producers have to pay the management costs when they outsource the production of intermediate goods. The labor input is required for management activities like supervision, quality control, communications costs, and contracting with the intermediate suppliers. When the final good producers buy the intermediate goods from Northern firms, they need the Northern labor input, f^N to manage these problems, and when they buy the intermediate goods from a Southern firm, they similarly need the Southern labor input f^S.

Furthermore, the final good producers in the North face problems of differences in language, laws, and customs between the countries when they outsource the production of intermediate goods to the South. Therefore, we assume that when the intermediate producers are located in the South, the labor input for the management tasks is higher than when they are located in the North, that is in their home country:

$$f^N < f^S. \tag{8}$$

By using (4), we can obtain the revenue of each final good producer as follows:

$$\begin{aligned} R(i) =& P(i)x(i) \\ =& X^{\mu-\alpha}\theta^{\alpha}m(i)^{\alpha}. \quad l = N \,\text{or}\, S,\ i \in [0, n]. \end{aligned} \tag{9}$$

We next consider the profit-maximization problem of the final good producers. Because the perfect competition prevails in the intermediate goods market, the price of intermediate goods is given by w_l ($l = N$ *or* S). Thus the final good producer i maximizes profits as follows:

$$\pi^l = R(i) - w_l m(i) - w_N f^l, \quad l = N \,\text{or}\, S,\; i \in [0, n]. \tag{10}$$

The profit-maximizing input of the intermediate good is given by:

$$m(i) = X^{\frac{\mu-\alpha}{1-\alpha}} \alpha^{\frac{1}{1-\alpha}} \theta^{\frac{\alpha}{1-\alpha}} w_l^{\frac{-1}{1-\alpha}},\; l = N \,\text{or}\, S. \tag{11}$$

By substituting (7) and (11) into (3), we obtain:

$$X = n^{\frac{1-\alpha}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} \theta^{\frac{1}{1-\mu}} w_l^{\frac{-1}{1-\mu}}, \qquad i \in [0, n]. \tag{12}$$

From (12), we see that X increases with the number of final good firms n and the productivity θ. On the other hand, X decreases with the wage rate w_l. Using (11) and (12), we can obtain the profit functions as follows:

$$\pi^l = n^{\frac{\mu-\alpha}{\alpha(1-\mu)}} \left(\frac{\alpha\theta}{w_l}\right)^{\frac{\mu}{1-\mu}} (1-\alpha) - w_N f^l, \qquad l = N \,\text{or}\, S,\; i \in [0, n]. \tag{13}$$

The profits of the final good producers decrease with the wage rate in the North w_N and the number of the final good firms n. On the other hand, the profit of the final good producer increases with the productivity θ.

2.3 R&D sector

The R&D activities of the present model follow the model of Grossman and Helpman (1991, Ch. 3). We assume that Southern firms cannot innovate and imitate a new variety of final good. The final good producers enter into the R&D race and finance the cost of R&D by issuing equity in the stock market. The equity is bought by individuals who live in either countries. The stock value of the final good producers at time t is equal to the present discounted value of its profit stream subsequent to t. Suppose that the final good producers change the intermediate firms from the North to the South at time s. Then, the stock value of the final good producers at time t is given by:

$$v^N = \int_t^s e^{-r(\tau)(\tau-t)} \pi^N d\tau + e^{-r(s)(s-t)} \int_s^\infty e^{-r(\tau)(\tau-s)} \pi^S d\tau. \tag{14}$$

Then, the stock value of the final good producers at time t, when they outsource the intermediate goods to the Southern firms, is given by:

$$v^S = \int_t^\infty e^{-r(\tau)(\tau - t)} \pi^S d\tau. \tag{15}$$

Differentiation of (14) and (15) with respect to time t yields the following no-arbitrage conditions:

$$\begin{aligned} \dot{v^N} &= -\pi^N + r(t) \left(\int_t^s e^{-r(\tau)(\tau-t)} \pi^N d\tau + e^{-r(s)(S-t)} \int_s^\infty e^{-r(\tau)(\tau-s)} \pi^S d\tau \right), \\ &= -\pi^N + r(t) v^N; \end{aligned} \tag{16}$$

$$\dot{v^S} = -\pi^S + r(t) v^S. \tag{17}$$

The final good producers hire labor to develop blueprints. We presume that there exist knowledge spillovers in R&D activities: the more innovations have been created previously, the lower the cost of innovating. We assume that L_A units of labor for R&D activity for a time interval dt produce a new variety of final good according to:

$$dn = \frac{L_A n}{a} dt. \tag{18}$$

The cost of the R&D activities is $w_N L_A dt$ because R&D sector is located only in the North. To produce innovative the blueprints creates value for the final good producers of $v^l dn$ since each blueprint has a market value of v^l. We assume that there is free entry into the R&D race. Therefore, the following free-entry condition must hold:

$$v^l \leq \frac{a w_N}{n}, \quad \text{with equality whenever } \dot{n} \equiv \frac{dn}{dt} > 0. \tag{19}$$

