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Design of a Social Security System: Pension System vs. Unemployment Insurance

Yusuke Kinai

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GCOE Secretariat
Graduate School of Economics
OSAKA UNIVERSITY
1-7 Machikaneyama, Toyonaka, Osaka, 560-0043, Japan

Design of a Social Security System: Pension System vs. Unemployment Insurance*

Yusuke KINAI†

Graduate School of Economics, Osaka University

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Abstract

This paper presents consideration of how the social security system evolves as the attributes of voters change. In our setting, policy determination is based on majority voting. The government has two components of social security policy: a pension system and unemployment insurance. When workers constitute most voters, the pension system is supported and when unemployed people are the majority, unemployment insurance is adopted. Under this setting, employing the concept of structure-induced equilibrium developed by Shepsle (1979), the present paper describes how the contents of the social security system evolve depending on the dynamics of capital accumulation and the unemployment rate, and demonstrates the possibility that one or the other social security system ceases to exist in certain instances.

Keywords: Social Security, Pension System vs. Unemployment Insurance, Majority Voting, Structure-induced equilibrium.

JEL Classification: E61, H53, H55.

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†Correspondence to: E-mail: ykinai@js8.so-net.ne.jp

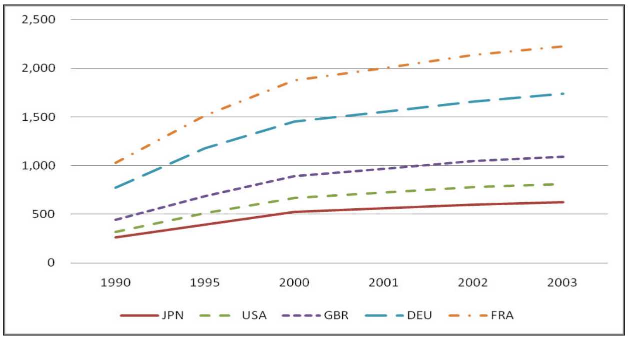

Figure 1 Expenditures on Pensions in Selected Countries: 1990–2005 (source: OECD Social Expenditure Database)

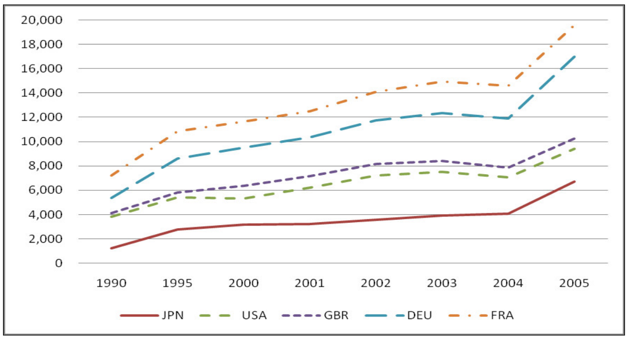

Figure 2 Recipients of Unemployment Insurance in Selected Countries: 1990–2005 (source: OECD Social Expenditure Database)

1 Introduction

The social security system includes medical care, public sanitation, social insurance (pension insurance, medical insurance, at-home care insurance, unemployment insurance, and workmen's compensation insurance) and other services and schemes. Among them, pensions and unemployment insurance are of particular importance. In Japan and some other countries, the increase in expenditures for social security are increasing as the population ages and as fewer babies are born (See Fig. 1.). On the other hand, from Fig. 2, it is apparent that the necessity for subsidies for unemployment (so-called "*NEET*", or "*Working poor*" in Japan) has also increased remarkably, and the expenditure in such a situation (i.e. unemployment insurance) is overwhelmingly large. Considering that the contributions to both pension and unemployment insurance are increasing, there might emerge a situation in which the government must prioritize either the pension system or policies for employment, although the government should carry out both policies.

The study examines the sustainability of social security systems. Although this is not an issue that remains confined to social security policy, the sustainability of economic policy is mainly based on political factors. In that regard, the determination of policy is based on the legislature (in Japan's case, the Diet) in developed countries, which are representative democracies. Given such a situation, what is important is consideration of the relative amounts of power of all voters when we analyze an economic policy in relation to a political issue. This paper is intended to model such a situation and to show how the scheme of social security varies as time passes from the viewpoint of political economy.

A sketch of our model is the following: First, households of two kinds are included in the model, including workers and unemployed people. The former hopes for a pay-as-you-go (PAYG) type pension system; the latter hopes for unemployment insurance. In that regard, decisions are based on majority voting. As time passes, the relative numbers of workers and unemployed people vary. For that reason, the contents of social security also vary. At this stage, its choice affects social welfare. In our setting, policy determination is based on majority voting and the government has social security policies of two kinds: a pension system and unemployment insurance. When workers constitute most voters, the pension system is supported. When unemployed people are the majority, unemployment insurance is adopted. Under such a situation, we show how the contents of the social security system evolve depending on the dynamics of capital accumulation and the unemployment rate, and show that the social security system ceases to exist in certain instances.

■Relation with the Literature Here, let us describe the relation of this paper with past studies in the following two respects. Some studies have specifically examined unemployment using an overlapping generations (hereinafter, OLG) model. Roughly speaking, two directions exist. One is a search model; the other incorporates trade unions[1)]. This paper takes the latter stance. Since Demmel and Keuschnigg (2000) and Corneo and Marquardt (2000), which were the first works to model the behavior of a trade union, some studies have modeled trade unions. For instance, Imoto (2003) extends the model of Corneo and Marquardt (2000)[2)] and shows that the existence of a trade union might cause a business cycle depending on the value of substitution between the volume of employment and the wage rate[3)]. Kaas and Thadden (2004) propose the other type of wage setting by a trade union in a similar model, and Ono (2007) specifically examines the interaction between pension and unemployment insurance and derives the unemployment dynamics dependent on social security policy. Bräuninger (2005) shows that the unemployment rate is constant under the assumption of an endogenous growth model and wage determination through Nash bargaining. What is common to those studies is that the kind of social security system is exogenous. To be more precise, how the policy is chosen is not considered. This paper extends these studies to endogenize the choice of social security system by introducing voting behavior.

On the other hand, since the seminal papers of Meltzer and Richard (1981) or Hu (1982), many studies have specifically examined the social security system in an OLG model in the context of political economy. These studies typically examine specifically how the ratio of voters varies as time passes and show how the social security system is altered. Recently, Hassler, Mora, Storesletten and Zilibotti (2003) and Conde-Ruiz and Galasso (2005) investigate how the contents of the social security system change depending on changes of the wealth distribution. Unlike these studies, we consider two combinations of redistribution schemes, pensions, and other redistribution policies as intergenerational and intragenerational redistributive schemes, respectively. This paper specifically examines unemployment insurance in the role of intragenerational policy.

To summarize, our paper specifically differs from the papers described above in the following two

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1) See also Galor and Lach (1990) or Bean and Pissarides (1993) introduces a search model into an OLG model.

2) More precisely, Imoto extends the objective function of the trade union in the model of Corneo and Marquardt into the CES-type function.

3) Coimbra, Lloyd-Braga and Modesto (2005) also implicitly derives the unemployment dynamics under the different model from Corneo and Marquardt (2000). The difference from those studies is the objective function of the trade union.

