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Marital Status and Derived Pension Rights: A Political Economy Model of Public Pensions with Borrowing Constraints

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Marital Status and Derived Pension Rights: A Political Economy Model of Public Pensions with Borrowing Constraints

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Abstract

This paper develops an overlapping-generation model featuring four types of households: single female, single male, one-breadwinner couple and two-breadwinner couple. The paper considers majority voting over public pension in the presence of derived pension rights for one-breadwinner couples. In an economy with a low intertemporal elasticity of substitution, borrowing-constrained one-breadwinner couples may prefer a lower tax rate than do other types of households, although the former attain a higher benefit-to-cost ratio of public pension than do others. Changes in the gender wage gap, the level of derived pension rights, and the fraction of two-breadwinner couples produce an inverse U-shaped relationship between the relevant variable and the tax rate.

Keywords: Borrowing constraint; Marital status; Gender wage gap; Derived pension rights; Political economy

JEL Classification: D72, H55, J12

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1 Introduction

Most OECD (Organization for Economic Co-operation and Development) countries offer pension benefits for non-working spouses and divorcees. The benefits, called derived pension rights, include (i) survivors' benefits for widows; (ii) benefits for divorced spouses; and (iii) spousal benefits as a supplement to a worker's benefit (Choi, 2006). These benefits imply that derived pension rights have an intra-generational redistribution component from working singles and two-breadwinner couples to one-breadwinner couples. Thus, recent pension reforms in many OECD countries that attempt to link contributions and benefits more closely (OECD, 2011) may provoke an intra-generational conflict over pension policy.

Despite the conflict among singles and couples caused by derived pension rights, there are few studies focusing on these rights in the political-economic literature. Exceptions are the works of Leroux and Pestieau (2011) and Leroux, Pestieau, and Racionero (2011). Leroux and Pestieau (2011) consider an economy composed of couples who maximize the joint lifetime utility of a husband and a wife. A husband always works regardless of his labor productivity, while a wife chooses whether or not to work depending on her reservation wage. Under this framework, Leroux and Pestieau (2011) demonstrate an interaction between a wife's labor supply decision and pension policy preferences, and they show that a pension system with derived pension rights is likely to emerge as a voting equilibrium outcome.

Leroux, Pestieau, and Racionero (2011) assume that the degree of derived pension rights is fixed. Instead, they allow for the presence of single males and females and examine how the degree of derived pension rights affects tax burden policy preferences for public pensions and thus a resulting pension system via voting. Their results are as follows: (1) a reduction of derived pension rights results in a smaller tax burden for public pension, and (2) an increase in the share of two-breadwinner couples have two opposing effects on the pension burden, where the net effect may be positive or negative depending on other economic factors.

The results in Leroux, Pestieau, and Racionero (2011) provide significant policy predictions for public pensions. However, their results heavily depend on the following two assumptions: quasi-linear utility and no borrowing constraints. The first assumption, which is often adopted in the political-economic analyses of social security (see, for example, Conde-Ruiz and Galasso, 2003, 2004, 2005; Borck, 2007), makes the analysis tractable, but draws attention away from the considerable effect of the intertemporal elasticity of substitution on a household's decisions concerning saving and voting (Casamatta et al., 2000; Conde-Ruiz and Profeta, 2007; Cremer et al., 2007; Arawatari and Ono, 2011). The second assumption allows for borrowing against future pension benefits, which is difficult to support from the empirical viewpoint (Diamond and Hausman, 1984; Mulligan and Sala-i-Martin, 1999). The aim of this paper is to relax these two assumptions in the framework of Leroux, Pestieau, and Racionero (2011) and to provide new insight into derived pension rights from a political-economic viewpoint.

For analytical purposes, we will extend the framework of Leroux, Pestieau, and Racionero (2011) in the following two ways. First, the preferences of each household are represented by a utility function with a constant intertemporal elasticity of substitution. Second, each household is unable to borrow against its future pension benefits. Under this extended framework, we show the following two results. First, in an economy where an intertemporal elasticity of substitution is below one, one-breadwinner couples who benefit from public pensions may prefer a lower, rather than higher, tax rate than single females who owe net burden, because of the presence of borrowing constraints. Borrowing-constrained one-breadwinner couples want to choose a low tax rate to keep their after-tax income level as high as possible. There is then a voting equilibrium, much like an ends-against-the-middle equilibrium, in which single females, along with the old, form a coalition against the others.

Secondly, when the intertemporal elasticity of substitution is below one, an inverse U-shaped relationship is created between the relevant variable and the tax rate due to the gender wage gap, the level of derided pension rights, and the ratio of double-breadwinner couples. Near the maximum of the inverse U-shaped curve, the decisive voter is borrowing-unconstrained on one side and borrowing-constrained on the other side. This two-toned effect, as well as the ends-against-the-middle equilibrium, both of which were not shown in Leroux, Pestieau, and Racionero (2011), are derived by the presence of a borrowing constraint associated with a low intertemporal elasticity of substitution.

The organization of this paper is as follows. Section 2 describes the economic environment. Section 3 demonstrates the utility maximization of singles, one-breadwinner couples, and two-breadwinner couples. Section 4 presents the political institution and pension policy preferences of the young and the old. Section 5 characterizes the political equilibrium. Section 6 performs a comparative statics analysis and shows how gender wage gap, derived pension rights, and the share of two-breadwinner couples affect the equilibrium pension policy. Section 7 provides concluding remarks. Proofs are provided in the Appendix.

2 The Economic Environment

Consider a discrete time economy in which time is denoted by $t = 0, 1, 2, \cdots$. The economy is comprised of overlapping generations of individuals, each of whom lives for two defined periods: youth and old age. Each generation is composed of a continuum of agents.

Specifically, in each generation, there are males and females; the size of each gender population is normalized to unity. Thus, the total population size of each generation is two.

Each generation consists of four different categories of households: single males, single females, one-breadwinner couples, and two-breadwinner couples. The fraction of singles is $1-\varphi \in (0,1)$, and the fraction of couples is φ. Among couples, a fraction $1-\mu \in (0,1)$ are to be considered one-breadwinner couples, and the remaining fraction μ is composed of two breadwinners. Therefore, in each generation, the population size of each type of household is as follows: the number of single males is $1-\varphi$; the number of single females is $1-\varphi$; the number of one-breadwinner couples is $2\varphi(1-\mu)$; and the number of two-breadwinner couples is $2\varphi\mu$. The fraction of each type of household is fixed over time. For simplicity, marriage decisions are not factored into the analysis.

Each agent is endowed with one unit of labor in youth and retires in old age. Males supply labor regardless of their marital status, while only females who are single or belong to two-breadwinner couples supply labor. Females who belong to one-breadwinner couples do not supply labor; instead, they devote their time to home production and leisure, both of which are assumed not to have an effect on utility or household income.

In this economy, there are two types of heterogeneity between males and females: wage and longevity. These types are characterized by the parameter pairs (w^m, π^m) for males and (w^f, π^f) for females, such that

$$\begin{cases} (w^m, \pi^m) = (w, \pi), \ \pi \in (0,1); \\ (w^f, \pi^f) = (\alpha w, 1), \ \alpha \in (0,1), \end{cases}$$

where w^i $(i = f, m)$ represents the wage, and π^i represents the probability of surviving in old age. The term $\alpha \in (0,1)$ represents the gender wage gap, and the term $\pi \in (0,1)$ represents the longevity difference between males and females. It is assumed that females have a longer life span than males but obtain a lower wage.

Individuals contribute to the pension system during youth and receive a pension benefits in old age. Following the convention in the literature, we present the efficiency loss of taxation by assuming convex costs of collecting taxes (see, for example, Casamatta, Cremer, and Pestieau, 2000; Bellettini and Berti Ceroni, 2007; Borck, 2007; Cremer et al., 2007). The actual tax revenue from the young is therefore given by

$$\tau(1-\tau) \cdot [w + \alpha w(1-\varphi) + \alpha w \varphi \mu],$$

where the terms w, $\alpha w(1-\varphi)$, and $\alpha w \varphi \mu$ in the square brackets correspond to the contributions by males, single females, and females who belong to two-breadwinner couples, respectively. The term $(1-\tau)$ is the distortionary factor. The assumption of distortionary taxation is made solely to ensure an interior solution to preferred tax rates and otherwise plays no role.

Let p denote pension benefits for contributors; let γp denote pension benefits for non-contributors, where $\gamma \in [0,1]$ represents the level of derived pension rights. The total pension payments are

$$p \cdot [\pi + (1-\varphi) + \varphi\mu + \gamma\varphi(1-\mu)].$$

The pension benefit for males is $p\pi$, rather than p, because their length of life in old age is assumed to be $\pi \in [0,1)$.

Under the assumption of a balanced budget, the government budget constraint becomes

$$p = w\chi(\cdot)\tau(1-\tau), \tag{1}$$

where

$$\chi(\cdot) \equiv \frac{1+\alpha(1-\varphi+\varphi\mu)}{\pi+1-\varphi+\varphi\mu+\gamma\varphi(1-\mu)}.$$

The tax rate τ is determined via majority voting, whereas the degree of derived pension rights γ is assumed to be fixed at the constitutional level. Voting over γ will be discussed in Section 7.

3 Economic Decisions

Let $j = f, m, c1$, and $c2$ denote single females, single males, one-breadwinner couples, and two-breadwinner couples, respectively. In this section, we illustrate the economic decisions on saving by singles and couples. Any old agent makes no economic decision because his/her saving is predetermined in youth.

3.1 Singles

Each single agent is assumed to receive utility from private consumption. The lifetime utility function of a type-j ($j = f, m$) single young agent is specified by:

$$U^j = \frac{(c^j)^{1-\sigma}-1}{1-\sigma} + \beta \cdot \frac{(d^j)^{1-\sigma}-1}{1-\sigma},$$

where c^j is consumption in youth, d^j is consumption in old age, $\beta \in (0,1)$ is a discount factor, and $\sigma(>0)$ is the inverse of the intertemporal elasticity of substitution. A lower $1/\sigma$ implies a lower intertemporal elasticity of substitution.[1]

Type-j's ($j = f, m$) individual budget constraints in youth and in old age are given by, respectively,

$$c^j + s^j \leq (1-\tau)w^j,$$

$$d^j \leq Rs^j + \pi^j p,$$

[1] For $j = m$, the second-period utility might be more appropriately written as $\pi\beta\{(d^j)^{1-\sigma}-1\}/(1-\sigma)$ since π is interpreted as the probability of surviving to the second period of life. Following Borck (2007), we assume away the effect of π on the second-period utility for the tractability of analysis.

where s^j is saving, τ is the income tax rate, $R(>1)$ is the gross interest rate, and p is the per capita pension benefit. If $j=f$, then $w^f=\alpha w$ and $\pi^f=1$; if $j=m$, then $w^m=w$ and $\pi^m=\pi$. Throughout this paper, we assume borrowing constraints, that is, $s^j \geq 0$. This constraint precludes the possibility of borrowing when young against future pension benefits (Diamond and Hausman, 1984; Mulligan and Sala-i-Martin, 1999).

