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Investment timing with fixed and proportional costs of external financing*

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Abstract

We develop a dynamic model in which a firm exercises an option to expand production with cash balance and costly external funds. While related papers explain their results only by numerical examples, we analytically prove the following results. In the presence of only a proportional cost of external financing, the firm with more cash balance invests earlier; however, the presence of both proportional and fixed costs leads to a non-monotonic relation between the investment time and cash balance. The firm with more cash balance invests later to save a fixed cost, particularly when the cash balance is close to the investment cost. Our results can potentially account for a variety of empirical results concerning the relation between investment volume and financing constraints.

JEL Classifications Code: G13; G31; G32.

Keywords: Real options; investment timing; costly external financing; growth option; optimal stopping.

1 Introduction

Modigliani and Miller (1958) showed that financing and investment decisions can be made independently in a frictionless market. Since their seminal work, a wide range of literature has focused on investigating financing and investment decisions in the presence of various frictions.[1] Recently, an increasing number of papers have analyzed investment timing decisions under financial frictions in the real options framework.[2]

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[1] An incomplete list includes Fazzari, Hubbard, and Petersen (1988), Hoshi, Kashyap, and Scharfstein (1991), Kaplan and Zingales (1997), Cleary (1999), Gomes (2001), and Hennessy and Whited (2005).

[2] For example, the literature examined the effects of liquidity constraints (Boyle and Guthrie (2003)), shareholders-debtholders conflicts (Mauer and Sarkar (2005), Sundaresan and Wang (2007), Morellec and

This paper investigates an investment timing decision with costs of external financing in the following model. A firm owns an option to expand the scale of production by a fixed rate, where the price of the output follows a stochastic process. The investment project is financed with cash balance and costly external funds. The cash balance gradually increases as the firm's existent production generates cash flows even before its expansion. If the firm waits for a sufficient level of cash balance, the project can be financed entirely with the cash balance. Otherwise, the firm must rely partially on costly external financing. Considering the trade-off, the firm determines the optimal financing and investment policy.

As in the standard real options literature (Dixit and Pindyck (1994)), our model assumes the irreversibility and indivisibility of investing as frictions. In addition, our model includes costs of external financing. The financing costs are regarded as one of the most influential frictions (e.g., Altinkilic and Hansen (2000), Hennessy and Whited (2007)). According to the pecking order hypothesis, asymmetric information problems associated with external funding generate higher costs; therefore, managers prefer internal over external finance (Myers (1984), Myers and Majluf (1984)). We examine the case with only a proportional cost and the case with both fixed and proportional costs. The former approximates investment by a large firm, whereas the latter approximates investment by a small firm (Hennessy and Whited (2007)).

Before describing the results, we emphasize that our results are analytically proved. Most of the related papers explain their results only by numerical examples for the reason that the complexity of the models precludes analytic results (e.g., Boyle and Guthrie (2003), Hirth and Uhrig-Homburg (2010a), Shibata and Nishihara (2012)). However, it is more important to derive analytic results in more complicated models; the complexity increases the possibility of computational errors and makes the parameter sensitivity unclear. In this paper, unlike related papers, we analytically prove interesting properties of the firm's optimal financing and investment policy by employing similar techniques to those developed in the mathematical finance literature (e.g., Broadie and Detemple (1997), Detemple (2006)). This paper contributes to the literature by demonstrating how to derive analytic results.

The results are summarized as follows. Costs of external financing reduce the option value and discourage the investment compared with the case with no financing costs. This result is consistent with the standard view from empirical and theoretical studies concerning costly external financing (e.g., Hennessy, Levy, and Whited (2007)). When costs of external financing are relatively low to the scale of the profit expansion, the firm may invest partially with external financing. Otherwise, the firm always waits until the cash balance reaches the investment cost so that the project can be financed entirely with internal funds. In this case, the firm receives a higher profit from saving costs of

Schürhoff (2011)), manager-shareholders-debtholders conflicts (Shibata and Nishihara (2010)), and debt capacities (Shibata and Nishihara (2012)).

external financing than a loss due to the distortion in investment timing. We derive a clear condition that forces the firm to invest entirely with internal funds. The condition is more likely to be satisfied in a situation involving a smaller-scale expansion, a lower investment cost, higher financing costs, a smaller firm, and more cash balance.

Whether there is a fixed cost of external financing greatly affects the relation between the investment time and cash balance. First, we explain the result in the case with only a proportional cost. In the absence of fixed cost, the firm with more cash balance invests earlier. The reasoning is as follows. An increase in the cash balance decreases the financing cost proportionally; hence, it decreases the threshold price above which the firm expands production partially with external financing. This monotonic relation is straightforwardly consistent with the classical view of underinvestment due to financing constraints (e.g., Fazzari, Hubbard, and Petersen (1988), Hubbard (1998)). In this view, a firm's investment volume has a monotonic relation with internal funds. This paper complements the literature by analytically proving that, in the presence of only a proportional cost, more internal funds accelerate investments in a dynamic model.[3]

Now, we consider the case with both fixed and proportional costs. A fixed cost, unlike a proportional cost, plays a role in discouraging the investment by the firm with more cash balance. The intuition is as follows. An increase in the cash balance decreases the time until the cash balance reaches the investment cost. With a shorter waiting time, the firm with more cash balance can invest entirely with internal funds and save a fixed cost. This fixed cost effect is opposite to the proportional cost effect described in the previous paragraph. The trade-off between the two effects determines the relation between the investment time and cash balance. We derive the result that the firm with more cash invests later if the cash balance is close to the investment cost. This non-monotonic relation is inconsistent with the conventional view of underinvestment due to financing constraints. However, our result can potentially explains empirical results against the conventional view (e.g., Kaplan and Zingales (1997), Cleary, Povel, and Raith (2007)) in terms of fixed and proportional costs of external financing.

The result is also consistent with recent papers in the real options literature. Boyle and Guthrie (2003) showed that a firm with less cash balance may invest earlier to avoid the risk of a cash shortfall. In their model, a liquidity constraint, rather than financing costs, plays a role in leading the non-monotonic relation.[4] Shibata and Nishihara (2012) concentrated on a debt capacity constraint instead of internal financing constraints and showed that investment thresholds have a U-shaped relation with a debt capacity constraint. Note that these related papers demonstrate the results only in numerical examples. We complement the literature by analytically proving the non-monotonic relation between

[3]Milne and Robertson (1996) showed that investment increases with cash holdings in a dynamic dividend and investment model. In the real options literature, Hirth and Uhrig-Homburg (2010a) and Nishihara and Shibata (2010) also showed that the investment threshold decreases with internal funds.

[4]Hirth and Uhrig-Homburg (2010b) extended Boyle and Guthrie (2003) to a case with financing costs.

the investment time and cash balance due to fixed and proportional costs of external financing.

The remainder of this paper is organized as follows. Section 2 presents the setup and the result in the case without financing costs. Section 3 presents the results in the cases with only a proportional cost as well as both fixed and proportional costs. Although the price is assumed to be a geometric Brownian motion, we discuss how the results can be extended in the case of geometric Lévy process. Section 4 presents numerical examples and examines the comparative statics with respect to the price volatility. Section 5 concludes the paper. All proofs appear in the appendix.