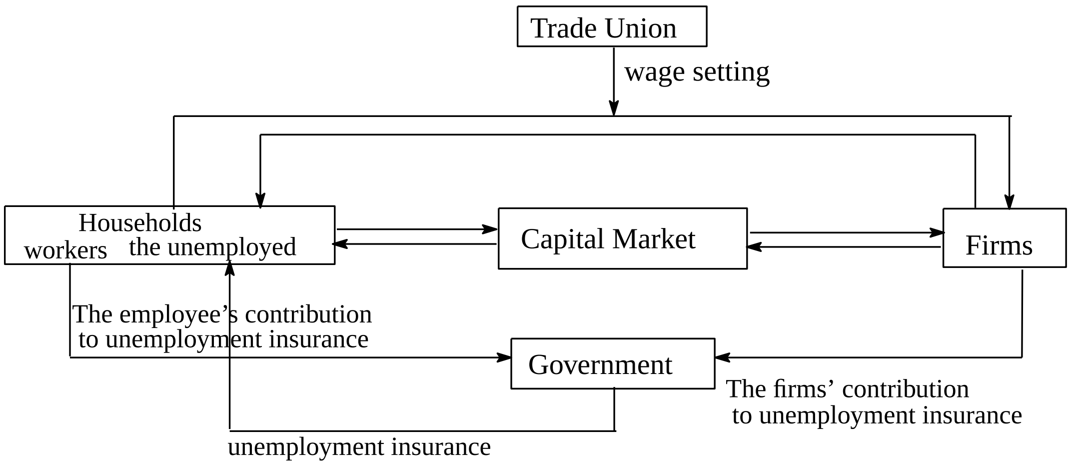


Figure 3 Structure of the model

Note: This figure depicts the intratemporal flow of goods, not intertemporal.

respects. First, unlike past studies under the first part, in our model, the policy determination is endogenized by introduction of the voting model. In that regard, we specifically examine the notion of issue-by-issue voting as a mode of policy determination. Second, we specifically examine unemployment insurance as an intragenerational redistribution scheme, which differs from the second part.

The remainder of this paper is organized as follows: Section 2 sets up the model. We investigate the dynamics of this economy in section 3. In section 4 section, we show how the contents of the social security system changes and show the possibility of annihilation of social security. Section 5 presents the conclusion.

2 The Model

We consider an infinitely-lived economy comprising households, firms, trade unions, and a government. Although our model is fundamentally similar to that of Kaas and Thadden (2004), our model differs from theirs in the following two respects. The first point is wage determination through bargaining other than Nash bargaining. The second point is that the determinant of the social security system is specifically examined in our model. Households exist for two periods: young and old periods. The population growth rate is μ, i.e., $N_{t+1} = (1+\mu)N_t$. To avoid complication, the model structure is depicted in Fig. 3, which summarizes the intratemporal, not intertemporal, flow of goods.

2.1 Behavior of Each Agent

2.1.1 Households

Households exist for two periods in a closed economy without a bequest motive. Dynasties derive utility from consumption in young and old periods. For simplicity, the preferences of the dynasty's cohort that survives at t period is described by the following additively separable function:

$$\begin{aligned}\max_{c_t^y, c_{t+1}^o} \quad U^i(\cdot) &= E_t[u(c_t^{yi}) + \frac{1}{1+\rho}u(c_{t+1}^{oi})] \\ &= l_t[u(c_t^{ye}) + \frac{1}{1+\rho}u(c_{t+1}^{oe})] + (1-l_t)[u(c_t^{yu}) + \frac{1}{1+\rho}u(c_{t+1}^{ou})].\end{aligned} \tag{1}$$

Therein, ρ denotes the discount factor, and $i = \{e, u\}$. Furthermore, c_t^{yi} and c_t^{oi} respectively denote consumption during young and old periods. Two subscripts "e" and "u" respectively denote employed people and unemployed people. We then specify the utility function as $u(\cdot) = \ln c$.

In Ono (2007), he assumes that whether households include workers or retired people is determined at birth, whereas we assume that households can be either workers or retired people at birth. Therefore, l_t denotes the employment rate defined by $l_t \equiv \frac{L_t}{N_t}$, which can be interpreted as the probability of obtaining jobs during the youth period.

The budget constraints of workers and unemployed people are shown, respectively, as follows.

- Case of being Employed:

  When people are young, they work and divide after-tax labor-income into savings, contributions to pensions, and consumption. When they are old, they consume savings and pension payments.

$$c_t^{ye} + s_t^e = (1 - \tau_t - \theta_{wt})w_t, \quad c_{t+1}^{oe} = R_{t+1}s_t^e + d_{t+1}^e, \tag{2}$$

  In those equations, τ and θ_w respectively signify the contributions to the pension and unemployment insurance. In addition, s_t and d_{t+1} respectively stand for savings and pension payments that old people receive. Furthermore, w_t and R_{t+1} denote the wage rate and the rental rate of capital stock. Maximization of the utility function of employed people under the constraint, eq. (2), yields

$$c_t^{ye} = \frac{1+\rho}{2+\rho}\left\{(1-\theta_{wt}-\tau_t)w_t + \frac{d_{t+1}^e}{R_{t+1}}\right\}, \tag{3a}$$

$$c_{t+1}^{oe} = \frac{1}{2+\rho}\left\{(1-\theta_{wt}-\tau_t)w_t + \frac{d_{t+1}^e}{R_{t+1}}\right\}, \tag{3b}$$

$$s_t^e = \frac{1}{2+\rho}\left\{(1-\theta_{wt}-\tau_t)w_t - (1+\rho)\frac{d_{t+1}^e}{R_{t+1}}\right\}. \tag{3c}$$

- Case of being unemployed:

  When they are young, people receive unemployment insurance (b_t) and divide it into savings, contributions to pensions (d_t^u), and consumption. When they are old, they consume savings and pension payments as

$$c_t^{yu} + s_t^u = (1-\tau_t)b_t, \quad c_{t+1}^{ou} = R_{t+1}s_t^u + d_{t+1}^u. \tag{4}$$

Maximization of the utility function of unemployed people under the constraint, eq. (4), yields

$$c_t^{yu} = \frac{1+\rho}{2+\rho}\left\{(1-\tau_t)b_t + \frac{d_{t+1}^u}{R_{t+1}}\right\}, \tag{5a}$$

$$c_{t+1}^{ou} = \frac{1}{2+\rho}\left\{(1-\tau_t)b_t + \frac{d_{t+1}^u}{R_{t+1}}\right\}, \tag{5b}$$

$$s_t^u = \frac{1}{2+\rho}\left\{(1-\tau_t)b_t - (1+\rho)\frac{d_{t+1}^u}{R_{t+1}}\right\}. \tag{5c}$$

Here, we assume the following.

A1. $d_t^e > d_t^u$ eliminates the trivial case in which all households choose unemployment.

A2. $R_t > 1+\mu$ means that the economy is *dynamically efficient*.

2.1.2 Firms

We assume that factor markets are perfectly competitive and that firms maximize their profits. Labor and capital stock are used for production; production technology is assumed to be neoclassical product function with constant returns to scale, $Y_t = F(K_t, L_t)$, where Y_t, K_t and L_t respectively represent output in aggregate terms, capital stock and the number of the workers at t period[4]. The firms' profit maximization problem is written as

$$\Pi_t = F(K_t, L_t) - R_t K_t - (1+\theta_{ft})w_t L_t,$$

where θ_{ft} represents the contributions to unemployment insurance and pensions which the firms bear. We then specify the production function as Cobb–Douglas type, $F(K_t, L_t) = K_t^{\alpha} L_t^{1-\alpha}$. Then, FOCs are derived as

$$\frac{\partial \Pi_t}{\partial K_t} = 0 \iff R_t = \alpha K_t^{\alpha-1} L_t^{1-\alpha}, \tag{6a}$$

$$\frac{\partial \Pi_t}{\partial L_t} = 0 \iff (1+\theta_{wt})w_t = (1-\alpha)K_t^{\alpha} L_t^{-\alpha}. \tag{6b}$$

Defining $\hat{k}_t \equiv \frac{K_t}{N_t} = \frac{K_t}{L_t}\frac{L_t}{N_t} = k_t l_t$, we obtain the following.

$$R_t = \alpha \hat{k}_t^{\alpha-1} l_t^{1-\alpha}, \tag{7a}$$

$$w_t = \frac{(1-\alpha)\hat{k}_t^{\alpha} l_t^{-\alpha}}{1+\theta_{wt}} \tag{7b}$$

If $L_t = N_t$ (perfect employment), then $l_t = 1$; the wage rate is derived as

$$(1+\theta_{wt})\bar{w}_t = (1-\alpha)\hat{k}_t^{\alpha}, \tag{8}$$

where $\bar{w}_t$ denotes the wage rate at perfect employment.

[4] Although Ono (2007) adopts the Grossman and Yanagawa (1993) type production function, we adopt the neoclassical growth model because our concern is not directed to the effect on growth rate.

