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Solution Concept for Intergenerational Conflict: the Role of Intergenerational Bargaining

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Solution Concept for Intergenerational Conflict: the Role of Intergenerational Bargaining*

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Abstract

This paper specifically examines intergenerational conflict and analyzes an overlapping generations model of public goods provision from the viewpoint of *time-consistency*. Public goods are financed through labor-income and capital-income taxation, thereby distorting savings and the labor supply. Taxes redistribute income across generations in the form of public goods. Under such a situation, there emerge dual intergenerational conflicts: the first is related to the amount of public goods and the second is the tax burden. We then contrast the politico-economic equilibrium with commitment allocation, and analyze the sources of conflict and *time-inconsistency*, and attempt to resolve such a conflict by introducing the concept of '*intergenerational bargaining*'. Our main findings are the following. First, taxation derived using Lagrange method fails to be *time-consistent*. Second, depending on bargaining-power, taxation based on intergenerational bargaining can be *time-consistent*. Third, we portray the properties of taxation and public goods provision rules based on intergenerational bargaining.

Keywords: Dual Intergenerational Conflicts; Intergenerational Bargaining;Time-inconsistency; Modified Public Goods Provision Rule.

JEL Classification: E61; E62; H41.

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1 Introduction

Traditionally, in analyzing economic policies such as fiscal or monetary policy, economists have assumed that the government is a monolithic organization that is intended mainly to maximize social welfare or the economic growth rate. In this regard, the following points should be emphasized: First, various entities (voters, bureaucrats, representatives, organizations, etc.) that are involved in policy determination foster conflicts. Therefore, it is impossible for the government to organize a policy based on only one situation or position. In other words, the government cannot avoid determining a policy that incorporates implications of numerous opinions. Second, even if a policy were derived that maximizes social welfare or the growth rate, carrying out such a policy with certainty would be difficult: it is hard to commit to such a policy[1)]. As a result, we cannot regard actual governments as monolithic organizations, as many economists have assumed. Similarly, looking at the voter side, various conflicts exist: young vs. old, unemployed vs. job-holders, rich vs. poor, and so forth[2)]. Given the existence of such conflicts, "political compromise" is inevitable; it is difficult to maintain the government commitment entirely. As countermeasures for such a situation, we find that it is necessary *ex post* to carry out some kind of coordination policy to attenuate inefficiencies that result from discretionary policy.

In light of the existence of such conflicts, we find that normative model analysis, in which the government is assumed to be benevolent in the sense that they maximize social welfare, is inadequate to determine policy implications because such a model cannot encompass the nature of the current situation. Therefore, this paper presents analysis of conflicts of interest among voters, especially intergenerational conflict. In addition, as a problem corresponding to such a conflict, it is necessary to consider the *time-inconsistency* problem[3)] because both this problem and such conflicts are concurrently related to government commitment. For that

[1)] Some conflicts exist even within actual governments. As just one example, the Ministry of Finance and the Ministry of Health, Labor and Welfare in Japan have frequent conflicts. These two organizations seek different objectives: the former seeks to decrease the deficit or debt, whereas the latter is responsible for promoting a social security system, even if it is very costly. Consequently, a contraposition of duties pertains between the two organizations.

[2)] As an example of conflicts among voters, we can indicate conflicts surrounding issues of social security, particularly pension systems.

[3)] For pioneering studies on this problem, see Kydland and Prescott (1977), Fischer (1980), and Chari, Kehoe and Prescott (1989).

reason, this paper describes the investigation of intergenerational conflict in connection with a *time-inconsistency* problem.

In turn, let us explain the relationship between this paper and past studies. We use a two-period version of an overlapping generations (OLG) mode and limit our survey to past studies that have used a similar model. In the context of intergenerational conflicts[4)], many studies center their scope on *intertemporal* conflict, as explained below. Here, we regard the *intertemporal* conflict case, in which a transfer from young people to elderly persons is done in each period, as in a pay-as-you-go type pension system. This can also be interpreted as a one-sided transfer case. In this regime, intergenerational conflict arises with the result that the burden that young people should bear varies depending on when they are born.

Among such studies that posit a one-sided transfer case, some studies such as Casamatta, Cremer and Pestieau (2000) (pension, transfer from elderly persons to young people) and Holtz-Eakin, Lovely and Tosun (2004) (education, a transfer from elderly persons to young people) address policy decisions based on majority voting, whereas Azariadis and Galasso (2002) investigates how cooperative solutions are attained, with specific attention to policy decision based on majority voting in combination with veto power. As another method of policy decision, Grossman and Helpman (1998) introduces lobbying between households and a myopic government. It is true that numerous studies have specifically addressed intergenerational transfer. However, as already described, we can categorize such studies as *intertemporal* conflict cases despite their associated methods of policy decision[5)].

On the other hand, to the best of our knowledge, few studies have analyzed *intratemporal* conflict, even though redistributive policies include two-sided transfers as well as one-sided transfers[6)]. Among such studies, Bassetto (2008) expands past studies described above into a

4) For a survey, see Breyer (1994), Galasso and Profeta (2002), and de Walque (2005). Breyer reviews precedent studies that specifically examined the assumptions of models and classifies the models accordingly. Some studies use the median voter theorem, based on the assumption of single-peaked utility, in determining policy. Related to this issue, see also Galasso and Profeta (2002) and Walque (2005).

5) For studies that incorporate intragenerational conflict as well as intergenerational conflict, we can point out Tabellini (1991, 2000), who examines substitution between social security (i.e. pension) and debt payment, using the altruistic model. That study shows that decisions reached by the older generation are not necessarily disadvantageous to young people, depending on the strength of the bequest motive. Lambertini and Azariadis (2003) specifically evaluates income redistribution from young people to elderly persons and compares open-loop equilibria (corresponding to the case with commitment) and closed-loop equilibria (without commitment) under the situation in which inequality pertains with regard to productive ability. They show that the amount of intergenerational and intragenerational transfer is determined according to the degree of inequality.

6) Here, we view a two-sided transfer as the case in which a transfer from young people to elderly persons and

two-sided transfer case by incorporating public goods in an OLG model[7]. In this approach, an intergenerational conflict arises depending on how the cost of public goods is assigned to young people and the older generation. Bassetto focuses on this point by incorporating public goods that serve in the role of intergenerational transfer (redistribution), and illustrates the possibility of multiple equilibria using numerical simulation[8]. Although the analysis in the present paper resemble those of Bassetto (2008) in the respect that an *intratemporal* conflict is emphasized, it differs absolutely from his analysis in other respects, as explained later.

In many of these studies, policy-making is assumed to be based on the median voter theorem or majority voting, which have the common disadvantage of abandoning an underdog or minority group[9]. Different from those studies, we elucidate intergenerational conflict and introduce the concept of ‘*intergenerational bargaining*’, as a solution to such a conflict, expanding the idea of Besley and Ghatak (2001). In this regard, some noteworthy points are the following: First, the government raises funds through taxation to procure public goods. Then the government redistributes tax revenues in the form of public goods, which affects the utility of both young and old generations[10]. Second, the younger generation has an incentive to decrease labor-income taxation, whereas elderly persons also have an incentive to decrease capital-income taxation because the labor supply is also endogenous in our model. In other words, an intergenerational conflict arises at this stage. Moreover, each generation decreases the tax burden without careful consideration, engendering a situation in which the amounts of public goods provision decrease. As a result, both generations decrease utility, creating a trade-off between utility derived through the use of public goods and the disutility that arises from the tax burden. Intergenerational conflict arises in terms of the tax burden.

its reverse are implemented during the same period.