A type-j young agent maximizes his/her utility subject to his/her budget constraint and the borrowing constraint. When $s^j>0$, the first-order condition for an interior solution is $d^j=(\beta R)^{1/\sigma}c^j$. This condition determines an interior solution of saving by a type-j agent. By taking the borrowing constraint into account, the saving function of a type-j agent becomes

$$s^j=\max\left\{0, \frac{(\beta R)^{1/\sigma}}{R+(\beta R)^{1/\sigma}}\left[(1-\tau)w^j-\frac{\pi^j p}{(\beta R)^{1/\sigma}}\right]\right\},\ j=f,m. \tag{2}$$

The saving function (2) and the government budget constraint (1) imply that there is a critical rate of tax such that

$$s^f>0 \Leftrightarrow \tau<\hat{\tau}^f \equiv \frac{\alpha(\beta R)^{1/\sigma}}{\chi(\cdot)};$$
$$s^m>0 \Leftrightarrow \tau<\hat{\tau}^m \equiv \frac{(\beta R)^{1/\sigma}}{\pi\chi(\cdot)}.$$

The critical rate for single females, $\hat{\tau}^f$, is lower than that for single males, $\hat{\tau}^m$, because single males obtain higher wages and live shorter than do single females.

With the saving function and the private and government budget constraints, we can obtain the consumption functions of a type-j $(=f,m)$ agent in youth and in old age. We use the functions to obtain the indirect utility function of type-j singles, denoted by $V^j(j=f,m)$:

$$V^j=\begin{cases} V^j_{s>0}\equiv \frac{1}{1-\sigma}\left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\cdot\left[(1-\tau)w^j+\frac{\pi^j w\chi(\cdot)}{R}\tau(1-\tau)\right]^{1-\sigma}-\frac{1+\beta}{1-\sigma} & \text{if}\quad \tau<\hat{\tau}^j; \\ V^j_{s=0}\equiv \frac{1}{1-\sigma}\left((1-\tau)w^j\right)^{1-\sigma}+\frac{\beta}{1-\sigma}\left\{\pi^j w\chi(\cdot)\tau(1-\tau)\right\}^{1-\sigma}-\frac{1+\beta}{1-\sigma} & \text{if}\quad \tau\geq\hat{\tau}^j. \end{cases}$$

The function $V^j_{s>0}$ $(j=f,m)$ denotes the indirect utility of a type-j young household when it saves some portion of income, and $V^j_{s=0}$ denotes the indirect utility when it is faced with a borrowing constraint and saves nothing. The term in square brackets in the equation of $V^j_{s>0}$ represents the lifetime income; the first and the second terms on the right-hand side in the equation for $V^j_{s=0}$ represent the utilities of consumption in youth and in old age, respectively; the constant term, $(1+\beta)/(1-\sigma)$, summarizes the parameters unrelated to the political decision on taxes.

3.2 Couples

We next consider consumption decisions by couples. Following Leroux, Pestieau, and Racionero (2011), we adopt the unitary model of a household that has only one set of preferences:

$$U^j = 2 \cdot \left\{ \frac{(c^j)^{1-\sigma} - 1}{1-\sigma} + \beta \cdot \frac{(d^j)^{1-\sigma} - 1}{1-\sigma} \right\}, j = c1, c2.$$

Under this specification, spouses play cooperatively and share their resources over their lifecycle.

A couple chooses consumption and saving to maximize the household utility subject to the budget constraints in youth and old age:

$$\begin{aligned} 2c^j + s^j &\leq (1-\tau)w^j, \\ 2d^j &\leq Rs^j + (\pi + \gamma^j)p, \end{aligned}$$

and the borrowing constraint, $s^j \geq 0$, where $w^{c1} = w$, $w^{c2} = (1+\alpha)w, \gamma^{c1} = \gamma$ and $\gamma^{c2} = 1$.

In the first period of life, a husband and/or wife work and earn the after-tax wage income, $(1-\tau)w^j$. A couple consumes a part of the after-tax wage and saves the rest for old-age consumption. In the second period of life, the couple obtains the return from savings, Rs^j, the pension benefit paid to the husband, πp, and that to the wife, $\gamma^j p$.

By taking the borrowing constraint into account, the saving function of the type-j couple becomes

$$s^j = \max \left\{ 0, \frac{(\beta R)^{1/\sigma}}{R + (\beta R)^{1/\sigma}} \left[(1-\tau)w^j - \frac{(\pi + \gamma^j)p}{(\beta R)^{1/\sigma}} \right] \right\}, j = c1, c2 \tag{3}$$

The saving function (3) and the government budget constraint (1) imply that there is a critical tax rate for couples such that

$$\begin{aligned} s^{c1} > 0 \Leftrightarrow \tau < \hat{\tau}^{c1} &\equiv \frac{(\beta R)^{1/\sigma}}{(\pi + \gamma)\chi(\cdot)}; \\ s^{c2} > 0 \Leftrightarrow \tau < \hat{\tau}^{c2} &\equiv \frac{(1+\alpha)(\beta R)^{1/\sigma}}{(\pi + 1)\chi(\cdot)}. \end{aligned}$$

With the saving function and the private and government budget constraints, we can obtain the consumption functions in youth and in old age. We use these functions to derive a type-j couples' indirect utility function:

$$V^j = \begin{cases} V^j_{s>0} & \text{if } \tau < \hat{\tau}^j; \\ V^j_{s=0} & \text{if } \tau \geq \hat{\tau}^j; \end{cases}$$

where:

$$V^j_{s>0} \equiv \frac{1}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma}\cdot\left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\cdot\left[(1-\tau)w^j+\frac{(\pi+\gamma^j)w\chi(\cdot)}{R}\tau(1-\tau)\right]^{1-\sigma}-\frac{2(1+\beta)}{1-\sigma};$$

$$V^j_{s=0} \equiv \frac{1}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma}\cdot\left((1-\tau)w^j\right)^{1-\sigma}+\frac{\beta}{1-\sigma}\left(\frac{1}{2}\right)^{-\sigma}\cdot\left\{(\pi+\gamma^j)w\chi(\cdot)\tau(1-\tau)\right\}^{1-\sigma}-\frac{2(1+\beta)}{1-\sigma}$$

The function $V^j_{s>0}$ denotes the indirect utility of a type-j couple when it saves in youth, and $V^j_{s=0}$ denotes the indirect utility when it is faced with a borrowing constraint and saves nothing in youth. The interpretation of each term in these equations follows that offered for singles.

4 The Political Institution and Policy Preferences

The tax rate τ is determined by individuals through a political process of majority voting. Elections take place every period and all living individuals, both young and old, cast a ballot over τ. The tax preferences of young individuals are represented by the indirect utility functions presented in the previous section. The tax preferences of old agents are determined by the size of the pension because their saving when young is predetermined and has no critical effect on voting behavior. Every individual has zero mass, and thus no individual vote can change the outcome of the election. We thus assume individuals vote sincerely.

The majority voting game is intrinsically dynamic because it describes the interaction among successive generations. To address this feature, we assume commitment, or in other words, once-and-for-all-voting. Here, voters determine the constant sequence of the parameters: $\tau_t = \tau_{t+1} = \tau$ for all t, where τ_t denotes the tax rate in period t (see, for example, Casamatta, Cremer and Pestieau, 2000; Cond-Ruiz and Profeta, 2007). We can view the full commitment solution as the solution that includes intergenerational interaction because the full commitment solution can be supported as the subgame perfect equilibrium in repeated voting (see, for example, Conde-Ruiz and Galasso, 2003, 2005; Poutvaara, 2006).

Given the stationary environment, the current model presents a static voting game. Therefore, the median voter theorem can be applied to the voting game. To find the voting equilibrium, we need to show that tax preferences of voters are single-peaked. As for the tax preferences of old voters, their objective is to maximize their pension benefits regardless of their marriage status, labor supply, and saving. Although the benefit levels differ between old agents, the factor related to political decision is common to all old agents and is specified by the Laffer curve $\tau(1-\tau)$. Thus, the tax preferences of the old are single-peaked; their preferred tax rate, denoted by τ^{oj}, is $\tau^{oj} = 1/2\ \forall j = f, m, c1, c2$.

4.1 Policy Preferences of the Young

Next, let us consider the preferences of the young. To show that the preferences of a young agent who belongs to a type-j ($j = f, m, c1, m2$) household are single peaked, we should note that the following three properties hold. First, $\partial^2 V^j_{s>0}/\partial\tau^2 < 0$ and $\partial^2 V^j_{s=0}/\partial\tau^2 < 0$ hold; that is, $V^j_{s>0}$ and $V^j_{s=0}$ are single peaked. Second, the indirect utility V^j of a young agent in a type-j household is continuous at $\tau = \hat{\tau}^j$: $V^j_{s>0}\big|_{\tau=\hat{\tau}^j} = V^j_{s>0}\big|_{\tau=\hat{\tau}^j}$, $j = f, m, c1, c2$. Third, the slope of $V^j_{s>0}$ at $\tau = \hat{\tau}^j$ is equivalent to that of $V^j_{s=0}$ at $\tau = \hat{\tau}^j$:

$$\left.\frac{\partial V^j_{s>0}}{\partial\tau}\right|_{\tau=\hat{\tau}^j} = \left.\frac{\partial V^j_{s=0}}{\partial\tau}\right|_{\tau=\hat{\tau}^j}, \ j = f, m, c1, c2.$$

The details of the calculation are given in Appendix A.1. The three properties imply that V^j has a unique local maximum. In what follows, we derive the conditions that determine the tax rates preferred by four types of households.

First, consider a preferred tax rate by a young single female agent. She chooses τ that satisfies $\partial V^f_{s>0}/\partial\tau = 0$ when she is borrowing-unconstrained; and τ that satisfies $\partial V^f_{s=0}/\partial\tau = 0$ when she is borrowing-constrained. After some calculation, we can find that the preferred tax rate by single females satisfies:

$$LHS \equiv 1 - 2\tau = RHS^f \equiv \begin{cases} \frac{\alpha}{\chi(\cdot)} R & \text{if} \quad \tau < \hat{\tau}^f, \\ \frac{1}{\beta}\left(\frac{\alpha}{\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{if} \quad \tau \geq \hat{\tau}^f. \end{cases} \tag{4}$$

The terms LHS and RHS^f represent the *marginal efficiency loss of taxation* and the *marginal cost-to-benefit ratio of taxation*, respectively. Single females choose the tax rate that balances these terms. In the next subsection, we provide an interpretation of the ratio and explain its role in the determination of a preferred tax rate by each type of agent.