2.1.3 Trade Union

Following Corneo and Marquardt (2000) or Ono (2007), the wage is determined by the monopolistic trade union. The trade union strives to maintain both high wages and low unemployment rates simultaneously. Following Imoto (2003), who extends the model of Corneo and Marquardt, the problem of the trade union is:

$$w_t \equiv \arg\max_{w_t} W(\cdot) \equiv \left[\gamma(w_t - \bar{w_t})^{-\sigma} + (1-\gamma)(l_t)^{-\sigma}\right]^{-\frac{1}{\sigma}}, \ \ \sigma \in (-1, \infty), \text{ and } \ \gamma \in (0,1), \tag{9}$$

under the constraint of eq. (7b). Two parameters α and γ are exogenous parameters. The type of bargaining is "Right-to-Management": firms accept the wage requested by the trade union and then decide the amount of employment to maximize their profit. On the other hand, the trade union requests the wage based on that condition to maximize its objective function, eq. (9). The trade union decides the wage rate to maximize their objective function for given $\bar{w_t}$ treated as the reference wage rate. The first order condition for this problem is written as

$$\frac{\partial W(\cdot)}{\partial w_t} = 0 \iff -\gamma\sigma(w_t - \bar{w_t})^{-\sigma-1} + \frac{\sigma(1-\gamma)}{\alpha}\left(\left(\frac{1-\alpha}{1+\theta_f}\right)^{\frac{1}{\alpha}}\hat{k}_t\right)^{-\sigma} w_t^{\frac{\sigma}{\alpha}-1} = 0. \tag{10}$$

Denoting $L \equiv \frac{\alpha\gamma}{V_1}(w_t - \bar{w_t})^{-(1+\sigma)}$ and $R \equiv w_t^{\frac{\sigma}{\alpha}-1}$, where $V_1 = (1-\gamma)\left(\left(\frac{1-\alpha}{1+\theta_f}\right)^{\frac{1}{\alpha}}\hat{k}_t\right)^{-\sigma}$. Determination of the wage rate w_t is shown in Fig. 4.

In what follows, we assume that the following equation (second order conditions) holds.

$$\frac{\partial^2 W(\cdot)}{\partial w_t^2} < 0$$

2.1.4 The Government

Finally, let us describe government behavior. It has redistribution schemes of two kinds: a PAYG-type pension system (intergenerational redistribution scheme) and unemployment insurance (intragenerational redistribution scheme). The budget constraint under each scheme is balanced and written as follows.

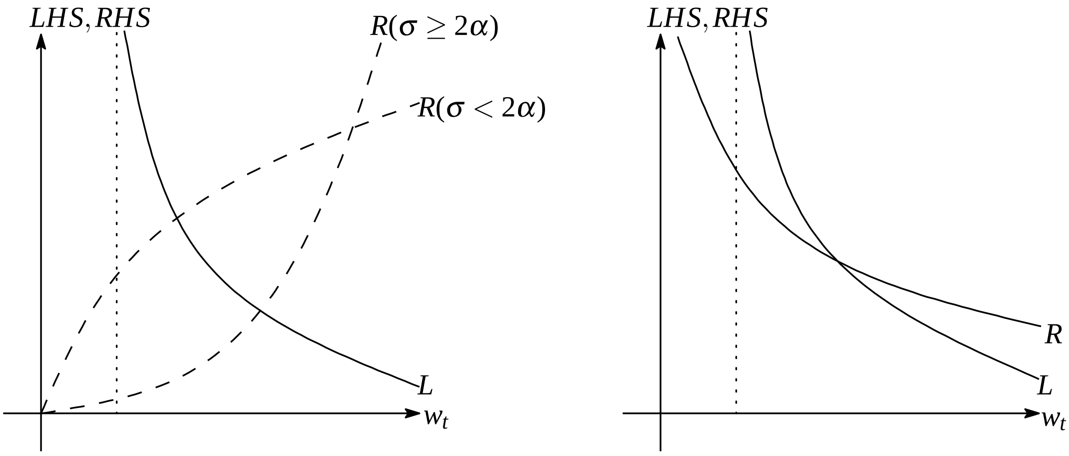
- PAYG-type pension system (intergenerational redistribution scheme)

  In the aggregate, the budget constraint under this scheme is written as

  $$L_t \times d_{t+1}^e + (N_t - L_t) \times d_{t+1}^u = \tau_{t+1} L_{t+1} w_{t+1} + \tau_{t+1}(N_{t+1} - L_{t+1}) b_{t+1},$$

  where b_t denotes the benefit from unemployment insurance. The left side term of the above equation denotes the pension entitlement. The first and second terms of the right side respectively denote contributions to pensions of workers, and unemployed people. Dividing both sides of the above equation with N_t yields

  $$l_t d_{t+1}^e + (1-l_t) d_{t+1}^u = (1+\mu)\{\tau_{t+1} w_{t+1} l_{t+1} + \tau_{t+1} b_{t+1}(1-l_{t+1})\}. \tag{11}$$



(a) $-1 < \sigma < 0$

(b) $0 < \sigma < \infty$

Figure 4 Wage Determination through Trade Union

- Unemployment insurance (intragenerational redistribution scheme)
  Similarly, in the aggregate, the budget constraint is written as

$$(N_t - L_t)b_t = \theta_{wt}L_t w_t + \theta_{ft}L_t w_t$$

.

The left side of the equation presented above signifies the entitlement of unemployment insurance; the first and second terms of the right side respectively denote contributions to unemployment insurance of workers and firms. Dividing both sides of the above equation with N_t yields

$$b_t = \frac{l_t}{1 - l_t}(\theta_{wt} + \theta_{ft})w_t. \tag{12}$$

Which policy is chosen is dependent on the voting constituency: if young unemployed people are in the majority, then unemployment insurance is supported. Old and young workers, if constituting the majority, will support the PAYG-type pension system. Therefore, we investigate the transitional change of voters in the next section.

2.2 Timing of Decision Making

Next, we summarize the sequence of decision-making (or political process). The decision-making sequence is also depicted in Fig. 5.

Stage 1. At the tth period, a new generation is born. Household members can be both employed and unemployed at this stage.

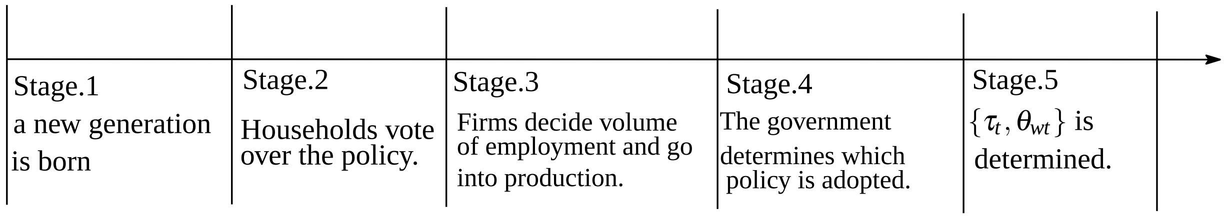


Figure 5 Decision-Making Sequence in the t-th period.

Stage 2. Households vote over the policy variables of social security system; the contribution to pension (τ_t) and unemployment insurance (θ_{wt}).

Stage 3. Then firms decide the volume of employment and go into production. At this stage, households are divided into two types, those with employed people and those with unemployed people.

Stage 4. The government determines which policy is adopted, that is, the contents of social security system is determined, based on the result of voting. In other words, the contribution to pension or unemployment insurance is also determined as $\{\tau_t, \theta_{wt}\}$. At the same time, θ_{ft} is also determined.

Stage 5. The $t+1$th generation is newly born.

2.3 Market Equilibrium

We finally formulate equilibrium conditions for each market.