7) For another study, see Renström (1996), and Pirttilä and Tuomala (2001), which incorporate public goods into the utility function in an OLG model. Renström derives optimal taxation, whereas Pirttilä and Tuomala derives the public goods provision rule using Mirrlees’ approach. Moreover, Kaas (2003) introduces public investment into an OLG model. However, those studies neglect the *time-inconsistency* problem.

8) Nevertheless, that result depends strongly on the specification of utility function. Moreover, this point also differs from our paper.

9) As another criticism of these means of policy-making, see Grossman and Helpman (1998, section 1). Those studies point out that this approach ignores the aspect of the parliamentary system, that is, coalition of voters. Moreover, the median voter theorem has some restrictions. For instance, this theorem is applicable only under a one-dimensional policy space.

10) From another perspective, we can regard this kind of public good as “intergenerational public goods”, according to the term used in Sandler and Smith (1976). For further examination of this point, see remark 1.

Moreover, regarding the amounts of public goods, an intergenerational conflict also arises because the amounts of public goods that young and older generations demand generally do not coincide. In other words, *dual* intergenerational conflicts emerge. Under such a trade-off, or because of intergenerational conflicts, how do we determine the public goods provision rule and taxation?[11] To answer such a question, this paper offers a new solution concept: '*intergenerational bargaining*'[12].

Our main findings are summarized as follows: First, taxation derived by the Lagrange method fails to be *time-consistent*. Second, depending on the value of bargaining power, such taxation based on intergenerational bargaining can be *time-consistent*. Third, we showed properties of taxation and the modified public goods provision rule based on intergenerational bargaining, which differs from Bassetto (2008).

The organization of this paper is as follows: section 2 describes the model and section 3 derives optimal taxation under the assumption that the government has commitment technology, and investigates the properties of such taxation. In section 4, we define the equilibrium concept and derive the modified public goods provision rule and taxation through intergenerational bargaining. Section 5 presents conclusions of this paper and future problems that require analysis.

2 The model

Our model resembles that of Renström (1996). We consider an overlapping generations economy with an endogenous labor supply in which the time horizons are infinite and discrete, as indexed by $t = 0, 1, 2, \cdots$. This economy has no uncertainty. We then present a brief description of the respective behaviors of households, firms and the government. Figure 1 depicts an intuitive structure of this model along with flows of goods and capital.

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[11] As another study that analyzes the public goods provision rule in an OLG model, we can point out Batina (1990). However, Batina expands the Samuelson rule under the assumption that public goods affect only the utility in the young period.

[12] As other studies that incorporate intergenerational bargaining, see Bassetto (2008) and Renström (2002). However, the former highlights the existence of equilibria indeterminacy and the model of the latter does not depict the situation of dual intergenerational conflict. Moreover, neither of the two papers highlight the *time-inconsistency* problem, as our paper does.

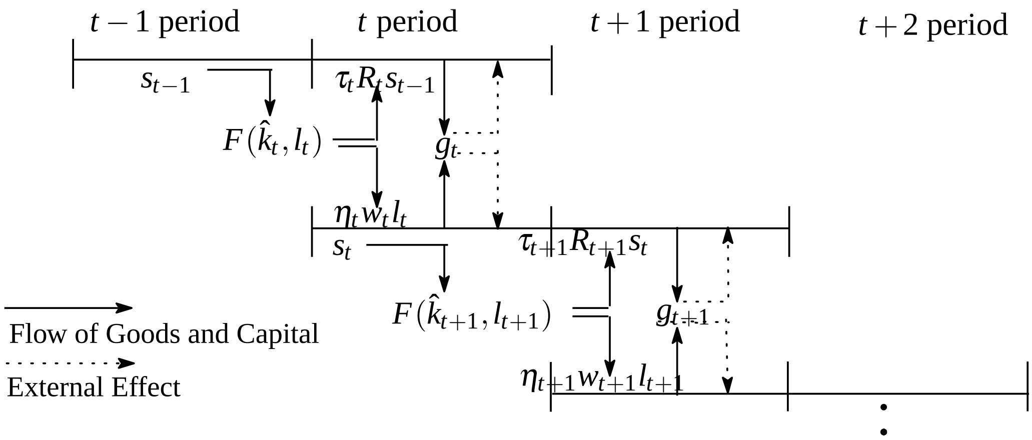

Figure 1 Structure of the model; External Effect and Flow of Goods and Capital

2.1 Preferences, Technology, and Policy

2.1.1 Production

First, we describe firms' behavior. We assume that factor markets are perfectly competitive and that firms maximize their profits. Labor and capital stock are used for production; production technology yields constant returns to scale. Therefore, production functions are expressed as $Y_t = F(K_t, L_t) : \Re_+^2 \to \Re_+$, where K_t, L_t, and Y_t respectively represent capital stock, labor, and output in aggregate terms. For definitions of capital stock and output per capita, l_t and $\hat{k}_t$ respectively denote $\frac{L_t^d}{N_t}$ and $\frac{K_t}{N_t}$. The maximization process yields

$$R_t = \frac{\partial F(\hat{k}_t, l_t)}{\partial \hat{k}_t} \equiv F_{\hat{k}}(\hat{k}_t, l_t), \quad w_t = \frac{\partial F(\hat{k}_t, l_t)}{\partial l_t} \equiv F_l(\hat{k}_t, l_t).$$

In those equations, R_t and w_t respectively indicate the rental rate of capital and the real wage rate.

2.1.2 Government

Next, let us describe government behavior. The government determines taxation and public expenditure to maximize social welfare. Some assumptions must be made. First, the government finances its expenditure with a tax on capital income and labor income; it is used to purchase public goods. Second, they cannot make use of lump-sum taxes and a consumption tax. Therefore, the government's budget constraint is expressed as

$$R_t b_t + g_t = b_{t+1} + R_t \tau_t s_{t-1} + w_t l_t \eta_t, \tag{1}$$

where b_t denotes the national debt.

2.1.3 Households

Finally, Households live two periods in a closed-economy with neither population growth nor a bequest motive. Each generation consists of only a *single* agent. Dynasties derive utility from public goods as well as consumption and leisure. For simplicity, preference of the dynasty's cohort that remain alive at t period is described by the following additively separable function:

$$U(c_t, d_{t+1}, 1-l_t, g_t, g_{t+1}) = u(c_t) + h(g_t) + v(1-l_t) + \frac{1}{1+\rho}\{u(d_{t+1}) + h(g_{t+1})\}, \tag{2}$$

where c_t, d_t, l_t, g_t and ρ respectively denote consumption in the younger period and old, the labor supply, the amount of public goods at period t, and the rate of time preference. We assume that the public goods at period t depreciate entirely at the end of that period. Furthermore, each function is twice differentiable, strictly increasing, strictly concave, and each satisfies the Inada condition. Two kinds of tax exist: a tax on labor income (η), and a tax on saving (τ). Households that form at date t divide after-tax labor-income into consumption (c_t) and saving (s_t), and consume (d_{t+1}) as after-tax savings when they are old. Consequently, the young-period and old-period budget constraints are shown respectively as

$$c_t + s_t = (1-\eta_t)w_t l_t, \ \ d_{t+1} = (1-\tau_{t+1})R_{t+1}s_t. \tag{3}$$

Each household is assumed to maximize his or her lifetime utility (2) by choosing a consumption plan and a level of labor supply, subject to the budget constraints of Eq. (3). Taking into consideration the budget constraint of the government, Eq. (1), the first order conditions are expressible as

$$U_s = -u'(c_t) + \frac{1}{1+\rho}\{\tilde{R}_{t+1}u'(d_{t+1}) + R_{t+1}\tau_{t+1}h'(g_{t+1})\} = 0, and \tag{4a}$$

$$U_l = \tilde{w}_t u'(c_t) - v'(1-l_t) + w_t\eta_t h'(g_t) = 0, \tag{4b}$$

where $\tilde{R}_{t+1} \equiv (1-\tau_{t+1})R_{t+1}$ and $\tilde{w}_t \equiv (1-\eta_t)w_t$. By solving these two equations, we obtain a savings and labor supply function as follows[13]:

$$s_t = s(\tilde{w}_t, \tilde{R}_{t+1}; g_t), \;\; l_t = l(\tilde{w}_t, \tilde{R}_{t+1}; g_t). \tag{5}$$