As in the case of a single female, we find that a preferred tax rate by single males satisfies:

$$LHS \equiv 1 - 2\tau = RHS^m \equiv \begin{cases} \frac{1}{\pi\chi(\cdot)} R & \text{if} \quad \tau < \hat{\tau}^m, \\ \frac{1}{\beta}\left(\frac{1}{\pi\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{if} \quad \tau \geq \hat{\tau}^m; \end{cases} \tag{5}$$

a preferred tax rate by one-breadwinner couples satisfies:

$$LHS \equiv 1 - 2\tau = RHS^{c1} \equiv \begin{cases} \frac{1}{(\pi+\gamma)\chi(\cdot)} R & \text{if} \quad \tau < \hat{\tau}^{c1}, \\ \frac{1}{\beta}\left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{if} \quad \tau \geq \hat{\tau}^{c1}; \end{cases} \tag{6}$$

and a preferred tax rate by two-breadwinner couples satisfies:

$$LHS \equiv 1 - 2\tau = RHS^{c2} \equiv \begin{cases} \frac{1+\alpha}{(\pi+1)\chi(\cdot)} R & \text{if} \quad \tau < \hat{\tau}^{c2}, \\ \frac{1}{\beta}\left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{if} \quad \tau \geq \hat{\tau}^{c2}. \end{cases} \tag{7}$$

4.2 The Role of Marginal Cost-to-benefit Ratio of Taxation

The conditions (4), (5), (6), and (7), determine the preferred tax rates by single females, single males, one-breadwinner couples, and two-breadwinner couples, respectively. Each type of young household chooses its preferred tax rate to equate the marginal efficiency loss of taxation, represented by the left-hand side, to the marginal cost-to-benefit ratio of taxation, represented by the right-hand side; each household prefers a lower tax rate as the ratio becomes higher.

To understand the reasoning behind the abovementioned property, let us consider the single female's condition, (4), as an example. To understand the role of the ratio, we focus on the parameter α, representing the gender wage gap, for illustrative purposes. There are two opposing effects of an increase in α on the ratio. First, given a tax rate, an increase in α imposes a further tax burden. This burden gives a single female an incentive to choose a lower tax rate, resulting in a negative effect on the preferred tax rate. Second, an increase in α augments wage income for single females, and thus pension benefits in old age, because the total wages on which the tax is levied are increased. This augmentation gives a young single female an incentive to choose a higher tax rate, resulting in a positive effect on the preferred tax rate via the term $\chi(\cdot)$.

When a young single female is borrowing-unconstrained, she can reallocate income freely across periods. Because of this intertemporal reallocation of income, the positive effect is compensated for by the negative effect regardless of the degree of intertemporal elasticity of substitution. Therefore, an increase in α results in a higher marginal cost-to-benefit ratio and thus a lower preferred tax rate when a young single female is borrowing-unconstrained. The result holds regardless of $1/\sigma$ because the objective for a borrowing-unconstrained household is to maximize lifetime income, which is independent of $1/\sigma$.

When a single female is borrowing-constrained, the positive effect is not necessarily compensated for by the negative one. The borrowing-constrained single female wants to consume more when young, but her demand is restricted by the borrowing constraint. In this situation, the borrowing-constrained individual attaches a large weight to the utility gain of an increase in her wage. This effect might lead to a situation in which the positive effect overcomes the negative one, resulting in a lower, rather than higher, marginal cost-to-benefit ratio and thus a higher preferred tax rate in response to an increase in α.

Which effect outweighs the other depends on the degree of intertemporal elasticity of substitution. A lower elasticity implies a stronger incentive for single females to smooth consumption over periods. Because of this incentive, borrowing-constrained single females attach a smaller weight to the positive effect on youthful consumption via a decrease in her preferred tax rate as the elasticity becomes lower. In other words, the negative effect on the preferred tax rate is more likely to be overcome by the positive one as the elasticity becomes lower.

The net effect depends on the degree of the intertemporal elasticity of substitution. When the elasticity is high, that is, $1/\sigma \geq 1$, the net effect on the tax is negative. An increase in α results in a lower preferred tax rate by borrowing-constrained single females. In contrast, when the elasticity is low, that is, $1/\sigma < 1$, the net effect becomes positive. An increase in α leads to a higher preferred tax rate.

In concluding this section, we note that the tax rates preferred by the young are lower than those preferred by the old who choose $\tau = 1/2$. The result of this phenomenon is that the decisive voter with respect to τ belongs to the young generation because the population size of the young is larger than that of the old given the death of some males in early life. Given this result, we focus on young agents' preferences over τ and consider the determination of τ in majority voting in the next section.

5 Political Equilibrium

This section characterizes the political equilibrium of the majority voting. In what follows, an "agent" implies a "young agent" if not otherwise specified.

5.1 Political Environment

We impose the following assumption to proceed with the analysis.

Assumption 1: (i) $1 < \alpha(1+\pi)$ and (ii) $\max\left\{\frac{1-\pi}{4(1-\mu)}, \frac{1-\pi}{4\mu}\right\} < \varphi < \frac{1+\pi}{2}$.

The first assumption, $1 < \alpha(1+\pi)$, enables us to illustrate a variety of cases. Under this assumption, there are two critical values of γ:

$$\gamma = \frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}.$$

With these two critical values, the order of critical tax rates that determine the status of saving becomes as follows:

$$\begin{array}{lll} \text{Case (a) } \hat{\tau}^f < \hat{\tau}^{c2} \leq \hat{\tau}^{c1} < \hat{\tau}^m & \text{if} & 0 \leq \gamma \leq \frac{1-\alpha\pi}{1+\alpha}, \\ \text{Case (b) } \hat{\tau}^f \leq \hat{\tau}^{c1} < \hat{\tau}^{c2} < \hat{\tau}^m & \text{if} & \frac{1-\alpha\pi}{1+\alpha} < \gamma \leq \frac{1-\alpha\pi}{\alpha}, \\ \text{Case (c) } \hat{\tau}^{c1} \leq \hat{\tau}^f < \hat{\tau}^{c2} < \hat{\tau}^m & \text{if} & \frac{1-\alpha\pi}{\alpha} < \gamma \leq 1. \end{array} \tag{8}$$

Case (c) is available if and only if Assumption 1(i) holds. We will investigate who becomes a decisive voter for each case.

The second assumption, $\max\left\{\frac{1-\pi}{4(1-\mu)}, \frac{1-\pi}{4\mu}\right\} < \varphi < \frac{1+\pi}{2}$, ensures that one who prefers the highest tax rate among the young becomes the decisive voter. To understand the argument stemming from Assumption 1(ii), suppose that a young agent who belongs to a type-j ($j = f, m, c1, c2$) household prefers the highest tax rate among the young. He/she becomes a decisive voter if the size of the old plus the size of young agents who belong to the type-j household are more than a half of the population. Based on the argument above, a young single female or male becomes a decisive voter if $(1-\varphi)+(1+\pi) > (3+\pi)/2$; a young agent who belongs to a one-breadwinner couple becomes a decisive voter if $2\varphi(1-\mu)+(1+\pi) > (3+\pi)/2$; and a young agent who belongs to a two-breadwinner couple becomes a decisive voter if $2\varphi\mu + (1+\pi) > (3+\pi)/2$.

For each condition, the first term on the left-hand side shows the number of young agents who prefer the highest tax rate among the young, the second term, $1+\pi$, shows the number of old agents, and the term on the right-hand side, $(3+\pi)/2$, shows a half of the population size. The above-mentioned three conditions are summarized as in Assumption 1(ii).

Hereafter, we will focus on the parameter σ, which represents the inverse of the intertemporal elasticity of substitution, and consider two cases separately: a high elasticity case ($1/\sigma \geq 1$) and a low elasticity case ($1/\sigma < 1$). We adopt this classification because the order of preferences for the tax rate depends critically on the degree of elasticity. Because the former case includes the case of Leroux, Pestieau, and Racionero (2011) as a special one, we leave it to Appendix A.6. In what follows here, we will focus exclusively on the latter case.

5.2 An Economy with $1/\sigma < 1$

To determine the decisive voter over τ when $1/\sigma < 1$, we recall the conditions (4), (5), (6), and (7), that determine the preferred tax rates by four types of households. The graphs of these conditions when $1/\sigma < 1$ are illustrated in Figure 1. The main feature is that RHS^j and RHS^k ($j \neq k$) intersect at some tax rate $\tau \in (0, 1/2)$. In an economy with $1/\sigma < 1$, the slope of RHS^j becomes steeper the lower the marginal cost-to-benefit ratio becomes when a type-j household is borrowing-constrained. To investigate in detail the property of the political equilibrium when $1/\sigma < 1$, we hereafter consider in turn three cases, (a), (b), and (c), classified according to the level of derived pension right as in (8), and then summarize the results of the three cases to provide a global characterization of the political equilibrium.

[**Figure 1** about here.]

5.2.1 Low Level of Derived Pension Rights: $0 \leq \gamma \leq \frac{1-\alpha\pi}{1+\alpha}$

Panel (a) of Figure 1 illustrates the conditions (4), (5), (6), and (7) that determine the preferred tax rates of the four types of households when the level of derived pension rights is low such that $\gamma \in [0, (1-\alpha\pi)/(1+\alpha)]$. As depicted in the figure, there are three critical values of τ, $\tilde{\tau}^{f,c2} \in \left(\hat{\tau}^{f}, \hat{\tau}^{c2}\right)$, $\tilde{\tau}^{c2,c1} \in (\hat{\tau}^{c2}, \hat{\tau}^{c1})$, and $\tilde{\tau}^{c1,m} \in (\hat{\tau}^{c1}, \hat{\tau}^{m})$, such that RHS^{f} and RHS^{c2} intersect at $\tau = \tilde{\tau}^{f,c2}$, RHS^{c2} and RHS^{c1} cross at $\tau = \tilde{\tau}^{c2,c1}$, and RHS^{c1} and RHS^{m} intersect at $\tau = \tilde{\tau}^{c1,m}$. By direct calculation, we obtain

$$\tilde{\tau}^{f,c2} \equiv \left(\frac{(1+\alpha)R\beta}{(\pi+1)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{\alpha}{\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}, \quad \tilde{\tau}^{c2,c1} \equiv \left(\frac{R\beta}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}},$$
$$\tilde{\tau}^{c1,m} \equiv \left(\frac{R\beta}{\pi\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}.$$

The tax rate preferred by a type-j household is determined by the crossing point of LHS and RHS^{j}. Given the assumption of household distribution in Assumption 1(ii), the decisive voter over τ is the one who prefers the highest tax rate among the young households. Based on the illustration in Panel (a) of Figure 1, the decisive voter over τ when $1/\sigma < 1$ and $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$ is determined as follows.

Lemma 1. *Suppose that $1/\sigma < 1$ and $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$ hold. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(j) *a type-f single female agent if $\chi(\cdot) \leq 2\left(\frac{(1+\alpha)R\beta}{\pi+1}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{(1+\alpha)R}{\pi+1}$;*

(ii) *a type-$c2$ agent who belongs to a two-breadwinner couple, otherwise.*

Proof. See Appendix A.2.

The result in Lemma 1 indicates that (i) a type-m or type-$c1$ agent never becomes a decisive voter; and (ii) the decisive voter is a type-f single female agent if the gender wage gap is small such that $\chi(\cdot) \leq 2\left(\frac{(1+\alpha)R\beta}{\pi+1}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{(1+\alpha)R}{\pi+1}$; otherwise, the decisive voter is a type-$c2$ agent who belongs to a two-breadwinner couple.

