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Relationships and Growth

Shingo Ishiguro

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OSAKA UNIVERSITY
1-7 Machikaneyama, Toyonaka, Osaka, 560-0043, Japan

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Shingo Ishiguro *
Graduate School of Economics
Osaka University

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Abstract

In this paper we present a dynamic general equilibrium model to investigate how different contracting modes based on formal and relational enforcements endogenously emerge and are dynamically linked with the process of economic development. Formal contracts are enforced by third party institutions (courts), while relational contracts are self-enforcing agreements without any third party involvement. The novel feature of our model is to demonstrate the co-evolution of these different enforcement modes and market equilibrium conditions, all of which are jointly determined. We then characterize the equilibrium paths of such dynamic processes and show the time structure of relational contracting (self-enforcing agreement) in the endogenous process of economic development. In particular we show that relational contracting fosters the emergence of the market-based economy in low development stages but its role declines as the economy grows and enters high development stages.

Keywords: dynamic general equilibrium, economic development, arm's length contract, relational contract
JEL Classification Numbers: D86, E10, O11

*Correspondence: Shingo Ishiguro, Graduate School of Economics, Osaka University. 1-7 Machikaneyama, Toyonaka, Osaka 560-0043, Japan. e-mail: ishiguro@econ.osaka-u.ac.jp

1 Introduction

Informal contract arrangements, which we call *relational contracting* in this paper, are common during the developing stages of economies. These arrangements are not based on formally written contracts, but rather on long term relationships, implicit agreements, and a reputation mechanism based on personal ties and connections, as typically observed in tribal and ancient societies[1] as well as in emerging and transition economies.[2]

Among other informal contract arrangements, one well-documented example is relationship (insider) lending based on personal relationships between borrowers and lenders, typically banks. Lamoreaux (1994) reported that, during the early 19th century, the New England banks lent a large portion of their funds to the board of directors and those who had close personal ties with these banks. Lamoreaux then found the evidence that such relationship lending contributed to the economic growth in New England during that period, in contrast to the view that relationship-based finance might be less efficient than market based finance.[3] Similar informal financial arrangements were also widely observed during preindustrial stages in other countries such as China, India, and Islamic Middle Eastern countries.[4]

Some authors argue that relational contracting, which is based on closed relationships among particular members such as kinship networks, inhibits economic development compared to the market based economy, in which trades are made in an impersonal manner. For example, one of the reasons that China, India, and the Islamic Middle East economically lagged behind Europe during the 19th century is thought to be their strong reliance on relational contracting, despite that these countries had technological advantages over Europe during the Middle Ages (see, for example, Kumar and Matsusaka (2006)). However, a contrasting view is that relational contracting is not a substitute but a complement to a market economy in that the former fosters the latter (see the papers contained in Aoki and Hayami (2000)).

The main objective of our paper is to provide a unified theoretical framework for understanding how and when relational contracting plays positive or negative roles in economic development. Then we characterize the time structures of relational contracting (self-enforcing agreement) in the endogenous process of economic development: when and in what stages of economic development does relational contracting have tighter self-enforcing agreement constraint? A novel feature of our model is to embed rela-

[1]See Levi-Strauss (1969), Malinowski (1961), and Mauss (1967) for anthropological studies on reciprocal exchange and gift exchange in tribal societies. See Greif (2006) and Milgrom, North and Weingast (1990) for a discussion on how the merchant trade system functioned as a reputation device in medieval times.

[2]See McMillan and Woodruff (1999) for a discussion of trade credits in Vietnam and Johnson, McMillan and Woodruff (2002) for a discussion of relational contracting in Russia.

[3]As Lamoreaux (1994) emphasized, insider lending was a phenomenon observed not only in New England but also in other U.S. states during the early 19th century before the U.S. markets expanded.

[4]See Kumar and Matsusaka (2009), Greif (2006) and North (1998) for historical evidence on this issue.

tional contracts that are supported by long term relationships into a dynamic general equilibrium model. Long term relationships have been mostly analyzed in the partial equilibrium frameworks in the literature of repeated games (see Mailath and Samulaeson (2006)): in a typical repeated game, players play the stage games repeatedly over time by assuming that the outside markets are exogenously given. However, despite much historical evidence that shows the important roles of long-term relationships in the process of economic development, there are few theoretical studies that consider the macroeconomic implications of these relationships. These implications are what we address in this paper.

In our model economy, producers who need to finance their capital investment have the incentive to default after they borrow the funds from lenders. We assume that there are two means by which producers can commit themselves not to default. The first type is the anonymous credit market, in which everyone can borrow and lend at a given market interest rate in the spot manner. When producers finance capital investment in the credit market, they must invest in the verification technology in advance, preventing themselves from defaulting. We call this type of contract an *arm's length (formal) contract* and call producers who engage in an arm's length contract *non-relationship producers*. The verification investment includes activities such as collecting evidence about accounting data, hiring lawyers and accounting professionals, and establishing the information disclosure system.

However, there is also a *local community* in our model economy: each producer in the community has a personal connection with a particular lender within the same community. The producer and the lender form a long term relationship over successive generations. We call this type of producer a *relationship producer*. Each relationship producer can engage in relational contracting with a particular lender for financing capital investment without using the outside credit market. This type of contract is what we call a relational contract. Because they interact with each other over time, the relationship producer and the lender can avoid strategic default via a self-enforcing agreement.

As is well known from the repeated games literature, an implicit agreement is self-enforceable if each party's deviation from honoring the agreement results in future losses larger than the one-time gains obtained by the deviation. The novel feature of our model is how it relates the self-enforceability of relational contracting to the endogenous process of economic development in a dynamic general equilibrium framework. The endogenous process of economic development determines the profit of an arm's length contract, which becomes the deviation payoff for each relationship producer when quitting the current relationship with a lender. This change then affects the self-enforcing condition of the relational contract. In turn, the change of the self-enforcing condition creates a feedback effect on the equilibrium determination of the profit of an arm's length contract through changing the market prices, such as wage and interest rates. These two-way interactions between the self-enforcing condition of a relational contract and market equilibrium conditions jointly determine the evolution processes of the economy.

The dynamic general equilibrium interactions mentioned above cause non-trivial effects on the determination of relational contracts over time. When capital accumulation proceeds over time, lowering the interest rates in the anonymous credit market, the profit of an arm's length contract becomes more attractive to producers, and the self-enforcing condition of a relational contract becomes tighter. However, a more stringent self-enforcing constraint devalues the relational contract, which then contributes less to the capital accumulation of the economy as a whole. Thus, in general, it is difficult to predict whether economic development positively affects and is affected by the sustainability of relational contracting.

By considering such dynamic general equilibrium effects, we characterize the equilibrium paths of the model economy and show the time structures of relational contracting in the endogenous process of development. In particular we demonstrate that equilibrium paths involve the structural change from a relationship-based system relying on relational contract to a market-based system relying on an arm's length (formal) contract. We show that in any equilibrium path there exists a unique switching period before which the self-enforcing constraint becomes slack but after which it becomes binding. In periods before the switching period, the economy is in low-developed stages such that the market interest rates in the credit market are high and the profit of arm's length contract is low, and hence only small fraction of non-relationship producers can finance their capital investments. After the economy passes the switching period, self-enforcing condition becomes tighter because interest rates fall, and hence the profit of an arm's length contract becomes larger as the anonymous credit market continues to expand. Thus, a relational contract contributes less to the capital accumulation process than non-relationship producers who use arm's length contract do in developed stages.

Our results are consistent with empirical and historical evidence, as described below. First, we show that the co-evolution processes of contract enforcement modes and market equilibrium conditions endogenously trigger the dynamic transformation from the relationship-based system, in which relational contracting in a local community contributes more to capital accumulation than does the anonymous credit market, to the market-based system, in which a relational contract exhibits a smaller contribution to capital accumulation than does an arm's length contract over time. This result captures an important argument made by Polanyi (1947): the Western societies experienced the "great transformation" from nonmarket systems to market-based systems when different goods and services were priced subject to the law of market mechanisms in the 19th century. Moreover, our result shows that the relationship-based system plays a positive role in fostering economic growth during low-development stages because it becomes less constrained by self-enforcing conditions during these stages, thus fostering the emergence of a market-based economy during subsequent stages of development. These results are consistent with the aforementioned historical evidence that relational contracting complements the rise of a market-based economy (Aoki and Hayami (2000) and Lamoreaux (1994)).

Second, we also show that relational contracting becomes further constrained by the self-enforcing condition as the economy grows and enters matured stages of development. As the economy becomes richer, it becomes cheaper to invest in the verification technology, thus preventing strategic default and easing the enforcement of formal contracts, while the role of the relational contract declines relative to that of the arm's length contract over time. This observation is consistent with the historical fact that the New England banks that lent to closely related persons (for example, directors of these banks) in the early 19th century eventually had begun to lend to "outside" borrowers, whom they did not personally know well, as the economy changed from capital poor stages to capital rich stages, thus expanding the anonymous credit market in the late 19th century (Lamoreaux (1994)). A related argument is that the relationship-based system was dominant in Asian countries such as Korea and Japan after World War II but it has been changing to the market-based system as capital markets became more integrated and open to the world (see Rajan and Zingales (2000)). Demirgüç-Kunt and Levine (2004) reported the related evidence that the ratio of bank finance relative to equity finance is negatively associated with per capita GDP levels across countries, suggesting that bank finance, which is often characterized as a long-term lending relationship between a particular bank and a firm, becomes less important and is replaced by market-based finance in developed countries.

Related literature Although several papers address relational contracting in partial equilibrium frameworks, [5] few studies have attempted to examine its macroeconomic implications via dynamic general equilibrium models. Some papers attempt to compare informal contracting enforcement, such as reputation, with formal and legal enforcement in random matching environments. Kranton (1996) focuses on the market-based monetary exchange and relational (self-enforcing) contracts that emerge in the Kiyotaki-Wright type of monetary search model. Dhilon and Rigolini (2006) make the comparison between reputation and legal enforcement by endogenizing the quality of enforcement institutions. Francois (2011) investigates the evolution of endogenous institutions, but his analysis focuses on the roles of social norms formed through the change in endogenous preferences. Francois and Roberts (2003) examine how relational contracting affects long-run economic growth in an R & D-based endogenous growth model. However, they focus on relational (self-enforcing) contracts but not on the choice between arm's length and relational contracts. They also address the steady state analysis of the long-run growth rate and the macroeconomic effects of productivity shocks on relational contracting between firms and workers. Fafchamps (2003) also addresses the dynamic issue of how markets spontaneously emerge in the repeated game setting. Our work here differs from all the research cited above because our main concern is the dynamic change in the contracting modes and its relation to the process of economic development. Our new insight is that both the sustainability of relational contracting and the evolution of economic development are endogenous and jointly determined through dynamic general equilibrium effects. Specifically, we

[5] See, for example, Baker, Gibbons and Murphy (2002), Itoh and Morita (2007), Levin (2003), MacLeod and Malcomson (1998), and Ramey and Watson (2003).

address the hitherto unaddressed issues of how and when relationship-based economic systems change to market-based systems during the endogenous process of economic development.

The remaining sections are organized as follows. In Section 2, we describe an overlapping generations model with a choice between arm's length and relational contracts. In Section 3, we derive a self-enforceable incentive compatibility condition for relational contracts. In Section 4, we define an equilibrium of the model economy. In Section 5, we characterize the set of equilibrium paths and show that relational contracting contributes more to economic growth in early stages of development but that its role becomes more limited as the economy enters mature stages of development. In turn we show that there exists an equilibrium path with such a feature. All proofs are relegated to the Appendix.

2 Model

2.1 Economic Environment

We consider an overlapping generations economy. Time is discrete and extends over infinity $t = 0, 1, 2, ...$ In the economy, there is a single final good, taken as a numèraire, which is used for both consumption and investment. The final good is produced by labor and a continuum of intermediate goods with one unit measure (see below for more specifications). Every period, a continuum of one unit mass of individuals is newly born and each individual lives for two periods: young and old. In each generation, one young individual is born from each old individual, and we use the notation i to denote both individual i and the dynasty to which individual i belongs. For simplicity, we assume that each individual is concerned with only his or her consumption when old.

The newly born individuals consist of workers and intermediate goods producers. We use the masculine pronoun for intermediate goods producers (or borrowers) and the feminine pronoun for workers (or lenders). Each young worker is endowed with one unit of labor and inelastically supplies it to the labor market to earn the market wage w_t. Because every young worker is concerned with her consumption level when old, she will lend all the wage income to borrowers. Thus, all young workers become lenders. No old workers are endowed with labor inputs, and hence they simply consume all the income saved when they are young.

On the other hand, intermediate good $i \in [0, 1]$ is produced by producer i, who possesses the specific knowledge to produce that intermediate good. Each intermediate good is produced by investing in capital one period in advance. Specifically, an intermediate good producer can produce one unit of his intermediate good when old if he invests in one unit of capital when young.

We assume that capital fully depreciates after one period and that young intermediate good producers are not endowed with labor so that they need to finance their investments when young. Thus, young intermediate goods producers become borrowers.

2.2 Preference

We assume that individual i (worker or producer) has an altruistic preference over the consumption level of his or her child. More specifically, consider an individual in dynasty i who was born in period $t-1$ and whose consumption of the final good when old (in period t) is denoted by $C_t^{t-1}(i) \geq 0$. Then, we assume that the utility $U^{t-1}(i)$ of an individual in dynasty i, born in period $t-1$, depends not only on his/her own consumption level when old (in period t), $C_t^{t-1}(i)$, but also on the consumption level of his/her child in period $t+1$, $C_{t+1}^t(i)$, as follows:

$$U^{t-1}(i) \equiv C_t^{t-1}(i) + \delta C_{t+1}^t(i), \tag{1}$$

where $\delta \in (0,1)$ represents the parameter value measuring the degree to which each individual is altruistic about his/her child.

We assume that there is no transfer technology of bequest across generations in the same dynasty. Then, each individual consumes all of his/her old income for himself/herself such that $C_t^{t-1}(i)$ is equal to the lifetime income level of individual i born in period $t-1$. We will discuss an extension of the basic model to allow bequests in Appendix B.

2.3 Final Good Market

A single final good Y_t is produced by a continuum of intermediate goods, each of which is indexed by $i \in [0,1]$ and labor L_t in the following manner:

$$Y_t = AL_t^{1-\alpha} \int_0^1 y_t(i)^\alpha di, \tag{2}$$

where $\alpha \in (0,1)$, $A > 0$ and $y_t(i)$ denotes the input demand for (equivalent to its output) intermediate good i in period t.

We assume that there is perfect competition in the final goods market. Then, the final good firm chooses the demand for labor L_t and intermediate inputs $y_t(i)$ to maximize its profit:

$$AL_t^{1-\alpha} \int_0^1 y_t(i)^\alpha di - w_t L_t - \int_0^1 p_t(i) y_t(i) di, \tag{3}$$

where wage rate w_t and price of intermediate good i, $p_t(i)$, are taken as given. The corresponding first-order conditions are as follows:

$$A\alpha L_t^{1-\alpha} y_t(i)^{\alpha-1} = p_t(i) \tag{4}$$

and

$$A(1-\alpha)L_t^{-\alpha} \int_0^1 y(i)^\alpha di = w_t. \tag{5}$$

2.4 Credit Market with Commitment Problem

Each young intermediate good producer needs to finance capital investment for production to be performed when old. However, we suppose that any borrowed amount is not verifiable. Then, each old intermediate good producer will always default a borrowed amount and refuse to repay lenders. Thus, there must be some commitment device impelling producers not to default because otherwise lenders would never lend to producers.

In the economy there are two places where intermediate goods producers finance their capital investments without default.

Credit Market for Arm's Length Contracts

One is the market place, which we call the credit market, where borrowing and lending are made at a given interest rate. The credit market is perfectly competitive in the sense that everyone takes the market interest rate $\rho_t \geq 0$ as given in any period t.[6] However, the credit market is not perfectly competitive in the sense that it involves a commitment problem regarding default, as we have already mentioned: it is not verifiable how much a producer (borrower) actually borrowed from lenders.[7] We assume that each borrower can, however, commit himself not to default his borrowed amount when he invests in the verification technology one period in advance.

Investment in the verification technology requires $I > 0$ units of the final good one period in advance. Here, I includes the costs of making the information disclosure credible, collecting hard evidence, hiring professionals such as lawyers and accountants who help write formal contracts, and using outside institutions such as courts. When he invests in the verification technology in advance, a producer can commit himself not to default when old.[8] In what follows, we call an intermediate good producer who finances capital investment from the anonymous credit market a *non-relationship producer*. We let $x_t \geq 0$ denote the capital investment (equivalently, the production level of an intermediate good) of a non-relationship producer. Then, if a non-relationship producer invests in capital x_t and in verification technology I in period $t-1$, he needs

[6] Here, ρ_t denotes the gross interest rate for a spot transaction in the credit market, which specifies that one unit of borrowing in period $t-1$ must result in the repayment of ρ_t units in the next period t. Thus, by the nature of spot transactions in the credit market, no borrower can roll a debt obligation forward in period $t-1$ to his child who would repay it in period $t+1$.

[7] Even when how much an intermediate good producer produced can be observed, it is not verifiable how much he actually borrowed. He might simply insist "I have prepared an amount x_t for capital investment by myself (e.g., from my own pocket or received freely from someone) but I have not borrowed it."

[8] We can also allow lenders to incur some verification cost when they lend to the credit market. For example, they monitor whether borrowers in the credit market renege on repayments to them. Specifically, we assume that $\gamma \in (0, 1)$ is the cost incurred by each lender to monitor one unit of lending. Thus, each lender obtains an interest return of $(1-\gamma)\rho_t$ from lending one unit to the credit market. Even when we introduce such verification (monitoring) costs on the side of lenders in the credit market, our main results are not substantially different. Thus, we assume away such costs from the model and simply set $\gamma = 0$ in what follows.

to borrow $x_t + I$ units of the final good in period $t-1$ and repay $\rho_t(x_t+I)$ in period t.

We call such spot contract made in the anonymous credit market an *arm's length contract*. This contract is formally written and enforced by the court.

Local Community

On the other hand, in the economy there is a local community where l ($l < 1$) intermediate producers and l workers (lenders) reside (thus the remaining $1-l$ intermediate producers and workers are outside the community.) The producers and lenders in the community are matched with each other in the initial period $t = 0$ in a one-to-one manner. Then, each of the matched l pairs forms a personal connection and relationship. We assume that such relationships formed in the initial period can be inherited over successive generations. We call an intermediate good producer in the community a *relationship producer* and a lender in the community a *relationship lender*. They can avoid the default problem by using long term relationships, as is shown below.

Any (young or old) relationship producer and lender in each pair can always exercise option to quit their relationship (*quitting option*) at any time by leaving the community. We will then assume that if a relationship producer (lender) quits the relationship, his (her) child cannot also form a relationship with the child of his (her) partner in the next period. In such a case, not only the current relationship producer and lender but also their children have no choice but to engage in an arm's length contract in the anonymous credit market. However, we do not assume the permanent dissolution; that is, we do not assume that once a relationship producer and a lender dissolve their relationship, all their descendants cannot form relationships forever. We only assume that it takes at least one period for a dissolved relationship to be re-formed. We later show that the assumption of one-period dissolution of a relationship is sufficient for each relationship pair to honor the agreed upon relational contracts over time.