■Remark 1 Here, as for the FOCs, Eqs. (4), the criticism might arise that the terms $h(g_t)$ and $h(g_{t+1})$ should be treated as given because g_t represents public goods and does not affect both generations' behavior. In our setting, we calculate FOCs in this manner because the government expenditure is dependent upon taxation, which affects households' behavior. Note that this calculation is based on the assumption that there is only single agent in each generation. From this viewpoint, we might regard public service rather than public goods as used by young and older people who are alive in the same period. Using the term in Sandler and Smith (1976), we can call these kinds of public services "intergenerational public goods" (IPG).[14]

2.2 Market Equilibrium

We then formulate equilibrium conditions for each market. Because $\hat{k}_t \equiv \frac{K_t}{N_t} = \frac{K_t}{L_t} \cdot \frac{L_t}{N_t} = k_t l_t$, where N is population, we can write this condition as follows:

1. Commodity market

$$c_t + d_t + g_t + k_{t+1} l_{t+1} = R_t k_t l_t + w_t l_t \tag{6}$$

2. Capital market

$$s_t - b_{t+1} = k_{t+1} l_{t+1} (= \hat{k}_{t+1}) \tag{7}$$

3. Labor market

   It is assumed that, in this economy, full employment is founded, meaning that a supply-demand balance is attained. That is, l_t^d denotes labor demand.

$$l_t^d = l_t \tag{8}$$

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13) To close the model, we must describe the behavior of the initial old. As in other periods, the initial old agent solves

$$\max \; \{u(d_1) + h(g_1)\}$$
$$s.t. \;\; d_1 = (1 - \tau_1) R_1 s_0,$$
$$\text{given } \; \tilde{R}_1, \; \tilde{w}_o, \; s_0, \text{ and } g_0.$$

14) Rangel (2005) cites environmental preservation or R&D as an example of IPG. Unlike this paper, he regards IPGs as goods that affect the utility of the next generation as well as the current generation.

2.3 Timing of Decisions

Finally, let us summarize the sequence of decision-making (or political process) as follows:

Stage 1. At period 0, the government determines the tax policy $\{\tau_t^*, \eta_t^*\}_{t=0}^{\infty}$ according to the tax sequence derived by the Lagrange Method. Note that this taxation is *ex ante* optimal. Similarly, $\{g_t^*\}_{t=0}^{\infty}$ is also determined.

Stage 2. At the tth period, a new generation is born.

Stage 3. Based on g_t^* determined at stage 1, the elderly and the young who are alive in the tth period start bargaining over the amount of public goods. Consequently, a new amount of public goods, $\bar{g_t}$ is determined.

Stage 4. After this kind of bargaining, taxation is also determined as $\{\bar{\tau_t}, \bar{\eta_t}\}$ in the same period through bargaining.

Stage 5. The $t+1$th generation is newly born.

After that, Stages 2–5 are repeated.

3 Dynamic Optimal Allocation

3.1 Benchmark

In this section, as a benchmark, we derive the dynamic optimal allocation (Ramsey allocation) under the assumption that the government implicitly has a commitment device, following traditional optimal taxation theory[15]. This taxation corresponds to Stage 1, as described in the 2.3 subsection. In this stage, the choice set of the planner is the set of sequences of taxes and public goods: $\{\tau_t, \eta_t, g_t\}_{t=0}^{\infty}$, in other words, $\{\tilde{R}_t, \tilde{w}_t, g_t\}_{t=0}^{\infty}$. In this section, we assume that public goods entail no congestion effect, so that both the younger and older generations can make use of public goods. Next, we define the social welfare function as

$$\mathrm{SW} \equiv \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + v(1-l_t) + h(g_t) + \frac{1}{1+\rho}\{u(d_{t+1}) + h(g_{t+1})\} \right\} + \frac{1}{1+\rho}\{u(d_0) + h(g_0)\},$$

[15] In other words, this concept is identical to the open-loop solution.

where $\beta(<1)$ stands for the social discount factor. Then, the constraints are summarized as follows:[16)]

1. First-order conditions: These equations can be interpreted as households' response functions.

$$\begin{cases} U_s = -u'(c_t) + \dfrac{1}{1+\rho}\{\tilde{R}_{t+1}u'(d_{t+1}) + (R_{t+1} - \tilde{R}_{t+1})h'(g_{t+1})\} = 0, \\ U_l = \tilde{w}_t u'(c_t) - v'(1-l_t) + (w_t - \tilde{w}_t)h'(g_t) = 0 \end{cases} \tag{4'}$$

2. Commodity market

$$c_t + d_t + g_t + k_{t+1}l_{t+1} = R_t k_t l_t + w_t l_t \tag{6}$$

3. Government budget constraint

   Based on the fact that $F(\cdot)$ is homogeneous of degree one, we can transform Eq. (1) as:[17)]

$$g_t = F(\hat{k}_t, l_t) - \tilde{R}_t k_t l_t - \tilde{w}_t l_t + k_t l_t + b_{t+1} - \tilde{R}_t b_t \tag{1'}$$

More formally, the problem of the government can be rewritten as

$$\begin{aligned} &\max_{\{\tilde{w}_t, \tilde{R}_t, b_t, g_t\}} \quad SW \\ &s.t. \;\; Eqs.\ (1'), (4'), (6), \;\; \text{given } \hat{k}_0 > 0. \end{aligned} \tag{IP$_\mathrm{G}$}$$

Setting $\{\mu_{1t}, \mu_{2t}, \kappa_t, \lambda_t\}$ as the Lagrange multiplier, let $\mathscr{L}$ be the Lagrange function as

$$\begin{aligned} \mathscr{L} = \sum_{t=0}^{\infty}\beta^t \Big[&\{u(c_t) + v(1-l_t) + h(g_t) + \frac{1}{1+\rho}\{u(d_{t+1}) + h(g_{t+1})\} \\ &+ \mu_{1t}\{-u'(c_t) + \frac{1}{1+\rho}\{\tilde{R}_{t+1}u'(d_{t+1}) + (R_{t+1} - \tilde{R}_{t+1})h'(g_{t+1})\}\} \\ &+ \mu_{2t}\{\tilde{w}_t u'(c_t) - v'(1-l_t) + (w_t - \tilde{w}_t)h'(g_t)\} \\ &+ \kappa_t \left\{g_t - F(\hat{k}_t, l_t) + \tilde{R}_t k_t l_t + \tilde{w}_t l_t - b_{t+1} + \tilde{R}_t b_t\right\} + \lambda_t (R_t k_t l_t + w_t l_t - c_t - d_t - g_t - k_{t+1}l_{t+1})\Big]. \end{aligned}$$

Note that κ_t and λ_t are interpreted respectively as the marginal social value and the marginal rate of substitution. Moreover, μ_{1t} and μ_{2t} can be interpreted respectively as the

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16) If we take the borrowing constraints into consideration, another condition, $k_t \geq 0$, is added, but we ignore it in this paper.