2 Preliminaries

2.1 Setup

Consider a risk-neutral firm that produces a commodity at a constant rate. The output is sold at the market price $X(t)$, which follows a geometric Brownian motion

$$\mathrm{d}X(t) = \mu X(t)\mathrm{d}t + \sigma X(t)\mathrm{d}B(t) \ (t > 0), \quad X(0) = x, \tag{1}$$

where $B(t)$ denotes the standard Brownian motion defined in a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ and $\mu, \sigma(> 0)$ and $x(> 0)$ are constants. For convergence, we assume that $r > \mu$, where r is a positive constant interest rate. Assume that the firm owns an option to expand production to the fixed scale $A(> 1)$ at any time. If the option is exercised at time τ, the firm pays a fixed cost at time τ and receives an instantaneous cash flow $AX(t)$ after time τ. Assume that the investment cost is $I(> 0)$ if the whole amount of the cost is internally financed. If part of the investment cost is externally financed, the firm pays a proportional cost $C(\geq 0)$ and a fixed cost $K(\geq 0)$ of external financing. The total investment cost is expressed as $I + C\max(I - Y(\tau), 0) + K1_{\{Y(\tau)<I\}}$, where $Y(\tau)$ denotes cash balance at time τ. Until the investment time τ, cash balance $Y(t)$ follows

$$\mathrm{d}Y(t) = rY(t)\mathrm{d}t + X(t)\mathrm{d}t, \ (0 < t < \tau) \quad Y(0) = y, \tag{2}$$

where $y(\geq 0)$ is a constant. Boyle and Guthrie (2003) assume dynamics of cash balance exogenously and consider an option to initiate a new project. In contrast, we relate cash balance $Y(t)$ to operating cash flows $X(t)$ more directly and consider the option to expand production. In the case of $C = 0$ and $K = 0$, the setup corresponds to a standard model of the growth option. For a comprehensive list of typical situations fitting the standard model, refer to Dixit and Pindyck (1994). Although the standard model presumes that the firm needs no costs of external financing (otherwise, it has sufficient internal funds), our model considers fixed and proportional costs of external financing. Unlike Boyle and

Guthrie (2003), who concentrated on a liquidity constraint,[5] we examine the effects of financing costs on optimal investment timing.

Our assumption of costly external financing is justified as follows. In the pecking order theory, a variety of agency and asymmetric information problems increase costs of external financing, which leads to a preference for internal over external finance (Myers (1984), Myers and Majluf (1984)). Practically, financing costs consist of a fixed cost (which is independent of the issue size) and a variable cost (which depends on the issue size). A fixed cost includes taxes, fees, and setup expenses. A variable cost increases with the issue size primarily because more underwriting services are required for more funds raised. In the standard view of the literature, a variable cost is convex with respect to the issue size (e.g., Altinkilic and Hansen (2000)). Hennessy and Whited (2007) estimated that proportional costs of equity financing are approximately 5% (10%) for large (small) firms. They argued that a significant level of fixed costs of equity financing may exist only for small firms, while the convexity is statistically insignificant for any firms. In considering their results, as well as preserving tractability of the model, we examine two cases: the case with only a proportional cost (Section 3.1) and the case with both fixed and proportional costs (Section 3.2).

2.2 Case with no financing costs

As a benchmark, this section briefly describes the case of $C = K = 0$. The firm solves the following problem:

$$\sup_{\tau \in \mathcal{T}} \mathbb{E}^x \left[\int_0^\tau \mathrm{e}^{-rt} X(t) \mathrm{d}t + \int_\tau^\infty \mathrm{e}^{-rt} X(t) \mathrm{d}t - \mathrm{e}^{-r\tau} I\right], \tag{3}$$

where $\mathcal{T}$ denotes the set of all stopping times and $\mathbb{E}^x[\cdot]$ denotes the expectation conditional on $X(0) = x$. In (3), τ represents the time to expand the scale of production. By the strong Markov property of $X(t)$, (3) can be easily reduced to

$$\frac{x}{r-\mu} + \underbrace{\sup_{\tau \in \mathcal{T}} \mathbb{E}^x\left[\mathrm{e}^{-r\tau}\left(\frac{A-1}{r-\mu}X(\tau) - I\right)\right]}_{=:V_{0,0}(x) \text{ the growth option value}},$$

where we denote the second term by $V_{0,0}(x)$. The standard argument (e.g., Dixit and Pindyck (1994)) proves that

$$V_{0,0}(x) = \begin{cases} \left(\dfrac{(A-1)x^*_{0,0}}{r-\mu} - I\right)\left(\dfrac{x}{x^*_{0,0}}\right)^\beta & (0 < x < x^*_{0,0}) \\ \dfrac{(A-1)x}{r-\mu} - I & (x \geq x^*_{0,0}), \end{cases} \tag{4}$$

[5] Hirth and Uhrig-Homburg (2010b) extended Boyle and Guthrie (2003) by considering both a liquidity constraint and financing costs.

where $\beta := 1/2 - \mu/\sigma^2 + \sqrt{(\mu/\sigma^2 - 1/2)^2 + 2r/\sigma^2}(> 1)$ is a positive characteristic root, and $x^*_{0,0} := \beta(r-\mu)I/\{(A-1)(\beta-1)\}$ is the threshold price above which the firm expands production.

3 Analytic Results

3.1 Case with a proportional cost

This subsection examines the case of $C > 0$ and $K = 0$. This assumption applies to investments by large firms (Hennessy and Whited (2007)). In this case, the growth option value, denoted by $V_{C,0}(x, y)$, is expressed as

$$V_{C,0}(x, y) = \sup_{\tau \in \mathcal{T}} \mathbb{E}^{x,y}[\mathrm{e}^{-r\tau} \left(\frac{A-1}{r-\mu} X(\tau) - I - C \max(I - Y(\tau), 0) \right)], \tag{5}$$

where where $\mathbb{E}^{x,y}[\cdot]$ denotes the expectation conditional on $(X(0), Y(0)) = (x, y)$. The term $C \max(I - Y(\tau), 0)$ means that a proportional cost is required when the firm is short of cash balance. The standard argument proves that the exercise region of the option is expressed as

$$S_{C,0} := \{(x, y) \in \mathbb{R}^2_+ \mid V(x, y) = (A-1)x/(r-\mu)x - I - C \max(I - y, 0)\}. \tag{6}$$

Now we analytically prove interesting properties of $V_{C,0}(x, y)$ and $S_{C,0}$.

Consider the following problems as approximations of $V_{C,0}(x, y)$:

$$V_U(x, y) := \sup_{\tau \in \mathcal{T}} \mathbb{E}^{x,y}[\mathrm{e}^{-r\tau} \left(\frac{A-1}{r-\mu} X(\tau) - I - C(I - Y(\tau)) \right)], \tag{7}$$

and

$$V_L(x, y) := \sup_{\tau \in \mathcal{T}} \mathbb{E}^{x}[\mathrm{e}^{-r\tau} \left(\frac{A-1}{r-\mu} X(\tau) - I - C(I - y) \right)]. \tag{8}$$

By the strong Markov property of $X(t)$, we can easily derive the closed-form solution of (7) as follows: If $A - C - 1 > 0$, we have

$$V_U(x, y) = \begin{cases} C\left(y + \dfrac{x}{r-\mu}\right) + \left(\dfrac{(A-1-C)x^*_U}{r-\mu} - (1+C)I\right)\left(\dfrac{x}{x^*_U}\right)^{\beta} & (0 < x_1 < x^*_U) \\ \dfrac{(A-1)x}{r-\mu} - I - C(I-y) & (x_1 \geq x^*_U), \end{cases} \tag{9}$$

where $x^*_U := \beta(r-\mu)(1+C)I/\{(\beta-1)(A-C-1)\}$ represents the threshold price above which the firm expands production. Otherwise, $V_U(x, y) = C(x/(r-\mu) + y)$ holds and the growth option will be never exercised. Note that in both cases the exercise policy is independent of $Y(t)$. In (8), $V_L(x, y)$ is the same as (4) replaced I with $I + C(I - y)$. Then, the threshold price, denoted by $x^*_L(y)$, is equal to

$$x^*_L(y) = \frac{\beta(r-\mu)(I + C(I-y))}{(\beta-1)(A-1)}. \tag{10}$$

First, we show several properties of the option value $V_{C,0}(x, y)$. The following proposition shows that $V_L(x, y)$ and $V_U(x, y)$ are closed-form bounds of $V_{C,0}(x, y)$.