- Commodity market
  In the aggregate, we can state this condition as $C_t^{ye} + C_t^{oe} + C_t^{yu} + C_t^{ou} + K_{t+1} = Y_t$, where C_t^i denotes the aggregate consumption of type i in t period. Dividing both sides of this equation with N_t yields

$$l_t c_t^{ye} + (1-l_t)c_t^{yu} + \frac{l_{t-1}}{1+\mu} c_t^{oe} + \frac{1-l_{t-1}}{1+\mu} c_t^{ou} + (1+\mu)k_{t+1} = y_t, \tag{13}$$

  where $y_t \equiv \frac{Y_t}{N_t}$.

- Capital market
  In the aggregate, we can state that $K_{t+1} = \varepsilon Y_t = L_t s_t^e + (N_t - L_t)s_t^u$, where ε is national savings

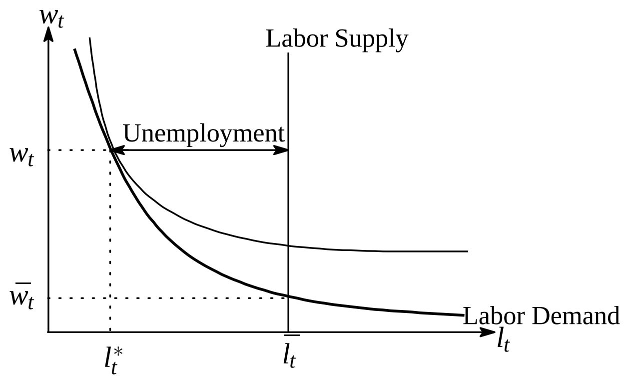


Figure 6

rate[5] We can write the following

$$(1+\mu)\hat{k}_{t+1} = l_t s_t^e + (1-l_t)s_t^u. \tag{14}$$

This equation determines the dynamics of capital accumulation in this economy.

- Labor market

  In this market, the demand for labor is expected to equal to the supply of labor. Therefore, combining the solution of eq. (6b) and that of eq. (10) yields the labor market equilibrium condition.

$$L_t = N_t l_t \tag{15}$$

The left and right side of the above equation respectively signify the labor demand and labor supply. Figure 6 depicts the situation of labor market equilibrium. In that figure, the heavy and middle lines respectively show the labor demand curve and the indifferent curve. The combination $(\bar{w_t}, \bar{l_t})$ denotes the wage and volume of employment at full employment. The direction between l_t^* (labor supply curve) and $\bar{l_t}$ denotes the amount of unemployment.

Finally, let us define the competitive equilibrium.

Defnition 1 ***(Competitive Equilibrium)*** *An economic equilibrium is a sequence* $\{c_t^i, c_{t+1}^i, s_t^i\}_{t=1}^{\infty}$, $i \in \{e,u\}$ *that accords with the following condition.*

(i) Given the sequence $\{\tau_t, \theta_{wt}, \theta_{ft}\}_{t=1}^{\infty}$, *each agent (employed or unemployed) determines policy variables that maximize their individual utility: the optimal policy variables meet the following*

[5] K_t is a constant that does not depend on policy variables of the social security system at t periods or $t+1$:

$$K_t = L_t s_{t-1}^e(\tau_{t-1}, \theta_{w,t-1}) + (N_t - L_t)s_{t-1}^u(\tau_{t-1}, \theta_{w,t-1}).$$

Therefore, we can treat the policy variables as given and treat the state variable as fixed. This point is related to the remark on an earlier page 16.

maximization problem:

$$\max\ \ln(c_t^{yi}) + \frac{1}{1+\rho}\ln(c_{t+1}^{oi}),\ \ i \in (e,u).$$

(ii) *Budget constraints of pension and unemployment insurance are balanced in each period.*

(iii) *Firms maximize their profit.*

(iv) *The condition for the trade union's wage setting holds.*

(v) *Finally, the commodity market clears, i.e. eq.* (13) *holds.*

3 Analysis

In this section, we consider the equilibrium dynamics treating the policy variable τ_t and θ_{wt} as constant. In other words, we assume that a government can commit to the policy once it is determined. The justification of this assumption is explained in the next section.

3.1 Dynamics of Capital Accumulation

From eqs. (3c), (5c), and (14), considering the case in which $\tau = \tau_t = \tau_{t+1}$ and $\theta_w = \theta_{wt} = \theta_{w,t+1}$, let us derive the dynamics of k_t as follows[6]:

$$\begin{aligned}\hat{k}_{t+1} &= \frac{1}{1+\mu}\{s_t^e \times l_t + s_t^u \times (1-l_t)\} \\ &= \frac{\alpha(1-(1+\theta_{wt})\tau_t + (1-\tau_t)\theta_{ft})(1+\theta_{ft})}{(1+\mu)(1+\theta_{ft})\}(2+\rho) + (1+\rho)(\tau_t(1+\theta_{wt}+\theta_{ft}))\}}\hat{k}_t^{\alpha} l_t^{1-\alpha}. \qquad (16)\end{aligned}$$

We can rewrite the equation above as $\hat{k}_{t+1} = \frac{1}{1+\mu}\varepsilon y_t$, and $\varepsilon \equiv \frac{\alpha(1-(1+\theta_{wt})\tau_t+(1-\tau_t)\theta_{ft})(1+\theta_{ft})}{(1+\mu)(1+\theta_{ft})\}(2+\rho)+(1+\rho)(\tau_t(1+\theta_{wt}+\theta_{ft}))\}}$ denotes the national saving rate. Here, subscript t is omitted. From this equation, we find that the increase in τ and θ_w causes decrease in the capital stock per capita at $t+1$ period because of the decrease in the saving rate.

3.2 Dynamics of the Employment Rate

Next, we must investigate the dynamics of the employment rate. From eqs. (10) and (16), we can derive the following equation:

$$l_t^{\sigma(1+\alpha)}(1-l_t^{\alpha})^{-(1+\sigma)} = \frac{1-\gamma}{\alpha\gamma}\left(\frac{1-\alpha}{1+\theta_f}\hat{k}_t^{\alpha}\right)^{\sigma}. \qquad (17)$$

This equation describes the relation $\hat{k}_t$ and l_t.

[6] For calculation details, see Appendix A.1.

Considering the labor and capital market equilibrium conditions in eqs. (15) and (14), the above equation can be rewritten as

$$\frac{l_{t+1}^{\frac{1+\alpha}{\alpha}}}{(1-l_{t+1}^{\alpha})^{\frac{1+\sigma}{\alpha\sigma}}} = \frac{\varepsilon}{1+\mu}\left(\frac{1-\gamma}{\alpha\gamma}\left(\frac{1-\alpha}{1+\theta_f}\right)^{\sigma}\right)^{\frac{1}{\alpha\sigma}(1-\alpha)}\frac{l_t^2}{(1-l_t^{\alpha})^{\frac{1+\sigma}{\sigma}}}. \tag{18}$$

This equation can be rewritten as

$$\frac{l_{t+1}^{1+\alpha}}{1-l_{t+1}^{\alpha}} = A^{\frac{1-\alpha}{\sigma}} l_t^{2\alpha} \tag{19}$$

where $A \equiv \frac{\varepsilon}{1+\mu}\left(\frac{1-\gamma}{\alpha\gamma}\left(\frac{1-\alpha}{1+\theta_f}\right)^{\sigma}\right)$.

Equation (19) implicitly shows the relation between l_{t+1} and l_t. From this equation, we can show the relation as

$$l_{t+1} = \phi(l_t), \tag{20}$$

which determines the dynamics of the employment rate. Depending on the value of σ, we have three cases. Figures 7(a)–7(c) depict the dynamic patterns of the employment rate. Unemployment characterizes this economy if the employment rate is less than half of the population.

Incidentally, we can derive the equilibrium at the steady state, which holds $\hat{k}_t = \hat{k}_{t+1} = \hat{k}^*$ or $l_t = l_{t+1} = l^*$. $\hat{k}^*$ and l^* meets the following equations:

$$\hat{k}^* = \frac{\alpha(1-(1+\theta_w)\tau+(1-\tau)\theta_f)(1+\theta_f)}{(1+\mu)(1+\theta_f)((2+\rho)+(1+\rho)(\tau(1+\theta_w+\theta_f)))}(\hat{k}^*)^{\alpha}(l^*)^{1-\alpha}, \tag{21a}$$

$$\frac{(l^*)^{1+\alpha}}{1-(l^*)^{\alpha}} = A^{\frac{1-\alpha}{\sigma}}(l^*)^{2\alpha}. \tag{21b}$$

We find that increases in τ and θ_w decrease the capital stock per capita at the steady state k^*. The increase in τ and θ_w decreases the employment rate at a steady state l^* if $\sigma \geq 0$. On the other hand, if $\sigma < 0$, then the increase in τ and θ_w increase l^*. In addition, the increase in μ engenders a similar result. Then, we establish the following proposition.