17) This equation is derived in the following manner.

$$\begin{aligned} g_t &= R_t \tau_t s_{t-1} + w_t l_t \eta_t + b_{t+1} - R_t b_t \\ &= R_t \tau_t (k_t l_t + b_t) + w_t l_t \eta_t + b_{t+1} - \tilde{R}_t b_t \quad (\because \text{eq.}(7)) \\ &= F(\hat{k}_t, l_t) - \tilde{R}_t k_t l_t - \tilde{w}_t l_t + b_{t+1} - \tilde{R}_t b_t \end{aligned}$$

sensitivity for the saving and labor supply. Noting that state variables are k_t and b_t, we can derive the first-order conditions (and the transversality condition) as follows[18]:

$$\tilde{w}_t \quad : \quad \mu_{2t}\{u'(c_t) + \frac{1}{1+\rho}(\frac{w_t}{\tilde{w}_t} - 1)h'(g_t)\} = \left(\kappa_t - \frac{w_t}{\tilde{w}_t}\lambda_t\right) l_t \tag{9a}$$

$$\begin{aligned}\tilde{R}_t \quad : \quad & \frac{1}{1+\rho}(\frac{R_t}{\tilde{R}_t} - 1)\mu_{1,t-1}h'(g_{t+1}) + \frac{\mu_{1,t-1}}{\beta}\frac{1}{1+\rho}u'(d_{t+1}) + \kappa_t(k_t l_t + b_t) \\ & + \lambda_t \frac{R_t}{\tilde{R}_t} k_t l_t = 0\end{aligned} \tag{9b}$$

$$b_t \quad : \quad -\kappa_t \tilde{R}_t + \frac{\kappa_{t-1}}{\beta} = 0 \tag{9c}$$

$$\begin{aligned}l_t \quad : \quad & -v'(1-l_t) + \mu_{2t}v''(1-l_t) + \kappa_t(\tilde{R}_t k_t + \tilde{w}_t - k_t) \\ & + \lambda_t(R_t k_t + w_t) - \frac{\lambda_{t-1}}{\beta}k_{t+1} = 0\end{aligned} \tag{9d}$$

$$\hat{k}_t \quad : \quad \kappa_t(-R_t + \tilde{R}_t) \ + \lambda_t R_t - \frac{\lambda_{t-1}}{\beta} = 0 \tag{9e}$$

$$\begin{aligned}g_t \quad : \quad & h'(g_t) + \frac{1}{\beta}\frac{1}{1+\rho}\{h'(g_{t+1}) + (R_t - \tilde{R}_{t+1})\mu_{1,t-1}h''(g_{t+1})\} \\ & + \mu_{2t}(1-\tilde{w}_t)h''(g_t) + \kappa_t - \lambda_t = 0\end{aligned} \tag{9f}$$

Finally, the transversality condition is

$$\lim_{t\to\infty} \beta^t \lambda_t k_t = 0.$$

[18] For the first-order condition with respect to $\tilde{w}_t$, we use the fact that $\frac{dw_t}{d\tilde{w}_t} = \frac{1}{1-\eta_t} = \frac{w_t}{\tilde{w}_t}$ because $\tilde{w}_t \equiv (1-\eta_t)w_t$. The same calculation applies for $\tilde{R}_{t+1}$.

At the steady state, setting $\hat{k}_t = \hat{k}_{t+1} = \cdots = \hat{k}$ for instance,

$$
\begin{aligned}
&(9a) &:& \quad \mu_2\{u'(c) + \frac{1}{1+\rho}(\frac{w}{\tilde{w}} - 1)h(g)\} = \left(\kappa - \frac{w}{\tilde{w}}\lambda\right) l &\quad (10a)\\
&(9b) &:& \quad \frac{1}{1+\rho}(\frac{R}{\tilde{R}} - 1)\mu_1 h'(g) + \frac{\mu_1}{\beta}u'(d) + \kappa(kl+b) + \lambda\frac{R}{\tilde{R}}kl = 0 &\quad (10b)\\
&(9c) &:& \quad -\beta\kappa\tilde{R} + \kappa = 0 &\quad (10c)\\
&(9d) &:& \quad -v'(1-l) + \mu_2 v''(1-l) + \kappa(\tilde{R}k + \tilde{w} - k) + \lambda_t(Rk + w_t) - \frac{\lambda}{\beta}k = 0 &\quad (10d)\\
&(9e) &:& \quad \kappa(-R+\tilde{R}) + \lambda R - \frac{\lambda}{\beta} = 0 &\quad (10e)\\
&(9f) &:& \quad h'(g) + \frac{1}{\beta}\frac{1}{1+\rho}\{h'(g) + (1-\tilde{R})\mu_1 h''(g)\} + \mu_2(1-\tilde{w})h''(g) + \kappa - \lambda = 0 &\quad (10f)\\
&(4') &:& \quad -u'(c) + \frac{1}{1+\rho}\{\tilde{R}u'(d) + (R-\tilde{R})\mu_1 h'(g)\} = 0 &\quad (10g)
\end{aligned}
$$

$$
\begin{aligned}
&(4') &:& \quad \tilde{w}u'(c) - v'(1-l) + (1-\tilde{w}) - h'(g) = 0 &\quad (10h)\\
&(1') &:& \quad g - F(\hat{k}, l) + \tilde{R}kl + \tilde{w}l - kl - b + \tilde{R}b = 0 &\quad (10i)\\
&(6) &:& \quad Rkl + wl - c - d - g \geq 0 &\quad (10j)
\end{aligned}
$$

From these equations, we obtain the following set of solution paths:

$$\{c_t^*, d_{t+1}^*, l_t^*, \tilde{R}_t^*, b_t^*, \tilde{w}_t^*, k_t^*, g_t^*, \mu_{1t}^*, \mu_{2t}^*, \kappa_t^*, \lambda_t^*\}_{t=0}^{\infty}.$$

In this paper, we designate solutions derived by Lagrangian method as "commitment solutions." From these equations, we obtain

$$\tau_t^* = 1 - \frac{d_t}{R_t \hat{k}_{t+1}}, \quad \eta_t^* = 1 - \frac{c_t + \hat{k}_{t+1}}{w_t}. \tag{11}$$

We then discuss some features of the Ramsey allocation in the following subsection.

3.2 Characterization of Commitment Solution

In this subsection, we characterize the equilibrium tax sequence, derived in the above way. Here, we show the following claim.

Proposition 1 *The commitment solution, derived by Lagrange method, has the following properties:*

1. *This taxation fails to be time-consistent.*

2. *This taxation presents intergenerational inequality of the amount of public goods.*

Proof *See appendix A.1.* ∎

■Remark 2 For the first result, we have three remarks. First, this result differs from Okuno and Yakita (1983)[19]. Their model assumes a naive expectation in relation to the amount of public goods provision, which means that the amount of public provision at period t equals that at $t+1$ period. Therefore, the *time-inconsistency* problem does not occur in their model. Second, the fact that the open-loop solution generally fails to be *time-consistent* is well known. However, Xie (1997) reported that there is a possibility that even an open-loop solution is *time-consistent*, so it is necessary to check whether the above solution is *time-inconsistent* or not. Third, intuitively, *time-inconsistency* occurs in this model for the following two reasons.

1. Cost allocation of public goods varies depending on when households are born.
2. Existence of the state variable, emphasized in this paper as physical capital.

The first point says that the source of *time-inconsistency* is conflict of interests, as indicated in Drazen (2000, pp.110–113). Furthermore, the second is more important. The property of variables changes from 'control variables' to a 'state variable' after some time has passed. In other words, such variables can be treated as a control variable until some period; subsequently, this can be treated merely as a state variable. In other words, this shows that the elasticity of a tax-base *ex ante* differs from that *ex post*.

■Remark 3 As for the second result, it is necessary to define a situation that has no intergenerational conflict. In this paper, a situation with intergenerational conflict is considered as follows. The amount of provided public goods differs from the level that each generation desires. In other words, taxation maximizes the sum of utility of young people and elderly persons as a fair policy. That is, at t period, a certain kind of taxation, $\{\tau_t, \eta_t, g_t\}$ does not agree with $\{\tau_t^*, \eta_t^*, g_t^*\}$, which maximizes the social welfare. Formally, we must show that the above commitment solution differs from that defined as follows:

[19] Moreover, in the model of Okuno and Yakita (1983), the labor supply is not endogenous, which differs from our model.