In what follows we let $z_t \geq 0$ denote the capital investment (equivalently, the production level of an intermediate good) of a relationship producer. Each young relationship producer, born in period $t-1$, can directly finance his investment z_t from his partner, a relationship lender, in exchange for making repayment R_t in period t. They can then save the verification cost $I > 0$ by such implicit agreement. However, because repayment R_t is not secured unless the investment in the verification technology I is made, such an agreement $\{z_t, R_t\}$ must be implicit and self-enforceable. We call this type of contract a *relational contract*.

2.5 Timing Within Each Period

Events in each period proceed in the local community as follows. First, at the beginning of each period, old relationship producer and lender of each relationship pair decide whether to exercise the quitting option. When they exercise the quitting option, their relationship is dissolved and their children will have no choice but to engage in an arm's length contract in the next period. When an old producer and an old lender in a relationship do not exercise the quitting option, in the same period their children (young relationship producer and lender) decide whether to exercise the quitting option. By exercising the quitting option, the young relationship producers and lenders can ensure at least the payoffs obtained by an arm's length contract in the credit market. Furthermore, their children born in the next period must engage in an arm's length contract because of our assumption that any dissolved relationship pair of producer and lender needs to wait at least one period for their descendants to re-forma relationship pair. When they do not exercise the quitting option, they agree on a relational contract, $\{z_t, R_t\}$, which specifies the capital investment level z_t and the repayment R_t to the relationship lender.[9] Because the producer cannot commit himself not to repay R_t, such a relational contract must be self-enforceable.

2.6 Labor Market

We assume that the labor market is perfectly competitive and that no commitment problems arise. We let $w_t \geq 0$ denote the competitive market wage in period t in the labor market.

2.7 Initial Period ($t = 0$)

In the initial period ($t = 0$), each old intermediate good producer (irrespective of non-relationship or relationship producer) owns an initial capital stock, $z_0 = x_0$, which is assumed to be historically given.[10] Because the old producers in this period do not need to raise funds for capital investment, they can supply intermediate goods $z_0 = x_0$ without any production and verification costs. There is also one unit mass of old lenders in the initial period.

3 Arm's Length and Relational Contracts

3.1 Arm's Length Contract Producers

Because every non-relationship producer faces the same demand function, (4), for his intermediate good, we will omit notation i hereafter.

Let π_t be the profit of an intermediate good producer who finances his investment $x_t + I$ via an arm's length contract in period t defined as follows:

$$\begin{aligned} \pi_t &\equiv p_t x_t - (x_t + I)\rho_t \\ &= A L_t^{1-\alpha} \alpha x_t^{\alpha} - (x_t + I)\rho_t, \end{aligned} \tag{6}$$

[9] Note here that $R_t = 0$ and $R_{t+1} > 0$ may be possible. Each old relationship producer (borrower) makes no repayment R_t in the current period, t; however, his child makes the repayment in the next period, $t + 1$, implying that each relationship producer (borrower) can roll his debt over to his child, an option unavailable in the case of borrowing by the arm's length contract.

[10] We here assume that all old producers in the initial period own the same amount of initial capital stock, $z_0 = x_0$, for simplifying the analysis. Our results are not substantially changed even when we allow $z_0 \neq x_0$.

where the price of an intermediate good, p_t, is given by (4). Here, note that every non-relationship producer has no other option but to borrow capital investment x_t using an arm's length contract in the credit market. As such, the non-relationship producer incurs borrowing cost $(x_t+I)\rho_t$ that includes investment I in the verification technology in addition to capital investment x_t for production. Because $L_t = 1$ holds in the labor market equilibrium, we set $L_t = 1$ in what follows.

Each young non-relationship producer born in period $t-1$ chooses capital investment level x_t to maximize his payoff $\pi_t + \delta\pi_{t+1}$. Here, note that his consumption level C_t^{t-1} is equal to the profit π_t he earned in period t, whereas the consumption level of his child is $C_{t+1}^t = \pi_{t+1}$. Thus, the payoff of a non-relationship producer is given by $C_t^{t-1} + \delta C_{t+1}^t = \pi_t + \delta\pi_{t+1}$.

We define by h_t a history of all the events observed by a non-relationship producer in a dynasty up to period t. Such a history includes the past investments $\{x_0, x_1, ..., x_{t-1}\}$ made in the same dynasty and the past market prices $\{w_s, \rho_s, p_s\}_{s=1}^{t-1}$. Let H_t be a set of all such histories up to period t. Then, a strategy for each non-relationship producer in each dynasty is defined as a mapping from the set of histories to capital investment levels: $\sigma_t : H_t \to [0, \infty)$.

3.2 Relationship Producers

Next, we consider a pair consisting of a relationship producer and a relationship lender who engage in relational contracting. Again, we omit notation i to index individuals because every relationship pair can be treated symmetrically.

Let J_t be the joint profit of a relationship pair, which is split between the relationship producer and lender after an intermediate good is produced. Because each young worker (who becomes a lender) earns market wage w_{t-1} in period $t-1$, the young relationship producer who wants to invest z_t units of capital in period $t-1$ can borrow this amount z_t directly from the young relationship lender matching him. He can then save on the verification cost $\rho_t I$, which would otherwise arise in an arm's length contract in the credit market. For the time being, assume that $w_{t-1} > z_t$. (Below, we will show that this is actually the case in equilibrium). Then, a relationship producer can borrow capital investment z_t via *insider lending* from the relationship lender instead of using an arm's length contract. The relationship lender then supplies the remaining amount, $w_{t-1} - z_t$, to the credit market to earn interest income $\rho_t(w_{t-1} - z_t)$ in the next period, t. Hence, the joint profit of a relationship pair in period t is given by the sum of the revenue from capital investment $p_t z_t$ and the interest income $\rho_t(w_{t-1} - z_t)$:

$$
\begin{aligned}
J_t &= p_t z_t + \rho_t(w_{t-1} - z_t) \\
&= \alpha A z_t^\alpha + \rho_t(w_{t-1} - z_t),
\end{aligned}
$$

where the price of intermediate good, p_t, is given by (4). The repayment R_t is used to split this joint profit between the relationship producer and lender.

In the following section we consider three constraints that limit the relational contract: The first constraint is the *incentive compatibility* (IC) condition, according to which the old relationship producer has no incentives to renege on the agreed upon repayment R_t. The second constraint is the *individual rationality condition for relationship producer* (IRP), according to which the young relationship producer is weakly better off by agreeing to a relational contract instead of exercising the quitting option. The third constraint is the *individual rationality condition for relationship lender* (IRL), according to which the young relationship lender is weakly better off agreeing to a relational contract instead of exercising the quitting option.

3.3 Incentive Compatibility

Consider any pair of a young relationship producer and a young relationship lender born in period $t-1$. Then, assume that they agree to a relational contract $\{z_t, R_t\}$: the young relationship producer promises to repay R_t to the relationship lender matching him when old (in period t) in exchange for borrowing z_t directly from her. Assume also that their relationship in period t is inherited by the next generation in period $t+1$ who will agree to a relational contract $\{R_{t+1}, z_{t+1}\}$. Assume then that the old relationship producer in period $t+1$ (who is the child of the old relationship producer in period t) does not exercise the quitting option but again honors the contracted agreement R_{t+1}.

Anticipating the outcome in period $t+1$ described above, the old relationship producer in period t makes the repayment R_t to the old relationship lender and does not exercise the quitting option only if the following incentive compatibility constraint, (IC$_t$), is satisfied:

$$p_t z_t - R_t + \delta\{p_{t+1}z_{t+1} - R_{t+1}\} \geq p_t z_t + \delta \max\{\pi_{t+1}, 0\} \qquad (\text{IC}_t),$$

where π_{t+1} denotes the profit an intermediate producer could obtain if he exercised the quitting option and financed his investment via an arm's length contract in period $t+1$.

We now explain (IC$_t$) in detail. The right-hand side of (IC$_t$) denotes the payoff the old relationship producer could obtain if he reneged on repayment R_t and exercised the quitting option. By doing so, he can save on repayment R_t and capture the whole revenue $p_t z_t$ from capital investment z_t, but in the next period, $(t+1)$, his child faces the dissolution of the relationship with the child of the current lender. In such a case, the child of such a deviating producer would obtain at least profit $\max\{\pi_{t+1}, 0\}$ via an arm's length contract in the credit market when old (in period $t+1$). Here, we use the operator $\max\{\pi_{t+1}, 0\}$ to denote whether profit π_{t+1} should be non-negative. This child's future payoff is evaluated using the altruistic parameter $\delta > 0$ from the viewpoint of the current producer. Thus, the sum of these payoffs can be guaranteed by the old relationship producer when he exercises the quitting option. On the other hand, the left-hand side of (IC$_t$) denotes the payoff of the old relationship producer when he makes contracted repayment R_t to the relationship lender matching him in period t expecting that the relationship is inherited by the next period generation, in which case his child also makes repayment R_{t+1} in period $t+1$. This future payoff is also evaluated using δ from the viewpoint of the current old producer in period t.

Thus, (IC_t) is necessary for the old relationship producer not to renege on repayment R_t in period t.

The following individual rationality constraint, (IRL_t), of the relationship lender must also be satisfied:

$$R_t + \rho_t(w_{t-1} - z_t) + \delta\{R_{t+1} + \rho_{t+1}(w_t - z_{t+1})\} \geq \rho_t w_{t-1} + \delta\rho_{t+1} w_t \qquad (\text{IRL}_t).$$

Otherwise, it is a strictly dominant strategy for the young relationship lender to exercise the quitting option in period $t-1$; by exercising the quitting option, the lender and her child lend their entire wage income to the credit market so as to earn the interest income corresponding to the payoffs on the right-hand side of (IRL_t). Here, the interest income of her child, $\rho_{t+1}w_t$, is evaluated by the altruistic parameter δ from the viewpoint of the current old lender. However, if the old relationship lender does not exercise the quitting option, she would obtain contracted repayment R_t in period t, which appears as the first term on the left-hand side of (IRL_t), in addition to interest income $\rho_t(w_{t-1} - z_t)$ from the savings on the remaining income $w_{t-1} - z_t$ after lending z_t to the relationship producer (note here that we are assuming that $w_{t-1} > z_t$). The child of the lender is paid the contracted amount R_{t+1} in addition to the interest income $\rho_{t+1}(w_t - z_{t+1})$ when old in period $t + 1$. Here, the payoff of the lender's child is evaluated by the altruistic parameter $\delta > 0$ again from the viewpoint of the current old lender.

Combining (IC_t) with (IRL_t), we can derive the following modified incentive compatibility condition, denoted by (IC_t^*):

$$\delta\{p_{t+1}z_{t+1} - \rho_{t+1}z_{t+1} - \max\{\pi_{t+1}, 0\}\} \geq \rho_t z_t \qquad (\text{IC}_t^*),$$

This condition is necessary for relationship pairs in period t to be sustained so that the relational contract is self-enforceable. Using (4), (IC_t^*) can also be expressed as

$$\delta\{\alpha A z_{t+1}^{\alpha} - \rho_{t+1}z_{t+1} - \max\{\pi_{t+1}, 0\}\} \geq \rho_t z_t \qquad (\text{IC}_t^*).$$

Next, we introduce the individual rationality constraint of the relationship producer (IRP_t):

$$p_t z_t - R_t + \delta\{p_{t+1}z_{t+1} - R_{t+1}\} \geq \max\{\pi_t, 0\} + \delta\max\{\pi_{t+1}, 0\} \qquad (\text{IRP}_t).$$

The condition (IRP_t) ensures that the relationship producer prefers continuing the relationship to dissolving it. Suppose that the producer exercises the quitting option when young. Then, he earns profit $\max\{\pi_t, 0\}$ via an arm's length contract when old (period t). Furthermore, his child earns profit $\max\{\pi_{t+1}, 0\}$ via an arm's length contract when old in period $t + 1$, which is evaluated by δ from the viewpoint of the current producer. The sum of these payoffs corresponds to the payoff that each non-relationship producer obtains via an arm's length contract, which thus becomes his outside option. However, the left-hand side of (IRP_t) denotes the payoff of the relationship producer who does not exercise the quitting option when young; by doing so, he earns $p_t z_t - R_t$ when old (period t) by making repayment R_t to the relationship lender. His child also continues the relationship and earns profit $p_{t+1}z_{t+1} - R_{t+1}$ by making the contracted repayment R_{t+1} to the relationship lender when old (period $t+1$). Thus, the young relationship producer obtains the sum of these payoffs by continuing the relationship.

Then, by subtracting the right-hand sides of (IRP$_t$) and (IRL$_t$) from their left-hand sides and using (4), the net total surplus of a pair of a relationship producer and a relationship lender born in period $t-1$ is defined as

$$\begin{aligned} TS_{t-1} &\equiv p_t z_t - \rho_t z_t - \max\{\pi_t, 0\} \\ &\quad + \delta\{p_{t+1}z_{t+1} - \rho_{t+1}z_{t+1} - \max\{\pi_{t+1}, 0\}\} \\ &= \alpha A z_t^{\alpha} - \rho_t z_t - \max\{\pi_t, 0\} \\ &\quad + \delta\{\alpha A z_{t+1}^{\alpha} - \rho_{t+1}z_{t+1} - \max\{\pi_{t+1}, 0\}\}. \end{aligned} \tag{7}$$

We impose the following condition, which ensures that each relationship pair born in period $t-1$ finds it optimal to sustain the relationship rather than dissolve it:

$$TS_{t-1} \geq 0 \qquad (\text{TS}_{t-1}).$$

We can then readily show that there exists a sequence of repayments $\{R_t\}_{t=1}^{\infty}$ that satisfy (IC$_t$), (IRL$_t$) and (IRP$_t$) for all $t \geq 1$ as long as (IC$_t^*$) and (TS$_{t-1}$) are satisfied for all $t \geq 1$.[11]

Although there are many possible equilibria sustained by different relational contracts, as in the Folk Theorem of repeated game theory, we will focus on the equilibrium, called the *Best Relational Contracting Equilibrium (BRCE)*, in which the initial generation of relationship pairs in each dynasty chooses a sequence of all future relational contracts $\{z_t, R_t\}_{t=1}^{\infty}$ so as to maximize the sum of joint payoffs of all generations in the same dynasty $\sum_{t=0}^{\infty} \delta^t J_t$ subject to the constraints $\{(\text{IC}_t^*), (\text{TS}_{t-1})\}_{t=1}^{\infty}$, given the future paths of all the market prices $\{\rho_t\}_{t=1}^{\infty}$ and $\{w_t\}_{t=0}^{\infty}$.[12] Along such an equilibrium path the relationship pair born in any subsequent period, say $T \geq 1$, has the incentive to follow the relational contract designed for period T, $\{z_T, R_T\}$, provided that all future generations in the same dynasty will also do so.[13]

[11] It is sufficient to set $R_t = \rho_t z_t$ for each $t \geq 1$.

[12] See Acemoglu, Golosov and Tsyvinski (2008) for a related treatment of self-enforcing agreements in a dynamic general equilibrium model, although they consider a different model from ours.

[13] This can be readily seen by the following fact: let $\{z_t, R_t\}_{t=1}^{\infty}$ be the optimal relational contracts that are the solutions to $\max \sum_t \delta^t J_t$ subject to $\{(\text{IC}_t^*), (\text{TS}_{t-1})\}_{t=1}^{\infty}$. If a relationship pair in some generation born in period $T-1$ would be better off by agreeing to a relational contract different from the original one, $\{z_T', R_T'\} \neq \{z_T, R_T\}$, it must satisfy (IC$_T^*$) and (TS$_{T-1}$) and have larger joint payoff $J_{T-1}' + \delta J_T'$ than the joint payoff obtained under the original relational contract $J_{T-1} + \delta J_T$. Then, if the relational contracts $\{z_t, R_t\}_{t=1}^{\infty}$ designed in the initial period are replaced by $\{z_t', R_t'\}_{t=1}^{\infty}$, where $(z_T', R_T') \neq (z_T, R_T)$ but $(z_t', R_t') = (z_t, R_t)$ for any $t \neq T$, the sum of the joint payoffs $\sum_{t=0}^{\infty} \delta^t J_t$ is improved. However, this result contradicts the fact that the relational contract $\{z_t, R_t\}_{t=1}^{\infty}$ maximizes $\sum_{t=0}^{\infty} \delta^t J_t$ subject to (IC$_t^*$) and (TS$_{t-1}$) for all $t \geq 1$.

4 Equilibrium Paths of the Economy

We let $\theta_t \in [0,1]$ denote the fraction of non-relationship producers who can produce their intermediate goods in period t. Because there is the verification cost $I > 0$ for each non-relationship producer to borrow in the credit market, some such producers may decide not to finance capital investment and hence decide to shut down production.

Now we provide a formal definition of an equilibrium path in this model economy:

Definition. *A sequence $\{x_t, z_t, w_t, \rho_t, \theta_t\}_{t=0}^{\infty}$ is said to be an equilibrium of the economy if the following conditions are satisfied:*

(i) *Each young non-relationship producer chooses a strategy $\sigma_t : H_t \to [0,\infty)$ that maps from the set of observed histories H_t to a capital investment level $x_t \geq 0$ so as to maximize his payoff $\pi_t + \delta\pi_{t+1}$,*

(ii) *The initial generation of the relationship pairs in each dynasty chooses a sequence of future relational contracts $\{z_t, R_t\}_{t=1}^{\infty}$ so as to maximize the sum of the joint payoffs of all generations in the same dynasty $\sum_{t=0}^{\infty} \delta^t J_t$ subject to $\{(IC_t^*), (TS_{t-1})\}_{t=1}^{\infty}$,*

(iii) *The labor market equilibrium (LME): $L_t = 1$, and the market wage w_t is determined by:*

$$w_t = A(1-\alpha)[lz_t^{\alpha} + \theta_t(1-l)x_t^{\alpha}],$$

(iv) *The credit market equilibrium (CME) is*

$$w_{t-1} = lz_t + \theta_t(1-l)(x_t + I),$$

where the initial capital $z_0 = x_0$ is given.

Here, condition (i) yields the optimal capital choice of each young non-relationship producer taking market prices w_t and ρ_t as given. Condition (ii) is the optimal choice with regard to relational contract $\{z_t, R_t\}$ for every relationship pair, as explained above. The third condition (LME) is the labor market clearing condition: market wage w_t is determined by clearing the labor market ($L_t = 1$) using the labor demand function (5) and that the labor supply is given by one unit mass of young workers in every period. The fourth condition (CME) is the credit market clearing condition. Each relationship producer lends $w_{t-1} - z_t$ to the anonymous credit market in period $t-1$ after she makes a relationship lending z_t to the relationship producer. In addition to this supply of credit, $1-l$ non-relationship lenders outside the community have nothing but to lend w_{t-1} to the credit market. Thus, the total supply in the anonymous credit market is given by $l(w_{t-1} - z_t) + (1-l)w_{t-1}$. Conversely, $1-l$ non-relationship producers finance their capital and verification investments $x_t + I$ from the credit market. However, only θ_t fraction of them can borrow the fund from the credit market. Thus, the total demand in the credit market is given by $\theta_t(1-l)(x_t+I)$. Then, the credit market clears if (CME) holds.