Proposition 1
The capital per capita at the steady state decreases if the contribution rate of pension and unemployment insurance which the worker bears and the population growth rate increase. Therefore, if the trade union prefers complementarity between wage rate and employment rate, then the employment rate decreases. On the other hand, under the opposite preferences, the employment rate increases.

Proof *We can derive the result presented above by differentiating eqs.* (21) *with respect to* τ *and* θ_w. ∎

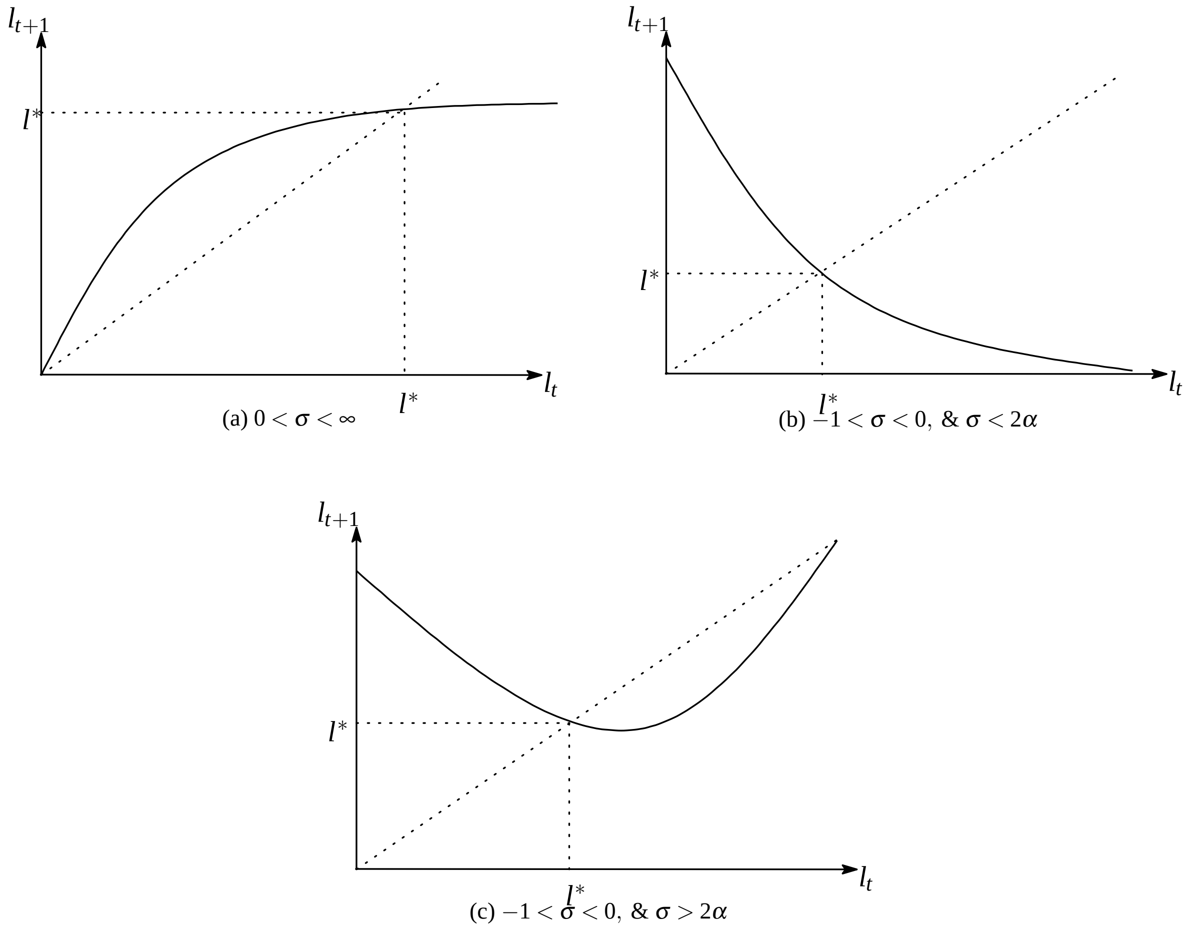


Figure 7　Dynamics of Employment Rate l_t under Commitment

4　Politico-Economic Equilibria

Given the discussion in the previous section, we advance the analysis by endogenizing policy choice. We then consider the voting behavior related to pension and unemployment insurance. We first assume that

A 1. Voting is held in each period, which means issue-by-issue voting under direct democracy.

A 2. Voting takes place *simultaneously* on contributions to pension and unemployment insurance.

A 3. Voters consist of young people (employed people and unemployed) and old people who are alive in the same period.

A 4. Policy determination is based on majority voting.

A 5. Voting is repeated among successive generations of voters.

4.1 Case with Commitment

To characterize the equilibria of this voting game, we first assume the government has commitment technology; that is, $\theta_{wt} = \theta_{w,t+1} = \theta_w$ and $\tau_t = \tau_{t+1} = \tau$, following Poutvaara (2006), Conde-Ruiz and Galasso (2005) and so forth. In other words, we assume that each generation alive in the same period considers τ_t and θ_{wt} chosen through election will be in place over its entire lifetime. From another angle, this assumption can be regarded as once-and-for-all voting or a static expectation. Under this case, voters determine the constant sequence of parameters of the welfare state. Under the existence of commitment, we can advance the analysis similarly to the case of static analysis[7)]

It is known that, generally, *no* Condorcet winner exists in voting over multiple issues such as a combination of policy of two kinds, without imposing additional conditions on voter's preference[8)]. To avoid such a problem, following Conde-Ruiz and Galasso (2005), we adopt the concept of a structure-induced equilibrium developed by Shepsle (1979)[9)]. We then investigate the preferences to each policy variable. The indirect utility functions of the worker and unemployed people are derived, respectively, as follows.

- Employed people:

$$V^e(\cdot) = \ln\left(\frac{1+\rho}{2+\rho}\right) + \frac{2+\rho}{1+\rho}\left[\ln(1-\tau_t-\theta_{wt})w_t + \frac{d_{t+1}}{R_{t+1}}\right] \tag{22}$$

- Unemployed people:

$$V^u(\cdot) = \ln\left(\frac{1+\rho}{2+\rho}\right) + \frac{2+\rho}{1+\rho}\left[\ln(1-\tau_t)b_t + \frac{d_{t+1}}{R_{t+1}}\right] \tag{23}$$

Therefore, the indirect utility function is written as shown below.

$$\begin{aligned} E[U(\cdot)] &= l_t V^e(\cdot) + (1-l_t)V^u(\cdot) \\ &= l_t\left\{\ln\left(\frac{1+\rho}{2+\rho}\right) + \frac{2+\rho}{1+\rho}\left[\ln(1-\tau_t-\theta_{wt})w_t + \frac{d_{t+1}}{R_{t+1}}\right]\right\} \\ &\qquad + (1-l_t)\left\{\ln\left(\frac{1+\rho}{2+\rho}\right) + \frac{2+\rho}{1+\rho}\left[\ln(1-\tau_t)b_t + \frac{d_{t+1}}{R_{t+1}}\right]\right\} \end{aligned} \tag{24}$$

We then have the following reaction functions of each agent by solving the following first order conditions.

$$\frac{\partial E[U(\cdot)]}{\partial \theta_w} = 0 \tag{25a}$$

$$\frac{\partial E[U(\cdot)]}{\partial \tau} = 0 \tag{25b}$$

---

7) Some readers might wonder whether this setting is considered to be commitment. Regarding this issue, see the remark of page 16.