2.4 Labor market

Labor market equilibrium requires labor supply to be equal to labor demand. We label the economy *Regime N* when the final good producers outsource the production of intermediate goods to suppliers in the North, and *Regime S* when they outsource to suppliers in the South. In *Regime N*, the demand for labor in the North comes from the intermediate sector, the R&D sector, and the management tasks. In the South, the demand for labor comes only from the homogeneous good sector. Therefore, the labor market equilibrium conditions become as follows:

$$
\begin{aligned}
L^N =& L_A + L_M^N + f^N n, \\
L^S =& x^N,
\end{aligned}
$$

where x^N is the labor demand for production of the homogeneous good in the South, L_M^N is the labor demand for production of the intermediate goods in the North, and $f^N n$ is the management tasks of the final good sector. On the other hand, in *Regime S*, the demand for labor in the North comes from the R&D sector and the management tasks. In the South, the demand for labor comes from the homogeneous good sector and the intermediate goods sector. Therefore, the labor market equilibrium conditions become as follows:

$$
\begin{aligned}
L^N =& L_A + f^S n, \\
L^S =& x^S + L_{M,S}^S,
\end{aligned}
$$

where x^S is the labor demand for production of the homogeneous good, L_M^S is the labor demand for production of the intermediate goods in the South, and $f^S n$ is the management input of the final good sector.

The production of one unit of the homogeneous good and of the intermediate goods requires one unit of labor each. Thus, production of the homogeneous good and the intermediate goods equals the labor demand. We can write the production of the homogeneous good by using the budget constraint, (4), (7), (11), and (12).

$$
\begin{aligned}
x^l &= E - \int_0^n P(i)x(i)di \\
&= E - X\left[\int_0^n x(i)^\alpha di\right] \\
&= E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}}\left(\frac{w_l}{\alpha\theta}\right)^{\frac{-\mu}{1-\mu}}. \qquad (20)
\end{aligned}
$$

We can derive the production of the intermediate goods by using (11) and (12) as follows:

$$
\begin{aligned}
L_M^l &= \int_0^n m(i)\ di \\
&= n^{\frac{\mu(1-\alpha}{\alpha(1-\mu)}}\alpha^{\frac{1}{1-\mu}}\theta^{\frac{\mu}{1-\mu}}w_l^{\frac{-1}{1-\mu}}, \quad l \in \{N, S\}. \qquad (21)
\end{aligned}
$$

In *Regime N*, we can rewrite the labor market clearing conditions in both countries using (18), (21), and (20) as follows:

$$\text{North} \qquad L^N = \frac{a\dot{n}}{n} + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} \theta^{\frac{\mu}{1-\mu}} w_N^{\frac{-1}{1-\mu}} + f^N n, \tag{22}$$

$$\text{South} \qquad L^S = E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \left(\frac{w_N}{\alpha\theta}\right)^{\frac{-\mu}{1-\mu}}. \tag{23}$$

Using (18), (21), and (20), we can rewrite the labor market clearing conditions in *Regime S* as follows:

$$\text{North} \qquad L^N = \frac{a\dot{n}}{n} + f^S n, \tag{24}$$

$$\text{South} \qquad L^S = E - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (\alpha\theta)^{\frac{\mu}{1-\mu}} + n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} \theta^{\frac{\mu}{1-\mu}}. \tag{25}$$

3 The Equilibrium Path

In this section, we examine the dynamics of the economy. We first consider the dynamic behaviors of *Regime N* and *Regime S* separately. Finally, we integrate these dynamic behaviors.

3.1 The dynamic behavior in *Regime N*

The equilibrium conditions are (6), the no-arbitrage condition, (16), the free-entry condition, (19), and the labor market clearing condition, (22). From the labor market clearing condition, we can derive the differential equation for the number of final goods, n, as follows:

$$\dot{n} = \frac{n}{a}\left[L^N - f^N n - n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} \alpha^{\frac{1}{1-\mu}} \theta^{\frac{\mu}{1-\mu}} w_N^{\frac{-1}{1-\mu}}\right]. \tag{26}$$

Using (6), (16), and (19), we can obtain the differential equation for the wage rate in the North, w_N, as follows:

$$\dot{w_N} = \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1}{a} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (\alpha\theta)^{\frac{\mu}{1-\mu}} w_N^{\frac{-\mu}{1-\mu}}. \tag{27}$$

These two equations, (26) and (27), constitute the dynamic system of *Regime N*.

Figure 2 depicts the phase diagram for this system on the (n, w_N) plane. The intersection point of the two curves $\dot{n} = 0$ and $\dot{w_N} = 0$ at point E is the steady state of this system. The equation for the $\dot{n} = 0$ locus is given by:

$$w_N = \left(L^N - f^N n\right)^{-(1-\mu)} \alpha\theta^{\mu} n^{\frac{\mu(1-\alpha)}{\alpha}}. \tag{28}$$

The $\dot{n} = 0$ locus is upward sloping and w_n approaches infinity as n tends to $\frac{L^N}{f^N}$. On the other hand, the equation for the $\dot{w_N} = 0$ locus is given by:

$$w_N = \left(a\rho + L^N\right)^{-(1-\mu)} (\alpha\theta)^{\mu} n^{\frac{\mu(1-\alpha)}{\alpha}}. \tag{29}$$

The $\dot{w_N} = 0$ locus is also upward sloping and concave. Furthermore, the slope of this locus diminishes monotonically toward zero as n approaches infinity. Thus, there exists t a unique steady state. The steady state value of n is given by:

$$n^* = \frac{(1-\alpha)L^N - \alpha a\rho}{f^N}. \tag{30}$$

We can show that the steady state E becomes a saddle point (see Appendix).