In this model economy, the initial condition is given by initial capital stock $z_0 = x_0$ owned by each initial old intermediate good producer. Thus, every initial old intermediate goods producer produces x_0 without any production and verification costs. Then, $\theta_0 = 1$ so that the market wage in the initial period ($t = 0$) is determined by LME: $w_0 = A(1-\alpha)x_0^{\alpha}$.

5 From Relationships to Markets

5.1 Characterization of Equilibrium Paths

In this section we show the characterization result that relational contracting contributes to economic growth in low development stages while its value declines as the economy grows and enters high development stages.

We begin by showing the three preliminary results that will be useful for characterizing equilibrium paths below.

First, Lemma 1 describes the optimal behavior of non-relationship producers. Because the utility function of each non-relationship producer depends on his child's consumption level, there may be intergenerational strategic interactions between a non-relationship producer's choice of capital investment x_t and his child's capital investment choice x_{t+1}. However, the following lemma shows that it is sufficient to focus only on the equilibrium in which each non-relationship producer acts to maximize only his own profit π_t no matter the history.

Lemma 1. *In any equilibrium, every non-relationship producer born in period $t-1$ chooses capital investment level x_t to maximize only his own profit π_t no matter the history observed up to period t.*

Thanks to Lemma 1, equilibrium investment of each non-relationship producer is determined by maximizing π_t. We then define such optimal investment as $x(\rho_t)$, which satisfies the following first-order condition:

$$A\alpha^2 x_t^{\alpha-1} = \rho_t. \tag{8}$$

Thus, we have

$$\pi(\rho) \equiv \max_x \pi = A\alpha(1-\alpha)x(\rho)^{\alpha} - \rho I. \tag{9}$$

Because $\max_x \pi$ is decreasing in ρ, we can find a unique $d > 0$ such that $\pi(d) = 0$. Then, the equilibrium interest rate in the credit market must be bounded above by d. Let $\underline{x} \equiv x(d)$. Because $\pi(d) = A\alpha(1-\alpha)\underline{x}^{\alpha} - dI = 0$ and $\alpha^2 A\underline{x}^{\alpha-1} = d$, we have $\underline{x} = \alpha I/(1-\alpha)$. Then, $\rho_t \leq d$ is equivalent to $x_t \geq \underline{x}$.

By (CME), $\rho_t = d$ becomes the equilibrium interest rate when $w_{t-1} - lz_t < (1-l)(\underline{x}+I)$ (see Figure 1). In this case, θ_t fraction of non-relationship producers decide to finance the investment $\underline{x} + I$, while the remaining fraction decide to shut down. In either case, each non-relationship producer obtains zero profit because $\pi(d) = 0$.

Second, Lemma 2 describes the optimal behavior of relationship pairs.

Lemma 2. *(i) Relational contract $\{z_t, R_t\}$ in period t is sustained (self-enforceable) if and only if (IC_s^*) and (TS_s) are satisfied for all $s \geq t$. (ii) $z_t \leq x(\rho_t)$ in any period t. (iii) $z_t < x(\rho_t)$ only if (IC_t^*) is binding.*

Lemma 2 (i) implies that a sequence of relational contracts $\{z_t, R_t\}_{t=1}^{\infty}$ is self-enforceable if and only if (IC_t^*) and (TS_{t-1}) are satisfied for all $t \geq 1$. Among all relational contracts that satisfy this requirement, the initial generation of relationship pairs in each dynasty chooses the relational contracts that maximize the sum of the joint payoffs of all generations in the same dynasty $\sum_{t=0}^{\infty} \delta^t J_t$. Lemma 2 (ii) and (iii) then show that the optimal relational contracts may involve a downward distortion of capital investment $z_t < x(\rho_t)$ relative to the one maximizing the joint payoff J_t without (IC_t^*). Such downward distortion occurs only if (IC_t^*) is binding.

Third, we show that in any equilibrium path each relationship producer never invests in capital more than the funds available to his matching lender w_{t-1} in any period t. Otherwise, $z_t > w_{t-1}$ holds in some period t, and hence each relationship producer needs to finance the remaining amount $z_t - w_{t-1}$ from the credit market after borrowing w_{t-1} directly from the relationship lender matching him. Then, the relationship producer must incur the verification cost $\rho_t I$, but this cannot be optimal relational contract because lowering capital investment from z_t to w_{t-1} can avoid the verification cost and improve the joint payoffs of some generations in the dynasty of relationship pairs.

Lemma 3. *In any equilibrium path $z_t \leq w_{t-1}$ must hold in any period t.*

Next, we turn to characterize equilibrium paths. The features of equilibrium paths depend on the altruistic parameter $\delta \in (0, 1)$, which represents how much each individual cares about his or her child. Here, δ plays a role similar to that of the discount factor in the repeated games. Thus, (IC_t^*) becomes less stringent as δ becomes larger, in which case the choice of capital investment by relationship pair z_t is never constrained and hence is given as the same level chosen by the non-relationship producer ($z_t = x_t = x(\rho_t)$).

As a benchmark, we first consider the case that (IC_t^*) is never binding in any period. Thus, the capital investment choice of any relationship pair is not constrained by (IC_t^*), and hence $z_t = x_t$, as shown in Lemma 2. Then, by using $w_t = (1-\alpha)[lAz_t^{\alpha} + (1-l)\theta_t A x_t^{\alpha}]$ together with (CME), we can readily verify that a sequence of capital investments and fractions of active non-relationship producers $\{x_t, \theta_t\}_{t=0}^{\infty}$ obeys the following dynamic equation:

$$(l + (1-l)\theta_t)(1-\alpha)Ax_t^{\alpha} = lx_{t+1} + (1-l)(\theta_{t+1}x_{t+1} + I) \tag{10}$$

where $\theta_t < 1$ occurs only when $x_t = \underline{x} = x(d)$.

We consider the following assumptions.

Assumption 1. $(1-\alpha)A\underline{x}^{\alpha} > \underline{x} + I$.

Assumption 2. *$x^* > \underline{x}$ where x^* is defined as a unique x satisfying $(1-\alpha)Ax^{\alpha} = x + (1-l)I$.*

Assumptions 1 and 2 state that the productivity of the economy A is so large that the total investment $\underline{x}+I$ can be covered and non-relationship producers can eventually escape from the lowest production level $\underline{x}$ as the economy reaches the steady state x^* without (IC_t^*). Without these requirements, the economy cannot take off from a low production level $\underline{x}$ for each non-relationship producer, even when we ignore (IC_t^*).

Now we can show the following result.

Proposition 1. *Suppose that Assumptions 1 and 2 hold. Then, if (IC_t^*) is never binding in any period t in an equilibrium path, the economy converges to a unique steady state x^* in that equilibrium.*

According to Proposition 1, if the altruistic parameter δ is so large that (IC_t^*) is not binding at all, the economy eventually converges to a unique steady state x^* in the long run. However, this scenario is never the case when δ is not large, as we will show in the following lemma.

Lemma 4. *Suppose that Assumptions 1 and 2 hold. Suppose also that $\delta < x^*/I$. Then, $z_t < x_t$ must hold so that (IC_t^*) must be binding in some period t in any equilibrium path.*

Lemma 4 is intuitive because if $z_t = x_t$ holds in any period t, the equilibrium path of the economy is unique and converges to the steady state x^* because of Proposition 1. However, then (IC_t^*) becomes $\delta\rho_{t+1}I \geq \rho_t x_t$ in a neighborhood of the steady state x^* because $z_t = x_t$ for all t and $\pi(\rho_{t+1}) > 0$ for all x_t close to x^*. Thus, (IC_t^*) can be expressed as $\delta x_{t+1}^{\alpha-1}I \simeq \delta(x^*)^{\alpha-1}I \geq x_t^{\alpha} \simeq (x^*)^{\alpha}$ for $x_{t+1} \simeq x_t \simeq x^*$, implying $\delta I \geq x^*$. This expression violates $\delta < x^*/I$, and hence (IC_t^*) must be binding in some period t in any equilibrium.

In what follows we rule out the trivial case that (IC_t^*) never binds in any period t. We thus assume that $\delta < x^*/I$, as Lemma 4 shows. We also suppose that δ is not so small that (IC_t^*) becomes slack when non-relationship producers operate at the lowest production level $\underline{x}$. Otherwise, we may reach the other polar case that (IC_t^*) is always binding at the lowest production level $z_t = \underline{x}$ in any period. What we will address in the following is to determine when (IC_t^*) changes from non-binding case to binding case or vice versa and how such timing interacts with the process of economic development. Thus, we will rule out the case that (IC_t^*) is always binding at the lowest production level $\underline{x}$. Let $x_{t+1} = x_t = \underline{x}$. Then, we can see that (IC_t^*) becomes slack at $z_{t+1} = z_t = \underline{x}$:

$$\delta\{\alpha A\underline{x}^\alpha - d\underline{x}\} > d\underline{x} \tag{11}$$

where the profit of non-relationship producer becomes $\pi(d) = 0$ at $x_{t+1} = \underline{x}$. The above inequality is equivalent to $\delta(1-\alpha)A\underline{x}^\alpha > \alpha^2 A\underline{x}^\alpha$, that is, $\delta > \alpha/(1-\alpha)$. When this condition is satisfied, (IC_t^*) will be not binding if non-relationship producers operate at the lowest production level $\underline{x}$.

Thus we assume the following:

Assumption 3. $\alpha/(1-\alpha) < \delta < x^*/I$.

We will also slightly strengthen Assumptions 1 and 2 as follows.

Assumption 4. $(1-l)(1-\alpha)A\underline{x}^\alpha > \underline{x} + I$.

Assumption 5. $\overline{x} > \underline{x}$ *where* $\overline{x}$ *is the value of* x *that is a unique solution to*

$$(\delta I/x^\alpha)^{\frac{1}{1-\alpha}} = (1-\alpha)Ax^\alpha - (1-l)I. \tag{12}$$

Assumption 4 means that the productivity of the economy measured by the parameter value A is so large that the economy can cover the minimum investment cost $\underline{x} + I$ even when only non-relationship producers operate a the lowest scale $\underline{x}$. We will show later that this assumption can ensure that the economy eventually takes off from the lowest production scale $\underline{x}$ as it develops over time. Assumption 5 is also satisfied when the productivity parameter A of the final good production is large relative to the verification cost I (note that $\underline{x} = \alpha I/(1-\alpha)$ and thus it is independent of A.)

It can be readily seen that $x^* > \overline{x}$ holds because $\delta < x^*/I$ under Assumption 3. Thus, Assumption 5 is stronger than Assumption 2 when Assumption 3 holds. Because Assumptions 4 and 5 imply Assumptions 1 and 2 as long as we keep Assumption 3, Proposition 1 and Lemma 4 still hold.

Now we proceed to show the general characterization result of equilibrium paths of the economy. To this end, we combine (LME) with (CME) to obtain the following dynamic equation:

$$(1-\alpha)A[lz_t^\alpha + (1-l)\theta_t x_t^\alpha] = lz_{t+1} + (1-l)\theta_{t+1}(x_{t+1} + I) \tag{CME_t}$$

where $\theta < 1$ occurs only when $x_t = \underline{x}$, and $x_t > \underline{x}$ implies that $\theta_t = 1$. Further, note that $x_t = x(\rho_t)$ holds by (8).

Additionally, the choice of capital investment z_t by a relationship pair must satisfy the following optimality condition in Lemma 2: $z_t = x_t$ holds if (IC_t^*)

$$\delta\{\alpha A z_{t+1}^\alpha - \rho_{t+1}z_{t+1} - \pi(\rho_{t+1})\} \geq \rho_t z_t \tag{IC_t^*}$$

is not binding, and $z_t \leq x_t$ otherwise.

These two dynamic conditions govern the evolution of $\{z_t, x_t, \rho_t, \theta_t\}_{t=0}^{\infty}$ given the initial condition $z_0 = x_0$. Before showing a formal result, it would be helpful to describe these equilibrium conditions diagrammatically. In Figure 2 (a), we depict possible combinations of x_{t+1} and z_{t+1} that satisfy both (CME$_t$) and (IC$_t^*$). To simplify argument, here we assume that $\theta_t = \theta_{t+1} = 1$ such that $x_t > \underline{x}$ and $x_{t+1} > \underline{x}$. The downward sloping straight line corresponds to (CME$_t$): $l(1-\alpha)Az_t^{\alpha} + (1-l)(1-\alpha)Ax_t^{\alpha} = lz_{t+1} + (1-l)(x_{t+1} + I)$ given (x_t, z_t). The upward sloping curve corresponds to (IC$_t^*$) with equality given (z_t, x_t): $\delta\{\alpha A z_{t+1}^{\alpha} - \rho_{t+1}z_{t+1} - \pi(\rho_{t+1})\} = \rho_t z_t$ (see more details about this result in Lemma A1 in Appendix). The shaded area in Figure 2 (a) is the region for possible (z_{t+1}, x_{t+1}) for which (IC$_t^*$) is satisfied. Then, a straight line BD corresponds to possible equilibrium values of z_{t+1} and x_{t+1}, given (z_t, x_t). If (IC$_t^*$) is not binding, the equilibrium must be the intersection between BD line and 45 degree line, a point B. If (IC$_t^*$) is binding, the equilibrium must be a point D. Once (z_{t+1}, x_{t+1}) is determined as shown above, (z_{t+2}, x_{t+2}) in the next period can be found in the similar way.

It seems that it is difficult to characterize equilibrium paths defined above. In general, we cannot rule out the case that there exist multiple paths of capital accumulation and that they are not monotonic over time. Due to such complication of equilibrium paths, it is not easy to see the time structures of relational contracting in equilibrium paths: in what stages of an equilibrium path do the incentive compatibility (self-enforcing agreement) constraints become binding or slack? Even so, we can show the following characterization result about the time structure of relational contracting.

Proposition 2. *Suppose that Assumptions 3-5 are satisfied. Then, in any equilibrium path there exists a unique switching period $T \geq 0$ such that*

(i) $z_t = x_t$ *holds for all $t \leq T$ and, in particular, (IC$_t^*$) is not binding in any period $t \leq T - 1$ (if $T \geq 2$), and*

(ii) *(IC$_t^*$) is binding so that $z_t < x_t$ for all $t \geq T + 1$*

where T satisfies $x_T \leq \overline{x}$ for $\overline{x}$ defined in Assumption 5.

Proposition 2 states that in any equilibrium path the capital investment chosen by relationship pairs must be *downward distorted* $z_t < x_t$ in subsequent stages of the equilibrium path ($t \geq T + 1$), but has no distortions $z_t = x_t$ in early stages ($t \leq T$). In other words, in any equilibrium path the time structure of relational contracting must possess a unique switching period that triggers the change from the relationship-based system, in which (IC$_t^*$) does not constrain capital accumulation, to the market-based system, in which (IC$_t^*$) becomes tighter, and hence the value of a relational contract becomes lower relative to arm's length contract ($z_t < x_t$). The switching period T must occur before the economy hits a unique critical production level $\overline{x}$, that is, $x_T \leq \overline{x}$. Above such critical production level, the equilibrium path must have downward distortion of the relational contract.

Although the formal proof of Proposition 2 is relegated to the Appendix, we explain the main logic of this result by using diagrams. The key to the result is the dynamic interactions between (CME) and (IC_t^*) over time.

First, we show that if downward distortion does not occur in some period t, $z_t = x_t$, then this is the case in any period before t, i.e., $z_s = x_s$ for all $s \leq t$. To see this result, simplify the argument by assuming that $\theta_t = \theta_{t+1} = 1$ (see the proof in Appendix for more details). Assume that $z_t = x_t$ holds in some period t. Then, (CME) in period t becomes

$$(1-\alpha)Ax_t^\alpha = lz_{t+1} + (1-l)(x_{t+1} + I) \qquad (\text{CME}_t)$$

When we set $z_t = x_t$ in Figure 2(a), (z_{t+1}, x_{t+1}) must lie on the segment of (CME_t), denoted by BD in Figure 2(a). Then, from Figure 2(a), we have $(\delta I\alpha^2 A/\rho_t x_t)^{1/(1-\alpha)} \geq (1-\alpha)Ax_t^\alpha - (1-l)I$, which is equivalent to $x_t \leq \overline{x}$. Next, we turn to the previous period $t-1$. Because $x^* > \overline{x}$ and $\overline{x} \geq x_t$, we can find a unique $\tilde{x}_{t-1}$ such that $(1-\alpha)A\tilde{x}_{t-1}^\alpha = x_t + (1-l)I$ (see Figure 3). Here, $\tilde{x}_{t-1} < \overline{x}$. Thus, $x_t = (1-\alpha)A\tilde{x}_{t-1}^\alpha - (1-l)I < (\delta I\alpha^2 A/\rho(\tilde{x}_{t-1})\tilde{x}_{t-1})^{1/(1-\alpha)}$. Also, because (CME) in period $t-1$ becomes

$$(1-\alpha)A(lz_{t-1}^\alpha + (1-l)x_{t-1}^\alpha) = x_t + (1-l)I, \qquad (\text{CME}_{t-1})$$

as well as $z_{t-1} \leq x_{t-1}$, we have $x_{t-1} \geq \tilde{x}_{t-1} \geq z_{t-1}$. Thus, $\rho_{t-1} \equiv \rho(x_{t-1}) \leq \rho(\tilde{x}_{t-1})$. Then, because $\rho_{t-1}z_{t-1} \leq \rho(\tilde{x}_{t-1})\tilde{x}_{t-1}$, ($\text{IC}_{t-1}^*$) must be slack when $z_t = x_t$ (a point E in Figure 2(b)):

$$\delta\{\alpha Ax_t^\alpha - \rho_t x_t - \pi(\rho_t)\} > \rho_{t-1}z_{t-1}.$$

Repeating this process backward, we can show that (IC_s^*) becomes slack in any period $s \leq t-1$ if capital investment of relationship pairs have no distortions ($z_t = x_t$) in some period t.

Second, by Lemma 3 we already know that any equilibrium path must possess the property that (IC_t^*) is binding in some period, for example $T+1$. By combining this with the above result, we can show that (IC_t^*) must be binding for all $t \geq T+1$, but is slack for all $t \leq T$. To see this last result, assume that (IC_m^*) is binding again after (IC_T^*) is binding but (IC_t^*) is slack for all t such that $T < t < m$. However, if (IC_t^*) is slack at t where $m < t$, downward distortion never occurs in period t ($z_t = x_t$), and then (IC_m^*) must be slack as well, resulting in a contradiction. Thus, there must exist a unique switching period T such that (IC_t^*) becomes slack in any period $t \leq T$ but must be binding in any period $t \geq T+1$.

Although Proposition 2 states that relational contracting becomes less constrained in the early stages of an equilibrium path than in subsequent stages, it does not address how the value of relational contracting is related to the development levels of the economy. To understand this implication, we make the following additional condition:

Assumption 6. $w_0 < (1-l)(\underline{x} + I)$.

Assumption 6 says that the initial wage w_0 is not so large enough that all non-relationship producers can finance their capital and verification investment $x_1 + I$ in period 1. Thus, only some fraction of these producers can finance the investment for future production, and the market interest rate is given by its upper bound $\rho_1 = d$ in period 1.