8) Regarding this issue, see Persson and Tabellini (2000), for instance.

9) This approach is also adopted in Konishi (2008), Bethencourt and Galasso (2008), Conde-Ruiz and Profeta (2007), Poutvaara (2006) and so forth.

Here, the situation is identical in the sense that the desired contribution pension is

$$\tau^{*,old} = 1$$

for both employed and unemployed when they are young because both kinds of elder generations desire to receive as much pension money as possible. Therefore, in the case in which old people are in the majority, unemployment insurance ceases to exist and only the pension system survives. Consequently, capital accumulation does not proceed. To eliminate such a trivial case, we advance the analysis assuming that old people desires the same amount of pension as young workers does. Then, it is necessary to consider the ratio of voters. The relative frequencies of young unemployed households, young people, employed households, and old people is $1 - l_t : l_t : \frac{1}{1+\mu}$. To support the pension system, the following equation is expected to hold: $l_t + \frac{1}{1+\mu} \geq \frac{1}{2}(1 + \frac{1}{1+\mu}) \iff l_t \geq \frac{\mu+2}{2+2\mu}$. Therefore, it is necessary to classify the analyses into three cases depending on the value of l_t.

Then, the preferences for unemployment insurance and pensions are given respectively as follows:

- contribution to unemployment insurance:

$$\theta_{wt}^* = \arg\max_{\theta_w \in [0,1]} E_t[V(\cdot)] = \begin{cases} \theta_w \text{ that satisfies (25a)} & \text{if } l_t \leq \frac{\mu+2}{2+2\mu} \\ 0 & otherwise \end{cases} \tag{26}$$

- contribution to pension:

$$\tau_t^* = \arg\max_{\tau \in [0,1]} E_t[V(\cdot)] = \begin{cases} \tau \text{ that satisfies (25b)} & \text{if } l_t \geq \frac{\mu+2}{2+2\mu} \\ 0 & otherwise \end{cases} \tag{27}$$

The optimal solution can be derived as an intersection of reaction functions of the following two kinds:

$$\theta_{wt} = \theta_{wt}(\tau) \tag{28a}$$
$$\tau_t = \tau_t(\theta_w) \tag{28b}$$

Depending on the patterns of intersections, three plausible cases exist[10]. As in the case of Fig. 8, both the pension and the unemployment insurance are adopted; either the pension or the unemployment insurance is supported as in the case of Fig. 9 or Fig. 10. Case 1 is that corresponding to the situation in which both pension and unemployment insurance survives. Cases 2 and 3 show the situation in which either the pension system or unemployment insurance survives. Case 2 is the situation in which the pension system does and case 3 is the case in which only unemployment insurance survives.

To summarize, depending on the dynamics of capital accumulation and the unemployment rate, the contents of social security system vary as in the following cases.

[10] In the Appendix A.4, we show the two response functions are downward-sloping.

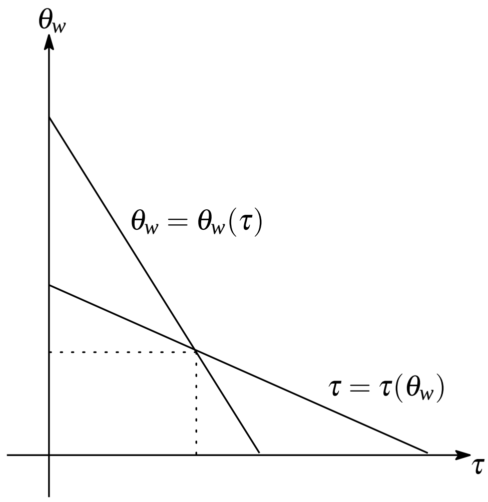


Figure 8 Case 1.

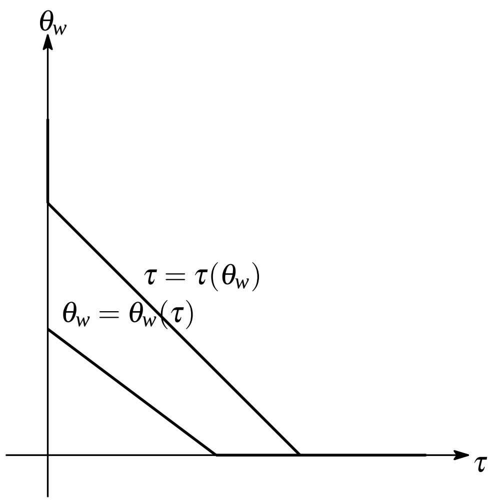


Figure 9 Case 2.

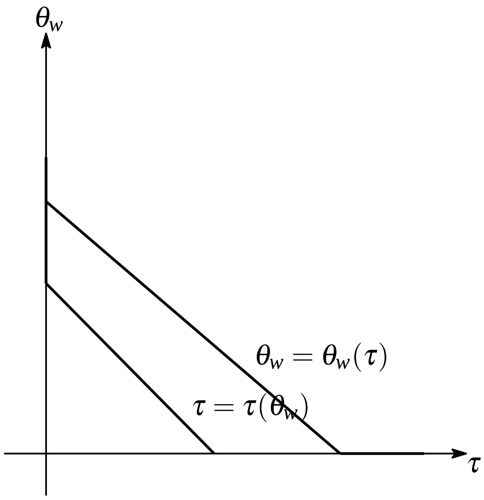


Figure 10 Case 3.

Case 1. Both pension and unemployment insurance policies survive.
Case 2. Only the pension policy survives.
Case 3. Only the unemployment insurance survives.

Here, let us explain the change of the social security system under each case, i.e., the case of $\sigma \geq 0$ and $\sigma \in (-1,0)$ (the case of $\sigma < 2\alpha$ or $\sigma > 2\alpha$). First, regarding the former case, until the time when unemployed people are in the majority, the social security is the same as in case 2 (Only pension policy survives). Subsequently, the situation is the same as in case 3 (Only unemployment insurance survives.). On the other hand, we can explain how the social security system varies under the case of Fig. 7(b). Under this regime, both pension and unemployment insurance policies survive as in case 1 when the level of capital accumulation is not so high. After that, as the employment rate rises, it is likely that only unemployment insurance is adopted.

Stage. 1 $l_t < \frac{\mu+2}{2+2\mu}$: Only unemployment insurance.
Stage. 2 $l_t = \frac{\mu+2}{2+2\mu}$: Both pension and unemployment insurance.
Stage. 3 $l_t > \frac{\mu+2}{2+2\mu}$: Only the pension system.
Stage. 4 After that, only the pension system survives.

Turning to the second case, depending on the value of l_0, both stages 1 and 2 emerge alternately and eventually; finally, either of the two policies is adopted.

Stage. 1 $l_t < \frac{\mu+2}{2+2\mu}$: Only unemployment insurance.
Stage. 2 $l_t = \frac{\mu+2}{2+2\mu}$: Both pension and unemployment insurance.
Stage. 3 $l_t > \frac{\mu+2}{2+2\mu}$: Finally, either a pension or unemployment insurance is adopted.

Finally, regarding the third case, the contents vary as follows: both stages 1 and 2 emerge alternately. Eventually, either of the two policies is adopted. The second and third cases are similar, but the patterns of fluctuation mutually differ.

Stage. 1 $l_t > \frac{\mu+2}{2+2\mu}$: Only the pension system is adopted.
Stage. 2 $l_t < \frac{\mu+2}{2+2\mu}$: Only unemployment insurance is adopted.
Stage. 3 $l_t = \frac{\mu+2}{2+2\mu}$: Both pension and unemployment insurance.
Stage. 4 Finally, either the pension or unemployment insurance is adopted.

To summarize the discussion presented above,

Proposition 2 *The patterns of policy change are summarized as follows.*

1. *Case of* $-1 < \sigma < 0$*:*
   For the last time, only the pension system survives.
2. *Case of* $0 < \sigma < 1,\ \&\ \sigma < 2\alpha$*:*
   The case in which only unemployment insurance survives and the case in which only the pension system survives emerge alternately.
3. *Case of* $0 < \sigma < 1,\ \&\ \sigma > 2\alpha$*:*
   The case in which only unemployment insurance survives and the case in which only the pension system survives emerge alternately. Eventually, either pension or unemployment insurance is adopted, depending on the value of the employment rate at the steady state.