Defnition 1 ***(Equilibrium with Intragenerational Equal Treatment)*** *The set of allocation with intragenerational equal treatment satisfies the following three conditions:*

(i) *Together with the situation in which both the generations' utility and firms' profits are maximized,*

(ii) *The allocation satisfies the following:*

$$\max_{\{\tilde{w}_t, \tilde{R}_t\}} \; U(c_t, g_t, 1-l_t) + U(d_t, g_t)$$
$$s.t. \;\; Eqs. \; (1), (3).$$

(iii) *The following markets clear.*

Commodity Market: Eq. (6), *Capital Market: Eq.* (7), *Labor Market: Eq.* (8)

For a more detailed proof, see the second half of A.1.

4 Intergenerational Bargaining

In the previous section, we have analyzed the taxation and public goods provision rule under the assumption of no congestion effect and the government has a commitment device. We have shown that taxation derived by the Lagrange method, which is determined at the initial point, fails to be *time-consistent*. Consequently, the government has an incentive to change the policy at some point[20]. In this section, we introduce the concept of '*intergenerational bargaining*', which is an alternative to Lagrange method or majority voting. In what follows, first, we define the equilibrium concept (i.e. politico-economic equilibrium). Second, we formulate a problem in which older and younger generations should solve and derive the modified public goods provision rule. Finally, we examine the properties of the modified public goods provision rule and explain the relationship using the Samuelson rule.

4.1 Equilibrium Definition

According to Krusell, Quadrini and Rios-Rull (1997), let us define the equilibrium concept (politico-economic equilibrium[21]).

[20] One might infer that the government should carry out a no-commitment policy. However, it is not desirable to do so in terms of credibility. Furthermore, it is extremely difficult to change the policy repeatedly because of the cost of policy-change. Third, this way of method does not show how the government should change the policy in concrete form.

[21] This concept corresponds to so-called Markov-perfect equilibrium. These conditions are dependent on the relationship between the t and $t+1$ period. Therefore, this concept meets the Markov property.

Defnition 2 ***(Politico-Economic Equilibrium)*** *A politico-economic equilibrium is a sequence* $\{\tilde{w}_t, l_t, c_t, d_t, \tilde{R}_{t+1}, g_t\}_{t=0}^{\infty}$ *that accords with the following.*

(i) Given the sequence $\{\tau_t^*, \eta_t^*, g_t^*\}_{t=1}^{\infty}$*, each agent (young or elderly) determines the policy variables that maximize their individual utility. That is, the optimal policy variables meet the following maximization problem:*

- *Young people:*

$$\max \left\{ u(c_t) + v(1 - l_t) + h(g_t) + \frac{1}{1+\rho} \{u(d_{t+1}) + h(g_{t+1})\} \right\}$$

- *Elderly persons:*

$$\max \left\{ \frac{1}{1+\rho} \{u(d_t) + h(g_t)\} \right\}$$

Based on the solutions of such problems, the tax policy by which both generations are alive in the same period as that in which demand is determined. Moreover, the policy $\{g_s\}_{s=t+1}^{\infty}$*, which is also achieved through the above method, is repeated after* $t+1$*-period.*

(ii) Based on the bargaining solutions, the government determines the policy.

(iii) Finally, the following markets clear.

Commodity Market: Eq. (6)*, Capital Market: Eq.* (7)*, Labor Market: Eq.* (8)

This equilibrium concept is superior to the Ramsey allocation discussed in the previous section in the following two respects: First, by its definition of Markov property, the above equilibrium meets the property of *time-consistency* and neither young nor elder agents have an incentive to deviate from the bargaining solution because it is based on the solution of utility maximization problem. Second, compared with the Ramsey allocation from the viewpoint of intergenerational fairness, this equilibrium does not engender much intergenerational inequality because this taxation is based on the behavior of living generations in the same period. As for this equilibrium concept, note that the government is assumed to be *myopic* in the sense that they ignore the long-run effect and emphasize only the effect of a single period. This situation can be interpreted as a one-period commitment. Hereafter, we search the public goods provision rule and the taxation that meets the above properties.

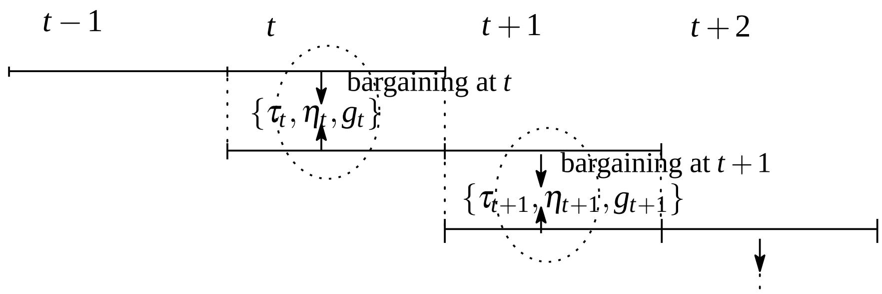

Figure 2 Schematic of intergenerational bargaining

4.2 Bargaining Problem between Elderly Persons and Young People

In this section, we formulate the concept of *intergenerational bargaining*. The intuitive structure of this bargaining is depicted in Fig. 2. In this paper, the amount of public goods provision as well as taxation is assumed to be determined by a kind of bargaining, especially Nash-bargaining. In more detail, we assume that both generations solve their own problems and determine the amount of public goods through bargaining. Subsequently, the government determines the public goods provision rule[22].

4.2.1 Optimal Choice of Young Individuals

First, the problem for young people is summarized as follows: The younger generation determines the amount of public goods to maximize the lifetime utility: the sum of utilities of younger and older people[23]. In summary, young people face the following problem:

$$\max_{g_t, \eta_t} \left\{ u(c_t) + v(1-l_t) + h(g_t) + \frac{1}{1+\rho}\{u(d_{t+1}) + h(g_{t+1})\} \right\} \qquad (\mathrm{IP}_{\mathrm{young}})$$
$$s.t. \ \ Eqs. \ (1').(3)$$

4.2.2 Optimal Choice of Old Individuals

Next, let us derive the amount of public goods for elderly persons. Note that elderly persons are myopic in the sense that they consider only their own utility in old age. In this case, we

[22] This type of bargaining is also adopted by Besley and Coate (2003).

[23] True of the old case that is discussed later, we specifically assess the amount of public goods, given the taxation, $\{\eta_t, \tau_t\}$.

can define the amount of public goods as the solution of the following problem:

$$\max_{g_t,\tau_t} \frac{1}{1+\rho}\{u(d_t)+h(g_t)\} \quad s.t.\ Eqs.\ (1'),(3) \tag{IP$_{\text{old}}$}$$

4.2.3 Determination of the Amount of Public Goods and the Tax Burden through Bargaining

As described in section 1, we do not adopt policy-making based on majority voting but rather that based on intergenerational bargaining. Especially, we adopt policy-making based on Nash-type bargaining[24]. Although situations in which bargaining does not progress are conceivable, we eliminate such cases from consideration in this paper. Each generation faces the following bargaining problem[25].