To understand the mechanism behind the first result, suppose that a type-m or type-$c1$ agent is a decisive voter. Based on the observation in Panel (a) of Figure 1, the equilibrium tax rate must be high so that $\tau > \tilde{\tau}^{c2,c1}$. For this high tax rate, the marginal cost-to-benefit ratios of taxation for type-m and type-$c1$ agents, which are increasing in the tax rate, are always larger than the marginal efficiency loss of taxation, $1-2\tau$, which is decreasing in the tax rate. The marginal ratio and efficiency loss of taxation are never balanced for $\tau > \tilde{\tau}^{c2,c1}$ for type-m and type-$c1$ agents. Therefore, neither of the agents becomes a decisive voter.

Next, to understand the mechanism behind the second result, recall the condition that produces an equilibrium in which a type-f single female agent becomes a decisive voter in Lemma 1. The condition is rewritten as

$$\frac{1-\frac{\alpha}{1+\alpha}\varphi(1-\mu)}{1-\frac{\varphi(1-\mu)(1-\gamma)}{\pi+1}} \leq 2\,(R\beta)^{\frac{1}{\sigma}} \cdot (\pi+1)^{1-\frac{1}{\sigma}} \cdot \left(\frac{\alpha}{1+\alpha}\right)^{\frac{\sigma-1}{\sigma}} + R, \tag{9}$$

where the left-hand side is decreasing in α, and the right-hand side is increasing in α. Therefore, the condition (9) states that a type-f agent is more likely to become a decisive voter when α is higher.

Single females owe greater tax burden as α becomes higher (i.e., as their wage becomes higher). However, an increase in α results in a lower marginal cost-to-benefit ratio and thus a higher preferred tax rate by single females when $1/\sigma < 1$ as demonstrated in Subsection 4.2. Therefore, type-f agents prefer a higher tax rate than type-$c2$ agents when α is high such that (9) holds.

5.2.2 Medium Level of Derived Pension Right: $\frac{1-\alpha\pi}{1+\alpha} < \gamma \leq \frac{1-\alpha\pi}{\alpha}$

Panel (b) of Figure 1 illustrates the conditions (4), (5), (6), and (7) that determine the preferred tax rates by the four types of households when the level of derived pension right is medium such that $\gamma \in ((1-\alpha\pi)/(1+\alpha), (1-\alpha\pi)/\alpha]$. As depicted in the figure, there are three critical values of τ, $\tilde{\tau}^{f,c1} \in (\hat{\tau}^f, \hat{\tau}^{c1})$, $\tilde{\tau}^{c1,c2} \in (\hat{\tau}^{c1}, \hat{\tau}^{c2})$, and $\tilde{\tau}^{c2,m} \in (\hat{\tau}^{c2}, \hat{\tau}^m)$, such that RHS^f and RHS^{c1} intersect at $\tau = \tilde{\tau}^{f,c1}$, RHS^{c1} and RHS^{c2} cross at $\tau = \tilde{\tau}^{c1,c2}$, and RHS^{c2} and RHS^m intersect at $\tau = \tilde{\tau}^{c2,m}$. By direct calculation, we obtain

$$\tilde{\tau}^{f,c1} \equiv \left(\frac{R\beta}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{\alpha}{\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}, \quad \tilde{\tau}^{c1,c2} \equiv \left(\frac{R\beta(1+\alpha)}{(\pi+1)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}},$$
$$\tilde{\tau}^{c2,m} \equiv \left(\frac{R\beta}{\pi\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}.$$

As in the previous case, we can characterize the political equilibrium for the case of $1/\sigma < 1$ and $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}\right]$ as follows.

Lemma 2. *Suppose that $1/\sigma < 1$ and $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}\right]$ hold. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(j) *a type-f single female agent if* $\chi(\cdot) \leq 2\left(\frac{R\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{R}{\pi+\gamma}$;

(ii) *a type-$c1$ agent who belongs to a one-breadwinner couple otherwise.*

Proof. See Appendix A.3.

The main difference from the previous case is that a type-$c2$ agent cannot be a decisive voter; instead, a type-$c1$ agent can be a decisive voter under a certain condition. A change of the decisive voter is due to that, in the current case, a type-$c1$ agent attains a higher pension benefit because of a higher level of derived pension right relative to the previous case. This relationship results in a higher marginal cost-to-benefit ratio of taxation for a type-$c1$ agent compared to a type-$c2$ agent. Therefore, a type-$c1$ agent prefers a higher tax rate than does a type-$c2$ agent.

In the current case, the decisive voter is a type-f or a type-$c1$ agent. The former becomes a decisive voter if the condition in Lemma 2 holds. The left-hand side of the condition is decreasing in α, while the right-hand side is increasing in α. An interpretation similar to that of the previous case applies to the current case. For a higher α, a type-f, single female agent is more likely to become a decisive voter.

5.2.3 High Level of Derived Pension Right: $\frac{1-\alpha\pi}{\alpha} < \gamma \leq 1$

Panel (c) of Figure 1 illustrates the conditions (4), (5), (6), and (7) that determine the preferred tax rates by four types of households when the level of derived pension rights is high such that $\gamma \in ((1-\alpha\pi)/\alpha, 1]$. As depicted in the figure, there are three critical values of τ, $\tilde{\tau}^{c1,f} \in \left(\hat{\tau}^{c1}, \hat{\tau}^{f}\right)$, $\tilde{\tau}^{f,c2} \in \left(\hat{\tau}^{f}, \hat{\tau}^{c2}\right)$, and $\tilde{\tau}^{c2,m} \in (\hat{\tau}^{c2}, \hat{\tau}^{m})$, such that RHS^{c1} and RHS^{f} intersect at $\tau = \tilde{\tau}^{c1,f}$, RHS^{f} and RHS^{c2} intersect at $\tau = \tilde{\tau}^{f,c2}$, and RHS^{c2} and RHS^{m} intersect at $\tau = \tilde{\tau}^{c2,m}$. By direct calculation, we obtain

$$\tilde{\tau}^{c1,f} \equiv \left(\frac{\alpha R\beta}{\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{\alpha}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}, \quad \tilde{\tau}^{f,c2} \equiv \left(\frac{(1+\alpha)R\beta}{(\pi+1)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{\alpha}{\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}},$$
$$\tilde{\tau}^{c2,m} \equiv \left(\frac{R\beta}{\pi\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}}.$$

As in the previous two cases, we can characterize the political equilibrium for the case of $1/\sigma < 1$ and $\gamma \in ((1-\alpha\pi)/\alpha, 1]$ as follows:

Lemma 3. *Suppose that $1/\sigma < 1$ and $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, 1\right]$ hold. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(j) *a type-$c1$ agent who belongs to a one-breadwinner couple if $\chi(\cdot) \leq 2\left(\alpha R\beta\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + \alpha R$;*

(ii) *a type-f single female agent otherwise.*

Proof. See Appendix A.4.

A set of possible decisive voters is unchanged from case (b): the decisive voter is a type-f or a type-$c1$ agent depending on α . However, the qualitative impact of these parameters on the determination of the decisive voter is completely reversed. In case (b), a type-f agent is more likely to be a decisive voter for a higher α. In contrast, she is more likely to be a decisive voter for a lower α in the current case.

The difference between the two cases is due to the different levels of derived pension right. The level of derived pension right in the current case is higher than that in case (b). This difference implies that when α is high enough so that both types of agents are borrowing-unconstrained, the marginal cost-to-benefit ratio of taxation for type-$c1$ agents is lower than that for type-f agents. Therefore, type-$c1$ agents prefer the highest tax rate among the young and become the decisive voter if α is high such that the condition in Lemma 3 holds. In contrast, when α is low enough so that both types of agents are borrowing-constrained, the order of the ratios is reversed because of a low intertemporal elasticity of substitution. The ratio for type-f agents becomes lower than that for the type-$c1$ agents; the former prefer the highest tax rate among the young and become decisive voters.

5.2.4 A Decisive Voter When $1/\sigma < 1$

The results established so far are summarized as follows.

Proposition 1. *Suppose that $1/\sigma < 1$ holds. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(i) *a type-$c1$ agent who belongs to a one-breadwinner couple if $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}\right]$ and $\chi(\cdot) > 2\left(\frac{R\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{R}{\pi+\gamma}$; or if $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, 1\right]$ and $\chi(\cdot) \leq 2(\alpha R\beta)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + \alpha R$;*

(ii) *a type-$c2$ agent who belongs to a two-breadwinner couple if $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$ and $\chi(\cdot) > 2\left(\frac{(1+\alpha)R\beta}{\pi+1}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{(1+\alpha)R}{\pi+1}$;*

(iii) *a type-f single female agent otherwise.*

Proof. It is immediate from Lemmas 1-3.

The fraction of two-breadwinner couples, denoted by μ, has a crucial effect on the determination of a decisive voter when $1/\sigma < 1$. The result in Proposition 1 states that for an economy with $1/\sigma < 1$, a type-$c2$ agent who belongs to a two-breadwinner couple becomes a decisive voter if (a) the level of derived pension right is set at a low level, and (b) the benefit-burden ratio of public pensions in the economy, denoted by $\chi(\cdot)$, is high. Because $\chi(\cdot)$ is increasing in μ, the latter condition implies that a type-$c2$ agent is more likely to be a decisive voter when the fraction of type-$c2$ agents is larger in the economy.

6 Gender Wage Gap, Derived Pension Rights and the Fraction of Two-breadwinner Couples

Given the characterization of the political equilibrium in Section 5, we investigate how the tax rate changes in response to recent trends in developed economies: a reduction of the gender wage gap, a reduction of derived pension rights, and an increase in the fraction of two-breadwinner couples. The aim of the analysis is to predict the direction of future change in the tax rate in response to these trends. The analysis also aims to explore the roles of the borrowing constraint and the intertemporal elasticity of substitution on the determination of the tax rate.

6.1 A Reduction of the Gender Wage Gap

First, we investigate the effect of a reduction of the gender wage gap on the determination of the tax rate.

Proposition 2: *In an economy with $1/\sigma < 1$ where the decisive voter is a type-j ($j = f, c1$, or $c2$) agent, a reduction of the gender wage gap (i.e., an increase in α) locally produces an inverse U-shaped relationship between α and τ around $\hat{\tau}^j$.*

Proof. See Appendix A.5.

To understand the mechanism behind the result in Proposition 2, we first consider the effect of an increase in α on the marginal cost-to-benefit ratio of taxation, denoted by $RHS^j (j = f, c1, c2)$, when an agent is borrowing-unconstrained. After some calculation, we obtain

$$\frac{\partial RHS^f}{\partial \alpha} > 0, \frac{\partial RHS^{c1}}{\partial \alpha} < 0, \frac{\partial RHS^{c2}}{\partial \alpha} > 0 \text{ if } s^j > 0 \ (j = f, c1, c2).$$

When an agent is borrowing-unconstrained, a reduction of the gender wage gap (i.e., an increase in α) increases the marginal cost-to-benefit ratio of taxation for type-f and type-$c2$ agents, whereas it decreases the ratio for type-$c1$ agents. The difference is due to the fact that single females and two-breadwinner couples owe an additional tax burden when there is an increase in females' wages, whereas one-breadwinner couples owe no additional burden. Because increased tax revenue is returned to all types of agents as lump-sum pension benefits, single females and two-breadwinner couples pay more than they receive, whereas one-breadwinner couples pay nothing but receive additional benefits. Therefore, a reduction of the gender wage gap results in a higher cost-to-benefit ratio of taxation and thus a lower preferred tax rate for type-f and type-$c2$ agents, whereas it results in a lower cost-to-benefit ratio and thus a higher preferred tax rate for type-$c1$ agents.