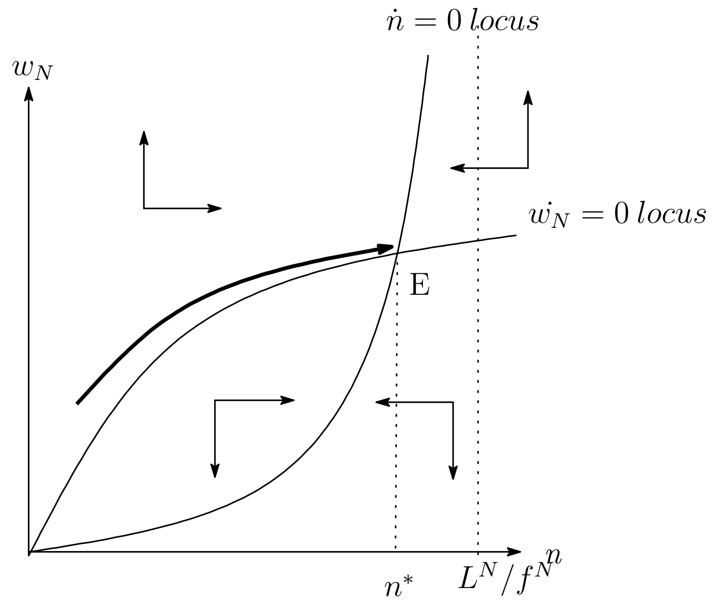

Figure 2: Phase diagram of *Regime N*

3.2 The dynamic behavior in *Regime S*

The equilibrium conditions are (6), the no-arbitrage condition, (17), the free-entry condition, (19), and the labor market clearing condition, (24). From the labor market clearing condition, we can derive the differential equation for the number of firms, n, as follows:

$$\dot{n} = \frac{n}{a}\left[L^N - f^S n\right]. \tag{31}$$

Using (6), (17), and (19), we can obtain the differential equation for the wage rate in the North, w_N, as follows:

$$\dot{w_N} = \left(\rho + \frac{L^N}{a}\right) w_N - \frac{1}{a} n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}} (\alpha\theta)^{\frac{\mu}{1-\mu}} (1-\alpha) \tag{32}$$

In the same way as in the former section, these two equations, (31) and (32), constitute the dynamic system of *Regime S*.

Figure 3 depicts the phase diagram for this system on the (n, w_N) plane. The intersection point of the two curves $\dot{n} = 0$ and $\dot{w_N} = 0$ at point $E^{'}$ is the steady state of this system. The equation for the $\dot{n} = 0$ locus is represented by:

$$n = \frac{L^N}{f^S}. \tag{33}$$

The $\dot{n} = 0$ locus is a vertical line at $n = \frac{L^N}{f^S}$. On the other hand, the equation for the $\dot{w_N} = 0$ locus is given by:

$$w_N = \frac{(\alpha\theta)^{\frac{\mu}{1-\mu}} (1-\alpha) n^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}}}{a\rho + L^N}. \tag{34}$$

The $\dot{w_N} = 0$ locus is also upward sloping and concave. Furthermore, the slope of this locus diminishes monotonically toward zero as n approaches to infinity. Thus, there exists a unique steady state. The steady state value of n is given by:

$$n^{**} = \frac{L^N}{f^S}. \tag{35}$$

From (34) and (35), we can obtain the steady state value of w_N:

$$w_N^{**} = \frac{(\alpha\theta)^{\frac{\mu}{1-\mu}} (1-\alpha)}{a\rho + L^N} \left(\frac{L^N}{f^N}\right)^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}}. \tag{36}$$

We can show that the steady state $E^{'}$ becomes a saddle point (see Appendix).

3.3 A boundary between *Regime N* and *Regime S*

When the final good firms produce, they have to outsource the intermediate goods to either the North or the South. The final good firms decide to buy the intermediate goods from the suppliers in the North (South) when they can obtain higher profits by buying them from the Northern (Southern) suppliers. We investigate the condition under which the profits of final good producers in *Regime N* become

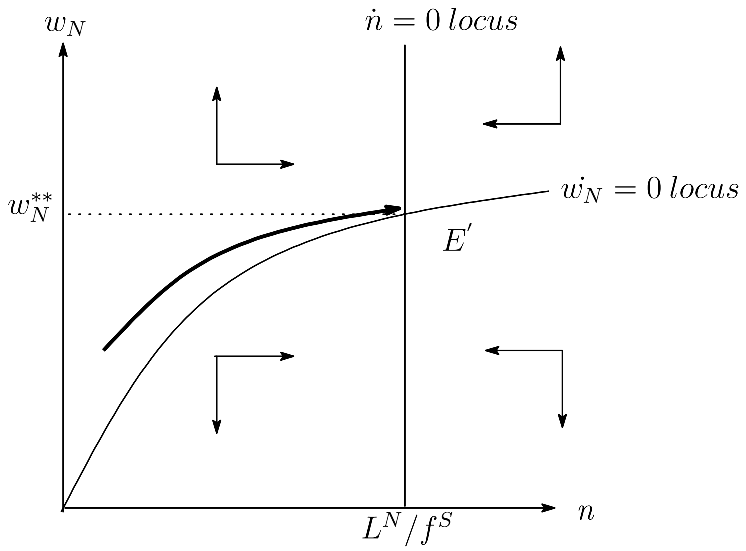

Figure 3: Phase diagram of *Regime S*

the same as the profits in *Regime S*, that is $\pi^N = \pi^S$. From (13), the boundary condition between *Regime N* and *Regime S* is given by:

$$n = \left(\frac{1-\alpha}{f^S - f^N}\right)^{\frac{\alpha(1-\mu)}{\alpha-\mu}} (\alpha\theta)^{\frac{\alpha\mu}{\alpha-\mu}} \left(\frac{1 - w_N^{\frac{-\mu}{1-\mu}}}{w_N}\right)^{\frac{\alpha(1-\mu)}{\alpha-\mu}}. \tag{37}$$