Proposition 1 If $y < I$, $V_L(x, y) \leq V_{C,0}(x, y) \leq V_U(x, y)$ is satisfied. Otherwise, $V_{C,0}(x, y) = V_{0,0}(x, y)$ holds.

Next, we show several properties of the exercise region $S_{C,0}$. It immediately follows from Proposition 1 that $S_{C,0}$ includes S_I which is defined by

$$S_I := \{(x, y) \in \mathbb{R}^2_+ \mid x \geq x^*_{0,0}, y \geq I\}. \tag{11}$$

Then, we examine the properties of $T_{C,0} := S_{C,0} \setminus S_I$. We can show the following lemma and proposition.

Lemma 1
Assume that $A - C - 1 > 0$. It holds that

$$0 \leq V_{C,0}(x + \Delta, y) - V_{C,0}(x, y) \leq \frac{(A - 1)\Delta}{r - \mu} \tag{12}$$

$$0 \leq V_{C,0}(x, y + \Delta) - V_{C,0}(x, y) \leq C\Delta \tag{13}$$

for any positive constant Δ.

Proposition 2
Case (a): $A - C - 1 > 0$
The exercise region $S_{C,0}$ is the disjoint union of S_I and

$$T_{C,0} = \{(x, y) \in \mathbb{R}^2_+ \mid x \geq x^*_{C,0}(y), y < I\}, \tag{14}$$

where $x^*_{C,0}(\cdot)$ is a continuous and decreasing function satisfying

$$x^*_L(y) \leq x^*_{C,0}(y) \leq x^*_U \tag{15}$$

and

$$\lim_{y \uparrow I} x^*_{C,0}(y) = \max\left(x^*_{0,0}, \frac{(C + 1)rI}{A - C - 1}\right) \tag{16}$$

Case (b): $A - C - 1 \leq 0$
The exercise region $S_{C,0}$ is equal to S_I, which means that $T_{C,0} = \emptyset$.

[Insert Figure 1 about here.]

Figure 1 illustrates the exercise region in each case.[6] Note that the exercise region is connected. The firm's financing and investment policy can be classified into two different types, depending on the relation between the scale of production expansion, $A - 1$, and

[6]In all figures in this paper, we set the axes in the same way as Boyle and Guthrie (2003) and Hirth and Uhrig-Homburg (2010b) for comparison.

a proportional cost of external financing, C. Consider Case (a) in which a proportional cost is relatively low. The firm invests partially with external financing when the output price, $X(t)$, exceeds the threshold $x^*_{C,0}(Y(t))$. Proposition 2 shows a monotonicity in the threshold $x^*_{C,0}(Y(t))$ with respect to cash balance $Y(t)$. The reason is as follows. An increase in $Y(t)$ decreases a financing cost, $C(I - Y(t))$, which enables the firm to invest earlier. Accordingly, the presence of the proportional cost of external financing leads to a straightforward result that the firm with more cash balance invests earlier.

In Case (b), the firm always waits until the cash balance reaches the whole amount of the investment cost. The reasoning is as follows. As long as $Y(t) < I$, the firm receives cost savings of $CX(t)\mathrm{d}t + r(1 + C)I\mathrm{d}t$ and loses $(A - 1)X(t)\mathrm{d}t$ by deferring investment by an infinitesimally short period $\mathrm{d}t$. If $A - C - 1 \leq 0$, we have $CX(t)\mathrm{d}t + r(1+C)I\mathrm{d}t - (A-1)X(t)\mathrm{d}t > 0$ for any $X(t)$, which means that the cost saving effect always dominates the loss. Then, the firm has no incentive to invest with external financing. Below, we describe major determinants of entirely internal financing. Clearly, low financing costs and much cash balance are the determinants. Since a small firm suffers from relatively higher financing costs (Hennessy, Levy, and Whited (2007)), a small firm is more likely to invest entirely with internal financing compared with a large firm. In addition, a small-scale expansion with low investment cost increases the possibility of entirely internal financing. For example, suppose that $C = 0.1$. While a large-scale investment with $A = 1.5$ and $I = 100$ leads to Case (a), a small-scale investment with $A = 1.1$ and $I = 20$ leads to Case (b).

It should be noted that we, unlike most of the related papers, analytically prove the existence of continuous and decreasing thresholds $x^*_{C,0}(Y(t))$. It is worth deriving the analytic results in a complicated problem involving multiple state variables because the complexity increases the possibility of computational errors. Although some of the techniques used in the proof are enlightened by the mathematical finance literature (e.g., Broadie and Detemple (1997), Detemple (2006)), the proofs are newly developed in this paper. While the mathematical finance studies analyzed the exercise regions of American options that involve a multi-dimensional geometric Brownian motion[7], the stochastic process $Y(t)$ in our model is not a geometric Brownian motion; instead, it is but defined by (2). Furthermore, the payoff function of problem (5) is not convex, which makes the proofs more difficult. This paper contributes to the literature from this technical viewpoint.

Proposition 2 shows that financing costs of external financing discourage the investment compared with the case with no financing costs. This is consistent with empirical results (e.g., Hennessy, Levy, and Whited (2007)). The monotonic relation between the investment time and cash balance is consistent with the conventional view of underinvestment due to financing constraints. Indeed, many empirical and theoretical papers have shown that a firm with less internal funds invests less than a firm with sufficient internal

[7]Nishihara (2011) showed the properties of multiple real options using similar techniques.

funds (e.g., Fazzari, Hubbard, and Petersen (1988), Hubbard (1998)). Milne and Robertson (1996) examined a firm's dynamic policy of dividend and investment; they showed that the investment level increases with cash holdings. Similar results are seen in the real options literature. Nishihara and Shibata (2010) showed that a firm delays the investment when it must rely more heavily on debt financing than the optimal level of the leverage. Hirth and Uhrig-Homburg (2010a) showed that the investment threshold is decreasing in the firm's liquid funds.

On the other hand, empirical findings against the monotonic relation between the investment volume and internal funds have been identified (e.g., Kaplan and Zingales (1997), Moyen (2004), Cleary, Povel, and Raith (2007)). These studies have shown that having more internal funds does not necessarily increase the investment level. In the next section, we will show that the inclusion of a fixed cost of external financing can lead to the non-monotonic relation between the investment time and cash balance.