Then we can show the following result.

Proposition 3. *Suppose that Assumptions 3-6 hold. Then, in any equilibrium path there must exist some periods $T^* \geq 0$ and $T^{**} \geq 0$ where $T^* \leq T^{**}$ such that*

- $z_t = x_t = \underline{x}$ *and* $\theta_t < 1$ *are satisfied in any period* $t \leq T^*$, *and*
- *(IC_t^*) is binding in any period* $t \geq T^{**}$ *in which* $z_t < x_t$, $x_t > \underline{x}$ *and* $\theta_t = 1$ *hold.*

Proposition 3 shows that relational contract becomes less constrained in low development stages of the economy than in developed stages. Proposition 3 thus captures the feature discussed by several historians (see for example Lamoreaux (1994)) that relational contracting contributes more to economic growth in low development phases, in which the anonymous market does not work well for all non-relationship producers to operate ($\theta_t < 1$ and $x_t = \underline{x}$), than in high development stages, in which the market well functions ($\theta_t = 1$ and $x_t > \underline{x}$). Also, this result is consistent with the argument that relational contracting is not a substitute for the market based economy but plays a role to complement it and foster economic development (see Aoki and Hayami (2000)): relational contracting has no downward distortion in low development stages, whereas all non-relationship producers cannot find a way to finance their capital investments. Thus, in these stages, a relational contract rather than an arm's length contract contributes to capital accumulation in the economy, which in turn lowers the interest rates over time, resulting in the increase of the profit of an arm's length contract. Thus, after the economy passes a critical development point, all non-relationship producers can start financing their capital investments.

The intuition behind Proposition 3 is as follows: Assumption 6 ensures that $\theta_t < 1$ holds for all small t, and hence non-relationship producers operate at the lowest level $\underline{x}$ in early stages of equilibrium path. In these stages, (IC_t^*) becomes more likely to be slack so that relational contract has no downward distortion (Proposition 2). Conversely, because the productivity of the economy A is large enough that the minimum scale of capital investment and the verification cost $\underline{x} + I$ can be covered under Assumption 4, we can also show that, as time goes by, the economy eventually takes off from the low production level $\underline{x}$. This result is due to the fraction of non-relationship producers who can finance capital investment tends to increase over time. Then, the increase in the profit of arm's length contract makes (IC_t^*) harder to satisfy so that relational contract eventually exhibits a downward distortion as the economy grows.

However, it is not obvious how much relational contracting becomes difficult sustained as the economy enters well-developed stages because the general equilibrium effect manifests: when (IC_t^*) becomes tighter, capital investment of relationship producers z_t declines, negatively affecting capital accumulation of the economy as a whole. However, this effect makes the interest rates larger, by which the profit of an arm's length contract becomes lower, and hence the self-enforcing condition of relational contract becomes more likely to be satisfied, contrary to the first effect that (IC_t^*) becomes tighter.

Even though this general equilibrium effect makes the long run features of equilibrium paths difficult to predict, in the next proposition we show that the downward distortion of capital investment choice by relationship producers persists and has a positive lower bound in the long run.

To see this effect, we define $\hat{x}$ as the largest value of x that satisfies

$$(1-l)(1-\alpha)A\hat{x}^{\alpha} = \hat{x} + (1-l)I. \tag{13}$$

By Assumption 4, we can verify that such $\hat{x}$ exists and $\hat{x} > \underline{x}$.

Then we can show the following result.

Proposition 4. *Suppose that Assumptions 3, 4 and $\delta < \hat{x}/I$ hold. Then in any equilibrium path, the downward distortion of capital investment by relationship pairs $x_t - z_t$ has the positive lower bound in the long run, that is,*

$$x_t - z_t \geq \hat{x} - \delta I > 0$$

for all large t.

Proposition 4 shows that the downward distortion of capital investment chosen by relationship producers never disappears and that it still persists in the long run. The degree to which relational contracting distorts capital investment depends on the altruistic parameter δ and the verification cost I. When δ (or/and I) is lower and thus the gain of supporting relational contract becomes smaller, the downward distortion becomes larger.

Proposition 4 confirms the empirical and historical fact that the relationship-based system, which relies on the use of personal ties, kinships, and informal connections, eventually declines and has serious limitations as the economy becomes richer: Lamoreaux (1994) found the evidence that insider lending practices, which had served an important financing device in New England in early 19th century, became less important and were eventually replaced by the market-based finance as the United States transitioned from a capital poor economy to a capital rich economy in late 19th century. Demirgüç-Kunt and Levine (2004) also reported the related fact that the bank finance, which is sometimes characterized as long term relationship between particular banks and firms, becomes less popular relative to the market-based finance, such as equity, in more developed country (see Rajan and Zingales (2000) for a related argument).

Although our characterization results above highlight the role of relational contracting in different phases of economic development, these results do not necessarily rule out the case that (IC_t^*) is binding in any period $t \geq 1$ (thus $T = 0$ holds in Proposition 2). To address this issue, we show that there exists an equilibrium path that has the following feature: there exists some $T \geq 1$ such that in early stages of development $t \leq T$ (IC_t^*) is not binding, and hence no downward distortion occurs ($z_t = x_t$), while in subsequent stages, $t \geq T + 1$ (IC_t^*) becomes binding, and hence the economy involves downward distortion $z_t < x_t$. We call this type of equilibrium path a *switching equilibrium path.*

5.2 Existence of Switching Equilibrium Path

To show the existence of the switching equilibrium path defined above, we make the following additional assumptions:

Assumption 7. $\min\{l, (1-l)\}(1-\alpha)A\underline{x}^\alpha > \underline{x} + (1-l)I$.

Assumption 8. $\overline{x}^* > \max\left\{\underline{x}, (\delta I/\underline{x}^\alpha)^{1/(1-\alpha)}\right\}$ *where* $\overline{x}^*$ *is defined as* x *satisfying*

$$(1-\alpha)Ax^\alpha - (1-l)I = (1-l)\left(\delta I/x^\alpha\right)^{1/(1-\alpha)}$$

Assumption 7 is essentially same as Assumption 4: the economy can eventually take off from the lowest production level $\underline{x}$ for each intermediate good of non-relationship producer. Assumption 8 seems complicated, but we can observe that this condition is satisfied when the verification cost I is not large (see Appendix for more details).

Then, we can show the following existence result.

Proposition 5. *Suppose that Assumptions 3 and 6-8 hold. Then, there exists the switching equilibrium path with* $T \geq 1$ *such that*

- *(IC_t^*) is not binding and* $z_t = x_t = \underline{x}$ *with* $\theta_t < 1$ *in any period* $t \leq T$*,*
- *(IC_t^*) is binding such that* $z_t < x_t$ *and* $x_t > \underline{x}$ *(*$\theta_t = 1$*) in any period* $t \geq T + 1$*.*

The proof of Proposition 5 is constructive: First we set $x_t = z_t = \underline{x}$ for all $t \leq T$ for some switching period $T \geq 1$. Thus, no downward distortion of relational contract occurs and the production level of non-relationship producers is given at the lowest scale $\underline{x}$. Put differently, the interest rate in the credit market hits its upper bound $\rho_t = d$ such that only some fraction of non-relationship producers can finance their capital investment in any period $t \leq T$. We can then see that (IC_t^*) becomes $\delta\{\alpha A\underline{x}^\alpha - d\underline{x}\} \geq d\underline{x}$ in any period $t \leq T$, which is satisfied by Assumption 3. After the switching period ($t \geq T + 1$), we sequentially construct a path $\{z_t, x_t\}_{t=T+1}^\infty$ as follows: (i) $x_t > \underline{x}$. (ii) (IC_t^*) holds as equality. (iii) (CME) holds with $\theta_t = \theta_{t+1} = 1$. Assumption 7 ensures that the economy can take off from the low production scale: $x_t > \underline{x}$ holds for all

$t \geq T+1$ when we take T to be large. By using Assumption 8, we can then find a pair (z_{t+1}, x_{t+1}) such that both (IC$_t^*$) as equality and (CME) in period t are satisfied, given (z_t, x_t).

6 Conclusion

In this paper, we have investigated a dynamic general equilibrium model that takes into account the dynamic change in the contract enforcement modes from relational contracts to arm's length contracts over time. We have shown that relational contracting plays an important role in sustaining production in the early stages of the development process in which arm's length contracting is not widely used. In subsequent periods, non-relationship producers find it profitable to use arm's length contracts because the economy is so well developed that the market size is large and the interest rate falls. However, as the economy enters its mature stages, relationship-based systems decline and may be partially replaced by market-based systems. We have focused on relational contracting between borrowers and lenders, which becomes valuable for relating our theoretical results to the historical evidence on relationship-based financing. Of course, this model is one of the modeling choices that capture relational contracting in dynamic general equilibrium frameworks. It is important to investigate how the development process is dynamically linked with long-term relationships in different contexts such as firm-worker relationships, inter-firm relationships, and government-public relationships.

We conclude the paper by discussing the role of bequest transfers between successive generations. In the main text we have assumed that each old individual has no technologies to give their children bequest at all. One might think that old individuals will bequeath their children when bequest is available because they care about the consumption levels of their children. The possibility of bequest allows each producer to finance a part of capital investment from the bequest he has received from his parent. Thus non-relationship producers can reduce the verification cost and relationship pairs can make (IC$_t^*$) more easily satisfied because the opportunity cost to use the arm's length contract becomes lower. However, we can show that no old individuals bequeath their children at all, as in our basic setting, as long as the altruistic parameter value δ belongs to a certain small range (a more detailed analysis is relegated to Appendix).

7 Appendix A: Proofs for Lemmas and Propositions

Proof of Lemma 1.

(i) For a relationship pair born in period $t-1$ to be inherited by the next generation, (IC$_s^*$) and (TS$_{s-1}$) must be satisfied for all $s \geq t$. Otherwise, (IC$_{\tilde{t}}^*$) or (TS$_{\tilde{t}-1}$) is violated in some period $\tilde{t} \geq t$, which implies that the relationship pair born in period $\tilde{t}-1$ is dissolved. By anticipating this and using the backward induction argument, every relationship pair born in any period before $\tilde{t}-2$ would not have the incentive to maintain the relationship. On the other hand, if both (IC$_s^*$) and (TS$_{s-1}$) are satisfied for all $s \geq t$, then we can show that there exists a subgame perfect equilibrium in which every relationship pair born in period $s \geq t-1$ honors the relational contract $\{z_s, R_s\}$, which satisfies all (IC_s), (IRP_s), and (IRL_s) for all $s \geq t$. To see this, assume that (IC^*_s) and (TS_{s-1}) hold for all $s \geq t$. Then, we can find a sequence of relational contracts $\{z_s, R_s\}_{s=t}^{\infty}$ such that all (IC_s), (IRP_s), and (IRL_s) are satisfied.[14] Given such a sequence of relational contracts $\{z_s, R_s\}_{s=t}^{\infty}$, we can show that the following strategies played by relationship producers and lenders can constitute a subgame perfect equilibrium in a continuation equilibrium from period t onward:

- Period $s \geq t$:
  - The young relationship producer and young relationship lender agree on a relational contract $\{z_{s+1}, R_{s+1}\}$. At this stage, the young relationship lender lends z_s to the relationship producer if their parents honored relational contract $\{z_s, R_s\}$ specified in the previous period. Otherwise, they exercise the quitting option.
- Period $s+1$:
  - The old relationship producer honors repayment R_{s+1} and does not exercise the quitting option, provided his and the relationship lender's parents did not exercise the quitting option in the previous period.
  - The old relationship lender does not exercise the quitting option, provided her and the relationship producer's parents did not exercise the quitting option in the previous period.

The above strategies specify the trigger-like feature in which the young relationship lender agrees on continuing the relationship provided the producer in the previous generation made the contractual repayment, R_s; if this was not the case, she would revert to the punishment strategy by exercising the quitting option. Here, note that we always have a continuation equilibrium in which any relationship producer and any relationship lender of any pair simultaneously exercise the quitting option. Then, we can readily see that, because (IC_s), (IRL_s), and (IRP_s) are satisfied in any period $s \geq t$, all relationship producers and lenders optimally follow these strategies from period t onward, ensuring that every relationship is inherited by the next generation.

(ii) Note first that $x(\rho_t)$ maximizes $\alpha A z_t^{\alpha} - \rho_t z_t$ over $z_t \geq 0$. Suppose now that $z_t > x(\rho_t)$ in some period t. Then, if we can slightly decrease z_t, we can still keep (IC^*_t) and (IC^*_{t-1}). Note that z_t affects only (IC^*_t) and (IC^*_{t-1}). The slight decrease of z_t makes (IC^*_t) easier to be satisfied while it increases the left hand side of (IC^*_{t-1}) because $x(\rho_t)$ maximizes the left hand side of (IC^*_{t-1}). But then such change increases the joint payoff $J_t = \alpha A z_t - \rho_t z_t + \rho_t w_{t-1}$ due to the definition of $x(\rho_t)$. Thus $z_t \leq x(\rho_t)$ holds.

[14] For example, we can set $R_s = \rho_s z_s$ in every period $s \geq t$, which satisfies all (IC_s), (IRP_s), and (IRL_s) with some z_s in every period $s \geq t$.

(iii) Suppose that $z_t < x(\rho_t)$ but (IC_t^*) is not binding in some period t. Then, if we can slightly increase z_t toward $x(\rho_t)$, we can still keep (IC_t^*) and (IC_{t-1}^*). The slight increase of z_t does not violate (IC_t^*) while it increases the left hand side of (IC_{t-1}^*) because of $z_t < x(\rho_t)$. But then such change increases the joint payoff J_t due to the definition of $x(\rho_t)$. Q.E.D.

Proof of Lemma 2.
Take an equilibrium path in which each non-relationship producer uses a strategy $\sigma_t : H_t \to [0, \infty)$ which maps from, H_t, the set of all previous observed histories up to period t to the current capital investment level $x_t \geq 0$.

Then we will show that $x_t = x(\rho_t)$ holds so that every non-relationship producer acts to maximize his own profit π_t in every period irrespective of observed histories $h_t \in H_t$. We denote by $\overline{\sigma}_t$ such strategy defined as $\overline{\sigma}(h_t) = x(\rho_t)$ for all $h_t \in H_t$.

To show this, suppose that there exists an equilibrium with $\sigma_t \neq \overline{\sigma}_t$ for some non–relationship producer in some period t. Thus $\sigma(h_t) \neq x(\rho_t)$ for some $h_t \in H_t$. We then denote by $\{x_s\}_{s=t}^{\infty}$ the equilibrium sequence of capital investments of non-relationship producers from period t onward according to the equilibrium strategies $\{\sigma_s\}_{s=t}^{\infty}$.

In the proof of Lemma 2, with a slight abuse of notation we will denote by $\pi(x_t)$ the profit of a non-relationship producer who chooses x_t in period t.

Since that non-relationship producer could always choose $x(\rho_t)$ in period t, it must be the case that

$$\pi(x_t) + \delta\pi(x_{t+1}) \geq \pi(x(\rho_t)) + \delta\pi(x'_{t+1})$$

where his child will choose x'_{t+1} in period $t+1$ following the choice of his parent $x(\rho_t)$ according to his equilibrium strategy σ_{t+1}. The above inequality yields

$$\delta\{\pi(x_{t+1}) - \pi(x'_{t+1})\} \geq \pi(x(\rho_t)) - \pi(x_t). \tag{A1}$$

Also, for x'_{t+1} to be the optimal choice by the young non–relationship producer in period $t+1$ following $x(\rho_t)$ chosen by his parent, we must have

$$\pi(x'_{t+1}) + \delta\pi(x''_{t+2}) \geq \pi(x(\rho_{t+1})) + \delta\pi(x'_{t+2})$$

because he could always choose $x(\rho_{t+1})$ following his parent choice $x(\rho_t)$ where σ''_{t+2} denotes the choice of his child in period $t+2$ following x'_{t+1}. Here x'_{t+2} is the choice in period $t+2$ following $x(\rho_{t+1})$. This then yields

$$\delta\{\pi(x''_{t+2}) - \pi(x'_{t+2})\} \geq \pi(x(\rho_{t+1})) - \pi(x'_{t+1}).$$

Since $\pi(x(\rho_{t+1})) \geq \pi(x_{t+1})$, we have

$$\delta\{\pi(x''_{t+2}) - \pi(x'_{t+2})\} \geq \pi(x_{t+1}) - \pi(x'_{t+1}). \tag{A2}$$

Combining inequalities (A1) with (A2), we have

$$\delta^2\{\pi(x''_{t+2}) - \pi(x'_{t+2})\} \geq \pi(x(\rho_t)) - \pi(x_t). \tag{A3}$$

In period $t+2$ the young non–relationship producer in the dynasty in question must choose x'_{t+2} following the choice $x(\rho_{t+1})$ by his parent in period $t+1$. Thus, since he could always choose x''_{t+2} instead of x'_{t+2} in period $t+2$, we must have

$$\pi(x'_{t+2}) + \delta\pi(x''_{t+3}) \geq \pi(x''_{t+2}) + \delta\pi(x'_{t+3})$$

where x''_{t+3} and x'_{t+3} denote the choices by the non–relationship producer in period $t+3$ following x'_{t+2} and x''_{t+2} respectively. Thus

$$\begin{aligned}\delta\{\pi(x''_{t+3}) - \pi(x'_{t+3})\} &\geq \pi(x''_{t+2}) - \pi(x'_{t+2}) \\ &\geq \pi(x''_{t+2}) - \pi(x'_{t+2})\end{aligned}$$

Combining this with (A3), we have

$$\delta^3\{\pi(x''_{t+3}) - \pi(x'_{t+3})\} \geq \pi(x(\rho_t)) - \pi(x_t)$$

Repeating this argument for all periods $s+t \geq t$, we obtain

$$\delta^s\{\pi(x''_{t+s}) - \pi(x'_{t+s})\} \geq \pi(x(\rho_t)) - \pi(x_t), \ \ \forall\ s \geq 0.$$

Since $\pi(x(\rho_t)) \neq \pi(x_t)$ by our supposition ($\sigma(h_t) = x_t \neq x(\rho_t)$), there exists some $\varepsilon > 0$ such that $\pi(x(\rho_t)) - \pi(x_t) \geq \varepsilon$. The left hand side of the above inequality is bounded above by $\delta^s\pi(\rho_{t+s})$ because $\pi(x_{t+s}) \geq 0$ and $\pi(x(\rho_{t+s})) = \max_x \pi(x)$ given ρ_{t+s}. Note here that $\pi(x_{t+s}) \geq 0$ because, by the spot transaction nature of arm's length contract in the anonymous credit market, each non-relationship producer must make the repayment $\rho_{t+s}(x_{t+s} + I)$ from what he earns $p_{t+s}x_{t+s}$ when he is old.