Let the bargaining power of young people and elderly persons at t period respectively represent α, $1-\alpha$ $(0<\alpha<1)$. Players maximize the following Nash products:

- Public goods : g_t

$$\bar{g_t} \equiv \arg\max_{g_t} \ (U(c_t,d_{t+1},1-l_t,g_t,g_{t+1}))^{\alpha}\cdot(U(d_t,g_t))^{1-\alpha} \tag{12}$$

- Tax burden : (τ_t,η_t)

$$(\bar{\eta_t},\bar{\tau_t}) \equiv \arg\max_{\eta_t,\tau_t} \ = (U(c_t,d_{t+1},1-l_t,g_t,g_{t+1}))^{\alpha}\cdot(U(d_t,g_t))^{1-\alpha} \tag{13}$$

Through transformation into logarithmic form, let us solve the following two problems:

$$\max_{g_t} \ \Phi_1 = \alpha\cdot\ln\{U(c_t,d_{t+1},1-l_t,g_t,g_{t+1})\}+(1-\alpha)\cdot\ln\{U(d_t,g_t)\} \quad s.t.\ (1'),(4'),(6), \tag{14}$$

and

$$\max_{\eta_t,\tau_t} \ \Phi_2 = \alpha\cdot\ln\{U(c_t,d_{t+1},1-l_t,g_t,g_{t+1})\}+(1-\alpha)\cdot\ln\{U(d_t,g_t)\} \quad s.t.\ (1'),(4'),(6). \tag{15}$$

Now, we are ready to present the following claim:

---

[24] As for the Nash Bargaining, see Binmore, Rubinstein and Wolinsky (1986). Note that the outside option of both elder generations and young persons is 0 under this setting.

[25] Before entering analysis, we explain the role of debt. Taxation through intergenerational bargaining does not necessarily cover the amount of public goods that is determined by bargaining. We introduce debt to cover that shortfall.

Proposition 2 $\{\bar{\tau}_t, \bar{\eta}_t, \bar{g}_t\}$ *meets the following properties.*

1. $\{\bar{\tau}_t, \bar{\eta}_t, \bar{g}_t\}$ *satisfies conditions of the politico-economic equilibrium.*
2. *Each solution achieved through intergenerational bargaining meets the following properties.*

$$\frac{\partial \bar{g}_t}{\partial \alpha} > 0, \qquad \frac{\partial \bar{\tau}_t}{\partial \alpha} > 0, \qquad \frac{\partial \bar{\eta}_t}{\partial \alpha} < 0 \tag{16}$$

Proof *See appendix A.2.* ∎

These second properties inform us that the amount of public goods provision and capital income tax increase as the bargaining power of young generation becomes stronger, while labor income tax decreases. The intuition behind these results is that, for the relationship between capital-income tax and bargaining power, as the bargaining power, α, becomes larger, which might imply that the voice of the younger generation becomes larger, they have an incentive to reduce the burden caused by labor income tax and impose a heavier burden on elderly persons; the older generation is compelled to bear a bigger burden because of the capital income tax, and it is the opposite. For the amount of public goods, the intuitive explanation is as follows: Because young persons live two periods and thereby live longer than the elderly group, they have more incentive to make use of public goods than do elderly persons.

Incidentally, is there a possibility that the solution derived by dynamic programming (DP) and the bargaining solution coincide? If so, the *time-inconsistency* problem is solvable by bargaining[26]. The following proposition answers such a question.

Proposition 3 *There exists "α" such that the solution derived by dynamic programming and that by intergenerational bargaining coincide. In other words, depending on the value of bargaining power, taxation derived through bargaining can meet the property of time-consistency.*

Proof *To prove this claim, we must show that these solutions through intergenerational bargaining coincide with the feedback solution. See also appendix A.3 for greater detail.* ∎

[26] Note that the DP solution meets the property of *time-consistency*.

4.3 Derivation of the Modified Samuelson Rule through Bargaining

In the previous subsection, we have investigated the properties of policy variables through bargaining. However, it is unlikely that households can determine all policy variables through bargaining. In this subsection, we then derive taxation rule based on intergenerational bargaining under the case that only the amount of public goods is determined through bargaining. Hereafter, we define that problem of government taking the discussion in the previous subsection into consideration. To sum up, the problem for the government is the following.

$$\begin{aligned} &\max \ \ \mathrm{SW} \\ &s.t. \ \ Eqs. \ (1'),(6),(30) \end{aligned} \tag{IP$_{\mathrm{G1}}$}$$

As in the previous section, we construct the following Lagrangian $\mathscr{H}$, setting $\{\kappa_t, \lambda_t, \phi_t\}$ as Lagrange multipliers:

$$\begin{aligned} \mathscr{H} = \sum_{t=0}^{\infty} \beta^t \Big[&\{u(c_t) + v(1-l_t) + h(g_t) + \frac{1}{1+\rho}\{u(d_{t+1}) + h(g_{t+1})\} \\ &+ \kappa_t \left\{ g_t - F(\hat{k}_t, l_t) + \tilde{R}_t k_t l_t + \tilde{w}_t l_t - b_{t+1} + \tilde{R}_t b_t \right\} + \lambda_t (R_t k_t l_t + w_t l_t - c_t - d_t - g_t - k_{t+1} l_{t+1}) \\ &+ \phi_t \{ \alpha \cdot \frac{h'(g_t) - u'(c_t) - v'(1-l_t)}{U(c_t, d_{t+1}, 1-l_t, g_t, g_{t+1})} + (1-\alpha) \cdot \frac{h'(g_t) - u'(d_t)}{U(d_t, g_t)} \} \Big] \end{aligned}$$

We then derive the following first-order conditions:

$$\begin{aligned} \tilde{w}_t \quad : \quad & -l_t u'(c_t) + h'(g_t) + \phi_t \left\{ \alpha \frac{h''(g_t)}{\{U(c_t, d_{t+1}, 1-l_t, g_t, g_{t+1})\}^2} + (1-\alpha) \frac{h''(g_t)}{\{U(d_t, g_t)\}^2} \right\} \\ & = \left(\kappa_t - \frac{w_t}{\tilde{w}_t} \lambda_t \right) l_t \end{aligned} \tag{17a}$$

$$\begin{aligned} \tilde{R}_t \quad : \quad & \frac{1}{1+\rho} u'(d_{t+1}) + \kappa_t (k_t l_t + b_t) + \lambda_t \frac{R_t}{\tilde{R}_t} k_t l_t \\ & + \phi_t \left\{ \alpha \frac{h''(g_t)}{\{U(c_t, d_{t+1}, 1-l_t, g_t, g_{t+1})\}^2} + (1-\alpha) \frac{h''(g_t)}{\{U(d_t, g_t)\}^2} \right\} = 0 \end{aligned} \tag{17b}$$

$$g_t \quad : \quad h'(g_t) + \kappa_t - \lambda_t + \phi_t \{ \alpha \frac{h''(g_t)}{\{U(c_t, d_{t+1}, 1-l_t, g_t, g_{t+1})\}^2} + \frac{(1-\alpha) h''(g_t)}{\{U(d_t, g_t)\}^2} \} = 0 \tag{17c}$$

$$b_t \quad : \quad -\kappa_t \tilde{R}_t + \frac{\kappa_{t-1}}{\beta} = 0 \tag{17d}$$

$$\hat{k}_t \quad : \quad \kappa_t (-R_t + \tilde{R}_t) + \lambda_t R_t - \frac{\lambda_{t-1}}{\beta} = 0 \tag{17e}$$

As in the previous section, we obtain the following conditions at the steady state.