3.2 Case with fixed and proportional costs

This section examines the case of $C > 0$ and $K > 0$. In this case, the option value, denoted by $V_{C,K}(x, y)$, is expressed as

$$V_{C,K}(x,y) = \sup_{\tau\in\mathcal{T}} \mathbb{E}^{x,y}[\mathrm{e}^{-r\tau}\left(\frac{A-1}{r-\mu}X(\tau) - I - C\max(I-Y(\tau),0) - K1_{\{Y(\tau)<I\}}\right)], \tag{17}$$

where $1_{\{Y(\tau)<I\}}$ denotes the defining function. Note that the exercise region of the option is

$$S_{C,K} := \{(x,y) \in \mathbb{R}^2_+ \mid V_{C,K}(x,y) = (A-1)x/(r-\mu) - I - C\max(I-y,0) - K1_{\{y<I\}}\}. \tag{18}$$

We can readily show the following proposition regarding the properties of $V_{C,K}(x, y)$.

Proposition 3 $V_{C,0}(x,y) - K \leq V_{C,K}(x,y) \leq V_{C,0}(x,y)$ is satisfied. If $A - C - 1 \leq 0$, $V_{C,K}(x,y) = V_{C,0}(x,y)$ is satisfied. If $y \geq I$, $V_{C,K}(x,y) = V_{0,0}(x,y)$ is satisfied.

Next, we analytically prove the properties of the exercise region $S_{C,0}$. Proposition 3 shows that $S_{C,0}$ includes S_I. Then, we concentrate on the properties of $T_{C,K} := S_{C,K} \setminus S_I$.

Proposition 4
Case (a-K): $A - C - K/I - 1 > 0$
The exercise region $S_{C,K}$ is the disjoint union of S_I and $T_{C,K}$ satisfying

$$T_{C,K} \subset T_{C,0} \cap \mathbb{R}_+ \times [0, I - K/(A-C-1)), \tag{19}$$

and

$$\{(x,y) \in \mathbb{R}^2_+ \mid x \geq x^*_{C+K/(I-y),0}(y), y < I - K/(A-C-1)\} \subset T_{C,K}, \tag{20}$$

where $x^*_{C+K/(I-y),0}(y)$ denotes the threshold of problem (5) replaced C with $C+K/(I-y)$. In particular, there exist $(x, y_1) \in S_{C,K}, (x, y_2) \notin S_{C,K}$, and $(x, y_3) \in S_{C,K}$ satisfying $y_1 < y_2 < y_3$.

Case (b-K): $A - C - K/I - 1 \leq 0$

The exercise region is equal to S_I.

[Insert Figure 2 about here.]

Figure 2 illustrates the exercise region in each case. The firm defers the investment so that the project can be financed entirely with internal funds if costs of external financing are sufficiently high (Case (b-K)). Otherwise, the firm may invest partially with costly external financing (Case (a-K)). A fixed cost $K > 0$ increases the possibility of Case (b-K) by the additional term K/I and discourages the investment, compared with the case with only a proportional cost. Similar to the case with only a proportional cost, a smaller-scale expansion, a lower investment cost, higher financing costs, a smaller firm, and more cash balance increase the possibility that the firm invests entirely with internal financing.

Below we explain Case (a-K) in which a fixed cost of external financing generates quite a different result from that of Case (a) in Proposition 2. The key result is that $\mathbb{R}_+ \times [I - K/(A - C - 1), I)$ is not included in the exercise region $S_{C,K}$. Before cash balance, $Y(t)$, reaches the critical level, $I - K/(A - C - 1)$, the firm may expand the scale of production for a sufficiently high price of the output. Once the cash balance reaches the critical level prior to the investment, the firm always defers the investment until the cash balance is equal to the whole amount of the investment cost.

A fixed cost provides a greater incentive for the firm with more cash balance to wait and invest entirely with internal funds. Indeed, the firm with more cash balance can save a fixed cost with a shorter deferment of the investment. On the other hand, there always exists the proportional cost effect; an increase in cash balance reduces a proportional cost of financing and increases the incentive to invest earlier. The two conflicting effects determine the relation between the investment time and cash balance. Proposition 4 shows that the fixed cost effect dominates the proportional cost effect when the cash balance is sufficiently close to the investment cost. Consequently, contrary to the conventional view (or the result in the case with only a proportional cost), the firm with more cash balance invests later in the situation. As easily seen, the fixed cost effect is weak when the cash balance is far from the investment cost. Thus, the presence of both fixed and proportional costs leads to the non-monotonic relation between the investment time and cash balance.

Proposition 4 has the potential to explain the non-monotonic relation observed in empirical studies (e.g., Kaplan and Zingales (1997), Cleary, Povel, and Raith (2007)), in terms of fixed and proportional costs of external financing. This result is also consistent with the following findings in the real options literature.[8], Boyle and Guthrie (2003)

[8] Apart from dynamic investment models, Cleary, Povel, and Raith (2007) showed both theoretically and empirically a U-shaped relation between the investment level and internal funds.

showed that a firm with less cash balance may invest earlier to avoid the risk of a cash shortfall. In their model, investment is possible only when a liquidity constraint is satisfied. The liquidity constraint leads to a V-shaped relation between the investment threshold and cash balance. Hirth and Uhrig-Homburg (2010b) extended Boyle and Guthrie (2003) to a case involving financing costs and showed the possibility of a variety of relations. Shibata and Nishihara (2012) showed that, in a dynamic investment and capital structure model, investment thresholds have a U-shaped relation with a debt issuance constraint. Although these papers explain their results only by numerical examples, we analytically prove the non-monotonic relation between the investment time and cash balance.

3.3 The output price following a geometric Lévy process

The results obtained in the previous subsections hold true when the output price follows a geometric Brownian motion, as well as when it follows a geometric Lévy process. The class of geometric Lévy processes, unlike a geometric Brownian motion, includes processes with jumps and can account for fat tails and skewness of probability distributions of the output price. Assume that the output price $X(t) := x\mathrm{e}^{Z(t)}$, where $Z(t)$ is a Lévy process (i.e., a process with stationary independent increments), satisfies the convergence condition: $\mathbb{E}^1[X(t)] = \mathrm{e}^{\mu t}$ with $\mu < r$. In the case with no financing costs, the results are seen in Boyarchenko (2004). Although the problem precludes a closed-form solution of $V_{0,0}(x)$ and $x^*_{0,0}$, we still obtain an investment threshold as proved in Mordecki (2002). A difficulty in extending the results in the previous subsections to the case of a geometric Lévy process is that the generating operator of $(X(t), Y(t))$ includes the integrals corresponding to the jumps (refer to (Øksendal and Sulem 2007)). However, we obtain the same relation, $\mathcal{L}f(x, y) - rf(x, y) \leq 0 \Leftrightarrow x \geq (C + 1)rI/(A - C - 1)$, as in the case with no jumps using the linearity of $f(x, y)$ defined by (23). As a result, we can trace all the proofs of the lemma and propositions in Sections 3.2 and 3.3. All technical details are omitted.

4 Numerical examples

As mentioned in Section 1, the main contribution of this paper is to analytically prove the properties of the firm's optimal financing and investment policy (Section 3). This section supplements the analytic results by presenting the comparative statics results with respect to the volatility σ in numerical examples. The base parameter values except for σ are set as follows:

$$r = 0.07, \mu = 0.03, I = 100, (X(0), Y(0)) = (10, 50), A = 1.5, C = 0.1, K = 1. \qquad (21)$$

Cases (a) and (a-K) are satisfied because $A - C - K/I - 1 = 0.39 > 0$ hold. In the computation, we make a tri-nomial lattice model that approximates to a a geometric Brownian motion (1), and we use a value function iteration algorithm.

[Insert Figure 3 about here.]