Then, if $\pi(x(\rho_{t+s}))$ is bounded above, the left hand side of the above inequality goes to zero by letting $s \to \infty$ and noting $\delta < 1$, which is a contradiction. Thus it suffices to show that $\pi(x(\rho_t)) < +\infty$ for any equilibrium interest rate $\rho_t \geq 0$. This is equivalent to the condition that $\rho_t \geq \underline{\rho}$ for all t for some $\underline{\rho} > 0$ because then $\pi(x(\rho)) \leq \max_x \alpha(Ax)^\alpha - \underline{\rho}x < +\infty$. Suppose that $\rho_t = 0$ for some period t or $\rho_t \to 0$. In either case $\rho_t < d$ which implies $\theta_t = 1$. Also $x(\rho_t)$ must go to infinity. Since $\pi(x_t) + \delta\pi(x_{t+1}) \geq \pi(x(\rho_t)) + \delta\pi(x'_{t+1})$ must hold in period t (see (A1)), either $\pi(x_t) \to \infty$ or $\pi(x_{t+1}) \to \infty$ or both must hold due to $\pi(x(\rho_t)) \to \infty$ and $\pi(x'_{t+1}) \geq 0$, which implies $x_t \to \infty$ or $x_{t+1} \to \infty$ with $\rho_{t+1} = 0$. In the former case ($x_t \to \infty$) (CME) must imply $w_{t-1} = lz_t + (1-l)(x_t + I) \to \infty$, which holds only when $w_{t-1} \to \infty$ so that $x_{t-1} \to \infty$ or $z_{t-1} \to \infty$. We can deal with the latter case $x_{t+1} \to \infty$ in a similar way. However, then $w_{t-2} \to \infty$ by (CME) in period $t-1$. Repeating this, $w_0 \to \infty$ which is however impossible. Thus $\rho_t \geq \underline{\rho}$ must hold in any period t for some $\underline{\rho} > 0$. This then establishes the lemma. Q.E.D.

Proof of Lemma 3.
Suppose contrary to the claim that $z_s > w_{s-1}$ in some period s in some equilibrium path. In such equilibrium each relationship producer must incur the verification cost $\rho_s I$ because all his capital investment z_s cannot be financed by the relationship lender

who owns only w_{s-1} and hence the remaining amount $z_s - w_{s-1}$ must be borrowed from the credit market.

There are two cases: (i) $z_{s-1} \leq w_{s-2}$ and (ii) $z_{s-1} > w_{s-2}$. In case (i), the relationship producer in period $s-2$ does not finance from the credit market. Thus, in case (i), (IC$_s$) for relationship producer and (IRL$_s$) for relationship lender should be modified to

$$p_{s-1}z_{s-1} - R_{s-1} + \delta\{p_s z_s - R_s - \rho_s(z_s - w_{s-1}) - \rho_s I\} \geq p_{s-1}z_{s-1} + \delta\pi_s \qquad (\text{IC}'_s)$$

and

$$R_{s-1} + \delta\rho_{s-1}(w_{s-2} - z_{s-1}) + \delta R_s \geq \rho_{s-1}w_{s-2} + \delta\rho_s w_{s-1} \qquad (\text{IRL}'_s)$$

Combining these conditions, (IC$^*_s$) must be changed to

$$\delta\{\alpha A z_s^\alpha - \rho_s(z_s + I) - \pi(\rho_s)\} \geq \rho_{s-1}z_{s-1}. \qquad (\text{IC}^*_s\text{-(i)})$$

In case (ii), the relationship producer in period $s-2$ finances the remaining amount $z_{s-1} - w_{s-2}$ from the credit market after borrowing w_{s-2} directly from the relationship lender. Thus, in case (ii), (IC$_s$) for relationship producer and (IRL$_s$) for relationship lender should be modified to

$$\begin{aligned} &p_{s-1}z_{s-1} - R_{s-1} - \rho_{s-1}(z_{s-1} - w_{s-2} - I) + \delta\{p_s z_s - R_s - \rho_s(z_s - w_{s-1}) - \rho_s I\} \\ &\geq p_{s-1}z_{s-1} - \rho_{s-1}(z_{s-1} - w_{s-2} - I) + \delta\pi_s \end{aligned}$$

and

$$R_{s-1} + \delta R_s \geq \rho_{s-1}w_{s-2} + \delta\rho_s w_{s-1} \qquad (\text{IRL}'_s)$$

Combining these conditions, (IC$^*_s$) must be changed to

$$\delta\{\alpha A z_s^\alpha - \rho_s(z_s + I) - \pi(\rho_s)\} \geq \rho_{s-1}w_{s-1}. \qquad (\text{IC}^*_s\text{-(ii)})$$

However, we show that the second case (ii) never happens in equilibrium. This is because the left hand side of (IC$^*_s$-(ii)) is bounded above by

$$\delta\{\alpha A x_s^\alpha - \rho(x_s + I) - \pi(\rho_s)\} = 0$$

and hence from (IC$^*_s$-(ii))we must have $\rho_{s-1}w_{s-1} = 0$, which is however impossible.

Thus we consider only the first case (i). Then, since the left hand side of (IC$^*_s$-(i)) is bounded above by zero, we must have $z_{s-1} = 0$ (due to $\rho_{s-1} > 0$) which happens only when $z_s = x_s$. Also, by (CME) in period $s-1$, we have $w_{s-1} = lz_s + (1-l)\theta_s(x_s + I) = lx_s + (1-l)\theta_s(x_s + I)$. Since $w_{s-1} < z_s$ and $x_s = z_s$, we must have $\theta_s < 1$ so that $x_s = \underline{x}$ and $\rho_s = d$. Thus $x_s = z_s = \underline{x}$ and the joint payoff of relationship pair in period s becomes $J_s = \alpha A \underline{x}^\alpha - d(\underline{x} + I) = \pi(d) = 0$. Next, since $z_{s-1} = 0$, from (IC$^*_{s-1}$-(i)) we have $-\delta\pi(\rho_{s-1}) \geq \rho_{s-2}z_{s-2}$ which implies that $\rho_{s-1} = d$ (hence $\pi(\rho_{s-1}) = 0$) and $z_{s-2} = 0$. Thus the joint payoff of relationship pair in period $s-1$ is given by $J_{s-1} = p_{s-1}z_{s-1} - dz_{s-1} = 0$. Repeating this process, we have $z_t = 0$ and $J_t = 0$ for all $t = 1, 2, ..., s$.

We then consider the deviation of relational contracts as follows:
Case A: $s \geq 2$. Then set $z'_s = w_{s-1} > 0$ and $z'_t = \varepsilon > 0$ for any t where $2 \leq t \leq s$ while $z'_t = z_t$ for all other t. We can then show that this deviation contract satisfies (IC^*_t) and (TS_{t-1}) for any period t. Since $\rho_t = d$ for all $t = 2, 3, ..., s$, (IC^*_s) holds when $\varepsilon > 0$ is small because

$$\delta\{\alpha A w^{\alpha}_{s-1} - d w_{s-1} - \pi(d)\} \geq d z_{s-1} = d\varepsilon$$

where $\pi(d) = 0$ and $w_{s-1} < z_s = x_s = \underline{x}$. Also, in any period t where $1 \leq t \leq s-1$, (IC^*_t) holds as well when $\varepsilon > 0$ is small because

$$\delta\{\alpha A \varepsilon^{\alpha} - d\varepsilon - \pi(d)\} \geq d\varepsilon$$

where $\pi(d) = 0$. Clearly, the joint payoffs of relationship pairs in period t ($2 \leq t \leq s$) are increased and hence (TS_{t-1}) are satisfied in these periods as well. Such deviation improves the payoffs of the generations from $t = 2$ to $t = s$ in a dynasty of relationship pairs. Thus the original relational contract cannot be optimal.
Case B: $s = 1$. Thus $w_0 < z_1$. If $\theta_1 = 1$, then we have from (CME) in period 1 that $w_0 = l z_1 + (1-l)(x_1 + I) < z_1$ and hence $z_1 > x_1 + I$ which is a contradiction. Thus $\theta_1 < 1$ so that $\rho_1 = d$. Then $J_1 = \alpha A z_1 - d(z_1 + I)$ which is not larger than $\pi(d) = 0$. Thus $J_1 \leq 0$. However, if we consider the deviation of relational contract as $z'_1 = w_0$, (IC^*_1) still holds because $\delta\{\alpha A z^{\alpha}_2 - \rho_2 z_2 - \pi(\rho_2)\} \geq \rho_1 z_1 > \rho_1 w_0$ while the joint payoff is increased to $J'_1 = \alpha A w^{\alpha}_0 - d w_0 > 0$ because $w_0 < z_1 \leq x_1 = \underline{x}$. Since ($TS_0$) is clearly satisfied, such deviation makes the initial and second generations in a dynasty of relationship pairs better off. Q.E.D.

Proof of Proposition 1.
Suppose that Assumption 1 and 2 are satisfied. Suppose also that (IC^*_t) is never binding in any period t. Then, $z_t = x_t$ holds in any period t. Thus the equilibrium path of capital accumulation $z_t = x_t$ follows

$$(l + (1-l)\theta_t)(1-\alpha)A x^{\alpha}_t = l x_{t+1} + (1-l)\theta_{t+1}(x_{t+1} + I), \quad t = 0, 1, 2... \tag{A4}$$

where $\theta_t < 1$ holds only when $x_t = \underline{x}$.

First we show that $\theta_s = 1$ for some period s. If this is not the case, $\theta_t < 1$ for all t. Then (A4) becomes

$$(l + (1-l)\theta_t)(1-\alpha)A\underline{x}^{\alpha} = l\underline{x} + (1-l)\theta_{t+1}(\underline{x} + I), \quad t = 0, 1, 2... \tag{A5}$$

Then, by Assumption 1 we can show that the sequence $\{\theta_t\}^{\infty}_{t=1}$ which satisfies (A5) must have $\theta_s > 1$ for some period s. This is a contradiction.

Second, we show that $\theta_s = 1$ implies $\theta_{s+1} = 1$. Suppose contrary to the claim that $\theta_s = 1$ but $\theta_{s+1} < 1$. Then we have from (A4) and $x_s \geq \underline{x}$ that

$$\begin{aligned}
(1-\alpha)A\underline{x}^{\alpha} &\leq l\underline{x} + (1-l)\theta_{s+1}(\underline{x} + I) \\
&< l\underline{x} + (1-l)(\underline{x} + I) \\
&= \underline{x} + (1-l)I \\
&< \underline{x} + I
\end{aligned}$$

which contradicts to Assumption 1.

By these results, we know that $\theta_t = 1$ for all $t \geq s$ for some period s. Thus, for large t, (A4) can be written by

$$(1-\alpha)Ax_t^\alpha = lx_{t+1} + (1-l)(x_{t+1} + I)$$

from which x_t converges to a unique steady state x^*. Q.E.D.

Proof of Lemma 4.
By Proposition 1, x_t converges to x^* when $z_t = x_t$ holds in any period t.

However, when t is so large that $x_t \simeq x^*$, (IC_t^*) can be written by

$$\delta\{\alpha A(x^*)^\alpha - \rho^*(x^*) - \pi(\rho^*) \geq \rho^* x^*$$

where ρ^* satisfies $x^* = x(\rho^*)$. However, such (IC_t^*) is modified as $\delta I \geq x^*$ which does not meet Assumption 3. Thus $z_t < x_t$ must hold in some period t. Q.E.D.

Proof of Proposition 2
We prove Proposition 2 by the following series of claims.

Lemma A1. *Suppose that $x_{T+1} > \underline{x}$ and $x_T > \underline{x}$ in some period T. Then, if $z_T = x_T$, we must have $x_T \leq \overline{x}$ where $\overline{x}$ is defined in Assumption 5.*

Proof. Let $x_{T+1} > \underline{x}$ and $x_T > \underline{x}$ in some period T. Suppose also that $z_T = x_T$. Then, since $\pi(\rho_{T+1}) > \pi(d) = 0$, ($\text{IC}_T^*$) can be written by

$$\delta\{\alpha A z_{T+1}^\alpha - \rho_{T+1} z_{T+1} - \pi(\rho_{T+1})\} \geq \rho_T x_T \tag{IC_T^*}$$

From this we can derive the relationship between z_{T+1} and x_{T+1} as follows:

$$\frac{dz_{T+1}}{dx_{T+1}} = -\frac{d\rho_{T+1}/dx_{T+1}(x_{T+1} - z_{T+1} + I)}{\alpha^2 A z_{T+1}^{\alpha-1} - \rho_{T+1}} > 0$$

for $z_{T+1} < x_{T+1}$ because $d\rho_{T+1}/dx_{T+1} = A\alpha^2(\alpha-1)x_{T+1}^{\alpha-2} < 0$ and the denominator is positive for $z_{T+1} < x_{T+1} = x(\rho_{T+1})$.

We also have (CME) in period T:

$$(1-\alpha)Ax_T^\alpha = lz_{T+1} + (1-l)(x_{T+1} + I)$$

because $\theta_T = \theta_{T+1} = 1$ and $z_T = x_T$.

Thus, combining (IC_T^*) with (CME) in period T, we can verify that z_{T+1} and x_{T+1} which satisfy both these conditions exist only if

$$(1-\alpha)Ax_T^\alpha - (1-l)I \leq \left(\frac{\delta\alpha^2 AI}{\rho_T x_T}\right)^{1/(1-\alpha)} \tag{A6}$$

where the right hand side can be written by $(\delta I / x_T{}^\alpha)^{1/(1-\alpha)}$ (see Figure 2 (a)). In other words inequality (A6) implies that $x_T \leq \overline{x}$ by definition of $\overline{x}$. Q.E.D.

Lemma A2. *Suppose that $z_T = x_T$ in some period T. Then (IC_t^*) is not binding so that $z_t = x_t$ holds in any period $t \leq T-1$.*

Proof. Let $z_T = x_T$. We obtain (IC_T^*) as follows:

$$\delta\{\alpha A z_{T+1}^\alpha - \rho_{T+1} z_{T+1} - \pi(\rho_{T+1})\} \geq \rho_T x_T$$

Then we show that (IC_t^*) is not binding in any period $t \leq T-1$ in what follows.

CASE 1: $x_T = \underline{x}$. Then $\rho_T = d$. Now we show that (IC_{T-1}^*) is not binding, i.e., $\delta\{\alpha A \underline{x}^\alpha - d\underline{x}\} \geq \rho_{T-1} z_{T-1}$ where note that $z_T = x_T = \underline{x}$.

We also have (CME) in period $T-1$:

$$l(1-\alpha)A z_{T-1}^\alpha + (1-l)\theta_{T-1} A x_{T-1}^\alpha = l\underline{x} + (1-l)\theta_T(\underline{x} + I).$$

From this we can show that $\theta_{T-1} < 1$. To see this, since $z_{T-1} \geq 0$ and $x_{T-1} \geq \underline{x}$, we have $(1-l)\theta_{T-1} A \underline{x}^\alpha \leq \underline{x} + (1-l)I$ which implies that $\theta_{T-1} \leq \frac{\underline{x}+(1-l)I}{(1-l)(1-\alpha)A\underline{x}^\alpha} < 1$ due to Assumption 4. Thus $\theta_{T-1} < 1$ which implies that $x_{T-1} = \underline{x}$ and hence $\rho_{T-1} = d$.

However, then by using Assumption 3 ($\delta > \alpha/(1-\alpha)$), we obtain

$$\begin{aligned} \delta\{\alpha A \underline{x}^\alpha - d\underline{x}\} &> d\underline{x} \\ &\geq d z_{T-1} \\ &= \rho_{T-1} z_{T-1} \end{aligned}$$

because $\underline{x} = x_{T-1} \geq z_{T-1}$. This shows that ($\text{IC}_{T-1}^*$) must be slack.

CASE 2: $x_T > \underline{x}$. Thus $\rho_T < d$. Now we will show that (IC_{T-1}^*) is not binding in what follows.

CASE 2-1: $x_{T+1} > \underline{x}$ (thus $\rho_{T+1} < d$). In this sub-case we have $x_{T+1} > \underline{x}$ and $x_T > \underline{x}$ (note that we are in CASE 2). Thus we can apply Lemma A1 so that $x_T \leq \overline{x}$.

We also have (CME) in period $T-1$ as $l(1-\alpha)A z_{T-1}^\alpha + (1-l)\theta_{T-1}(1-\alpha)A x_{T-1}^\alpha = x_T + (1-l)I$ because $z_T = x_T > \underline{x}$ by our supposition in Lemma A2 so that $\theta_T = 1$.

We will consider further two sub-cases:

Case (A): $\theta_{T-1} = 1$. We then define $\tilde{x}_{T-1}$ as

$$(1-\alpha)A\tilde{x}_{T-1}^\alpha = x_T + (1-l)I \tag{A7}$$

Since $z_{T-1} \leq x_{T-1}$, by using (CME) in period $T-1$ above, we can see that $x_{T-1} \geq \tilde{x}_{T-1}$ and $z_{T-1} \leq \tilde{x}_{T-1}$. Then, by using $x_T \leq \overline{x} < x^*$, we can show that $\tilde{x}_{T-1} < x_T$ and hence $\tilde{x}_{T-1} < \overline{x}$. Also we have $\rho_{T-1} = \rho(x_{T-1}) \leq \rho(\tilde{x}_{T-1})$.

Since $\tilde{x}_{T-1} < \overline{x}$, the following inequality is satisfied:

$$\left(\frac{\delta I}{\tilde{x}_{T-1}^{\alpha}}\right)^{1/(1-\alpha)} > (1-\alpha)A\tilde{x}_{T-1}^{\alpha} - (1-l)I$$

which is equivalent to the condition that $z_T = x_T$ satisfies

$$\delta\{\alpha A x_T^{\alpha} - \rho_T x_T - \pi(\rho_T)\} > \rho(\tilde{x}_{T-1})\tilde{x}_{T-1}.$$

Then we can show that

$$\begin{aligned}\delta\{\alpha A z_T^{\alpha} - \rho_T z_T - \pi(\rho_T)\} &= \delta\{\alpha A x_T^{\alpha} - \rho_T x_T - \pi(\rho_T)\} \\ &> \rho(\tilde{x}_{T-1})\tilde{x}_{T-1} \\ &\geq \rho_{T-1} z_{T-1}\end{aligned}$$

because $\rho(\tilde{x}_{T-1}) \geq \rho(x_{T-1}) = \rho_{T-1}$ and $z_{T-1} \leq \tilde{x}_{T-1}$. Thus ($\text{IC}^*_{T-1}$) is not binding.
CASE (B): $\theta_{T-1} < 1$. In this case $x_{T-1} = \underline{x}$. Thus (CME) in period $T-1$ becomes $l(1-\alpha)Az_{T-1}^{\alpha} + (1-l)\theta_{T-1}A\underline{x}^{\alpha} = x_T + (1-l)I$ which implies

$$\begin{aligned}x_T &= l(1-\alpha)Az_{T-1}^{\alpha} + (1-l)\theta_{T-1}(1-\alpha)A\underline{x}^{\alpha} - (1-l)I \\ &\leq (1-\alpha)A\underline{x}^{\alpha} - (1-l)I\end{aligned}$$

because $z_{T-1} \leq x_{T-1} = \underline{x}$. Thus

$$x_T \leq (1-\alpha)A\underline{x}^{\alpha} - (1-l)I. \tag{A8}$$

On the other hand, by noting that $z_T = x_T$ by our supposition, (IC^*_{T-1}) can be also written by

$$\begin{aligned}\delta\rho_T I &= \delta\{\alpha A x_T^{\alpha} - \rho_T x_T - \pi(\rho_T)\} \\ &\geq d z_{T-1}\end{aligned}$$

because $\rho_{T-1} = d$. If this is binding, we must have $\delta\rho_T I = dz_{T-1} \leq dx_{T-1} = d\underline{x}$. Thus $x_T \geq (\delta I/\underline{x}^{\alpha})^{1/(1-\alpha)}$ is satisfied due to $\rho_T = \alpha^2 A x_T^{\alpha-1}$.