■Remark Regarding the assumption of commitment, we have two remarks. First, once the government determines a policy based on voting, we assume that this policy lasts for periods when some generation is alive. It is possible to consider this situation as a steady state. However, strictly speaking, this differs from the steady state. The difference between the steady state is explained as follows: In the steady state case, all variables are constant through time t. In marked contrast, assuming a commitment by the government in this paper means that we assume that policy variables are constant while some generation is alive.

Secondly, some readers might wonder why the tax rate can be treated as constant despite the existence of the state variable. They consider that the tax rate is dependent on the state variable, as $\tau_t = \tau(k_t)$, and that the tax rate cannot be treated as constant as long as it is dependent on the state variable. We avoid such a question by assuming that the voting is held only once. Our answer to such a question is the following. From the capital market condition, we have

$$K_s = L_t s^e_{s-1}(\tau_{s-1}, \theta_{w,s-1}) + (N_t - L_t) s^u_{s-1}(\tau_{s-1}, \theta_{w,s-1}).$$

This equation shows that capital at s period is dependent only on the past policy variables. Therefore, we can treat K_s as constant because K_s is not dependent on policy variables at s period and thereafter. Conversely, it is apparent that the policy variables at s period do not depend on the state variable. Therefore, we can avoid the effects of a change in the state variables. Most studies which apply the structure-induced equilibrium (Conde-Ruiz and Galasso (2005), Konishi (2008), Bethencourt and Galasso (2008), for instance) avoid this criticism by dropping state variables (i.e. capital) from their model. However, it is the case with commitment that the situation in which policy variables are constant while some generation is alive in Conde-Ruiz and Profeta (2007), Poutvaara (2006), and the present paper, as long as capital is not taxed.

4.2 Case without Commitment

In this subsection, the assumption of commitment over future social security policies is relaxed. Before reviewing the analyses, let us define the equilibrium concept. Then, we investigate whether each agent has an incentive to deviate or not.

First, in the spirit of Krusell, Quadrini and Rios-Rull (1997), let us define the equilibrium concept (politico-economic equilibrium[11)]).

Defnition 2 ***(Politico-Economic Equilibrium)*** *A (Markov perfect) politico-economic equilibrium is defined as a pair of functions* $\{c^{iy}_t, c^{io}_t, \tau_t, \theta_{wt}\}^{\infty}_{t=0}$ *that accords with the following.*

(i) Given the sequence $\{\theta_{ft}\}^{\infty}_{t=1}$*, each agent (employed people and unemployed people) determines policy variables that maximize their individual utility. The optimal policy variables meet the following maximization problem.*

$$\max_{\tau,\theta_w} E_t[U(\cdot)], \text{ subject to } K_{t+1} = \Psi(K_t, \tau_t, \theta_{wt})$$

11) This concept corresponds to so-called Markov-perfect equilibrium. These conditions are dependent on the relation between the t and $t+1$ period. Therefore, this concept meets the Markov property. See also Forni (2005) who specifically examines the Markov-perfect equilibrium in an OLG model.

(ii) Budget constraints of pension and unemployment insurance are balanced in each period.

(iii) Firms maximize their profits.

(iv) The condition of the trade union's wage setting holds.

(v) Finally, the following markets clear.
Commodity Market, eq. (13)*; Capital Market, eq.* (14)*; Labor Market, eq.* (15)

We then formally define the voting game. The public history of the game at t period, $h_t = \{(\tau_0, \theta_{w0}), (\tau_1, \theta_{w1}), ..., (\tau_{t-1}, \theta_{w,t-1})\} \in H_t$ is the sequence of social security system (pension and unemployment insurance). In fact, H_t is the set of all possible history at time t. An action profile for the employee is, $\{\tau_t, b_t\} \in [0,1] \times [0,1]$. Analogously, an action for unemployed individual at time t is $\{\tau_t, b_t\} \in [0,1] \times [0,1]$.

Then, a strategy for the employee is at t period is a mapping from the history of the game into the action space, i.e., $\sigma^e : h_t \rightarrow \{\tau_t, \theta_{wt}\}$. Analogously, a strategy for unemployed people is at t period is $\sigma^u : h_t \rightarrow \{\tau_t, \theta_{wt}\}$. The strategy profile played by both individuals at t period is denoted as $\sigma_t \equiv \sigma_t^e \cup \sigma_t^u$.

At t periods, the objective function for each young player ($i \in \{e, u\}$) is

$$V_t^i(\sigma_0^i, \sigma_1^i, ...\sigma_t^i, \sigma_{t+1}^i...) = V_t^i(\tau_t, \theta_{wt}, \tau_{t+1}, \theta_{w,t+1}).$$

Regarding old agents,

$$V_t(\sigma_0, \sigma_1, ...\sigma_t, \sigma_{t+1}...) = V_t^i(\tau_t, \theta_{wt}).$$

These solutions describe the relation between the policy at t period and the one at $t+1$ period.

Moreover, we describe the definition of equilibrium.

Defnition 3 *(Definition of Markovian Structure-Induced Equilibrium)*

1. *σ meets the property of Markov perfect equilibrium.*
2. *For all t, at t period, the equilibrium outcome associated to σ_t is a structure-induced equilibrium of the static game with commitment.*

As contrasted with the analysis in the previous subsection, we assume that the government has no commitment technology in this subsection. Then, let us define the history of the game H_t as

$$H_t^0 \equiv \{h_t \in H_t | \theta_{w,t} = \theta_w*, \ \ t \in \{0, 1, ...\}\},$$

and

$$H_t^\sigma \equiv \{h_t \in H_t | \theta_{wk} = 0, \ k = 0, 1, ...t_0, \ and \ \theta_{wt} = 0, \ t \geq t_0.\}.$$

Moreover, the strategy profiles of employed people and unemployed people are respectively denoted as σ_t^e and σ_t^u. We then investigate whether each player has an incentive to deviate from the solution under full commitment, as discussed in the previous subsection. Under this setting, we first verify that unemployed people have no an incentive to deviate from the strategy. We assume that unemployed people adopt the following strategy: $\theta_{t0,w}^{deviate} > \theta_w^*$ and $\tau_{t0}^* < \tau_t^{deviate}$. However, employed people do not obtain an additional payoff by deviation because employed people punish employed people by reducing the payment of contributions to the pension system, τ, which exerts negative effects on the welfare of both agents. Therefore, it is apparent that unemployed people do not have an incentive to deviate from the commitment solution.

Regarding employed people, presuming that unemployed people deviate from equilibrium, i.e. they avoid paying contributions to pensions, then the workers will punish unemployed people by not paying contributions to unemployment insurance. Unemployed people would pay contributions to pensions to avoid being punished. Therefore, it is apparent that they have no incentive to deviate. To summarize, neither workers nor unemployed people have an incentive to deviate.

From the discussion, we have:

Proposition 3
Policies discussed in the previous subsection (with commitment case) coincide with those without commitment. In other words, the strategies with commitment are time-consistent.

5 Conclusion

This paper describes how the social security system evolves as voter attributes change. In our setting, policy determination is based on majority voting. The government has social security policy mechanisms of two kinds: a pension and unemployment insurance. When younger workers and old people constitute most voters, the pension system is supported. When young unemployed people are in the majority, unemployment insurance is adopted. Under such a situation, we show how the contents of the social security system evolve depending on the dynamics of capital accumulation and the unemployment rate, and show that the social security system ceases to exist in certain instances. This result might explain the future of social security policy in developed countries, including Japan.

Finally, we conclude this paper by mentioning problems to be solved in the future. First, when unemployment insurance (intragenerational redistribution scheme) is supported, the following conditions are needed: extremely highly population growth and/or a high unemployment rate. These parameters must be more concrete. Second, it is necessary to demonstrate the possibility that neither the pension nor unemployment insurance is adopted. Although the condition for the existence of such a situation is not derived, the present paper might be absorbing by deriving the possibility that the social security system vanishes completely. Finally, we presented the pattern of fluctuation of the social security system, but it might be necessary to fortify persuasion of our obtained result using a numerical simulation.