The opposite result holds when an agent is borrowing-constrained because the effect of α on RHS^j is reversed as demonstrated in Subsection 4.2. A reduction of the gender wage gap results in a lower RHS^j ($j = f, c2$) and thus in a higher preferred tax rate by type-f and type-$c2$ agents, whereas it results in a higher RHS^{c1} and thus in a lower preferred tax rate by type-$c1$ agents. Therefore, there is an inverse U-shaped relationship between α and τ^j around the critical value of the tax, $\hat{\tau}^j$, that divides the status of saving (see Figure 2).

[**Figure 2** about here.]

6.2 A Reduction of Derived Pension Rights

Next, we will consider the effect of a reduction of derived pension rights on the determination of the tax rate.

Proposition 3: *In an economy with* $1/\sigma < 1$ *where the decisive voter is a type-j* $(j = f, c1,$ *or* $c2)$ *agent, a reduction of derived pension rights* (*i.e., a decrease in* γ) *produces an inverse U-shaped relationship between* γ *and* τ *around* $\hat{\tau}^j$*.*

Proof. See Appendix A.5.

To understand the result in Proposition 3, we first consider the effect of the derived pension rights on the marginal cost-to-benefit ratio of taxation when an agent is borrowing unconstrained:

$$(-1)\frac{\partial RHS^f}{\partial \gamma} < 0, (-1)\frac{\partial RHS^{c1}}{\partial \gamma} > 0, (-1)\frac{\partial RHS^{c2}}{\partial \gamma} < 0 \text{ if } s^j > 0 \ (j = f, c1, c2)$$

We multiply the derivatives by (-1) to demonstrate the qualitative effect of a decrease in γ. A reduction of derived pension rights (i.e., a decrease in γ) increases the pension benefits for type-f and type-$c2$ agents, lowers the cost-to-benefit ratio of taxation, and thus raises their preferred tax rate. In contrast, such a reduction decreases the pension benefits for type-$c1$ agents, raises their cost-to-benefit ratio of taxation and thus lowers their preferred tax rate. However, the opposite result holds when an agent is borrowing-constrained as demonstrated in the previous subsection. Therefore, the two opposing effects result in an inverse U-shaped relationship between γ and τ^j around the critical value of the tax, $\hat{\tau}^j$.

6.3 An Increase in the Fraction of Two-breadwinner Couples

Finally, we examine the effect of the share of two-breadwinner couples on the equilibrium tax rate.

Proposition 4: *In an economy with* $1/\sigma < 1$ *where the decisive voter is a type-j* $(j = f, c1,$ *or* $c2)$ *agent, an increase in the fraction of two-breadwinner couples* (*i.e., an increase in* μ) (a) *locally produces an inverse U-shaped relationship between* μ *and* τ *around* $\hat{\tau}^j$ *if* $\gamma \neq (1-\alpha\pi)/(1+\alpha)$*;* (b) *has no effect on the equilibrium tax if* $\gamma = (1-\alpha\pi)/(1+\alpha)$.

Proof. See Appendix A.5.

The result in Proposition 4 can be understood by focusing on the effects of μ on the marginal cost-to-benefit ratio of taxation, denoted by RHS^j. When agents are borrowing unconstrained, the effects are summarized as follows:

$$\frac{\partial RHS^f}{\partial \mu} \gtreqless 0, \frac{\partial RHS^{c1}}{\partial \mu} \gtreqless 0, \frac{\partial RHS^{c2}}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}.$$

The marginal cost-to-benefit ratio of taxation, RHS^j ($j = f, c1, c2$), is affected by an increase in μ via the benefit-to-burden ratio of public pension, denoted by $\chi(\cdot)$, in the following two ways. On one hand, an increase in μ results in an increase of tax revenue from the working females and thus in an increase per capita pension benefit. This effect is observed in the numerator of $\chi(\cdot)$ in (1), which yields a negative effect on RHS^j. On the other hand, an increase in μ implies a larger size of two-breadwinner couples and a smaller size of one-breadwinner couples. Because the level of pension rights is larger for working females than for non-working females, an increase in μ results in a smaller per capita pension benefits. This effect is observed in the denominator of $\chi(\cdot)$ in (1), which yields a positive effect on RHS^j.

As observed in (1), the former effect is independent of γ, whereas the latter effect is dependent of γ. Specifically, the latter effect becomes larger as γ becomes smaller. Therefore, the latter effect overcomes the former one when the level of derived pension rights is low such that $\gamma < (1-\alpha\pi)/(1+\alpha)$. That is, RHS^j is increased by an increase in μ if $\gamma < (1-\alpha\pi)/(1+\alpha)$. The opposite result holds if $\gamma > (1-\alpha\pi)/(1+\alpha)$. The two opposing effects are offset by each other if $\gamma = (1-\alpha\pi)/(1+\alpha)$. These effects are reversed when an agent is borrowing-constrained as demonstrated in Subsection 4.2. Therefore, there is an inverse U-shaped relationship between μ and τ^j around the critical value of the tax, $\hat{\tau}^j$.

7 Concluding Remarks

This paper developed an overlapping-generation model based on that of Leroux, Pestieau and Racionero (2011). The model includes four types of households: single female, single male, one-breadwinner couple and two-breadwinner couple. The paper introduced a borrowing constraint into their model and generalized the model by assuming a utility function with a constant intertemporal elasticity of substitution. Under this generalized framework, we consider majority voting over public pension policy in the presence of derived pension rights, and investigate how the borrowing constraint and intertemporal elasticity of substitution affect the preferences of each household over pension and the resulting equilibrium pension policy.

The paper showed the following two results. First, in an economy where an intertemporal elasticity of substitution is below one, one-breadwinner couples may prefer a lower, rather than higher, tax rate than do single females because of the presence of borrowing constraints. There is an equilibrium, much like an ends-against-the-middle equilibrium, where the old and single females form a coalition against the others.

Second, the gender wage gap, the level of derived pension rights, and the fraction of two-breadwinner couples create an inverse U-shaped relationship between the relevant variable and the tax rate when the intertemporal elasticity of substitution is below one. This two-toned effect was derived via a borrowing constraint associated with a low intertemporal elasticity of substitution.

Throughout the analysis, we assumed that the degree of derived pension rights is fixed. This assumption can be relaxed by assuming a structure-induced Nash equilibrium of voting (for example, Conde-Ruiz and Galasso, 2003; 2005; Casamatta, Cremer and Pestieau, 2005; Conde-Ruiz and Profeta, 2007; Bethencourt and Galasso, 2008). In this voting equilibrium, one-breadwinner couples prefer a full derived pension right, whereas others prefer no right. Thus, the full derived pension right is realized if the number of one-breadwinner couples is larger than a half of the population; no derived pension right is realized otherwise. However, in the real world, the degree of the derived pension right is set between these two extreme solutions. There is a need to add an institutional feature to demonstrate a more realistic situation: this task is left as future work.

A Appendix

A.1 Single-peaked Preferences of the Young

In this appendix, we prove that preferences of a type-f young agent are single peaked. The proof applies to other types of young agents.

The proof proceeds as follows. First, we show that both $V^f_{s>0}$ and $V^f_{s=0}$ are single peaked over τ. Then we demonstrate that $\partial V^f_{s>0}/\partial\tau = \partial V^f_{s=0}/\partial\tau$ and $V^f_{s>0} = V^f_{s=0}$ hold at $\tau = \hat{\tau}^f$, implying that V^f has a unique local maximum over the whole range of τ and thus that V^f is single peaked over τ.

The first and the second derivatives of $V^{y,j}_{s>0}$ and $V^{y,j}_{s=0}$ with respect to τ are

$$
\begin{aligned}
\frac{\partial V^f_{s>0}}{\partial\tau} &= \left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\cdot\left[(1-\tau)\alpha w+\frac{w\chi(\cdot)}{R}\tau(1-\tau)\right]^{-\sigma}\cdot\left\{-\alpha w+\frac{w\chi(\cdot)}{R}(1-2\tau)\right\};\\
\frac{\partial^2 V^f_{s>0}}{\partial\tau^2} &= \left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\cdot(-\sigma)\cdot\left[(1-\tau)\alpha w+\frac{w\chi(\cdot)}{R}\tau(1-\tau)\right]^{-\sigma-1}\cdot\left\{-\alpha w+\frac{w\chi(\cdot)}{R}(1-2\tau)\right\}^2\\
&\quad+\left(\frac{R}{R+(\beta R)^{1/\sigma}}\right)^{-\sigma}\cdot\left[(1-\tau)\alpha w+\frac{w\chi(\cdot)}{R}\tau(1-\tau)\right]^{-\sigma}\cdot(-2)\frac{w\chi(\cdot)}{R}\\
&\quad<0;\\
\frac{\partial V^f_{s=0}}{\partial\tau} &= [(1-\tau)\alpha w]^{-\sigma}\cdot(-\alpha w)+\beta\cdot[w\chi(\cdot)\tau(1-\tau)]^{-\sigma}\cdot w\chi(\cdot)\cdot(1-2\tau);\\
\frac{\partial^2 V^f_{s=0}}{\partial\tau^2} &= (-\sigma)\cdot[(1-\tau)\alpha w]^{-\sigma-1}\cdot(\alpha w)^2\\
&\quad-\sigma\beta\cdot[w\chi(\cdot)\tau(1-\tau)]^{-\sigma-1}\cdot(w\chi(\cdot))^2\cdot(1-2\tau)^2\\
&\quad-2\beta\cdot[w\chi(\cdot)\tau(1-\tau)]^{-\sigma}\cdot w\chi(\cdot)\\
&\quad<0.
\end{aligned}
$$

The functions $V^f_{s>0}$ and $V^f_{s=0}$ are single peaked over τ because the second derivatives are negative.