By combining this with (A8), we obtain

$$(\delta I/\underline{x}^{\alpha})^{1/(1-\alpha)} \leq x_T \leq (1-\alpha)A\underline{x}^{\alpha} - (1-l)I$$

so that $(\delta I/\underline{x}^{\alpha})^{1/(1-\alpha)} \leq (1-\alpha)A\underline{x}^{\alpha} - (1-l)I$ which implies that $\underline{x} \geq \overline{x}$. This contradicts to Assumption 5. Thus (IC^*_{T-1}) is not binding.

CASE 2-2: $x_{T+1} = \underline{x}$. In this case $\rho_{T+1} = d$.

(CME) in period T becomes $(1-\alpha)Ax_T^{\alpha} = lz_{T+1} + (1-l)\theta_{T+1}(\underline{x} + I)$ because of $z_T = x_T > \underline{x}$ by our supposition and hence $\theta_T = 1$. Then, since $z_{T+1} \leq x_{T+1} = \underline{x}$ and $x_T > \underline{x}$, this can be written by $(1-\alpha)A\underline{x}^{\alpha} < \underline{x} + (1-l)I$ which contradicts to

Assumption 4. Q.E.D.

We have thus established that $z_t = x_t$ holds in any period $t \leq T$ if $z_T = x_T$. In particular we have shown that (IC_t^*) is not binding in any period $t \leq T-1$. ((IC_T^*) may bind even when $z_T = x_T$.) Also, by Lemma 4 we know that $z_t < x_t$ must hold in some period t. Then we can show that, if $z_{T+1} < x_{T+1}$ holds so that (IC_t^*) is binding in some period $T+1$, $z_t < x_t$ holds in any period $t \geq T+1$. If this is not the case, there exists some period m such that $T+1 < m$ and $z_m = x_m$. However, by Lemma A2, if $z_m = x_m$, then $z_t = x_t$ holds as well for all $t < m$. Thus $z_{T+1} = x_{T+1}$, a contradiction.

Thus we have shown that there exists a unique T such that $z_t = x_t$ holds in any period $t \geq T$ whereas $z_t < x_t$ holds in any period $t \geq T+1$. Q.E.D.

Proof of Proposition 3.
We will first show the following lemma.

Lemma A4. *Suppose that Assumption 4 holds. Then in any equilibrium path there exists some period $\tau \geq 1$ such that $x_t > \underline{x}$ and hence $\theta_t = 1$ hold for all $t \geq \tau$.*

Proof.
(CME) in period t:

$$l(1-\alpha)Az_t^{\alpha} + (1-l)\theta_t(1-\alpha)Ax_t^{\alpha} = lz_{t+1} + (1-l)\theta_{t+1}(x_{t+1}+I) \qquad \text{(CME)}$$

where $\theta_t < 1$ only when $x_t = \underline{x}$.

First we show that $\theta_s = 1$ in some period s. This is a similar result to Proposition 1 but we need some modification because $z_t \neq x_t$ may happen in this case.

Suppose contrary to the claim that $\theta_t < 1$ for all t. Then $x_t = \underline{x}$ (and hence $\rho_t = d$) for all t. Since by Proposition 2 we know that (IC_t^*) becomes binding for all $t \geq T$ for some T, we then have

$$\delta\{\alpha A z_{t+1}^{\alpha} - dz_{t+1}\} = dz_t$$

for all $t \geq T$. From this, z_t monotonically decreases over time.

We define a unique $\overline{z}$ such that $(1-\alpha)A\overline{z}^{\alpha} = \overline{z}$. Then, by using (CME) above, we have

$$\theta_t(1-\alpha)A\underline{x}^{\alpha} \leq \theta_{t+1}(\underline{x}+I). \qquad \text{(A10)}$$

for all large t because, if t is large, then $z_{t+1} < z_t < \overline{z}$ and hence $(1-\alpha)Az_t^{\alpha} - z_{t+1} \geq (1-\alpha)A\overline{z}^{\alpha} - \overline{z} = 0$ for all large t. From (A10) and Assumption 4, we can verify that θ_t unboundedly increases over time and hence $\theta_t > 1$ for some period t, a contradiction. Thus $\theta_s = 1$ holds in some period s.

Second, we show that $\theta_s = 1$ implies $\theta_{s+1} = 1$. Suppose that $\theta_s = 1$ but $\theta_{s+1} < 1$. Then, by substituting $x_{s+1} = \underline{x} \geq z_{s+1}$ into (CME) in period s, we have

$$\begin{aligned} l(1-\alpha)Az_s^\alpha + (1-l)(1-\alpha)Ax_s^\alpha &= lz_{s+1} + (1-l)\theta_{s+1}(\underline{x}+I) \\ &\leq l\underline{x} + (1-l)\theta_{s+1}(\underline{x}+I) \\ &\leq l\underline{x} + (1-l)(\underline{x}+I) \\ &\leq \underline{x} + (1-l)I \end{aligned}$$

Then, since $z_s \geq 0$ and $x_s \geq \underline{x}$, we obtain

$$(1-l)(1-\alpha)A\underline{x}^\alpha \leq \underline{x} + (1-l)I$$

which contradicts to Assumption 4. Thus $\theta_s = 1$ implies $\theta_{s+1} = 1$.

Then we have established the result that there exists some period $s \geq 1$ such that $\theta_t = 1$ for all $t \geq s$ (hence $x_t > \underline{x}$ for all $t \geq s$). Q.E.D.

Now we move back to the proof of Proposition 3.

Suppose that Assumption 6 holds. Then, if $\theta_1 = 1$, we have $w_0 = lz_1 + (1-l)(x_1 + I) \geq (1-l)(\underline{x}+I)$ which contradicts to Assumption 6. Thus $\theta_1 < 1$ must hold. By combining this with Proposition 2, we can find some $T^* \geq 0$ such that $z_t = x_t$ and $\theta_t < 1$ hold in any period $t \leq T^*$. On the other hand, Lemma A4 shows that $\theta_t = 1$ for all large t. Then, by Proposition 2 we can find some $T^{**} \geq 0$ such that $T^{**} \geq T^*$, (IC_t^*) is binding and $\theta_t = 1$ in any period $t \geq T^{**}+1$. Q.E.D.

Proof of Proposition 4

By Lemma A4, we know that $\theta_t = 1$ when t is large. Thus $\pi(\rho_t) > 0$ for large t. In what follows we take t to be large so that $\theta_t = 1$. When t is so large enough that $\pi_t > 0$, (IC_t^*) implies $\delta\{\pi(\rho_{t+1}) + \rho_{t+1}I - \pi(\rho_{t+1})\} \geq \delta\{\alpha Az_{t+1}^\alpha - \rho_{t+1}z_{t+1} - \pi(\rho_{t+1})\} \geq \rho_t z_t$ and hence $\delta\rho_{t+1}I \geq \rho_t z_t$. This can be further re-written by $\delta\left(x_t/x_{t+1}\right)^{1-\alpha} I \geq z_t$ due to $\rho_t = \alpha^2 Ax_t^{\alpha-1}$.

On the other hand, due to (CME) in period t, we have

$$l(1-\alpha)Az_t^\alpha + (1-l)(1-\alpha)Ax_t^\alpha = lz_{t+1} + (1-l)(x_{t+1}+I)$$

which implies that $x_{t+1} \geq (1-l)\{(1-\alpha)Ax_t^\alpha - I\}$ because of $z_t \geq 0$ and $x_{t+1} \geq z_{t+1}$.

By combining this with $\delta\left(x_t/x_{t+1}\right)^{1-\alpha} \geq z_t$, we can show that

$$\psi(x_t) \equiv \delta\left(\frac{x_t}{(1-l)[(1-\alpha)Ax_t^\alpha - I]}\right)^{1-\alpha} I \geq z_t$$

Here $x_t - \psi(x_t)$ is increasing in x_t because ψ is decreasing.

Let $\hat{x}$ be the value of x satisfying $(1-l)(1-\alpha)A\hat{x}^\alpha = \hat{x} + (1-l)I$. From (CME) for large t we obtain $(1-l)(1-\alpha)Ax_t^\alpha \leq x_{t+1} + (1-l)I$. Then, since $x_t > \underline{x}$ for large t (Lemma A4) and Assumption 4 holds, we can verify that x_t goes beyond $\hat{x}$ as t tends to be large enough. Thus $x_t \geq \hat{x}$ for large t. Then, since $x_t - \psi(x_t)$ is increasing, we obtain $x_t - z_t \geq x_t - \psi(x_t) \geq \hat{x} - \psi(\hat{x}) = \hat{x} - \delta I > 0$ for large t. Q.E.D.

Proof of Proposition 5.
We construct the switching equilibrium path described in the proposition as follows.

First, we set $z_1 = x_1 = \underline{x}$ and $\theta_1 < 1$ such that

$$w_0 = l\underline{x} + (1-l)\theta_1(\underline{x} + I)$$

Such $\theta_1 < 1$ exists under Assumption 6. Then, given the initial value θ_1 defined in this way, we sequentially define a sequence of $\{\theta_t\}_{t=1}^{T-1}$ such that

$$(l + (1-l)\theta_{t-1})(1-\alpha)A\underline{x}^\alpha = l\underline{x} + (1-l)\theta_t(\underline{x} + I), \quad t = 2, 3, ..., T-1$$

where T is defined such that $\theta_{T-1} \leq 1$ but $\theta_T > 1$. Under Assumption 7 the above sequence $\{\theta_t\}_{t=1}^{T-1}$ is increasing over time and $\theta_T > 1$ for some T. Then we define an equilibrium path $\{z_t, x_t, \theta_t\}_{t=1}^{T-1}$ from $t = 1$ to $t = T-1$ such that $z_t = x_t = \underline{x}$ and $\theta_t < 1$ satisfying the above equation. We call the phase with $t \leq T-1$ *low development phase*. In the low development phase (IC_t^*) is slack under Assumption 3 ($\delta > \alpha/(1-\alpha)$): $\delta\{\alpha A\underline{x} - d\underline{x}\} > d\underline{x}$. Also ($TS_{t-1}$) is satisfied in any period $t \leq T-1$ because $\pi(d) = 0$ and $\alpha A\underline{x}^\alpha - d\underline{x} > 0$ for all $t \leq T-1$.

Second, we consider the switching period $t = T$. Define $\theta_T = 1$ and set (z_T, x_T) satisfying both (IC_{T-1}^*) as equality

$$\delta\{\alpha A z_T^\alpha - \rho_T z_T - \pi(\rho_T)\} = d\underline{x} \qquad (IC_{T-1}^*)$$

and (CME) in period $T-1$:

$$(l + (1-l)\theta_{T-1})(1-\alpha)A\underline{x}^\alpha = l z_T + (1-l)(x_T + I) \qquad (CME_{T-1})$$

To show the existence of (z_T, x_T) such that both (CME_{T-1}) and (IC_T^*) as equality hold with $\underline{x} < x_T < \overline{x}^*$ and $z_T < x_T$, note that the following inequality is satisfied

$$\underline{x} < (l + (1-l)\theta_T)(1-\alpha)A\underline{x}^\alpha - (1-l)I < (1-l)\left(\delta I/\underline{x}^\alpha\right)^{1/(1-\alpha)}. \qquad (A11)$$

To see this, note first that under Assumption 7 the first inequality of (A11) holds. Second, due to Assumption 8 $\underline{x} < \overline{x}^*$ holds so that, by definition of $\overline{x}^*$, we have $(1-\alpha)A\underline{x}^\alpha - (1-l)I < (1-l)\left(\delta I/\underline{x}^\alpha\right)^{1/(1-\alpha)}$. Since $\theta_T < 1$, the second half of inequality (A11) also holds.

Thus, from (A11) we can see that $\underline{x} < x_T < (\delta I/\underline{x}^\alpha)^{1/(1-\alpha)}$ (see Figure 4 (a)). By Assumption 8 we have $(\delta I/\underline{x}^\alpha)^{1/(1-\alpha)} \leq \overline{x}^*$. Then we can find a unique (z_T, x_T) such that $z_T < x_T$ and $\underline{x} < x_T < \overline{x}^*$ with (CME_{T-1}) and (IC_{T-1}^*) as equality are satisfied.

Next we consider period $T+1$. Given (z_T, x_T) defined above, we will find (z_{T+1}, x_{T+1}) which satisfies (CME) in period T:

$$l(1-\alpha)Az_T^\alpha + (1-l)(1-\alpha)Ax_T^\alpha = l z_{T+1} + (1-l)(x_{T+1} + I) \qquad (CME_T)$$

and (IC_T^*) with equality:

$$\delta\{\alpha A z_{T+1}^{\alpha} - \rho_{T+1} z_{T+1} - \pi(\rho_{T+1})\} = \rho_T z_T. \qquad (\mathrm{IC}_T^*)$$

Since $x_T < \overline{x}^*$ holds by definition of $\overline{x}^*$, we have $(1-\alpha)Ax_T^{\alpha} - (1-l)I < (1-l)\left(\delta I/x_T^{\alpha}\right)^{1/(1-\alpha)}$ which implies that

$$\begin{aligned} x_{T+1} &< \frac{1}{1-l}\{l(1-\alpha)Az_T^{\alpha} + (1-l)Ax_T^{\alpha} - (1-l)I\} \\ &< \frac{1}{1-l}\{(1-\alpha)Ax_T^{\alpha} - (1-l)I\} \\ &< (\delta I/x_T^{\alpha})^{1/(1-\alpha)} \end{aligned}$$

and hence

$$\begin{aligned} x_{T+1} &< (\delta I/x_T^{\alpha})^{1/(1-\alpha)} \\ &= (\delta I \alpha^2 A/\rho_T x_T)^{1/(1-\alpha)} \\ &< (\delta I \alpha^2 A/\rho_T z_T)^{1/(1-\alpha)} \end{aligned}$$

due to $z_T < x_T$. Thus such (z_{T+1}, x_{T+1}) satisfying the above two equations (CME_T) and (IC_T^*) exists where $z_{T+1} < x_{T+1}$ (see Figure 4 (b)). We can also show that $\underline{x} < x_{T+1}$ because Assumption 7 implies

$$\begin{aligned} \underline{x} &\leq (1-l)(1-\alpha)A\underline{x}^{\alpha} - (1-l)I \\ &\leq l(1-\alpha)Az_T^{\alpha} + (1-l)(1-\alpha)Ax_T^{\alpha} - (1-l)I \\ &= lz_{T+1} + (1-l)x_{T+1} \\ &< x_{T+1} \end{aligned}$$

Thus we can find (z_{T+1}, x_{T+1}) which satisfies (CME_T) and (IC_T^*) with equality where $z_{T+1} < x_{T+1}$ and

$$\underline{x} < x_{T+1} < (\delta I/x_T^{\alpha})^{1/(1-\alpha)} < (\delta I/\underline{x}^{\alpha})^{1/(1-\alpha)}.$$

Thus $x_{T+1} < \overline{x}^*$ due to Assumption 8.

Repeating this process for all $t \geq T$, we can find a sequence of $\{z_t, x_t\}_{t=T}^{\infty}$ such that (CME) in period t and (IC_t^*) with equality are satisfied as well as $z_t < x_t$ and $\underline{x} < x_t$ hold. We also verify that these paths satisfy (TS_{t-1}) for all $t \geq T$ as follows: (TS_{T-1}) holds because $J_{T-1} = \alpha A\underline{x}^{\alpha} - d\underline{x} > \pi(d) = 0$ and $J_T = \alpha A z_T^{\alpha} - \rho_T z_T > \pi(\rho_T)$ due to (IC_T^*). Similarly, since $J_t > \pi(\rho_t)$ holds for all $t \geq T+1$, (TS_{t-1}) is satisfied for all $t \geq T+1$.

We have thus established that a sequence $\{z_t, x_t, w_t, \rho_t, \theta_t\}_{t=1}^{\infty}$ exists such that (IC_t^*), (TS_{t-1}) and (CME) are all satisfied in any period t. We can then find a sequence of repayments $\{R_t\}_{t=1}^{\infty}$ which satisfies (IC_t), (IRL_t) and (IRP_t) in any period t, given the constructed market variables $\{w_t, \rho_t, \theta_t\}_{t=1}^{\infty}$. From this, we can readily show that the trigger-like strategy of relationship producers and lenders as described in the proof of Lemma 1 (i) sustains the constructed relational contracts $\{z_t, R_t\}_{t=1}^{\infty}$ as a subgame perfect equilibrium.

Finally we can show that $w_{t-1} \geq z_t$ holds in any period t in the equilibrium path constructed above. In the low development phases ($t \leq T-1$) we have $w_{t-1} = (1-\alpha)A\underline{x}^{\alpha} \geq \underline{x} = z_t$ due to Assumption 7. On the other hand, in any period $t \geq T$ we have $w_{t-1} = lz_t + (1-l)(x_t + I)$ which implies that $w_{t-1} > z_t + (1-l)I > z_t$. Q.E.D.

8 Parametric Conditions on Assumption 8

Let $X \equiv (\delta I/\underline{x}^{\alpha})^{\frac{1}{1-\alpha}}$. Then Assumption 8 requires (i) $\overline{x}^* \geq X$ and (ii) $\overline{x}^* > \underline{x}$.

(i) $\overline{x}^* \geq X$. To see this, note that such inequality is equivalent to $(1-\alpha)AX^{\alpha} - (1-I) \leq (1-l)(\delta I/X^{\alpha})^{\frac{1}{1-\alpha}}$. By substituting $\underline{x} = (\alpha/(1-\alpha))I$ and $X = (\delta I/\underline{x}^{\alpha})^{\frac{1}{1-\alpha}}$ into this inequality, we obtain

$$(1-\alpha)A\delta^{\frac{\alpha}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\right)^{-\frac{\alpha^2}{1-\alpha}} I^{\alpha} \leq I + \delta\left(\frac{\alpha}{1-\alpha}\right)^{\frac{\alpha^2}{1-\alpha}} I.$$

Here the left hand is increasing and concave with respect to I while the right hand side is linear with I. Thus we can find some $\overline{I} > 0$ such that the inequality is satisfied for all $I < \overline{I}$.

(ii) $\overline{x}^* > \underline{x}$. By definition of $\overline{x}^*$, this inequality is equivalent to $(1-\alpha)A\underline{x}^{\alpha} - (1-l)I < (\delta I/\underline{x}^{\alpha})^{\frac{\alpha}{1-\alpha}}$. By substituting $\underline{x} = (\alpha/(1-\alpha))I$ into this condition, we have

$$(1-\alpha)A\left(\frac{\alpha}{1-\alpha}\right)^{\alpha} I^{\alpha} < I + \delta^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{1-\alpha}\right)^{-\frac{\alpha}{1-\alpha}} I.$$

Here the left hand side is increasing and concave with respect to I while the right hand side is linear with I. Thus we can find some $\tilde{I} > 0$ such that the inequality is satisfied for all $I < \tilde{I}$.