Figure 4 depicts this boundary condition. The boundary line has the inverse-C shape where n takes a maximum value at $w_N^{max} = (1-\mu)^{\frac{\mu-1}{\mu}}$. As w_N approaches infinity, n approaches zero. The inequality $\pi^N < \pi^S$ holds in the shaded area of Figure 4, thus *Regime S* prevails in the shaded area and the economy follows the equilibrium condition of *Regime S*. The other area corresponds to $\pi^N > \pi^S$. This means that the economy is in *Regime N*. Therefore, in this area, the economy follows the equilibrium condition of *Regime N*. In the next section, these three figures are integrated into one figure.

We can give the intuition of Figure 4 as follows. Suppose that the number of firms n takes a constant value. When the wage rate in the North is small, the final good producers obtain the intermediate goods from the North. As the wage rate in the North increases, the intermediate goods made in the North become expensive compared with the intermediate goods made in the South. Therefore, the final good producers outsource the production of intermediate goods to the South. However, the greater the North-South wage rate difference, the more expensive the management cost of outsourcing production to the South. Thus, the final good producers cannot pay the management cost, give up outsourcing the production of intermediate inputs to the South, and obtain them instead from the Northern firms.

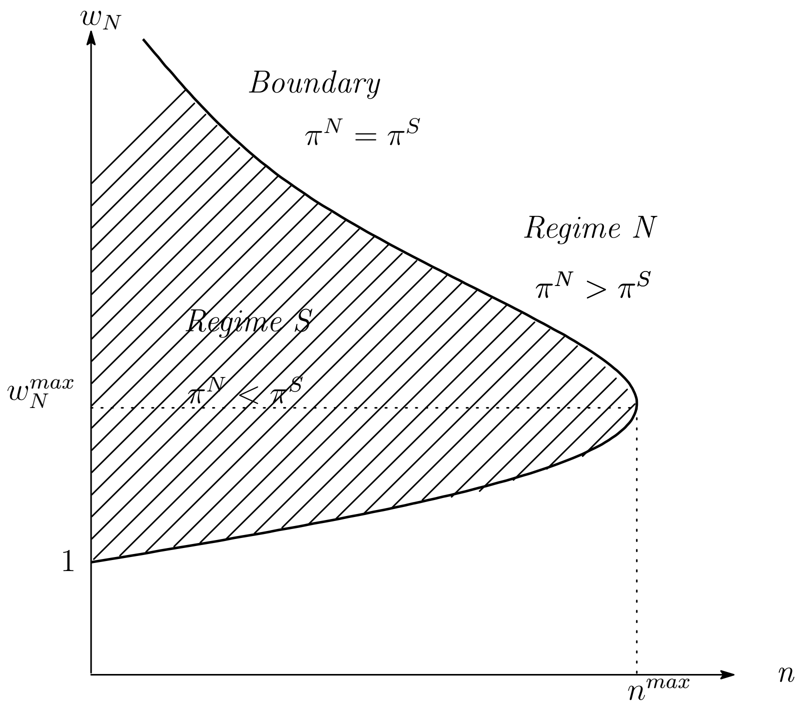

Figure 4: Boundary condition between *Regime N* and *Regime S*

3.4 Equilibrium path from *Regime N* to *Regime S*

In the previous subsections, we explored the dynamic behaviors in *Regime N* and *Regime S* and showed the boundary condition between *Regime N* and *Regime S*. We now consider the relative positions of the $\dot{n} = 0$ and $\dot{w_N} = 0$ loci in *Regime N* and *Regime S* in order to integrate the two phase diagrams of into one.

We first compare the curve $\dot{n} = 0$ in *Regime N* with that in *Regime S*. From $\frac{L^N}{f^N} > \frac{L^N}{f^S}$, there exists an intersection between the curve $\dot{n} = 0$ in *Regime N* and that in *Regime S*. Next, we compare the locus of $\dot{w_N} = 0$ in *Regime N* with that in *Regime S*. The intersection of the curve $\dot{w_N} = 0$ in *Regime N* with that in *Regime S*, $\tilde{n}$, is:

$$\tilde{n} = \left(a\rho + L^N\right)^{\frac{\alpha(1-\mu)}{\mu(1-\alpha)}} (\alpha\theta)^{\frac{-\alpha}{1-\alpha}} (1-\alpha)^{\frac{-\alpha(1-\mu)}{\mu^2(1-\alpha)}} . \tag{38}$$

When the inequality $n < \tilde{n}$ holds, the $\dot{w_N} = 0$ locus in *Regime N* is above that in *Regime S*. On the other hand, when the inequality $n > \tilde{n}$ holds, the $\dot{w_N} = 0$ locus of *Regime S* is above the $\dot{w_N} = 0$ locus of *Regime N*.