Next, we show that $\partial V^f_{s>0}/\partial\tau = \partial V^f_{s=0}/\partial\tau$ and $V^f_{s>0} = V^f_{s=0}$ hold at $\tau = \hat{\tau}^f$. By direct calculation, we have:

$$
\begin{aligned}
V^{y,j}_{s>0}\Big|_{\tau=\hat{\tau}^f} &= V^{y,j}_{s=0}\Big|_{\tau=\hat{\tau}^f}\\
&= \frac{1}{1-\sigma}\cdot\frac{R+(\beta R)^{1/\sigma}}{R}\cdot\left(1-\frac{\alpha(\beta R)^{1/\sigma}}{\chi(\cdot)}\right)^{1-\sigma}\cdot(\alpha w)^{1-\sigma}-\frac{1+\beta}{1-\sigma},\\
\left.\frac{\partial V^{y,j}_{s>0}}{\partial\tau}\right|_{\tau=\hat{\tau}^f} &= \left.\frac{\partial V^{y,j}_{s=0}}{\partial\tau}\right|_{\tau=\hat{\tau}^f}\\
&= \left(1-\frac{\alpha(\beta R)^{1/\sigma}}{\chi(\cdot)}\right)^{-\sigma}\cdot(\alpha w)^{1-\sigma}\cdot\left[-1+\frac{\chi(\cdot)}{\alpha R}+\frac{2}{R}(\beta R)^{1/\sigma}\right].
\end{aligned}
$$

With this result and the single-peakedness of $V_{s>0}^{y,j}$ and $V_{s=0}^{y,j}$ over τ, we can conclude that V^f has a unique local maximum with respect to τ over the whole range of τ. Specifically, V^f is maximized at $\tau = \arg\max V_{s>0}^f$ if $\arg\max V_{s>0}^f < \hat{\tau}^f$; it is maximized at $\tau = \arg\max V_{s=0}^f$ otherwise.

■

A.2 Proof of Lemma 1

1st step. We first show that a type-j ($j = c1, m$) agent cannot be a decisive voter. To show this, suppose that an agent who belongs to a type$-c1$ or type$-m$ household is a decisive voter. From Panel (a) of Figure 1, it must be that $LHS > RHS^{c2}$ at $\tau = \tilde{\tau}^{c2,c1}$; that is,

$$1 - 2\tilde{\tau}^{c2,c1} > \frac{R}{(\pi+\gamma)\chi(\cdot)}.$$

By rearranging this condition, we obtain:

$$\begin{aligned} 1 - 2\left(\frac{R\beta}{(\pi+\gamma)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1+\alpha}{(\pi+1)\chi(\cdot)}\right)^{\frac{\sigma-1}{\sigma}} &> \frac{R}{(\pi+\gamma)\chi(\cdot)} \\ \Leftrightarrow (\pi+\gamma)\cdot\chi(\cdot) > 2(\pi+\gamma)\cdot\left(\frac{R\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1+\alpha}{\pi+1}\right)^{\frac{\sigma-1}{\sigma}} &+ R. \end{aligned} \tag{10}$$

The left-hand side of (10), $(\pi+\gamma)\chi(\cdot)$, is less than one under the current assumption of $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$.[2] Conversely, the right-hand side is greater than one under the assumption of $R > 1$. Therefore, the condition (10) fails to hold, implying that an agent who belongs to a type$-j$ ($j = c1, m$) household never becomes a decisive voter.

2nd step. Given the result in the first step, a candidate for the decisive voter is an agent who belongs to a type$-f$ or type$-c2$ household. From Panel (a) of Figure 1, a type-f agent becomes a decisive voter if and only if

$$1 - 2\tilde{\tau}^{f,c2} \leq \frac{(1+\alpha)R}{(\pi+1)\chi(\cdot)},$$

that is, if and only if

$$\chi(\cdot) \leq 2\left(\frac{(1+\alpha)R\beta}{\pi+1}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{(1+\alpha)R}{\pi+1}.$$

Otherwise, a type-$c2$ agent becomes a decisive voter.

■

[2] The condition $(\pi+\gamma)\chi(\cdot) \gtrless 1$ is rewritten as

$$\begin{aligned} &(\pi+\gamma)\chi(\cdot) \gtrless 1 \\ &\Leftrightarrow [(-1)(1-\pi\alpha) + \gamma(1+\alpha)] \cdot [1 - \varphi(1-\mu)] \lesseqgtr 1 \end{aligned}$$

where the left-hand side in the second line is negative because $(-1)(1-\pi\alpha) + \gamma(1+\alpha) < 0$ holds by the current assumption of $\gamma \in \left[0, \frac{1-\alpha\pi}{1+\alpha}\right]$. Thus, we obtain $(\pi+\gamma)\chi(\cdot) < 1$.

A.3 Proof of Lemma 2

1st step. We first show that a type-j ($j = c2, m$) household cannot be a decisive voter. To show this, suppose that a type$-j(= c2, m)$ be a decisive voter. From Panel (b) of Figure 1, it must be that $LHS > RHS^{c1}$ at $\tau = \tilde{\tau}^{c1,c2}$, i.e.,

$$1 - 2\tilde{\tau}^{c1,c2} > \frac{R(1+\alpha)}{(\pi+1)\chi(\cdot)}.$$

By rearranging this condition, we obtain

$$\frac{(\pi+1)\chi(\cdot)}{1+\alpha} > \frac{\pi+1}{1+\alpha} \cdot 2\left(\frac{R\beta(1+\alpha)}{\pi+1}\right)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{\pi+\gamma}\right)^{\frac{\sigma-1}{\sigma}} + R. \tag{11}$$

The left-hand side of (11) is less than one under the current assumption of $\gamma \in \left(\frac{1-\alpha\pi}{1+\alpha}, \frac{1-\alpha\pi}{\alpha}\right]$, while the right-hand side is greater than one under the assumption of $R > 1$. Therefore, (11) fails to hold, implying that a type$-j$ ($j = c2, m$) agent never becomes a decisive voter.

2nd step.

Given the result in the first step, a decisive voter is an agent who belongs to a type-f or type-$c1$ household. From Panel (b) of Figure 1, a type-f agent becomes a decisive voter if and only if

$$1 - 2\tilde{\tau}^{f,c1} \leq \frac{R}{(\pi+\gamma)\chi(\cdot)},$$

or

$$\chi(\cdot) \leq 2\left(\frac{R\beta}{\pi+\gamma}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} + \frac{R}{\pi+\gamma}.$$

Otherwise, a type$-c1$ agent becomes a decisive voter.

■

A.4 Proof of Lemma 3

1st step.

We first show that a type$-j(= c2, m)$ agent cannot be a decisive voter. To show this, suppose that a type$-j(= c2, m)$ agent is a decisive voter. From Panel (c) of Figure 1, it must be that $LHS > RHS^{f}$ at $\tau = \tilde{\tau}^{f,c2}$, i.e.,

$$1 - 2\tilde{\tau}^{f,c2} > \frac{(1+\alpha)R}{(\pi+1)\chi(\cdot)},$$

which is rewritten as

$$\frac{(\pi+1)\chi(\cdot)}{1+\alpha} > 2\left(\frac{(1+\alpha)R\beta}{(\pi+1)\chi(\cdot)}\right)^{\frac{1}{\sigma}} \cdot (\alpha)^{\frac{\sigma-1}{\sigma}} \frac{\pi+1}{1+\alpha} + R. \tag{12}$$

The left-hand side of (12) is less than one under the assumption of $\gamma \in \left(\frac{1-\alpha\pi}{\alpha}, 1\right]$, while the right-hand side is greater than one under the assumption of $R > 1$. Therefore, (12) fails to hold, implying that a type$-j(= c2, m)$ agent never becomes a decisive voter.

2nd step.

Given the result in the first step, a decisive voter is a type$-j$ $(= c1, f)$ agent. A type-$c1$ agent becomes a decisive voter if and only if

$$1 - 2\tilde{\tau}^{c1,f} \leq \frac{\alpha R}{\chi(\cdot)},$$

or

$$\chi(\cdot) \leq 2 (\alpha R \beta)^{\frac{1}{\sigma}} \cdot \left(\frac{1}{\pi + \gamma}\right)^{\frac{\sigma-1}{\sigma}} + \alpha R.$$

Otherwise, a type$-f$ agent is a decisive voter.

■

A.5 Proof of Propositions 2-4

First, we focus on the terms $\alpha/\chi(\cdot), 1/(\pi+\gamma)\chi(\cdot)$ and $(1+\alpha)/(\pi+1)\chi(\cdot)$ that affect the marginal cost-to-benefit ratios of taxation for type-$f, c1$ and $c2$ agents, respectively: see equations (4), (6) and (7). We denote these terms as:

$$\widetilde{RHS}^f \equiv \frac{\alpha}{\chi(\cdot)}, \widetilde{RHS}^{c1} \equiv \frac{1}{(\pi+\gamma)\chi(\cdot)}, \widetilde{RHS}^{c2} \equiv \frac{1+\alpha}{(\pi+1)\chi(\cdot)}.$$

After some calculation, we obtain the following properties of $\widetilde{RHS}^j$ $(j = f, c1, c2)$:

$$\frac{\partial \widetilde{RHS}^f}{\partial \alpha} > 0, \frac{\partial \widetilde{RHS}^{c1}}{\partial \alpha} < 0, \frac{\partial \widetilde{RHS}^{c2}}{\partial \alpha} > 0; \tag{13}$$

$$\frac{\partial \widetilde{RHS}^f}{\partial \gamma} > 0, \frac{\partial \widetilde{RHS}^{c1}}{\partial \gamma} < 0, \frac{\partial \widetilde{RHS}^{c2}}{\partial \gamma} > 0; \tag{14}$$

$$\frac{\partial \widetilde{RHS}^f}{\partial \mu} \gtreqless 0, \frac{\partial \widetilde{RHS}^{c1}}{\partial \mu} \gtreqless 0, \frac{\partial \widetilde{RHS}^{c2}}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}. \tag{15}$$

In what follows, we denote τ^j $(j = f, c1, c2)$ as the tax rate preferred by a type-j agent.

A.5.1 The case of $1/\sigma < 1$

Consider the political equilibrium in an economy with $1/\sigma < 1$. The decisive voter in the current case is a type-f, type-$c1$ or type-$c2$ agent (Proposition 1). Because we here consider the effects of α, γ and μ around the critical value $\hat{\tau}^j$ defined in Section 3, we calculate the effects of these parameters on $\hat{\tau}^j$ as follows:

$$\frac{\partial \hat{\tau}^f}{\partial \alpha} > 0, \frac{\partial \hat{\tau}^{c1}}{\partial \alpha} < 0, \frac{\partial \hat{\tau}^{c2}}{\partial \alpha} > 0; \tag{16}$$

$$\frac{\partial \hat{\tau}^f}{\partial \gamma} > 0, \frac{\partial \hat{\tau}^{c1}}{\partial \gamma} < 0, \frac{\partial \hat{\tau}^{c2}}{\partial \gamma} > 0; \tag{17}$$

$$\frac{\partial \hat{\tau}^f}{\partial \mu} \gtreqless 0, \frac{\partial \hat{\tau}^{c1}}{\partial \mu} \gtreqless 0, \frac{\partial \hat{\tau}^{c2}}{\partial \mu} \gtreqless 0 \text{ if and only if } \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}. \tag{18}$$

(i) The effect of α on the equilibrium tax rate: Proof of Proposition 2 Consider first the equilibrium where the decisive voter is a type-f agent. Suppose that α is initially given such that type-f's preferred tax rate is $\tau = \hat{\tau}^f$. We denote α^f as the α that makes a type-f young agent choose $\tau = \hat{\tau}^f$.