Figure 3 plots the exercise regions with varying levels of σ in the cases with only a proportional cost and with both fixed and proportional costs. We can see from Figure 3 that a higher σ decreases the exercise regions in both cases. This implies that the investment threshold and the option value increase with σ. Thus, the effect of the volatility σ on the investment policy remains unchanged from that of the standard model with no financing costs. In the upper panel of Figure 3, there is a gap between $\lim_{y\uparrow I} x^*_{C,0}(y) = (C+1)rI/(A-C-1)$ and $x^*_{0,0}$ for $\sigma = 0.1, 0.15$, whereas $\lim_{y\uparrow I} x^*_{C,0}(y)$ is equal to $x^*_{0,0}$ for $\sigma = 0.2, 0.25, 0.3$. Since $x^*_{0,0}$ is increasing with σ, a higher σ reduces a gap between $\lim_{y\uparrow I} x^*_{C,0}(y)$ and $x^*_{0,0}$ (cf. Propostion 2). In the lower panel of Figure 3, the boundary of $T_{C,K}$ has an asymptote $y = I - K/(A-C-1)$. As proved in Propostion 4, the option is not exercised for $Y(t) \in [0, I - K/(A-C-1))$.

5 Conclusion

This paper investigated a firm's option to expand the scale of production by a fixed rate. We assumed that the project is financed with cash balance, which is increasing with time, and external funds that may require proportional and fixed costs. We, unlike most of the related papers, analytically proved the properties of the firm's optimal financing and investment policy. The results are summarized as follows.

Costs of external financing reduce the option value and discourage the investment compared with the case with no financing costs. When costs of external financing are relatively low to the scale of the profit expansion, the firm may invest partially with external financing. Otherwise, the firm always waits until the cash balance reaches the investment cost so that the project can be financed entirely with internal funds. The entirely internal financing is likely to be adopted in a small-scale expansion by a small firm with more cash balance. In the presence of only a proportional cost, the firm with more cash balance invests earlier; however, the presence of both proportional and fixed costs leads to a non-monotonic relation between the investment time and cash balance. Our results can potentially account for ambiguous results in empirical studies regarding the relation between investment volume and financing constraints.

Acknowledgment

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A Proof of Proposition 1

Since $Y(t)$ monotonically increases from the initial value y, we have for $y < I$

$$I - Y(t) \leq \max(I - Y(t), 0) \leq I - y$$

at any time t. This implies that $V_L(x, y) \leq V_{C,0}(x, y) \leq V_U(x, y)$ holds for $y < I$. Clearly we have $V_{C,0}(x, y) = V_{0,0}(x, y)$ for $y > I$.

B Proof of Lemma 1

First, we show (12). Note that $\mathbb{E}^1[\cdot]$ represents the expectation with $X(t)$ starting from $X(0) = 1$. For any positive constant Δ, we have

$$\begin{aligned}
&V_{C,0}(x+\Delta, y)\\
=&\sup_{\tau\in\mathcal{T}} \mathbb{E}^1[\mathrm{e}^{-r\tau}\left(\frac{(A-1)(x+\Delta)}{r-\mu}X(\tau) - I - C\max(I - \mathrm{e}^{r\tau}y - \int_0^\tau \mathrm{e}^{r(\tau-s)}(x+\Delta)X(s)\mathrm{d}s, 0)\right)]\\
\leq&\sup_{\tau\in\mathcal{T}} \mathbb{E}^1[\mathrm{e}^{-r\tau}\left(\frac{(A-1)x}{r-\mu}X(\tau) - I - C\max(I - \mathrm{e}^{r\tau}y - \int_0^\tau \mathrm{e}^{r(\tau-s)}xX(s)\mathrm{d}s, 0)\right.\\
&\left.+\frac{(A-1)\Delta}{r-\mu}X(\tau) + C\Delta\int_0^\tau \mathrm{e}^{r(\tau-s)}X(s)\mathrm{d}s\right)]\\
\leq&\underbrace{\sup_{\tau\in\mathcal{T}} \mathbb{E}^1[\mathrm{e}^{-r\tau}\left(\frac{(A-1)x}{r-\mu}X(\tau) - I - C\max(I - \mathrm{e}^{r\tau}y - \int_0^\tau \mathrm{e}^{r(\tau-s)}xX(s)\mathrm{d}s, 0)\right)]}_{=V_{C,0}(x,y)}\\
&+\sup_{\tau\in\mathcal{T}} \mathbb{E}^1[\mathrm{e}^{-r\tau}\left(\frac{(A-1)\Delta}{r-\mu}X(\tau) + C\Delta\int_0^\tau \mathrm{e}^{r(\tau-s)}X(s)\mathrm{d}s\right)]\\
=&V_{C,0}(x,y) + \sup_{\tau\in\mathcal{T}} \mathbb{E}^1[\mathrm{e}^{-r\tau}\left(\frac{(A-1)\Delta}{r-\mu}X(\tau)\right) + C\Delta\left(\int_0^\infty \mathrm{e}^{-rs}X(s)\mathrm{d}s - \int_\tau^\infty \mathrm{e}^{-rs}X(s)\mathrm{d}s\right)]\\
=&V_{C,0}(x,y) + \frac{C\Delta}{r-\mu} + \frac{(A-1-C)\Delta}{r-\mu}\underbrace{\sup_{\tau\in\mathcal{T}} \mathbb{E}^1[\mathrm{e}^{-r\tau}X(\tau)]}_{=1} \qquad (22)\\
=&V_{C,0}(x,y) + \frac{(A-1)\Delta}{r-\mu},
\end{aligned}$$

where in (22) $\sup_{\tau\in\mathcal{T}} \mathbb{E}^1[\mathrm{e}^{-r\tau}X(\tau)] = 1$ follows from $\mu < r$.

Next, we show (13). For any positive constant Δ, we have

$$\begin{aligned}
&V_{C,0}(x, y+\Delta)\\
=&\sup_{\tau\in\mathcal{T}} \mathbb{E}^x[\mathrm{e}^{-r\tau}\left(\frac{(A-1)}{r-\mu}X(\tau) - I - C\max(I - \mathrm{e}^{r\tau}(y+\Delta - \int_0^\tau \mathrm{e}^{r(\tau-s)}X(s)\mathrm{d}s, 0)\right)]\\
\leq&\sup_{\tau\in\mathcal{T}} \mathbb{E}^x[\mathrm{e}^{-r\tau}\left(\frac{(A-1)}{r-\mu}X(\tau) - I - C\max(I - \mathrm{e}^{r\tau}y - \int_0^\tau \mathrm{e}^{r(\tau-s)}X(s)\mathrm{d}s, 0) + \mathrm{e}^{r\tau}C\Delta\right)]\\
=&\underbrace{\sup_{\tau\in\mathcal{T}} \mathbb{E}^x[\mathrm{e}^{-r\tau}\left(\frac{(A-1)}{r-\mu}X(\tau) - I - C\max(I - \mathrm{e}^{r\tau}y - \int_0^\tau \mathrm{e}^{r(\tau-s)}X(s)\mathrm{d}s, 0)\right)]}_{=V_{C,0}(x,y)} + C\Delta\\
=&V_{C,0}(x,y) + C\Delta.
\end{aligned}$$

C Proof of Proposition 2

First, consider the case of $A - C - 1 > 0$. Fix $(x, y) \in T_{C,0}$ and (x', y') satisfying $x \leq x'$ and $y \leq y' < I$. Using Lemma 1, we have

$$\begin{aligned} V_{C,0}(x', y') &= V_{C,0}(x', y') - V_{C,0}(x, y') + V_{C,0}(x, y') - V_{C,0}(x, y) + V_{C,0}(x, y) \\ &\leq \frac{(A-1)(x'-x)}{r-\mu} + C(y'-y) + V_{C,0}(x, y) \\ &= \frac{(A-1)(x'-x)}{r-\mu} + C(y'-y) + \frac{(A-1)x}{r-\mu} - I - C(I-y) \\ &= \frac{(A-1)x'}{r-\mu} - I - C(I-y'), \end{aligned}$$

where the last inequality implies $(x', y') \in T_{C,0}$. This proves that $T_{C,0}$ is expressed as (14) with the decreasing function $x^*_{C,0}(\cdot)$. By Proposition 1, we immediately obtain inequality (15).