We next focus on the equilibrium path that the economy evolves from *Regime N* to *Regime S*. There are three sufficient conditions that guarantee the existence of the equilibrium path that the economy evolves. The first condition is that the steady state in *Regime S* exists in the shaded area in Figure 4 and that the equilibrium path doesn't converge to the steady state in *Regime N*. In the steady state in *Regime S*, the wage rate in the North is w_N^{**}. When the wage rate in the North is w_N^{**}, let n_1 be the profits of final good producers in *Regime N*, which become the same as the profits in *Regime S*. Thus, the condition that the steady state in *Regime S* exists in the shaded area in Figure 4 is $n^{**} < \hat{n}_1$. This inequality can be expressed as follows:

$$\left(\frac{1-\alpha}{f^S - f^N}\right)^{\frac{\alpha(1-\mu)}{\alpha-\mu}} (\alpha\theta)^{\frac{\alpha\mu}{\alpha-\mu}} \left\{\frac{1 - w_N^{**\frac{-\mu}{1-\mu}}}{w_N^{**}}\right\}^{\frac{\alpha(1-\mu)}{\alpha-\mu}} > \frac{L^N}{f^S}. \tag{39}$$

The second condition should satisfy the following two preconditions: there exists an intersection of the $\dot{w_N} = 0$ locus in *Regime N* with the boundary line; and the equilibrium path does not converge to the steady state in *Regime N*. First, we show the former precondition. If an intersection exists, we can find the real numbers that satisfy the following equation from (29) and (37):

$$\hat{n}_2 = \left(\frac{1-\alpha}{f^S - f^N}\right)^{\frac{\alpha(1-\mu)}{\alpha-\mu}} (\alpha\theta)^{\frac{\alpha\mu}{\alpha-\mu}} \left\{\frac{1 - \left[(a\rho + L^N)^{-(1-\mu)}(\alpha\theta)^{\mu}\hat{n}_2^{\frac{\mu(1-\alpha)}{\alpha}}\right]^{\frac{-\mu}{1-\mu}}}{(a\rho + L^N)^{-(1-\mu)}(\alpha\theta)^{\mu}\hat{n}_2^{\frac{\mu(1-\alpha)}{\alpha}}}\right\}^{\frac{\alpha(1-\mu)}{\alpha-\mu}}, \tag{40}$$

where $\hat{n}_2$ is defined as the smallest real numbers satisfying this equation. If $\hat{n}_2$ exists, this implies the existence of an intersection between the $\dot{w_N} = 0$ locus in *Regime N* and the boundary line. The second precondition is that $\hat{n}_2$ is smaller than the number of final good firms in the steady state in *Regime N*, (30). That is:

$$\hat{n}_2 < \frac{(1-\alpha)L^N - a\alpha\rho}{f^N}. \tag{41}$$

Note that if $\hat{n}_2$ is larger than the steady state in *Regime N*, then the equilibrium path will remain in *Regime N*.

Finally, the third condition is that the intersection point of the $\dot{w_N} = 0$ locus in *Regime S* with (37) is below the intersection point of the two $\dot{w_N} = 0$ loci in both regimes, that is, $\hat{n}_2 > \tilde{n}$. Therefore, the condition is represented by:

$$\hat{n}_2 > \left(a\rho + L^N\right)(\alpha\theta)^{\frac{-\alpha}{1-\alpha}}(1-\alpha)^{\frac{-\alpha(1-\mu)}{\mu^2(1-\alpha)}}. \tag{42}$$

When the three necessary conditions, (39), (41), and (42), are satisfied, we can depict the phase diagram for that system on the (n, w_N) plane. Figures 2, 3, and 4 are integrated into Figure 5. In Figure 5, the arrow shows the transitional dynamics. These results are stated as the following proposition.

Proposition 1. *Suppose that parameters satisfy conditions* (39)*,* (41)*, and* (42)*. Then there exists the equilibrium path along which the economy evolves from Regime N to Regime S. Moreover, the steady state is a saddle point.*

Suppose that the initial number of firms satisfies $n(0) < \frac{L^N}{f^S}$. At the initial point, the economy is in *Regime N*. Then n and w_N increases and eventually the trajectory crosses the boundary line, (37). At the intersection point of the trajectory with the boundary line, the final good firms can outsource the intermediate inputs to either the Northern firms or the Southern firms. After the crossing, the economy turns from *Regime N* into *Regime S* and follows the stable path towards the steady state E'. Therefore, the final good producers change the region to which they outsource the production of intermediate inputs from the North to the South as the number of firms increases and the wage gap between the North and the South increases.

4 Comparative Statics

In this section, we conduct comparative statics analysis of the steady state.

4.1 A Lower the management cost in *Regime S*

A decrease in the management cost in *Regime S* is expressed by a decrease in f^S. For example, if the number of people in the South who can speak the language of the North increases, the number of lawyers in the South increases, and the amount of bureaucratic corruption decreases, then the management cost can decrease.