With the property of $\widetilde{RHS}^f$ in (13) and the property of $\hat{\tau}^f$ in (16), we find a positive real number $\varepsilon(> 0)$ around α^f such that the type-f agent is borrowing-constrained for $\alpha \in (\alpha^f - \varepsilon, \alpha^f)$ and borrowing-unconstrained for $\alpha \in [\alpha^f, \alpha^f + \varepsilon)$. The equilibrium tax rate satisfies the following condition:

$$LHS \equiv 1 - 2\tau = RHS^f \equiv \begin{cases} \frac{1}{\beta}\left(\frac{\alpha}{\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{for} \quad \alpha \in (\alpha^f - \varepsilon, \alpha^f], \\ \frac{\alpha}{\chi(\cdot)}R & \text{for} \quad \alpha \in \left(\alpha^f, \alpha^f + \varepsilon\right). \end{cases}$$

Given the properties in (13) and (16), we can illustrate the effects of an increase in α on RHS^f and $\hat{\tau}^f$ as in Figure 2. The illustration leads to the following result:

$$\begin{array}{ll} \frac{\partial \tau^f}{\partial \alpha} > 0 & \text{for} \quad \alpha \in (\alpha^f - \varepsilon, \alpha^f), \\ \frac{\partial \tau^f}{\partial \alpha} < 0 & \text{for} \quad \alpha \in \left(\alpha^f, \alpha^f + \varepsilon\right). \end{array}$$

The result shows that an increase in α locally produces an inverse U-shaped relationship between α and τ^f around $\tau = \hat{\tau}^f$.

The analysis and result apply to the equilibrium in which the decisive voter is a type-$c2$ agent because the effects of α on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ are qualitatively similar between the two types of agents, as demonstrated in (13) and (16).

Next, consider the equilibrium where the decisive voter is a type-$c1$ agent. Suppose that α is initially given such that type-$c1$'s preferred tax rate is $\tau = \hat{\tau}^{c1}$. We denote α^{c1} as the α that makes a type-$c1$ young agent choose $\tau = \hat{\tau}^{c1}$.

With the property of $\widetilde{RHS}^{c1}$ in (13) and the property of $\hat{\tau}^{c1}$ in (16), we find a positive real number $\varepsilon(> 0)$ around α^{c1} such that the type-$c1$ agent is borrowing-unconstrained for $\alpha \in (\alpha^{c1} - \varepsilon, \alpha^{c1})$ and borrowing-constrained for $\alpha \in [\alpha^{c1}, \alpha^{c1} + \varepsilon)$. The equilibrium tax rate satisfies the following condition:

$$LHS \equiv 1 - 2\tau = RHS^{c1} \equiv \begin{cases} \frac{1}{(\pi+\gamma)\chi(\cdot)}R & \text{for} \quad \alpha \in (\alpha^{c1} - \varepsilon, \alpha^{c1}); \\ \frac{1}{\beta}\left(\frac{1}{(\pi+\gamma)\chi(\cdot)}\right)^{1-\sigma} \cdot (\tau)^\sigma & \text{for} \quad \alpha \in [\alpha^{c1}, \alpha^{c1} + \varepsilon). \end{cases}$$

Given the properties in (14) and (17), we obtain the following result:

$$\frac{\partial \tau^{c1}}{\partial \alpha} > 0 \quad \text{for} \quad \alpha \in (\alpha^{c1} - \varepsilon, \alpha^{c1}),$$
$$\frac{\partial \tau^{c1}}{\partial \alpha} < 0 \quad \text{for} \quad \alpha \in (\alpha^{c1}, \alpha^{c1} + \varepsilon).$$

The result shows that an increase in α locally produces an inverse U-shaped relationship between α and τ^{c1} around $\tau = \hat{\tau}^{c1}$.

(ii) The effect of γ on the equilibrium tax rate: Proof of Proposition 3 Suppose that the decisive voter is a type-j ($j = f, c1, c2$) agent. Suppose that γ is initially given such that type-j's preferred tax rate is $\tau = \hat{\tau}^j$. We denote γ^j as the γ that makes a type-j agent choose $\tau = \hat{\tau}^j$.

Under the abovementioned situation, suppose that an *increase* in γ around γ^j locally produces an inverse U-shaped relationship between γ and type-j's preferred tax rate. This assumption implies that a *decrease* in γ around γ^j locally produces an inverse U-shaped relationship between γ and type-j's preferred tax rate. Therefore, it is sufficient to show the effect of an increase in γ on the preferred tax rate by the decisive voter.

The analysis of the effect of α applies to the current analysis because the effects of γ on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ are qualitatively similar to those of α on $\widetilde{RHS}^j$ and $\hat{\tau}^j$. Therefore, we obtain the result described in Proposition 3.

(iii) The effect of μ on the equilibrium tax rate: Proof of Proposition 4 Suppose that $\gamma < (1 - \alpha\pi)/(1 + \alpha)$ holds. The decisive voter is a type-f or type-$c2$ agent (Lemma 1). The effects of μ on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ ($j = f, c2$) are qualitatively similar to those of α on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ ($j = f, c2$). We can apply the analysis and result in Proposition 2 to the current case.

Next, suppose that $\gamma > (1 - \alpha\pi)/(1 + \alpha)$ holds. The decisive voter is a type-f or a type-$c1$ agent (Lemmas 2 and 3). The effects of μ on $\widetilde{RHS}^j$ and $\hat{\tau}^j$ ($j = f, c1$) are qualitatively similar to those of α on $\widetilde{RHS}^{c1}$ and $\hat{\tau}^{c1}$. We can apply the analysis and result in Proposition 2 to the current case.

Finally, suppose that $\gamma = (1 - \alpha\pi)/(1 + \alpha)$ holds. The parameter μ has no effect on $\widetilde{RHS}^j$ and $\hat{\tau}^j$. A change in μ has no effect on the equilibrium tax rate.

■

A.6 The Case of a High Intertemporal Elasticity of Substitution: $1/\sigma \geq 1$

Figure 3 illustrates the conditions (4), (5), (6), and (7) that determine the preferred tax rates by single females, single males, one-breadwinner couples, and two-breadwinner couples, respectively, for the case of $1/\sigma \geq 1$. Specifically, panels (a), (b), and (c) correspond to the cases (a), (b), and (c) in (8), respectively. The left-hand side of each condition, denoted by LHS, is decreasing in τ, independent of the type, and featured with $LHS|_{\tau=0} = 1$ and $LHS|_{\tau=1/2} = 0$. In contrast, the right-hand side of each condition, denoted by RHS^j ($j = f, m, c1$ and $c2$), is non-decreasing in τ, dependent on the type of a household, and characterized by

$$\begin{array}{lll} \text{Case (a):} & RHS^f \leq RHS^{c2} \leq RHS^{c1} \leq RHS^m & \text{if} \quad 0 \leq \gamma \leq \frac{1-\alpha\pi}{1+\alpha}, \\ \text{Case (b):} & RHS^f \leq RHS^{c1} \leq RHS^{c2} \leq RHS^m & \text{if} \quad \frac{1-\alpha\pi}{1+\alpha} < \gamma \leq \frac{1-\alpha\pi}{\alpha}, \\ \text{Case (c):} & RHS^{c1} \leq RHS^f \leq RHS^{c2} \leq RHS^m & \text{if} \quad \frac{1-\alpha\pi}{\alpha} < \gamma \leq 1, \end{array}$$

where equality holds if and only if $1/\sigma = 1$ and $s^j = 0$. The kink point of $\tau = \hat{\tau}^j$ implies that a type-j household can save part of its income if $\tau < \hat{\tau}^j$ and nothing if $\tau \geq \hat{\tau}^j$.

[**Figure 3** about here.]

The crossing point of LHS and RHS^j determines the tax rate preferred by a young agent who belongs to a type-j household. Under Assumption 1(ii), the decisive voter is the one who prefers the highest tax rate among the young. From the observation in Figure 3, we can conclude that the decisive voter is a single female ($j = f$) agent if $\gamma \leq (1 - \alpha\pi)/\alpha$ (see panels (a) and (b)); this voter is an agent who belongs to a one-breadwinner couple ($j = c1$) if $\gamma > (1 - \alpha\pi)/\alpha$ (see panel (c)).

Proposition A1. *Suppose that $1/\sigma \geq 1$ holds. There exists a unique equilibrium of the voting game with $\tau \in (0, 1/2)$. The decisive voter over τ is*

(i) *a type-f, single female agent if $\gamma \leq \frac{1-\alpha\pi}{\alpha}$;*

(ii) *a type-$c1$ agent who belongs to a one-breadwinner couple otherwise.*

The result established in Proposition A1 has the following two features. First, a single male or an agent who belongs to a two-breadwinner couple does not become a decisive voter. Such agents' marginal cost-to-benefit ratios of taxation in terms of utility are always higher than those of the other two types of households. This result implies that single males and two-breadwinner couples prefer a lower tax rate than do other two types of young agents. The decisive voter therefore is a single female or an agent who belongs to a one-breadwinner couple.

Second, which household becomes a decisive voter depends on α, π and γ that represent the gender wage gap, life expectancy of men, and the fraction of derived pension rights, respectively. Suppose that the gender wage gap is high (i.e., α is low), the life expectancy of men (π) is low, and the level of derived pension rights (γ) is low such that $\gamma \leq$

$(1-\alpha\pi)/\alpha$. Then, the marginal cost-to-benefit ratio of taxation in terms of utility for single females is lower than that for one-breadwinner couples because the former owe less tax burden whereas the latter receive lower pension benefits. Therefore, single females prefer a higher tax rate than do one-breadwinner couples and thus become decisive voters if $\gamma \leq (1-\alpha\pi)/\alpha$.

A.6.1 Comparative Statics Analysis

Consider the political equilibrium in an economy with $1/\sigma \geq 1$. Suppose that $\gamma \leq (1-\alpha\pi)/\alpha$ holds: the decisive voter is a type-f agent (Proposition A1). The optimality condition for a type-f agent, given by (4), indicates that a higher $\widetilde{RHS}^f$ results in a lower preferred tax rate except for the case of $1/\sigma = 1$ and $s^f = 0$:

$$\begin{cases} \frac{\partial \tau^f}{\partial \widetilde{RHS}^f} = 0 \text{ if } 1/\sigma = 1 \text{ and } s^f = 0, \\ \frac{\partial \tau^f}{\partial \widetilde{RHS}^f} < 0 \text{ otherwise.} \end{cases}$$

With the property of $\widetilde{RHS}^f$ in (13) - (15), we obtain the following result:

$$\frac{\partial \tau^f}{\partial \alpha} \leq 0, \frac{\partial \tau^f}{\partial \gamma} \leq 0,$$

and

$$\frac{\partial \tau^f}{\partial \mu} \lesseqgtr 0 \Leftrightarrow \gamma \lesseqgtr \frac{1-\alpha\pi}{1+\alpha}.$$

Next, suppose that $\gamma > (1-\alpha\pi)/\alpha$ holds: the decisive voter is a type-$c1$ agent (Proposition A1). The optimality condition for a type-$c1$ agent, given by (6), indicates that a higher $\widetilde{RHS}^{c1}$ results in a lower preferred tax rate except the case of $1/\sigma = 1$ and $s^{c1} = 0$:

$$\begin{cases} \frac{\partial \tau^{c1}}{\partial \widetilde{RHS}^{c1}} = 0 \text{ if } 1/\sigma = 1 \text{ and } s^{c1} = 0, \\ \frac{\partial \tau^{c1}}{\partial \widetilde{RHS}^{c1}} < 0 \text{ otherwise.} \end{cases}$$