Next, we will show (16). Clearly we have $\lim_{y\uparrow I} x^*_{C,0}(y) \geq x^*_{0,0}$. Denote the payoff function of problem (7) as

$$f(x, y) := \frac{(A-1)x}{r-\mu} - I - C(I-y). \tag{23}$$

We have

$$\begin{aligned} &\mathcal{L}f(x, y) - rf(x, y) \leq 0 \\ \Leftrightarrow &\frac{(A-1)\mu x}{r-\mu} + C(x + ry) - r\left(\frac{(A-1)x}{r-\mu} - I - C(I-y)\right) \leq 0 \\ \Leftrightarrow &x \geq \frac{(C+1)rI}{A-C-1}, \end{aligned} \tag{24}$$

where $\mathcal{L}$ denotes the generating operator of $(X(t), Y(t))$. Since the general theory of optimal stopping ensures $\mathcal{L}V_{C,0}(x, y) - rV_{C,0}(x, y) \leq 0$ (refer to Peskir and Shiryaev (2006)), by (24) $f(x, y)$ is not equal to $V_{C,0}(x, y)$ for $x < (C+1)rI/(A-C-1)$ and $y < I$. In other words, the option is not exercised in the region $\{(x, y) \in \mathbb{R}^2_+ \mid x < (C+1)rI/(A-C-1), y < I\}$. This proves that $\lim_{y\uparrow I} x^*_{C,0}(y) \geq (C+1)rI/(A-C-1)$.

[Insert Figure 4 about here.]

Now, suppose that $(C+1)rI/(A-C-1) \leq x^*_{0,0} < \lim_{y\uparrow I} x^*_{C,0}(y)$ (see Figure 4). We can lead to contradiction as follows. Consider problem (7) with a finite maturity T. Generally, the exercise region of an American option converges to the region $\mathcal{L}f - rf \leq 0$, where $\mathcal{L}$ is the generating operator and f is the payoff function, when the remaining life of the option goes to zero (refer to Detemple (2006)). Then, because of (24), the exercise region of problem (7) with a finite maturity T converges to $\{(x, y) \in \mathbb{R}^2_+ \mid x \geq (C+1)rI/(A-C-1)\}$ when $T \downarrow 0$. Consider the exercise region of problem (5) for a fixed x satisfying $x^*_{0,0} < x < \lim_{y\uparrow I} x^*_{C,0}(y)$ and $y \uparrow I$. Note that $\tau_I := \inf\{t \geq 0 \mid$

$X(t) \geq x^*_{0,0}, Y(t) \geq I\}$ converges to 0 as $y \uparrow I$. Accordingly, the exercise region of the problem (5) for the fixed x and $y \uparrow I$ converges to that of problem (7) with $T \downarrow 0$. This implies that $\lim_{y\uparrow I} x^*_{C,0}(y) = (C+1)rI/(A-C-1)$, which contradicts the assumption of $(C+1)rI/(A-C-1) < \lim_{y\uparrow I} x^*_{C,0}(y)$. Similarly we can lead to contradiction if $x^*_{0,0} < (C+1)rI/(A-C-1) < \lim_{y\uparrow I} x^*_{C,0}(y)$ is supposed. Thus, we complete the proof of (16).

We can show the continuity of $x^*_{C,0}(\cdot)$ as follows. By Lemma 1 we have the continuity of $V_{C,0}(x,y)$. Since $V_{C,0}(x,y)$ and $(A-1)x/r-\mu-I-C\max(I-y,0)$ are both continuous, $S_{C,0}$ is a closed set. Then, we have $\lim_{\epsilon\downarrow 0}(x^*_{C,0}(y+\epsilon), y+\epsilon) \in S_{C,0}$, which leads to $\lim_{\epsilon\downarrow 0} x^*_{C,0}(y+\epsilon) \geq x^*_{C,0}(y)$. We have $\lim_{\epsilon\downarrow 0} x^*_{C,0}(y+\epsilon) \leq x^*_{C,0}(y)$ because $x^*_{C,0}(\cdot)$ is decreasing. Thus, we obtain the right-continuity of $x^*_{C,0}(\cdot)$. Now, suppose that there exists $y(<I)$ satisfying $x^*_{C,0}(y) < \lim_{\epsilon\downarrow 0} x^*_{C,0}(y-\epsilon)$. We can lead to contradiction as the same method as the proof of (16). Consider the exercise region of problem (5) for a fixed x satisfying $x^*_{C,0}(y) < x < \lim_{\epsilon\downarrow 0} x^*_{C,0}(y-\epsilon)$ and $y-\epsilon$. Note that $\inf\{t \geq 0 \mid X(t) \geq x^*_{C,0}(Y(t))\}$ converges to 0 as $\epsilon \downarrow 0$. Then, the exercise region converges to that of problem (7) with $T \downarrow 0$. This implies that $\lim_{\epsilon\downarrow 0} x^*_{C,0}(y-\epsilon) = (C+1)rI/(A-C-1)$, which contradicts $(C+1)rI/(A-C-1) \leq x^*_{C,0}(y) < \lim_{\epsilon\downarrow 0} x^*_{C,0}(y-\epsilon)$. Thus, we obtain the left-continuity of $x^*_{C,0}(\cdot)$.

Lastly, consider the case of $A-C-1 \leq 0$. In this case, we have for any $(x,y) \in \mathbb{R}^2_+$

$$\mathcal{L}f(x,y) - rf(x,y) = -(A-C-1)x + (C+1)rI > 0,$$

where $\mathcal{L}$ is the generating operator of $(X(t), Y(t))$ and $f(x,y)$ is defined by (23). Since $\mathcal{L}V_{C,0}(x,y) - rV_{C,0}(x,y) \leq 0$ holds by the general theory of optimal stopping, $f(x,y)$ does not agree with $V_{C,0}(x,y)$. This implies that $T_{C,0} = \emptyset$ and $S_{C,0} = S_I$.

D Proof of Proposition 3

By $0 \leq K1_{\{Y(t)<I\}} \leq K$ we have $V_{C,0}(x,y) - K \leq V_{C,K}(x,y) \leq V_{C,0}(x,y)$. Suppose that $A-C-1 \leq 0$. It follows from Proposition 2 and the optimality of $V_{C,K}(x,y)$ that

$$V_{C,0}(x,y) = \mathbb{E}^{x,y}[\mathrm{e}^{-r\tau_I}\left(\frac{A-1}{r-\mu}X(\tau_I) - I\right)] \tag{25}$$

$$\leq V_{C,K}(x,y), \tag{26}$$

where τ_I denotes the first hitting time to $S_I = \{(x,y) \in \mathbb{R}^2_+ \mid x \geq x^*_{0,0}, y \geq I\}$. Then, we obtain $V_{C,0}(x,y) = V_{C,K}(x,y)$. Clearly we have $V_{C,K}(x,y) = V_{0,0}(x,y)$ for $y \geq I$.