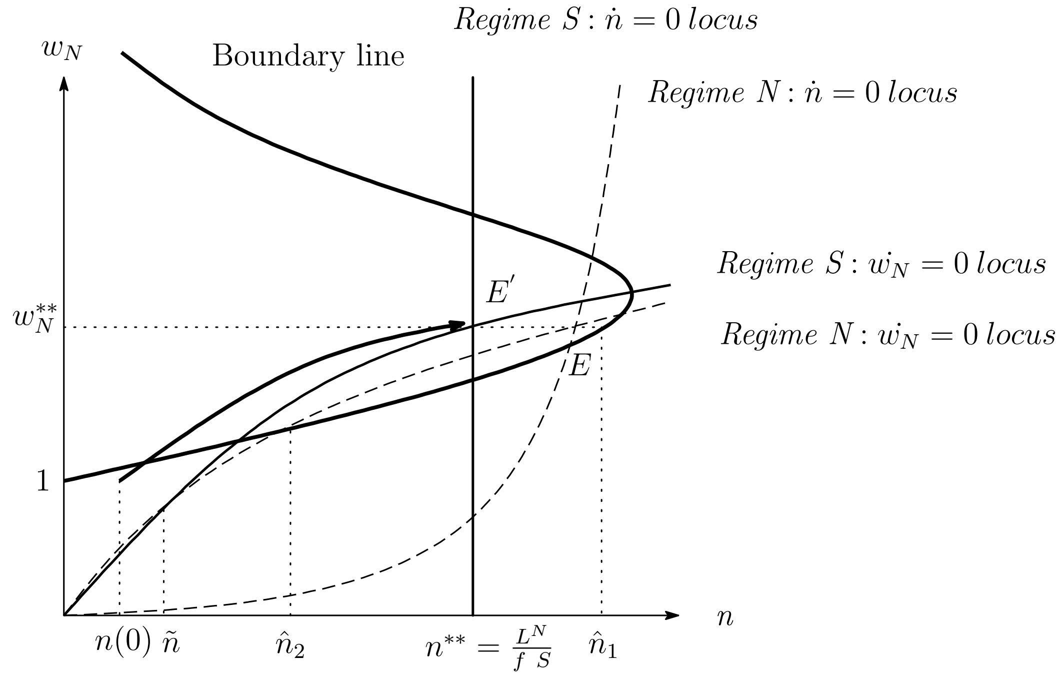

Figure 5: The phase diagram from *Regime N* to *Regime S*

In the steady state, a decrease in f^S affects the number of firms and the wage rate in the North as follows:

$$\frac{\partial n^{**}}{\partial f^S} = -\frac{L^N}{(f^S)^2} < 0, \tag{43}$$

$$\frac{\partial w_N^{**}}{\partial f^S} = -\frac{\mu(1-\alpha)}{\alpha(1-\mu)}(a\rho + L^N)(\alpha\theta)^{\frac{\mu}{1-\mu}}(1-\alpha)(\frac{L}{f^S})^{\frac{\mu(1-\alpha)}{\alpha(1-\mu)}}\frac{1}{f^S} < 0. \tag{44}$$

In the steady state, a decrease in f^S increases the number of firms and the wage rate in the North. Intuitively, when the management cost in *Regime S* decreases in the steady state, the labor force used in the management sector decreases and that in the R&D sector increases. Therefore, the number of final good firms increases. When the number of final good firms increases, R&D activities become much easier, from (18), and the labor demand for the R&D sector increases. Therefore, the wage rate in the North increases. Thus, we can summarize these results as the following proposition.

Proposition 2. *A decrease in the management cost in Regime S promotes obtaining the intermediate goods from the South and the number of firms and relative wage rate in the North.*

4.2 A higher productivity of R&D

An increase in the productivity of R&D is expressed by a decrease in a. The effect of an increase in the productivity of R&D is given by:

$$\frac{\partial n^{**}}{\partial a} = 0 \tag{45}$$

$$\frac{\partial w_N^{**}}{\partial a} = -\frac{\rho(\alpha\theta)^{\frac{\mu}{1-\mu}}(1-\alpha)n^{**}}{(a\rho + L^N)^2} < 0 \tag{46}$$

Therefore, an increase in the productivity of R&D positively affects the wage rate in the North but does not affect the number of the firms. When an increase in the productivity of R&D occurs, from (18), R&D activity is easier, and the labor demand for the R&D sector increases. Therefore, the wage rate in the North increases. In the steady state in *Regime S*, the number of final good firms increases until all the labor in the North are hired in the management sector. Hence, the number of final good firms does not change when the productivity of R&D increases. We can summarize the above analysis as the following proposition.

Proposition 3. *An improvement in the productivity of R&D promotes the wage rate in the North and does not affect the number of final good firms.*

4.3 A larger labor endowment

An increase in the labor services of both regions is expressed by increases in L^N and L^S. In the steady state, an increase in the labor services of the South does not affect the number of the final good firms, n^{**}, nor the wage rate in the North, w_N^{**}. The labor demand of the intermediate sector does not change when the labor services in the South increase in the steady state. People who are not hired in the intermediate goods sector are hired in the homogeneous sector. However, since the homogeneous good is chosen to be the numeraire, both the wage rate in the North and the number of final good firms do not change. On the other hand, in the steady state, the effects of an increase in the labor services of the North are as follows:

$$\frac{\partial n^{**}}{\partial L^N} = \frac{1}{f^S} > 0 \tag{47}$$

$$\frac{\partial w_N^{**}}{\partial L^N} = -\frac{(\alpha\theta)^{\frac{\mu}{1-\mu}}(1-\alpha)}{(a\rho + L^N)^2} n^{**\frac{\mu-\alpha}{\alpha(1-\mu)}} \left[L^N - (a\rho + L^N)\frac{\mu(1-\alpha)}{\alpha(1-\mu)} \right]. \tag{48}$$