$$(17a) \quad : \quad -lu'(c)+h'(g)+\left(\kappa-\frac{w}{\tilde{w}}\lambda\right)l+\phi\left\{\alpha\frac{h''(g)}{\{U(c,d,1-l,g,g)\}^2}+\frac{(1-\alpha)h''(g)}{\{U(d,g)\}^2}\right\}=0 \tag{18a}$$

$$(17b) \quad : \quad \frac{1}{1+\rho}u'(d)+\kappa(kl+b)+\lambda\frac{R}{\tilde{\tilde{R}}}kl+\phi\left\{\alpha\frac{h''(g)}{\{U(c,d,1-l,g,g)\}^2}+\frac{(1-\alpha)h''(g)}{\{U(d,g)\}^2}\right\}=0$$

$$(17c) \quad : \quad h'(g)+\kappa-\lambda+\phi\{\frac{\alpha h''(g)}{\{U(c,d,1-l,g,g)\}^2}+\frac{(1-\alpha)h''(g)}{\{U(d,g)\}^2}\}=0 \tag{18b}$$

$$(17d) \quad : \quad -\beta\kappa\tilde{R}+\kappa=0 \tag{18c}$$

$$(17e) \quad : \quad \kappa(-R+\tilde{R}-1)\ +\lambda R-\frac{\lambda}{\beta}=0 \tag{18d}$$

We define the solution by $\star$ as follows.

$$\{c_t^\star,d_{t+1}^\star,l_t^\star,\tilde{R}_t^\star,b_t^\star,\tilde{w}_t^\star,k_t^\star,g_t^\star,\kappa_t^\star,\lambda_t^\star,\phi_t^\star\}_{t=0}^{\infty}$$

We refer to the above solution as “modified public goods provision rule”. Then, we show the following claim:

Proposition 4 *The above modified optimal taxation to finance public goods in the long run is characterized as*

$$\frac{u'(c)}{h'(g)}+\frac{u'(d)}{h'(g)}=1+\mu_2\frac{u'(c)}{h'(g)}-\kappa(k+\frac{b}{l})-\lambda\frac{R}{\tilde{\tilde{R}}}k. \tag{19}$$

■Remark 4 The left side of the equation shown above (19) denotes the sum of the marginal rate of substitution between consumption and public goods. Regarding the right side of this equation, we can infer that this result reflects the following channels.

- the cost of public goods distribution, which corresponds to the first term of R.H.S. of eq.(19).
- Pigou effect – The sum effect of $(i)+(ii)$, which corresponds to the second and third term of R.H.S. of eq.(19).
  - (i) the negative effect of labor-income tax – distortion in labor supply
  - (ii) the negative effect of capital-income tax – distortion in saving
- the positive effect of public goods, which corresponds to the fourth term of R.H.S. of eq.(19).

Especially, the third effect has a positive effect on the utility of both young and elderly persons. Moreover, we can regard this result in terms of the Samuelson Rule, which means that $\sum_i \mathrm{MRS}_i = MRT$, where MRS and MRT respectively show the marginal rate of substitution and marginal rate of transformation.

Proof *From eq.* (18c)*, we obtain*

$$\beta \tilde{R} = 1.$$

Eliminating Φ *from eq.*(18a) *and eq.*(18b)*, we have*

$$lu'(c) + \frac{1}{1+\rho}u'(d) = h'(g) + \left(\kappa - \frac{w}{\tilde{w}}\lambda\right) l - \kappa(kl+b) - \lambda \frac{R}{\tilde{R}}kl$$

Dividing both sides of the above equation by $l \times h'(g)$*, we have*

$$\frac{u'(c)}{h'(g)} + \frac{1}{1+\rho}\frac{u'(d)}{h'(g)} = 1 + \left(\kappa - \frac{w}{\tilde{w}}\lambda\right) - \kappa(k + \frac{b}{l}) - \lambda \frac{R}{\tilde{R}}k \tag{20}$$

Here, using the following equations, those are the Eqs. (18a)*-(18c) as the first order conditions,*

$$u'(c) - \{\mu_1 - \mu_2 \tilde{w} l_t\} u''(c) - \lambda = 0 \tag{21a}$$

$$u'(d) - \lambda \tilde{R} u''(d) - \lambda = 0 \tag{21b}$$

$$\mu_2 u'(c) = \left(\kappa - \frac{w}{\tilde{w}}\lambda\right) l \tag{21c}$$

We can rewrite eq.(20) *as*

$$\frac{u'(c)}{h'(g)} + \frac{u'(d)}{h'(g)} = 1 + \mu_2 \frac{u'(c)}{h'(g)} - \kappa(k + \frac{b}{l}) - \lambda \frac{R}{\tilde{R}}k.$$

This concludes the proof. ∎

■Remark 5

1. In relation to this proposition, we can adopt the following perspective. Considering that the structure of intergenerational bargaining proceeds after the $t+1$ period, we can regard this model as having identical structure to an ordinary repeated game. Therefore, we can conclude that the cooperative solution is attainable by applying Kandori (1992)'s discussion. Thereby, the *time-consistent* solution is obtainable.
2. As discussed in Atkeson, Chari and Kehoe (1999), taxing capital income is not preferable from the viewpoint of efficiency. However, we showed that this way of fund-raising of public goods is justifiable from the viewpoint of intergenerational conflict.

4.4 Discussion

In this subsection, let us point out some remarks as for intergenerational bargaining proposed in this section.

■the Specific Example of Bargaining Power

Applying the concept of intergenerational bargaining to reality, bargaining power (α) can be regarded as the strength of the coalition within the congress in this paper or simply stated, the political power of young agents. More concretely speaking, in the congress (or for instance, the national diet), the bargaining power corresponds to the ratio of assembly members, to whom young persons give support. If the voter turnout of young persons is high, there is a greater possibility that the assembly members, to whom young persons lend support, can win an election. Therefore, from another perspective, the bargaining power also corresponds to the approval rating among young persons for the policy that the government determines.

■How can we discover the optimal value of bargaining power?

On the other hand, proposition 3 says that depending on the value of the bargaining power, the bargaining solution coincides with the DP solution. However, another question remains to be discussed as follows: How can we find the optimal value of α that satisfies such a property? Our proposal is as follows: If the government seeks to attain such a value of bargaining power, they should merely adjust the difference of preferences to the tax burden and the amount of public goods between elderly and young persons by introducing a proper policy that is supportable by young people.

5 Concluding Remarks

In this paper, we have presented analysis of dynamic optimal taxation and a public goods provision rule under the presence of intergenerational conflict by introducing a solution concept for such a conflict as an alternative to majority voting or lobbying. What we have shown in this paper is summarized as the following: First, taxation derived using the Lagrange Method fails to be *time-consistent*. Second, depending on bargaining-power, taxation based on intergenerational bargaining can be *time-consistent*. Third, we investigated properties of taxation and a public goods provision rule based on intergenerational bargaining.

Finally, we conclude this paper by pointing out possible directions for extending its analysis and implications. First, in Nash-bargaining, bargaining power is given exogenously; for that reason, we are compelled to endogenize it as a next step. Second, considering the existence of inequality related to ability, income, and labor supply, we must also analyze intragenerational conflict[27]. Finally, with regard to policy-making, we must also consider other political decision-making modalities including ideology, veto-power, rent-seeking, and so forth.

Appendix A

A.1 Proof of Proposition 1

■Proposition 1.1 First, we begin to prove Proposition 1.1. This fashion of proof is based on that of Kinai (2005). To prove this claim, we must demonstrate that the solution derived using the Lagrange method (open-loop solution) differs from that by dynamic programming (closed-loop solution). Next, we rewrite the maximization problem that the government should solve.

$$\max_{\{\tilde{w}_t, \tilde{R}_t, g_t\}} \sum_{t=0}^{\infty} \beta^t \left\{ u(c_t) + v(1 - l_t) + h(g_t) + \frac{1}{1+\rho} \{u(d_{t+1}) + h(g_{t+1})\} \right\}$$

Using Eq. (5), the budget constraints of the government can be written as: [28]

$$b_{t+1} = -F(s(\tilde{w}_{t-1}, b_t) - b_t, l(\tilde{w}_t, \tilde{R}_{t+1})) + \tilde{R}_t\{s(\tilde{w}_{t-1}, b_t) - b_t\} + \tilde{w}_t l(\tilde{w}_t, \tilde{R}_{t+1}) + \tilde{R} b_t + g_t. \tag{22}$$

Now, let us define q_t as follows by differentiating Eq. (22) with respect to b_{t+1}.

$$1 - (w_t - \tilde{w}_t) \frac{\partial l_t}{\partial \tilde{R}_{t+1}} \frac{\partial \tilde{R}_{t+1}}{\partial b_{t+1}} \equiv q_t \tag{23}$$

Hereafter, using q_t, we advance our calculation as follows: Using Eq. (5), the budget

[27] As an existing study that presents a similar viewpoint, see Hassler, Storesletten and Zilibotti (2007).