E Proof of Proposition 4

First, suppose that $A - C - K/I - 1 > 0$. By Proposition 3 we have for any $(x, y) \in T_{C,K}$

$$\begin{aligned} V_{C,0}(x,y) &\leq V_{C,K}(x,y) + K \\ &= \frac{(A-1)x}{r-\mu} - I - C(I-y) - K + K \\ &= \frac{(A-1)x}{r-\mu} - I - C(I-y), \end{aligned}$$

where the last inequality implies $(x, y) \in T_{C,0}$. This proves that $T_{C,K} \subset T_{C,0}$.

Fix any $(x, y) \in \mathbb{R}_+^2$ satisfying $I - K/(A - C - 1) \leq y < I$. Consider the first hitting time to S_I, denoted by τ_I. Since $Y(t)$ monotonically increases from the initial point $Y(0) = y$, we have

$$\begin{aligned} V_{C,K}(x,y) &\leq \underbrace{\sup_{\tau\in\mathcal{T}} \mathbb{E}^{x,y}[\mathrm{e}^{-r\tau}\left(\frac{A-1}{r-\mu}X(\tau) - I - \left(C + \frac{K}{I-y}\right)\max(I - Y(\tau), 0)\right)]}_{=V_{C+K/(I-y),0}(x,y)} \\ &= \mathbb{E}^{x,y}[\mathrm{e}^{-r\tau_I}\left(\frac{A-1}{r-\mu}X(\tau_I) - I\right)] \qquad (27) \\ &\leq V_{C,K}(x,y), \end{aligned}$$

where by Proposition 2 we have (27) because of $A - C - K/(I - y) - 1 \leq 0$. The last inequality implies that $(x, y) \notin T_{C,K}$, which leads to $T_{C,K} \subset \mathbb{R}_+ \times [0, I - K/(A - C - 1))$. This completes the proof of (19).

Next, fix any $(x, y) \in \mathbb{R}_+^2$ satisfying $x \geq x^*_{C+K/(I-y),0}(y)$ and $y < I - K/(A - 1 - C)$. Since $Y(t)$ monotonically increases from the initial point $Y(0) = y$, we have

$$\begin{aligned} V_{C,K}(x,y) &\leq \underbrace{\sup_{\tau\in\mathcal{T}} \mathbb{E}^{x,y}[\mathrm{e}^{-r\tau}\left(\frac{A-1}{r-\mu}X(\tau) - I - \left(C + \frac{K}{I-y}\right)\max(I - Y(\tau), 0)\right)]}_{=V_{C+K/(I-y),0}(x,y)}, \\ &= \frac{(A-1)x}{r-\mu} - I - \left(C + \frac{K}{I-y}\right)(I-y) \\ &= \frac{(A-1)x}{r-\mu} - I - C(I-y) - K, \end{aligned}$$

where the last inequality implies $(x, y) \in T_{C,K}$, and, therefore, we obtain (20).

Lastly, consider the case of $A - C - K/I - 1 \leq 0$. We use the same technique as the proof of (19). Fix any $(x, y) \in \mathbb{R}_+^2$ satisfying $y < I$ and denote by τ_I the first hitting time to S_I. Because $Y(t) \geq 0$ holds for any time t, we have

$$\begin{aligned} V_{C,K}(x,y) \leq& \underbrace{\sup_{\tau\in\mathcal{T}} \mathbb{E}^{x,y}[\mathrm{e}^{-r\tau}\left(\frac{A-1}{r-\mu}X(\tau) - I - \left(C + \frac{K}{I}\right)\max(I - Y(\tau), 0)\right)]}_{=V_{C+K/I,0}(x,y)} \\ =&\mathbb{E}^{x,y}[\mathrm{e}^{-r\tau_I}\left(\frac{A-1}{r-\mu}X(\tau_I) - I\right)] \qquad (28) \\ \leq& V_{C,K}(x,y), \end{aligned}$$

where by Proposition 2 we have (28) because of $A - C - K/I - 1 \leq 0$. The last inequality proves that $(x, y) \notin T_{C,K}$, and, then, we have $T_{C,K} = \emptyset$ and $S_{C,K} = S_I$.

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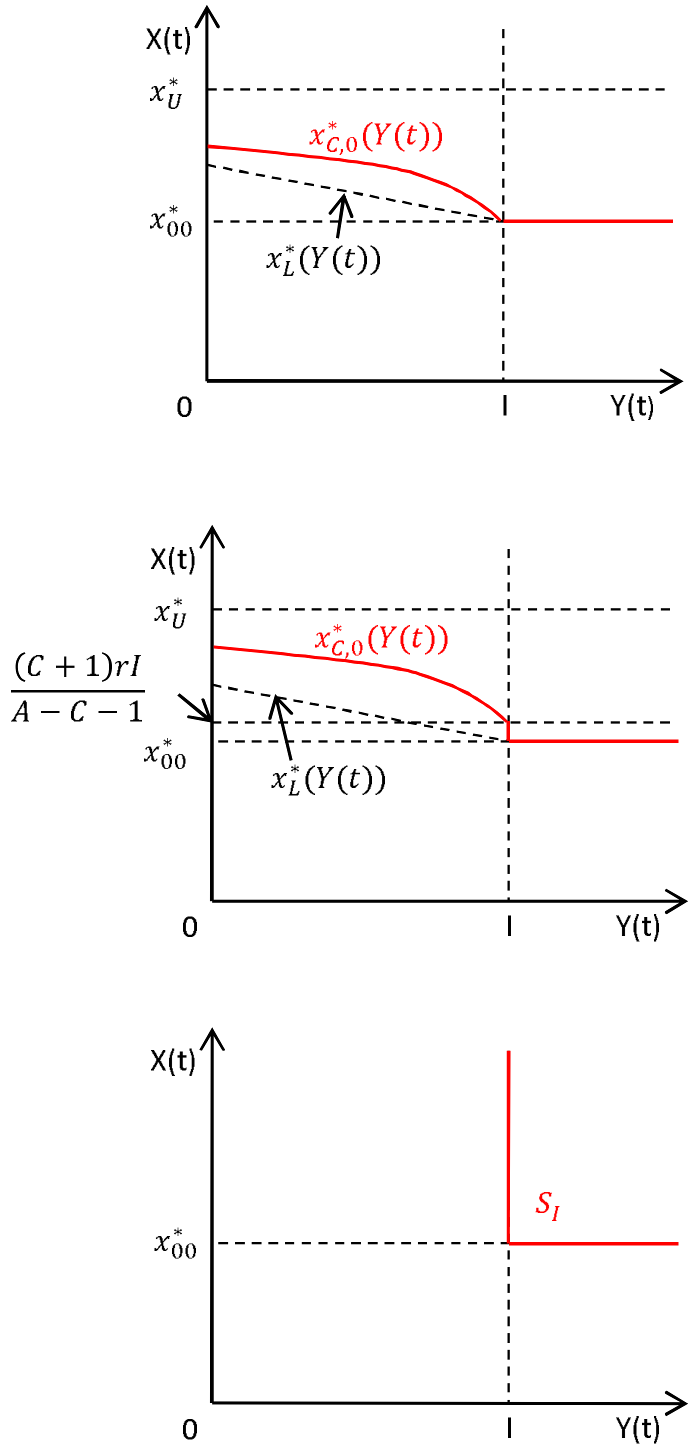

Figure 1: The exercise region in the case with only a proportional cost. The upper panel illustrates Case (a) satisfying $x_{0,0}^* \geq (C+1)rI/(A-C-1)$. The middle panel illustrates Case (a) satisfying $x_{0,0}^* < (C+1)rI/(A-C-1)$. In this case, there is a gap between $\lim_{y \uparrow I} x_L^*(y)$ and $x_{0,0}^*$. The lower panel illustrates Case (b).