An increase in the labor services of the North increases the number of the final good firms whereas, the effect of the wage rate in the North is ambiguous. The change in L^N affects w_N^{**} through two channels: a direct effect and a change in n^{**}. "A direct effect" means that the expansion of the labor services of the North reduces the wage rate in the North. The first term in the parentheses in (48) represents this direct effect. On the other hand, when the labor supply increases, the number of firms increases. Since the expansion of the number of the final good firms increases the Northern labor demand in the management sector, the wage rate in the North rises. The second term in the parentheses in (48) represents this effect. Therefore, the two effects work in the opposite direction on the wage rate in the North. When the size of the labor endowment of the North is larger than $\frac{\mu(1-\alpha)a\rho}{\alpha-\mu}$, a direct effect is larger than a change in n^{**} and the wage rate in the North decreases. On the other hand, when the size of the labor endowment of the North is smaller than $\frac{\mu(1-\alpha)a\rho}{\alpha-\mu}$, a direct effect is smaller than a change in n^{**}, and the wage rate in the North increases. This can be summarized as follows.

Proposition 4. *The increase in the size of the labor force in the North promotes a higher number of final good firms and a higher wage rate in the North if the population in the North is smaller ($L^N < \frac{\mu(1-\alpha)a\rho}{\alpha-\mu}$). The increase in the population in the South does not affect the number of the final good firms nor the wage rate in the North.*

5 Indeterminacy of the Equilibrium Path

In this section, we indicate that the equilibrium path can be indeterminate. There are two sufficient conditions for the equilibrium path to be indeterminate. The first condition is that the steady state in *Regime S* exists in the shaded area of Figure 4, where *Regime S* prevails and the equilibrium path can converges to the steady state in *Regime S*. The other condition is that the steady state in *Regime N* exists in the *Regime N* area in Figure 4, and the equilibrium path can also converge to the steady state in *Regime N*. The former condition is given by (39), and the latter condition is that equation (40) does not have real solution. When these two sufficient conditions are satisfied, the equilibrium path is indeterminate and we can depict the phase diagram for that system on the (n, w_N) plane in Figure 6. At the initial point, if individuals expect that the equilibrium path will converge to the steady state in *Regime N*, the economy converges to the steady state in *Regime N* and the wage rate in the North is determined. On the other hand, at the initial point, if individuals expect that the equilibrium path will converge to the steady state in *Regime S*, the economy converges to the steady state in *Regime S* and the wage rate in the North is determined. Therefore, the equilibrium path is determined by expectations of individuals. These results are stated as the following proposition.

Proposition 5. *Suppose that the parameters satisfy the conditions of* (39) *and that the equation* (40) *have no real solution. Then there exist two equilibrium paths, that the economy evolves from Regime N to Regime S and that the economy remains in Regime N, and the equilibrium path is indeterminate.*