With the property of $\widetilde{RHS}^{c1}$ in (13) - (15) and the assumption of $\gamma > (1-\alpha\pi)/\alpha$, we obtain the following result:

$$\frac{\partial \tau^{c1}}{\partial \alpha} \geq 0, \frac{\partial \tau^{c1}}{\partial \gamma} \geq 0, \frac{\partial \tau^{c1}}{\partial \mu} \geq 0.$$

■

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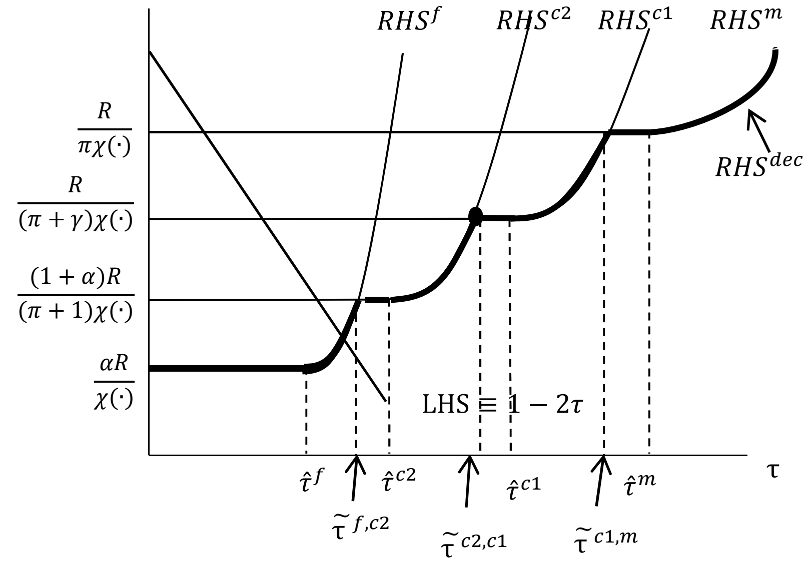
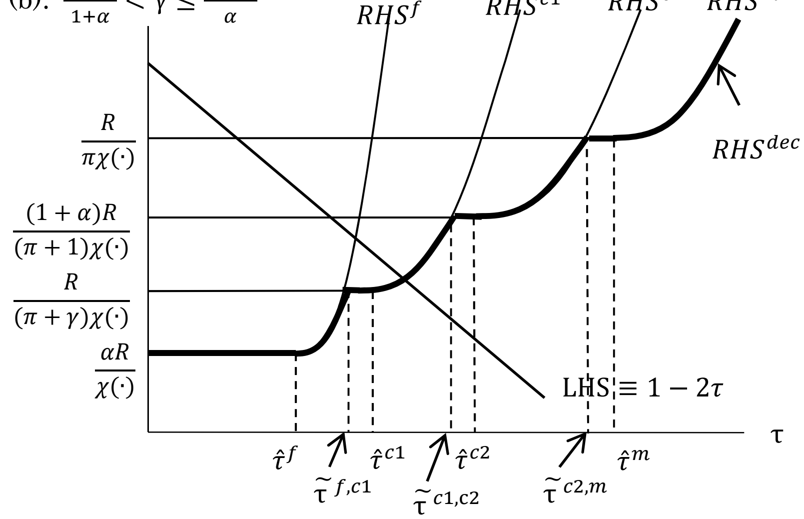
Panel (a). $0 \leq \gamma \leq \frac{1-\alpha\pi}{1+\alpha}$



Panel (b). $\frac{1-\alpha\pi}{1+\alpha} < \gamma \leq \frac{1-\alpha\pi}{\alpha}$



Panel (c). $\frac{1-\alpha\pi}{\alpha} < \gamma \leq 1$

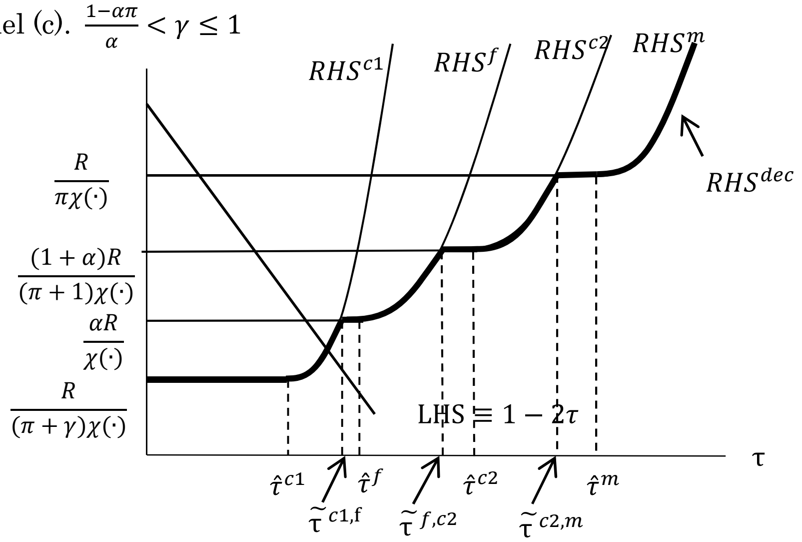


Figure 1. The preferred tax rates when $\frac{1}{\sigma} < 1$. Panel (a) is the case of $0 \leq \gamma \leq (1-\alpha\pi)/(1+\alpha)$; panel (b) is the case of $(1-\alpha\pi)/(1+\alpha) < \gamma \leq (1-\alpha\pi)/\alpha$; and panel (c) is the case of $(1-\alpha\pi)/\alpha < \gamma \leq 1$.

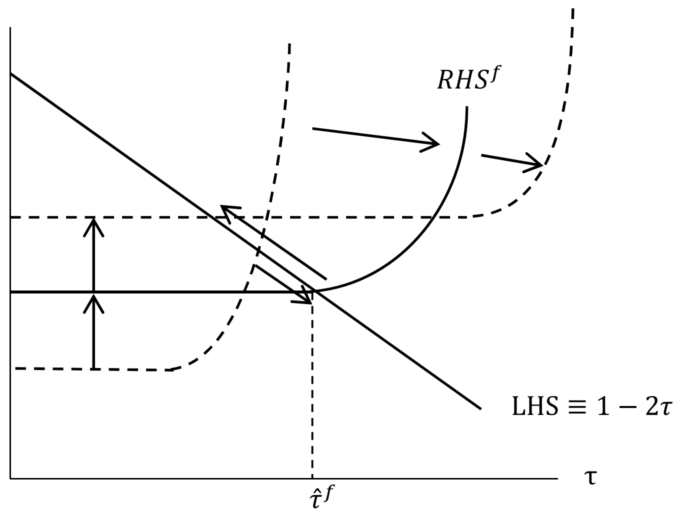


Figure 2. The figure illustrates the effect of an increase in α on the equilibrium tax rate in the case of $1/\sigma < 1$ with a decisive voter $j = f$.

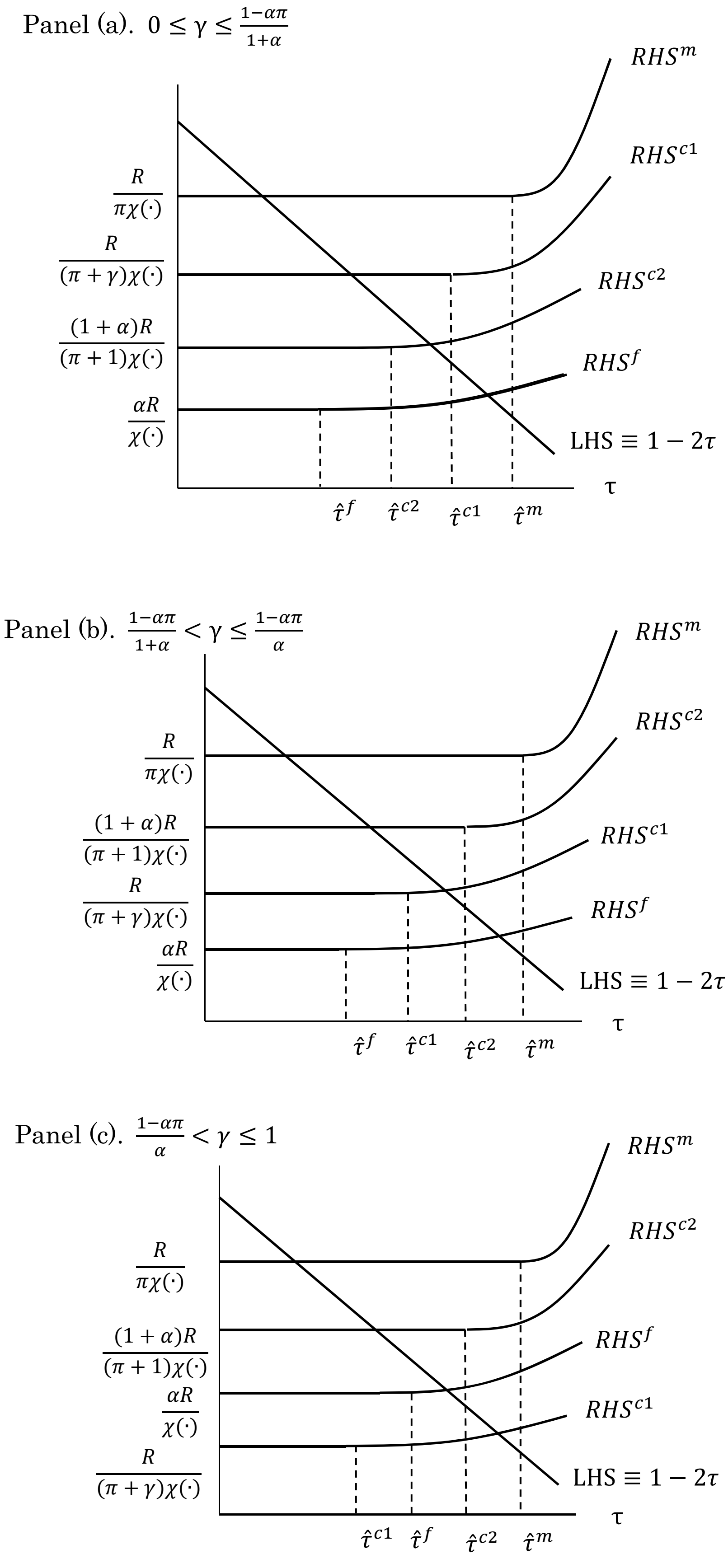


Figure 3. The preferred tax rates when $1/\sigma \ge 1$. Panel (a) is the case of $0 \le \gamma \le (1-\alpha\pi)/(1+\alpha)$; panel (b) is the case of $(1-\alpha\pi)/(1+\alpha) < \gamma \le (1-\alpha\pi)/\alpha$; and panel (c) is the case of $(1-\alpha\pi)/\alpha < \gamma \le 1$.