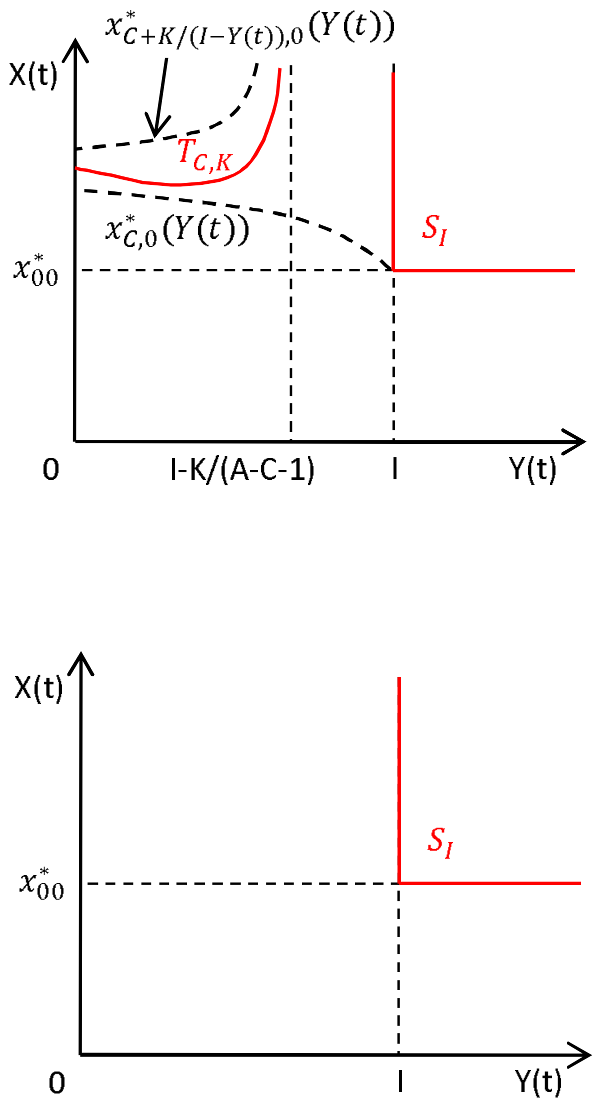

Figure 2: The exercise region in the case with both fixed and proportional costs. The upper panel illustrates Case (a-K). The lower panel illustrates Case (b-K).

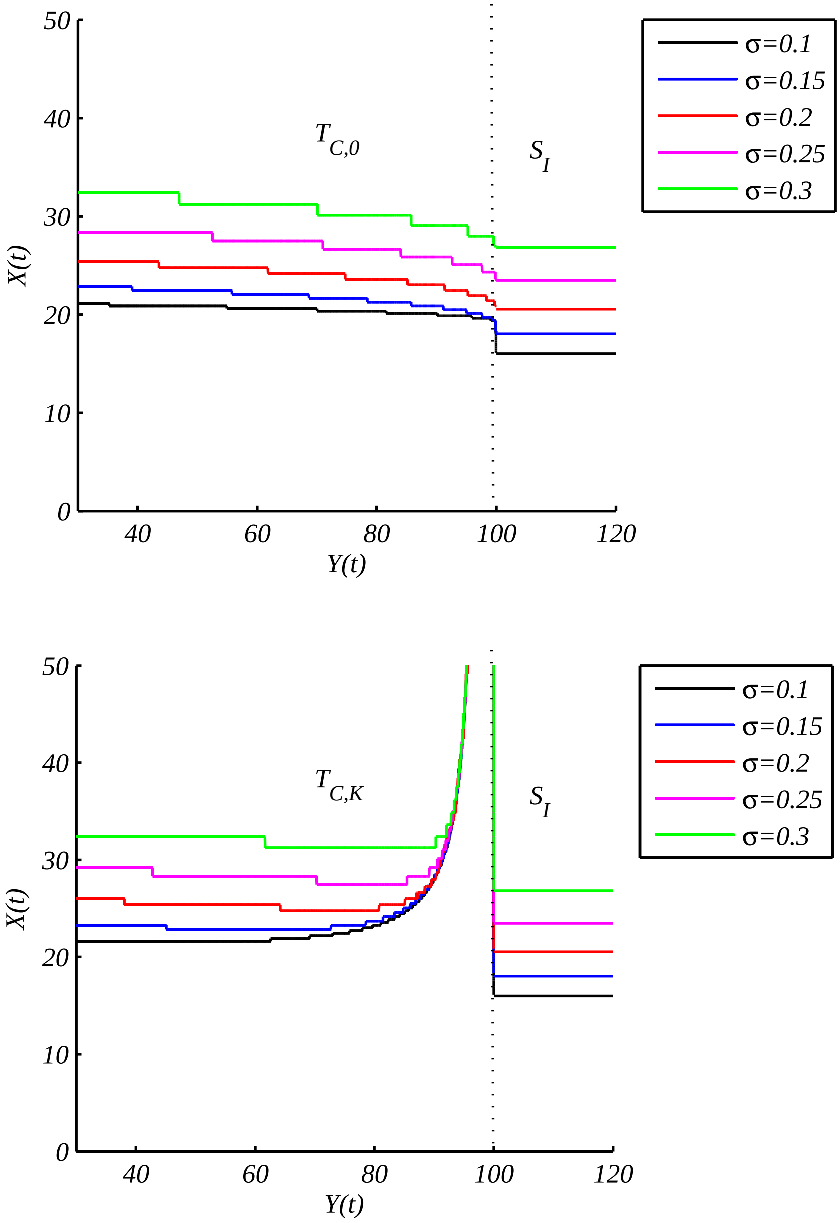

Figure 3: The comparative statics with respect to the volatility σ. The upper and lower panels illustrate the cases with only a proportional cost and both fixed and proportional costs, respectively. The parameter values other than σ are set at the base case (21).

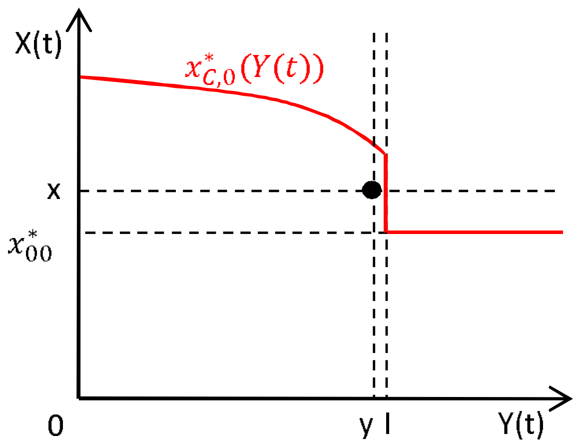

Figure 4: The assumption of $x^*_{0,0} < \lim_{y\uparrow I} x^*_{C,0}(y)$. The dot represents the initial point (x, y) satisfying $x^*_{0,0} < x < \lim_{y\uparrow I} x^*_{C,0}(y)$ and $y \approx I$.