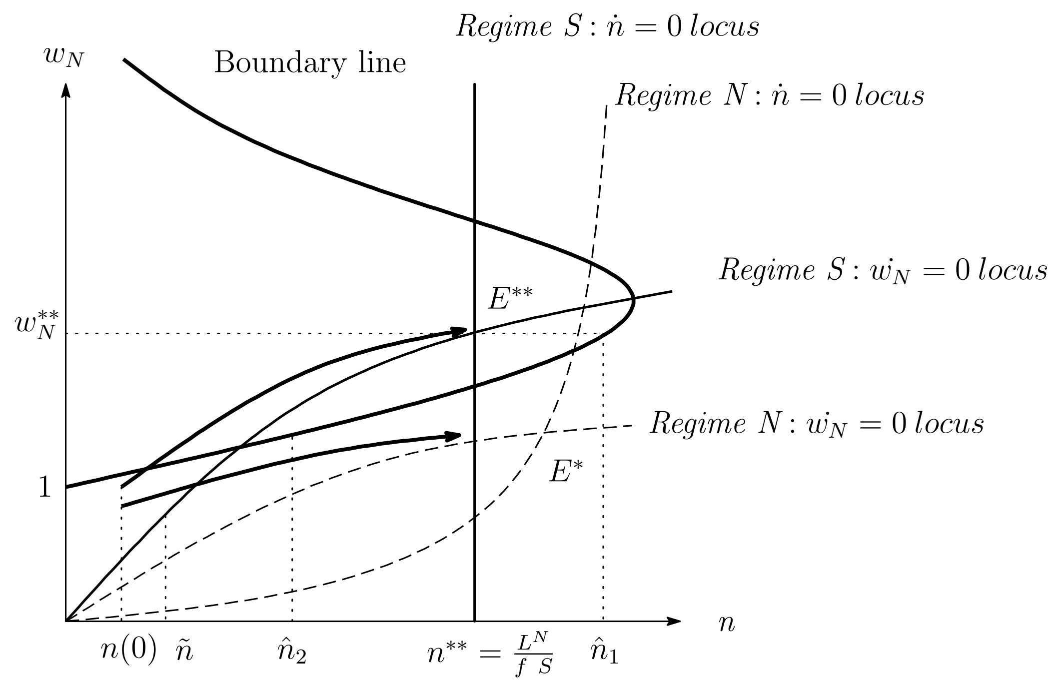

Figure 6: The phase diagram when the equilibrium path is indeterminate

6 Conclusion

This paper proposes a dynamic North-South endogenous growth model in which final good producers in the North determine to outsource the production of intermediate goods to either the North or the South. In this model, the final good producers are only in the North, where the wage rate is high, and they buy intermediate goods either from the North or from the South. When they outsource to the North, the price of intermediate goods is expensive, whereas the management cost is low. On the other hand, when they outsource to the South, the price of the intermediate goods is low, whereas the management cost is expensive. This model shows that as the economy develops, the wage rate in the North becomes higher. Therefore, the final good producers change the region to which they outsource production from the North to the South. We show that in the transition process, there exists a time at which the final good producers change the location to which they outsource the production of intermediate inputs from the North to the South. This paper proves that the steady state in the economy is a saddle point. In the steady state, a decrease in the management cost paid by the final good producers to obtain the intermediate goods from Southern firms causes an increase in the North of both the number of firms and the wage rate in the North. Indeed, an increase in the productivity of R&D raises the wage rate in the North. An increase in the labor endowment in the North also increases the number of firms.

Appendix

In this appendix, we derive that the steady states in *Regime N* and *Regime S* are saddle points. The linearized system of (26) and (27) around the steady state is given by:

$$\begin{pmatrix} \dot{n} \\ \dot{w_N} \end{pmatrix} = \begin{pmatrix} b_1 & b_2 \\ b_3 & b_4 \end{pmatrix} \begin{pmatrix} n - n^* \\ w_N - w_N^* \end{pmatrix},$$

where

$$b_1 = \frac{(L^N - 2f^N n^*)}{a} - \frac{1}{a}\left(\frac{\mu(1-\alpha)}{\alpha(1-\mu)} + 1\right)\left(L^N - f^N n^*\right),$$

$$b_2 = \frac{1}{a(1-\mu)}\frac{n^*}{w_N^*}\left(L^N - f^N n^*\right),$$

$$b_3 = -\frac{\mu(1-\alpha)}{\alpha(1-\mu)}\frac{w_N^*}{n^*}\left(\rho + \frac{L^N}{a}\right),$$

and

$$b_4 = \frac{1}{1-\mu}\left(\rho + \frac{L^N}{a}\right).$$

The determinant of the characteristic matrix is as follows:

$$-\frac{\rho + \frac{L^N}{a}}{a(1-\mu)} f^N n^* < 0.$$

Thus, the determinant takes a negative value. This condition implies that the two eigenvalues of the system have opposite signs and the steady state of *Regime N* is a point.

We next show that the steady state in *Regime S* displays saddle-path stability. The linearized system of (31) and (32) around the steady state is:

$$\begin{pmatrix} \dot{n} \\ \dot{w_N} \end{pmatrix} = \begin{pmatrix} -\frac{L^N}{a} & 0 \\ -\frac{w_N^*}{n^*}\left(\rho + \frac{L^N}{a}\right) & \rho + \frac{L^N}{a} \end{pmatrix} \begin{pmatrix} n - n^* \\ w_N - w_N^* \end{pmatrix},$$

The determinant of the characteristic matrix become

$$-\frac{L^N}{a}\left(\rho + \frac{L^N}{a}\right) < 0.$$

Thus, the determinant takes a negative value. This condition implies that the two eigenvalues of the system have opposite signs and the steady state of *Regime S* is a saddle point.

References

Aghion, Philippe and Peter Howitt, (1992),‘ A Model of Growth through Creative Destruction,’ *Econometrica*, 60:2, 323-351.

Antràs, Pol and Elhanan Helpman, (2004),‘ Global soucing,’ *Journal of Political Economy*, 112:3, 552-580.

Campa, Josè M. and L. S. Goldberg, (1997),‘ The Evolving External Orientation of Manufacturing: A Profile of Four Countries,’ *Federal Reserve Bank New York Econmic Policy Review*, 3:2, 53-81.

Feenstra Robert C. and Gordon H. Hanson, (1996),‘ Globalization, Outsourcing, and Wage Inequality,’ *American Economic Review*, 86(May): 240-245.

Gao, Ting, (2007),‘ Trade Costs, International Production Shifting, and Growth,’ *European Economic Review*51: 317-335.

Grossman, Gene M. and Elhanan Helpman, (1991), *Innovation and Growth in the Global Economy*, Cambridge, MA: MIT Press.

Grossman, Gene M. and Elhanan Helpman, (2002),' Integration versus Outsourcing in Industry Equilibrium,' *Quarterly Journal of Economics*, 117:1, 85-120.

Hanson, Gordon H., Raymond J. Mataloni and Matthew J. Slaughter, (2001), ' Expansion Strategies of U.S. Multinational Firms,' in D. Rodrik and S. M. Collins, eds., *Brookings Trade Forum 2001.*

Hanson, Gordon H., Raymond J. Mataloni and Matthew J. Slaughter, (2005), ' Vertical Production Networks in Multinational Firms,' *Review of Economics and Statistics*, 87:4, 664-678.

Hummels, David, Jun Ishii and Kei-Mu Yi, (1998), ' The Nature and Growth of Vertical Specialization in World Trade,' *Journal of International Economics*, 54:1, 75-96.

Naghavi, Alireza and Gianmarco Ottaviano, (2008),' Offshoring and Product Innovation,' *Economic Theory*, forthcoming.

Yeats, Alexander J., (2001), ' Just How Big Is Global Production Sharing?' in S. W. Arndt and H. Kierzkowski, eds., *New Production Patterns in the World Economy*, Oxford: Oxford University Press.