Therefore Assumption 8 is satisfied for all $I \leq \min\{\overline{I}, \tilde{I}\}$.

9 Appendix B: Bequest transfer

We have so far assumed that any individual cannot give his/her child bequest at all. We now allow each individual to bequeath his/her child when old. The introduction of bequest may affect the analysis because the bequest can be used to finance capital investment. However, in what follows we will show that non–relationship producers have no incentives to bequeath their children in *any* equilibrium when the altruistic preference parameter $\delta > 0$ is in some small range. Then we can show that the results we have already established in the main text remain true even when we introduce the bequest technology into the basic model as long as δ is in a certain small range.

Consider a non–relationship producer born in $t-1$ who receives bequest b_{t-1} from his parent and wants to invest x_t in capital. Suppose also that he gives his child bequest $b_t \geq 0$. Then let (x_t, b_t) be a choice of a non–relationship producer born in period $t-1$. Given this, the profit of a non–relationship producer is given as follows:

$$\pi(x_t, b_t | b_{t-1}) = \begin{cases} p_t x_t + (b_{t-1} - x_t)\rho_t - b_t & \text{if } b_{t-1} \geq x_t \\ p_t x_t - (x_t - b_{t-1})\rho_t - b_t - \rho_t I & \text{if } b_{t-1} < x_t \end{cases} \tag{B1}$$

When $b_{t-1} \geq x_t$, the bequest b_{t-1} can be used for financing x_t and the remaining amount $b_{t-1} - x_t$ can be saved to earn $\rho_t(b_{t-1} - x_t)$. In this case the verification cost $\rho_t I$ is not necessary. When $b_{t-1} < x_t$, the non-relationship producer needs to use the external financing and incur the verification cost $\rho_t I$. We suppose that each individual observes the past capital investments and bequest transfers made in the dynasty to which he or she belongs to as well as he or she observes the past market prices. Let H_t be a set of all these histories observed to a non-relationship producer up to period t. A strategy of a non-relationship producer is then defined as a mapping from H_t the set of observed histories up to period t to capital investment level $x_t \geq 0$ and bequest transfer to his child $b_t \in [0, \overline{b}_t]$ where $\overline{b}_t \equiv \pi(x(\rho_t), 0 | b_{t-1})$. Let $\sigma_t : H_t \to [0, \overline{b}_t] \times [0, \infty)$ be such strategy. Note that $\sigma_t = (x_t, b_t)$. Here each non-relationship producer cannot give his child bequest more than the maximum current profit.

With a slight abuse of notation, we will denote by $\pi(\sigma_t | b_{t-1})$ the profit of non-relationship producer who uses a strategy σ_t in period t, given the bequest b_{t-1} received from his parent. Then a non–relationship producer born in period $t-1$ obtains his payoff as $\pi(\sigma_t | b_{t-1}) + \delta\pi(\sigma_{t+1} | b_t)$ and chooses his strategy σ_t to maximize this payoff subject to $b_t \in [0, \overline{b}_t]$ (the bequest must be paid from the realized income), given the previous period choice b_{t-1}.

Then we show the following result:

Proposition B1. *Suppose that $\delta < 1/(1+d)$. Then in any equilibrium every non–relationship producer born in period $t-1$ chooses $\sigma_t = \overline{\sigma}_t \equiv (x(\rho_t), 0)$ which maximizes only his own profit $\pi(x_t, b_t | 0)$ given no bequest of his parent $b_{t-1} = 0$ where $x(\rho_t) \equiv \arg\max_{x_t} \pi(x_t, 0 | 0) = \alpha(Ax_t)^\alpha - \rho_t x_t$ no matter the history.*

Proof. Suppose contrary to the claim that there exists an equilibrium with $\{\sigma_t\}_{t=1}^{\infty}$ in which some non–relationship producer born in some period $t-1$ chooses $\sigma_t \neq \overline{\sigma}_t$. For σ_t to be an equilibrium choice, it must be that

$$\pi(\sigma_t | b_{t-1}) + \delta\pi(\sigma_{t+1} | b_t) \geq \pi(\overline{\sigma}_t | b_{t-1}) + \delta\pi(\sigma'_{t+1} | 0) \tag{B2}$$

where σ'_{t+1} denotes the strategy chosen by the child of the non–relationship producer who deviates from his equilibrium play σ_t to $\overline{\sigma}_t = (x(\rho_t), 0)$.

Let $\hat{\sigma}_t \equiv (x_t, 0)$ be the strategy defined by setting bequest transfer equal to zero in the original strategies $\sigma_t = (x_t, b_t)$. Let also $\hat{\sigma}'_{t+1} \equiv (x'_{t+1}, 0)$.

Then, by using (B1), (B2) can be written by

$$\begin{aligned} -(1-\delta\rho_{t+1})b_t - \delta b_{t+1} &\geq \{\pi(\overline{\sigma}_t|b_{t-1}) - \pi(\hat{\sigma}_t|b_{t-1})\} \\ &\quad + \delta\{\pi(\hat{\sigma}'_{t+1}|0) - \pi(\hat{\sigma}_{t+1}|0)\} - \delta b'_{t+1} \\ &\geq \{\pi(\overline{\sigma}_t|0) - \pi(\hat{\sigma}_t|0)\} \\ &\quad + \delta\{\pi(\hat{\sigma}'_{t+1}|0) - \pi(\overline{\sigma}_{t+1}|0)\} - \delta b'_{t+1} \\ &= \Delta_t - \delta\Delta_{t+1} - \delta b'_{t+1} \end{aligned}$$

where $\Delta_t \equiv \pi(\overline{\sigma}_t|0) - \pi(\hat{\sigma}_t|0) \geq 0$ and $\Delta_{t+1} \equiv \pi(\overline{\sigma}_{t+1}|0) - \pi(\hat{\sigma}'_{t+1}|0)$. Since $\sigma_t \neq \overline{\sigma}_t$, $\Delta_t > 0$ or/and $b_t > 0$ must hold.

Since $b_{t+1} \geq 0$, the above inequality implies

$$-(1-\delta\rho_{t+1})b_t - \Delta_t \geq -\delta\Delta_{t+1} - \delta b'_{t+1} \tag{B3}$$

For σ'_{t+1} to be the strategy chosen by the child of the non–relationship producer after the latter chose $\overline{\sigma}_t$, it must be that

$$\pi(\sigma'_{t+1}|0) + \delta\pi(\sigma''_{t+2}|b'_{t+1}) \geq \pi(\overline{\sigma}_{t+1}|0) + \delta\pi(\sigma'_{t+2}|0) \tag{B4}$$

where σ''_{t+2} and σ'_{t+2} denote the strategies chosen by the child of the non–relationship producer in period $t+1$ following σ'_{t+1} and $\overline{\sigma}_{t+1}$ respectively. Then, by a similar argument to (B3), we can show that

$$-(1-\delta\rho_{t+2})b'_{t+1} - \Delta_{t+1} \geq -\delta\Delta_{t+2} - \delta b'_{t+2}. \tag{B5}$$

The left hand side of (B5) cannot be larger than $-\delta b'_{t+1} - \Delta_{t+1}$ due to $\delta \leq 1 - \delta d \leq 1 - \delta\rho_{t+2}$. By using this fact and combining (B3) with (B5), we can derive

$$-(1-\delta\rho_{t+1})b_t - \Delta_t - (1-\delta)\Delta_{t+1} \geq -\delta\Delta_{t+2} - \delta b'_{t+2}. \tag{B6}$$

When we apply (B5) to period $t+2$, we derive

$$-b'_{t+2} \geq \frac{1}{1-\delta\rho_{t+3}}\{\Delta_{t+2} - \delta\Delta_{t+3} - \delta b'_{t+3}\} \tag{B7}$$

where $1 > \delta d \geq \delta\rho_{t+3}$ due to $\delta < 1/(1+d)$ and $\rho_{t+3} \leq d$. Then, by using this and (B6), we obtain

$$-(1-\delta\rho_{t+1})b_t - \Delta_t - (1-\delta)\Delta_{t+1} \geq \frac{\delta^2\rho_{t+3}}{1-\delta\rho_{t+3}}\Delta_{t+2} - \frac{\delta^2}{1-\delta\rho_{t+3}}(\Delta_{t+3} + b'_{t+3})$$

which implies

$$-(1-\delta\rho_{t+1})b_t - \Delta_t - (1-\delta)\Delta_{t+1} \geq -\frac{\delta^2}{1-\delta d}(\Delta_{t+3} + b'_{t+3}) \tag{B8}$$

because $\Delta_{t+2} \geq 0$ and $1 > \delta d \geq \rho_{t+3}$. Then, by applying (B7) to any period $T \geq t$ and using $\Delta_{t+i} \geq 0$ for any $i = 1, 2, ...$, the right hand side of (B8) cannot be smaller than

$$-\delta \left(\frac{\delta}{1 - \delta d} \right)^T (\Delta_{t+T+1} + b'_{t+T+1}).$$

Since T is arbitrary and Δ_s and b_s are bounded above for all $s \geq 0$,[15] this term converges to zero by taking $T \to \infty$, given $\delta/(1 - \delta d) < 1$ which is equivalent to $\delta < 1/(1 + d)$. Thus the right hand side of (B8) is bounded below from zero. However, since the left hand side of (B8) is strictly negative because $1 > \delta d \geq \delta\rho_{t+1}$ and $b_t > 0$ or/and $\Delta_t > 0$, this is a contradiction. Q.E.D.

We also show that every non-relationship lender has no incentives to give their children positive bequest in any equilibrium when $\delta < 1/(1 + d)$.

Proposition B2. *Suppose that $\delta < 1/(1 + d)$. Then non-relationship lenders leave no bequest to their children at all in any equilibrium.*

Proof. Suppose that $\delta < 1/(1 + d)$. Suppose also contrary to the claim of that there exists some equilibrium in which some non-relationship lender in some dynasty gives a positive bequest $b_t > 0$ in some period t where $b_t \in [0, w_t]$. For this to be an equilibrium choice, the equilibrium payoff of that lender in period t

$$\rho_t(w_{t-1} + b_{t-1}) - b_t + \delta\{\rho_{t+1}(w_t + b_t) - b_{t+1}\}$$

must not be less than

$$\rho_t(w_{t-1} + b_{t-1}) + \delta\{\rho_{t+1}w_t - b'_{t+1}\}$$

which can be obtained by offering no bequest $b'_t = 0$ which is then followed by the choice of his child b'_{t+1} in the next period $t + 1$. This condition thus yields

$$-(1 - \delta\rho_{t+1})b_t - \delta b_{t+1} \geq -\delta b'_{t+1} \tag{B9}$$

For b'_{t+1} to be an optimal choice by the child of the deviating lender in period $t+1$, we must have that

$$\begin{aligned} &\rho_{t+1}w_t - b'_{t+1} + \delta\{\rho_{t+2}(w_{t+1} + b'_{t+1}) - b'_{t+2}\} \\ \geq\ &\rho_{t+1}w_t + \delta\{\rho_{t+2}w_{t+1} - b''_{t+2}\} \end{aligned}$$

where b'_{t+2} is the bequest choice by the lender in $t+2$ following his parent's choice b'_{t+1} whereas b''_{t+2} is the choice by the same lender following no bequest given by his parent. This is simplified to

$$-(1 - \delta\rho_{t+2})b'_{t+1} \geq -\delta b''_{t+2} \tag{B10}$$

[15] Note here that $b_s \leq \pi(x(\rho_s), 0|0) < +\infty$ and $\Delta_s \leq \pi(x(\rho_s), 0|0) < +\infty$.

By combining (B9) with (B10), we obtain

$$\begin{aligned} -(1-\delta\rho_{t+1})b_t &\geq -\delta b'_{t+1} \\ &\geq -\frac{\delta^2}{1-\delta\rho_{t+2}} b''_{t+2} \end{aligned}$$

where the first inequality follows from $b_{t+1} \geq 0$ and the second inequality from (B10) respectively where $1 > \delta d \delta \rho_{t+2}$ due to $1/(1+d) > \delta$. Repeating the similar argument over period $t+3$, the right hand side of the above inequality is bounded below from

$$-\frac{\delta^3}{(1-\delta\rho_{t+2})(1-\delta\rho_{t+3})} b''_{t+3} \tag{B11}$$

Repeating this process for any period $t+T \geq t$, (B11) is bounded below from

$$-\frac{\delta^T}{\Pi_{i=1}^{T-1}(1-\delta\rho_{t+i})} b''_{t+T}$$

which is bounded below from

$$-(1-\delta d)\left(\frac{\delta}{1-\delta d}\right)^T b''_{t+T}$$

because of $d \geq \rho_{t+i}$. Since T can be arbitrary and $b''_{t+T} \leq w_{t+T} < +\infty$, this converges to zero by taking $T \to \infty$. Thus the right hand side of (B9) must be zero but its left hand side is strictly negative because of $b_t > 0$, which is a contradiction. Q.E.D.

Next we show that introducing bequest transfer never expands the set of incentive compatible relational contracts. Then the optimal relational contract which maximizes the sum of all the generations in each dynasty uses no bequest transfers at all.

Proposition B3. *Suppose that $\delta < 1/(d+1)$. Then the optimal relational contract involves no bequest transfers at all even when bequest transfer is allowed.*

Proof. We consider the incentive compatibility condition for an old relationship producer to honor an agreed upon repayment R_t in period t, called (IC-B$_t$):

$$\begin{aligned} & p_t z_t - R_t + \rho_t b^p_{t-1} - b^p_t + \delta\{p_{t+1}z_{t+1} - R_{t+1} + \rho_{t+1}b^p_t - b^p_{t+1}\} \\ \geq\ & p_t z_t + \rho_t b^p_{t-1} + \delta \max\{\pi_{t+1}, 0\} \end{aligned}$$

where $b^p_t \geq 0$ denotes the bequest transferred from the relationship producer to his child in period t. The relationship producer is given a bequest b^p_{t-1} from his parent in period $t-1$, lends it to the credit market and receives the corresponding interest income $\rho_t b^p_{t-1}$ in period t. By honoring an agreed upon repayment R_t and making a bequest b^p_t to his child in period t, the relationship producer obtains the payoff of the left hand side of the above inequality while he can obtain the payoff of its right hand side by reneging on the repayment R_t, making no bequest transfer $b_t^p = 0$ to his child and exercising the quitting option. In the latter case the relationship will be dissolved in period $t+$ in which his child will obtain the payoff corresponding to the arm's length contract $\max\{\pi_{t+1}, 0\}$. As we have shown in Proposition B1, the continuation payoff π_{t+1} after the deviation is equal to $\pi_{t+1} = \pi(x_{t+1}, 0|0)$ because $b_t = 0$ is chosen by any non-relationship producer who engages in arm's length contract in any period t.

Also, we consider the individual rationality constraint that a young relationship lender agrees on a relational contract $\{z_t, R_t\}$, called (IRL-B$_t$):

$$\begin{aligned} &R_t + \rho_t(w_{t-1} + b_{t-1}^l - x_t) - b_t^l + \delta\{R_{t+1} + \rho_{t+1}(w_t + b_t^l - z_{t+1}) - b_{t+1}^l\} \\ &\geq\ \rho_t(w_{t-1} + b_{t-1}^l) + \delta\rho_{t+1}w_{t+1} \end{aligned}$$

where $b_t^l \geq 0$ denotes the bequest transferred from an old relationship lender to her child in period t. The relationship lender is given a bequest b_{t-1}^l from her parent in period $t-1$ and thus owns total income $w_{t-1} + b_{t-1}^l$ in period $t-1$. That lender then lends x_t to the relationship lender matching her and the remaining amount $w_{t-1} + b_{t-1}^l - z_t$ to the credit market which gives her $\rho_t(w_{t-1} + b_{t-1}^l - z_t)$. The right hand side of the above inequality means the payoff of the relationship lender by exercising the quitting option and making no bequest transfer to her child. As we have shown in Proposition B2, in any continuation equilibrium after an old lender engages in arm's length contract, all her descendants will make no bequest transfers to the future generations. Thus no bequest appears in the last term of the right hand side of the above inequality.

By combining (IC-B$_t$) with (IRL-B$_t$) and defining total bequest $\tilde{b}_t \equiv b_t^p + b_t^l$, we obtain

$$-\tilde{b}_t + \delta\{p_{t+1}z_{t+1} - \rho_{t+1}z_{t+1} - \max\{\pi_{t+1}, 0\} + \rho_{t+1}\tilde{b}_t - \tilde{b}_{t+1}\} \geq \rho_t z_t \qquad \text{(IC-B}^*\text{)}$$

Here, if $1 > d\delta$ holds, then the left hand side of (IC-B*) is bounded above by

$$\delta\{p_{t+1}z_{t+1} - \rho_{t+1}z_{t+1} - \max\{\pi_{t+1}, 0\}\}$$

because $-\tilde{b}_t(1 - \delta\rho_{t+1}) \leq -\tilde{b}_t(1 - \delta d) \leq 0$ (by $\rho_{t+1} \leq d$) and $\tilde{b}_{t+1} \geq 0$ are satisfied. Thus (IC-B*) implies the incentive compatibility condition (IC$_t^*$) given in the main text without bequest transfer. This shows that the set of relational contracts satisfying (IC$_t^*$) is not expanded even when we allow bequest transfers if $\delta < 1/(1+d)$.

Next we consider the maximization of $\sum_{t=0}^{\infty} \delta^t J_t$ subject to (IC$_t^*$) and (TS$_{t-1}$) for all $t \geq 1$ when allowing bequest transfers b_t^l and b_t^p for relationship producer and lender. Here the joint payoff of a relationship producer and a relationship lender J_t is given by

$$J_t = p_t z_t + \rho_t(w_{t-1} - z_t + \tilde{b}_{t-1}) - \tilde{b}_t + \delta\{p_{t+1}z_{t+1} + \rho_{t+1}(w_t - z_{t+1} + \tilde{b}_t)\}$$

where $\tilde{b}_t = b_t^l + b_t^p$ denotes the total bequest transfer of the relationship producer and lender. However, if $\delta < 1/(1+d)$, then maximizing J_t with respect to $\tilde{b}_t$ yields $\tilde{b}_t = 0$.

Also, we already know that the set of possible relational contracts satisfying (IC_t^*) is not expanded by allowing bequest transfers as we have shown above. In addition to this, since J_t does not increase in $\tilde{b}_t$, the net total surplus TS_{t-1} does not increase in $\tilde{b}_t$ as well. Thus setting $\tilde{b}_t = 0$ makes both (IC_t^*) and (TS_{t-1}) easier to be satisfied. From this, the optimal relational contract must have $b_t^l = b_t^p = 0$ in any period t. Q.E.D.

Proposition (B1)-(B3) show that our characterization results given in the main text (Proposition 2-4) still remain true even when we introduce bequest transfer into the basic model if the altruistic preference δ is so small that $\delta < 1/(1+d)$ holds.