AppendixA

A.1 Derivation of eq. (16)

$$
\begin{aligned}
\hat{k}_{t+1} &= \frac{1}{1+\mu}\{s_t^e \times l_t + s_t^u \times (1-l_t)\} \\
&= \frac{1}{1+\mu}\Big[\frac{1}{2+\rho}\left\{(1-\theta_{wt}-\tau_t)w_t-(1+\rho)\frac{d_{t+1}^e}{R_{t+1}}\right\}\times l_t + \frac{1}{2+\rho}\left\{(1-\tau_t)b_t-(1+\rho)\frac{d_{t+1}^u}{R_{t+1}}\right\}\times(1-l_t)\Big] \\
&= \frac{1}{1+\mu}\frac{1}{2+\rho}\{(1-\theta_{wt}-\tau_t)w_t-(1+\rho)\frac{d_{t+1}^e}{R_{t+1}}\}\times l_t+(1-\tau_t)b_t-(1+\rho)\frac{d_{t+1}^u}{R_{t+1}}\times(1-l_t)\} \\
&= \frac{1}{1+\mu}\frac{1}{2+\rho}\{\underbrace{(1-\theta_{wt}-\tau_t)w_t l_t+(1-\tau_t)b_t(1-l_t)}_{(*)}-\frac{1+\rho}{R_{t+1}}\underbrace{\{(l_t d_{t+1}^e)+((1-l_t)d_{t+1}^u)\}}_{(**)}
\end{aligned}
$$

Here, using eqs.(11) and (12), the term $(*)$ and the second term $(**)$ are rewritten respectively as $(1-\theta_{wt}-\tau_t)w_t l_t+(1-\tau_t)b_t\}(1-l_t)$ and $(1+\mu)l_{t+1}w_{t+1}\left\{\tau_{t+1}(1+\theta_{wt+1}+\theta_f)\right\}$. Moreover, using eq. 7a and 7b, we obtain the following.

$$
\begin{aligned}
\hat{k}_{t+1} &= \frac{\alpha(1-(1+\theta_{wt})\tau+(1-\tau_t)\theta_{ft})(1+\theta_{ft})}{(1+\mu)(1+\theta_{ft})((2+\rho)+(1+\rho)(\tau_t(1+\theta_{wt}+\theta_{ft})))}\hat{k}_t^\alpha l_t^{1-\alpha} \\
&= \varepsilon\hat{k}_t
\end{aligned}
$$

A.2 Derivation of eq. (20); the dynamics of l_t

At the steady state, we have

$$
(l)^{\frac{\sigma-\alpha}{\sigma}}(1-l^\alpha)^{\frac{1-\sigma}{\sigma}} = A^{\frac{\alpha}{1-\alpha}}. \tag{29}
$$

Letting the right side of the equation above be $\psi(l)$, we obtain the expression shown below.

$$
\psi'(l) = [(1-\frac{\alpha}{\sigma})-\alpha\frac{1-\sigma}{\sigma}l^\alpha(1-l^\alpha)^{-1}](l)^{-\frac{\alpha}{\sigma}}(1-l^\alpha)^{\frac{1-\sigma}{\sigma}}
$$

■Case of $0<\sigma<\infty$ In this case, $\psi'(l)$ is positive. By totally differentiating eq. (19), we have:

$$
\frac{dl_{t+1}}{dl_t}\Big|_{l_{t+1}=l_t=l} = \frac{\psi^{\frac{1}{\alpha}}+\psi(1-l)\psi'}{\psi^{\frac{1}{\alpha}}+\psi(1-l)\psi'} < 1 \tag{30}
$$

Therefore, the steady state equilibrium is locally stable.

■Case of $-1<\sigma<0,\ \&\ \sigma<2\alpha$ We investigate the relation between $|\phi'(l)|$ and 1. In this case, $\psi'(l)<0$ and $\psi(0)=\infty$ and $\psi(1)=0$. We then obtain

$$
\frac{dl_{t+1}}{dl_t}\Big|_{l_{t+1}=l_t=l} = \frac{\psi^{\frac{1}{\alpha}}+\psi(1-l)\psi'}{\psi^{\frac{1}{\alpha}}+\psi(1-l)\psi'} \tag{31}
$$

At $l = 1$, we have

$$\frac{dl_{t+1}}{dl_t}\Big|_{l_{t+1}=l_t=l=1} = 0$$

and at $l = 0$, we have, by L'Hôpital's rule,

$$\frac{dl_{t+1}}{dl_t}\Big|_{l_{t+1}=l_t=l=0} = \frac{\{\psi^{\frac{1}{\alpha}} + \psi(1-l)\psi'\}'}{\{\psi^{\frac{1}{\alpha}} + \psi(1-l)\psi'\}'}$$

Then, we have

$$\frac{\alpha\psi^{\frac{1}{\alpha}} + \frac{1}{\alpha}\psi^{\frac{1}{\alpha}-1}\psi'(1-l)\psi' + \psi^{\frac{1}{\alpha}}(1-l)\psi''}{\alpha\psi^{\frac{1}{\alpha}} + \frac{1}{\alpha^2}\psi^{\frac{1}{\alpha}-1}\psi'(1-l)\psi' + \frac{1}{\alpha}\psi^{\frac{1}{\alpha}}(1-l)\psi''} > 0$$

. Therefore, the dynamics of l_t is depicted as Fig. 7(b).

■Case of $-1 < \sigma < 0$, & $\sigma > 2\alpha$ We then have

$$\frac{dl_{t+1}}{dl_t}\Big|_{l_{t+1}=l_t=l} = \frac{\psi^{\frac{1}{\alpha}} + \psi(1-l)\psi'}{\psi^{\frac{1}{\alpha}} + \psi(1-l)\psi'}. \tag{32}$$

For the interval $0 < l < 1$, there exists $\bar{l}$ such that $\frac{dl_{t+1}}{dl_t}\Big|_{l_{t+1}=l_t=l} = 0$m because $\phi'(\cdot) < 0$. Therefore, for the interval $0 < l < 1$, there exists at least one solution that satisfies $[0, \bar{l}]$, $\frac{dl_{t+1}}{dl_t} < 0$ and for $[\bar{l}, 1]$, $\frac{dl_{t+1}}{dl_t} > 0$. The dynamics under this case is depicted as Fig. 7(c).

A.3 Shapes of the Response functions

Here, we show the two response functions are downward-sloping. Then, we investigate the sign of $\frac{\partial\tau}{\partial\theta_w}$ and $\frac{\partial\theta_w}{\partial\tau}$. Each type of individual determines the contributions to unemployment insurance, θ_w and pension, τ to maximize personal utility. Preferences to θ_w and τ are derived by solving the following first order conditions:

$$\frac{\partial E_t[V_t]}{\partial\theta_w} = 0 \iff l_t\left(\frac{2+\rho}{1+\rho}\right)\{\frac{-w_t}{1-\tau-\theta_w}\} = 0 \tag{33a}$$

$$\frac{\partial E_t[V_t]}{\partial\tau} = 0 \iff l_t\left(\frac{2+\rho}{1+\rho}\right)\{\frac{-w_t}{1-\tau-\theta_w}\} + (1-l_t)(\frac{2+\rho}{1+\rho})\{\frac{-b_t}{1-\tau}\} = 0 \tag{33b}$$

First, we define the LHS of (33a) as

$$G(\cdot) \equiv l_t\left(\frac{2+\rho}{1+\rho}\right)\{\frac{-w_t}{1-\tau-\theta_w}\}.$$

From the implicit function theorem, we find

$$\frac{\partial\tau}{\partial\theta_w} = -\frac{\frac{\partial G(\cdot)}{\partial\theta_w}}{\frac{\partial G(\cdot)}{\partial\tau}} < 0$$

Similarly, we have

$$\frac{\partial\theta_w}{\partial\tau} < 0,$$

which shows that the two response functions are downward-sloping.

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