Next we will show that there actually exists an equilibrium in which all non–relationship producers and relationship pairs choose zero bequest if δ is small. Thus Proposition 5 still holds. To show this, we take the equilibrium described in Proposition 4. Then, first we consider non–relationship producers and their strategies as follows: every young non-relationship producer born in period $t-1$ always chooses zero bequest $b_t = 0$ and the optimal investment $x_t = x(\rho_t)$ defined by $\alpha^2 A^\alpha x_t^{\alpha-1} = \rho_t$ whatever choices of his or her parent. Then, given such strategy of all future generations, each non–relationship producer in the current period, say $t-1$, will choose $b_t = 0$ and x_t to maximize his own profit $\pi(x_t, 0|b_{t-1})$ whatever choice $\sigma_{t-1} = (b_{t-1}, x_{t-1})$ by his parent in the previous period. To show this, we define by $\tilde{\pi}(b_{t-1})$ the maximum of $\pi(x_t, 0|b_{t-1})$ over $x_t \geq 0$. We then show that each non–relationship producer chooses the capital investment x_t as follows: (i) $x_t = x(\rho_t)$ where $\alpha^2 Ax(\rho_t)^{\alpha-1} = \rho_t$ if $b_{t-1} < \underline{b}_{t-1}$ or if $b_{t-1} > \bar{b}_{t-1}$. Here $\underline{b}_{t-1}$ is defined as b_{t-1} such that $\alpha Ab_{t-1}^\alpha = \max_{x_t} p_t x_t - (x_t - b_{t-1})\rho_t - \rho_t I$ and $\bar{b}_{t-1}$ is defined as $\bar{b}_{t-1} \equiv x(\rho_t)$. (ii) $x_t = b_{t-1}$ if $b_{t-1} \in [\underline{b}_{t-1}, \bar{b}_{t-1}]$. In summary we obtain

$$\tilde{\pi}(b_{t-1}) \equiv \begin{cases} A\alpha(1-\alpha)x(\rho_t)^\alpha + b_{t-1}\rho_t - \rho_t I & \text{if } b_{t-1} < \underline{b}_{t-1} \\ A\alpha b_{t-1}^\alpha & \text{if } b_{t-1} \in [\underline{b}_{t-1}, \bar{b}_{t-1}] \\ A\alpha(1-\alpha)x(\rho_t)^\alpha + b_{t-1}\rho_t & \text{if } b_{t-1} > \bar{b}_{t-1} \end{cases}$$

Each old non–relationship producer in period $t-1$ chooses $b_{t-1} \geq 0$ to maximize his payoff $-b_{t-1} + \delta\tilde{\pi}(b_{t-1})$. Then, we can show that the optimal bequest level b_{t-1} which maximizes $-b_{t-1} + \delta\tilde{\pi}(b_{t-1})$ becomes zero if (i) $\delta\rho_t - 1 \leq 0$ for $b_{t-1} \geq \bar{b}_{t-1}$ or $b_{t-1} \leq \underline{b}_{t-1}$ and (ii) $\delta\alpha^2\delta^{\alpha-1} - 1 \leq 0$ for $b_{t-1} \in [\underline{b}_{t-1}, \bar{b}_{t-1}]$. Since $\rho_t \leq d$, condition (i) is reduced to $\delta d \leq 1$. Condition (ii) holds if $\delta\alpha^2\underline{b}_{t-1}^{\alpha-1} \leq 1$. Here, since $\underline{b}_{t-1}$ is given by b satisfying

$$\alpha Ab^\alpha = \alpha(1-\alpha)Ax(\rho_{t-1})^\alpha + \rho_{t-1}b - \rho_{t-1}I \tag{B12}$$

where the right hand side is equal to $\max_{x_{t-1}} A\alpha x_{t-1}^\alpha - \rho_{t-1}(x_{t-1} - b + I)$. We will write $\underline{b}(\rho_{t-1})$ for $\underline{b}_{t-1}$. Then we can verify that

$$\frac{\partial \underline{b}_{t-1}}{\partial \rho_{t-1}} = \frac{-(x(\rho_{t-1}) - \underline{b}_{t-1}) - I}{\alpha^2 A\underline{b}_{t-1}^{\alpha-1} - \rho_{t-1}} < 0$$

because $\underline{b}$ is the smallest root of (B9) so that $x(\rho) > \underline{b}$ and the denominator is positive. Thus $\underline{b}_{t-1} = \underline{b}(\rho_{t-1}) \geq \underline{b}(d)$ because $\rho_{t-1} \leq d$. Then we have

$$\begin{aligned} \delta\alpha^2 A\underline{b}_{t-1}^{\alpha-1} - 1 &\leq \delta\alpha^2 A^\alpha \underline{b}(d) - 1 \\ &\leq 0 \end{aligned}$$

if δ is so small that $\delta\alpha^2 A^\alpha \underline{b}(d)^{\alpha-1} \leq 1$. To be consistent with Assumption 3, we need to have (B12) Since $\underline{b}(d)$ satisfies $\underline{b}(d)^{\alpha-1} = d/\alpha(1-\alpha)A$, (B12) holds for some δ if $\alpha/(1-\alpha) < 1/\alpha^2 A\underline{b}(d)^{\alpha-1} = A^{1-\alpha}/d$. The latter condition is satisfied for small α or/and large A.

In what follows we thus assume

$$\frac{\alpha}{1-\alpha} \leq \delta \leq \min\left\{\frac{A^{1-\alpha}}{d}, \frac{1}{1+d}\right\} \tag{B13}$$

Given these parametric conditions, we have established the result that all non–relationship producers give no bequest at all. Similarly, we can show that every non–relationship lender never bequeaths to her child. To see this, note that the payoff of non–relationship lender who bequeaths b_{t-1} to her child is given by $\delta\rho_t b_{t-1} - b_{t-1}$ because the child obtains b_{t-1} to earn the additional interest income $\rho_t b_{t-1}$ which is evaluated by δ in the view point of the current period non–relationship lender. Maximizing $\delta\rho_t b_{t-1} - b_{t-1}$ with respect to $b_{t-1} \geq 0$ yields $b_{t-1} = 0$ because of $\delta d \leq 1$.

Next we consider any pair of a relationship producer and a relationship lender. Then suppose that they play the following strategy: they agree on a relational contract $\{z_t, R_t\}$ defined in the main text but leave no bequest to their children at all whatever bequest level they have received from their parents in the previous period. Then we can show that they will actually choose zero bequest when δ is small as follows: Let $\tilde{b}_{t-1} \geq 0$ be the total bequest given by the relationship producer and lender to their children in period $t-1$. Then we have $\tilde{b}_{t-1} + w_{t-1} > x_t$ due to $w_{t-1} > x_t$ and $\tilde{b}_{t-1} \geq 0$ so that the total profit of a relationship pair born in period $t-1$ is given by

$$J(z_t; \tilde{b}_{t-1}) \equiv p_t z_t + \rho_t(\tilde{b}_{t-1} + w_{t-1} - z_t)$$

where they do not need to incur the verification cost $I > 0$. Since the young relationship producer and lender in the next generation of period $t+1$ will choose x_{t+1} whatever $\tilde{b}_t$, the current old relationship pair chooses the bequest $\tilde{b}_t$ to maximize their joint payoffs $\delta J(z_{t+1}; \tilde{b}_t) - \tilde{b}_t$ which gives $\tilde{b}_t = 0$ again, provided δ is so small that $\delta d \leq 1$. Thus, the equilibrium path described in Proposition 5 still exists where no bequest transfers occur as long as (B13) is satisfied.

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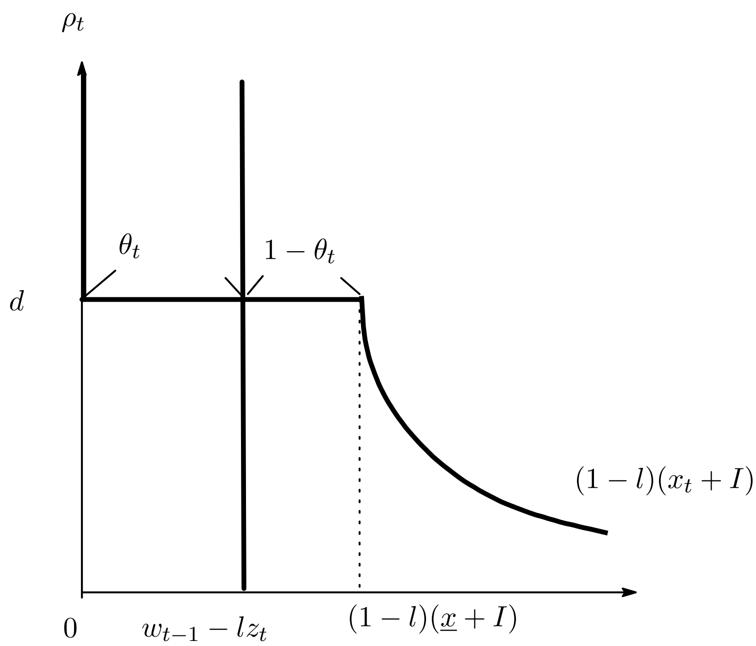

Figure 1: Credit Market Equilibrium

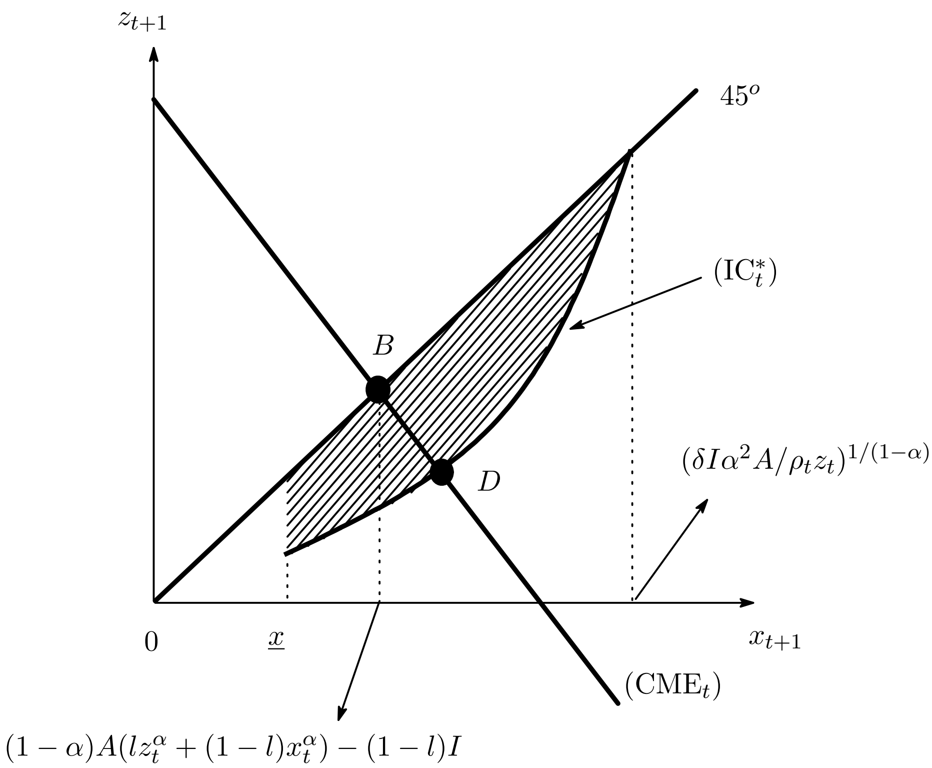

Figure 2 (a): Equilibrium Determination of (x_{t+1}, z_{t+1})

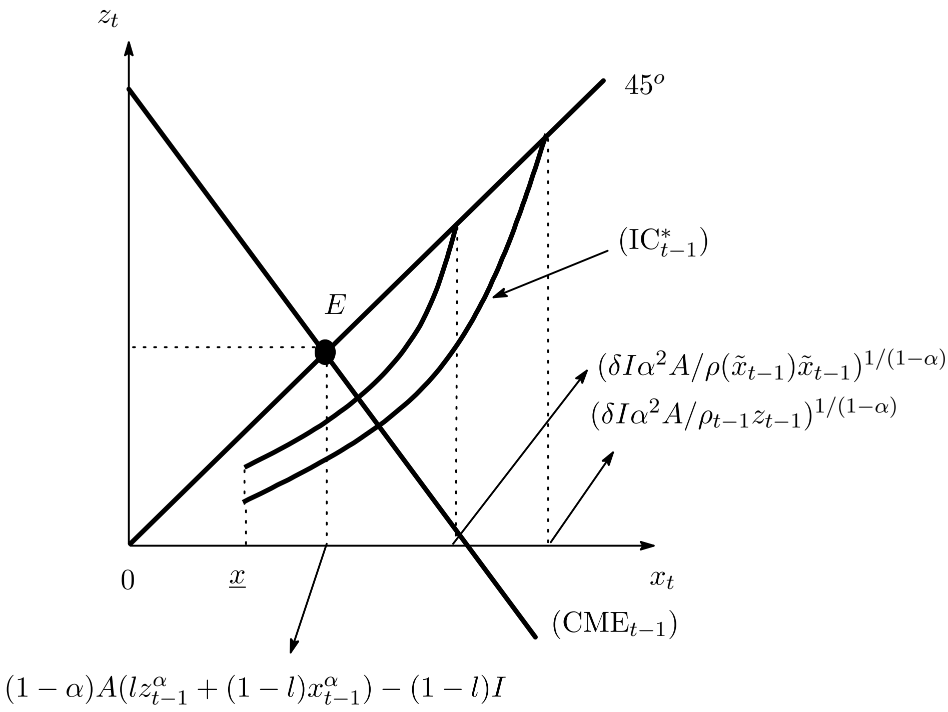

Figure 2 (b): (IC^*_{t-1}) is not binding when $z_t = x_t$.

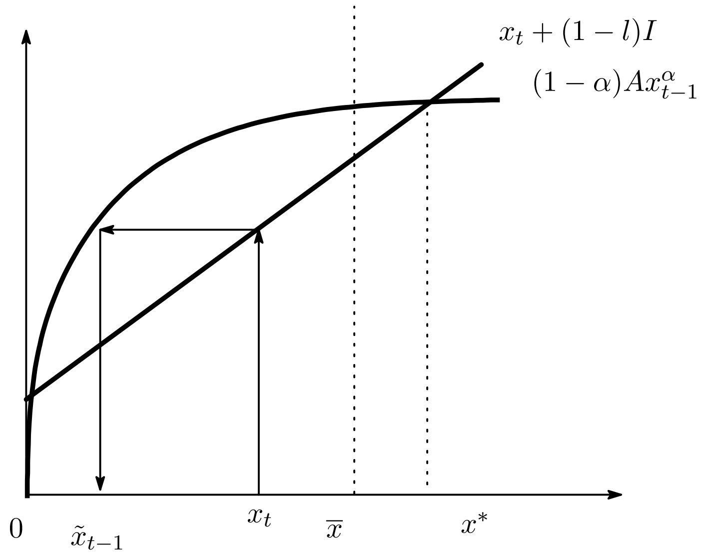

Figure 3

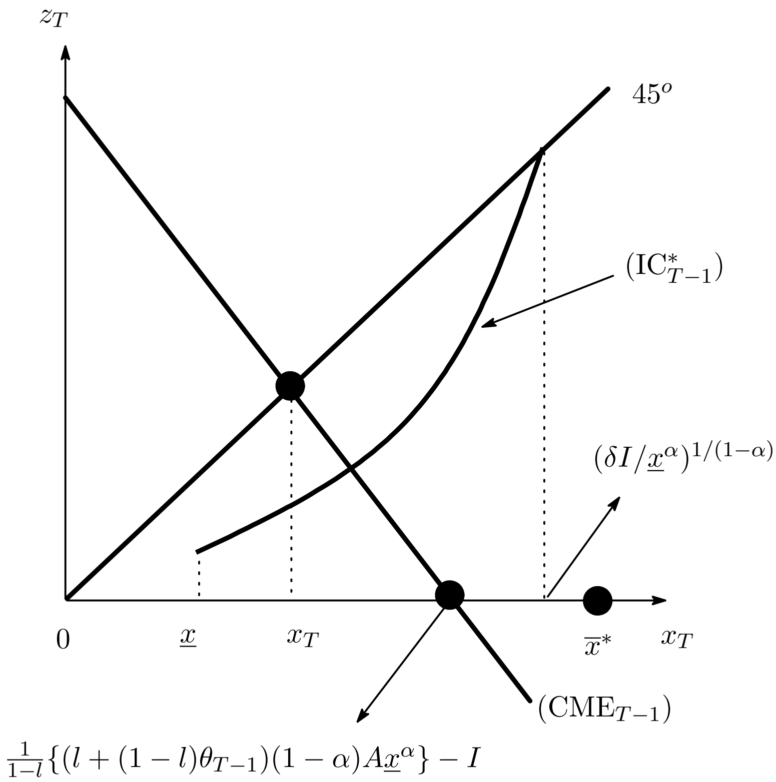

Figure 4 (a): Construction of Switching Equilibrium Path

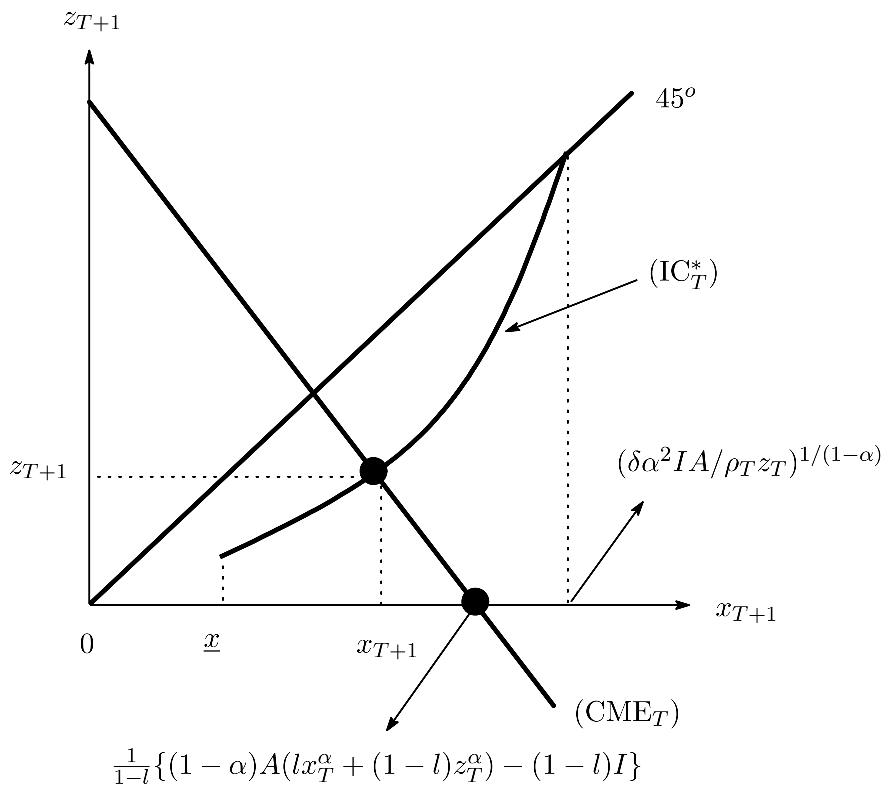

Figure 4 (b): Construction of Switching Equilibrium Path