[28] This transformation is based on the fact that we can rewrite $s(\tilde{w}_{t-1}, b_t)$ as $F(s(\tilde{w}_{t-1}, b_t))$, using the production function, $F(k_t, L_t)$.

constraints of the government can be written as:

$$q_t \cdot \frac{\partial b_{t+1}}{\partial \tilde{w}_t} = l_t + (\tilde{w}_t - w_t)\left(\frac{\partial l_t}{\partial \tilde{w}_t} + \frac{\partial l_t}{\partial \tilde{R}_{t+1}}\frac{\partial \tilde{R}_{t+1}}{\partial b_{t+1}}\right) \tag{24a}$$

$$q_t \cdot \frac{\partial b_{t+1}}{\partial \tilde{R}_t} = s_t \tag{24b}$$

$$q_t \cdot \frac{\partial b_{t+1}}{\partial g_t} = 1 \tag{24c}$$

Next, we obtain the following equations by differentiating Eq. (22) with respect to two state variables: b_t and $\tilde{w}_{t-1}$.

$$q_t \cdot \frac{\partial b_{t+1}}{\partial b_t} = (\tilde{R}_t - R_t)\frac{\partial s_t}{\partial \tilde{R}_t}\frac{\partial \tilde{R}_t}{\partial b_t} + 1 \tag{24d}$$

$$q_t \cdot \frac{\partial b_{t+1}}{\partial \tilde{w}_{t-1}} = (\tilde{R}_t - R_t)\left[\frac{\partial s_t}{\partial \tilde{w}_{t-1}} + \frac{\partial s_t}{\partial \tilde{R}_t}\frac{\partial \tilde{R}_t}{\partial \tilde{w}_{t-1}}\right] \tag{24e}$$

Setting the value function as $\Omega(k_t, g_{t-1})$, we establish the Bellman Equation as follows.

$$\begin{aligned}\Omega_t(k_t, g_{t-1}) = \max_{\{\tilde{w}_t, \tilde{R}_t, g_t\}} \Big[&\{u(\tilde{w}_t l_t - s(\tilde{w}_{t-1}, b_t)) + v(1-l_t) + h(g_t)\} + \frac{1}{1+\rho}\{u(\tilde{R}_{t+1}s(\tilde{w}_{t-1}, b_t)) + h(g_{t+1})\} \\ &+ \beta\Omega_{t+1}(k_{t+1}, g_t)\Big] \\ &s.t.\ \ Eq.(22)\end{aligned} \tag{25}$$

From Eq. (25), we derive the first-order conditions with respect to $\tilde{w}_t$, $\tilde{R}_t$, and g_t.

$$q_t l_t u'(c_t) + \beta\frac{\partial \Omega_t(k_{t+1}, g_{t+1})}{\partial b_{t+1}}\left\{l_t + (\tilde{w}_t - w_t)\left(\frac{\partial l_t}{\partial w_t} + \frac{\partial l_t}{\partial l_t}\frac{\partial \tilde{R}_{t+1}}{\partial w_t}\right)\right\} = 0 \tag{26a}$$

$$q_t\frac{1}{1+\rho}\frac{\partial \tilde{R}_{t+1}}{\partial \tilde{R}_t}u'(d_{t+1}) + \beta\frac{\partial \Omega_{t+1}(g_{t+1}, \tilde{w}_t)}{\partial b_{t+1}}s_t = 0 \tag{26b}$$

$$q_t\frac{\partial \tilde{R}_{t+1}}{\partial g_t}\{h'(g_t) + \frac{1}{1+\rho}h'(g_{t+1})\} + \beta\frac{\partial \Omega_{t+1}(g_{t+1}, \tilde{w}_t)}{\partial g_{t+1}} = 0 \tag{26c}$$

Using the envelope theorem yields the Benveniste-Scheinkman condition as[29)]

$$q_t \cdot \frac{\partial \Omega_t}{\partial b_t} = q_t \left\{ -u'(c_t)\frac{\partial s_t}{\partial b_t} + \frac{1}{1+\rho}u'(d_{t+1})\tilde{R}_{t+1}\frac{\partial s_t}{\partial b_t} \right\} + \beta \frac{\partial \Omega_{t+1}}{\partial b_t} \left\{ (\tilde{R}_t - R_t)\frac{\partial s_t}{\partial \tilde{R}_t}\frac{\partial \tilde{R}_t}{\partial b_t} + R_t \right\} \tag{26d}$$

$$q_t \cdot \frac{\partial \Omega_t}{\partial \tilde{w}_{t-1}} = q_t \left\{ -u'(c_t)\frac{\partial s_t}{\partial b_t}\frac{\partial b_t}{\partial \tilde{w}_{t-1}} + \frac{1}{1+\rho}u'(d_{t+1})\tilde{R}_{t+1}\frac{\partial s_t}{\partial b_t}\frac{\partial b_t}{\partial \tilde{w}_{t-1}} \right\} + \beta \frac{\partial \Omega_{t+1}}{\partial b_{t+1}}(\tilde{R}_t - R_t)\left[\frac{\partial s_t}{\partial \tilde{R}_t} + \beta \frac{\partial s_t}{\partial \tilde{R}_t}\frac{\partial \tilde{R}_t}{\partial \tilde{w}_{t-1}} \right] \tag{26e}$$

Now we show that the solution derived in this manner differs from that obtained using Lagrange method. From Eq. (10c), we obtain

$$\beta \tilde{R} = 1. \tag{27}$$

Substituting Eq. (23), and Eqs. (26a)–(26d) into the above equations yields the following.

$$(23) \quad : \quad 1 - (w - \tilde{w})\frac{\partial l}{\partial \tilde{R}}\frac{\partial \tilde{R}}{\partial b} \equiv q \tag{28a}$$

$$(26a) \quad : \quad q \cdot l u'(c) + \beta \frac{\partial \Omega(b, \tilde{w})}{\partial b} \left\{ l + (\tilde{w} - w)\left(\frac{\partial l}{\partial w} + \frac{\partial l}{\partial \tilde{R}}\frac{\partial \tilde{R}}{\partial b} \right) \right\} = 0 \tag{28b}$$

$$(26b) \quad : \quad q \cdot \frac{1}{1+\rho}\frac{\partial \tilde{R}}{\partial \tilde{R}}u'(d) + \beta \frac{\partial \Omega(b, \tilde{w})}{\partial b}s = 0 \iff q\frac{1}{1+\rho}u'(d) + \beta \frac{\partial \Omega}{\partial b}s = 0 \tag{28c}$$

$$(26c) \quad : \quad q\{h'(g) + \frac{1}{1+\rho}h'(g)\} + \beta \frac{\partial \Omega}{\partial g} = 0 \tag{28d}$$

$$(26d) \quad : \quad q \cdot \frac{\partial \Omega}{\partial b} = q\left\{ -u'(c)\frac{\partial s}{\partial b} + \frac{1}{1+\rho}u'(d)\tilde{R}\frac{\partial s}{\partial b} \right\} + \beta \frac{\partial \Omega}{\partial b}\left\{ (\tilde{R} - R)\frac{\partial s}{\partial \tilde{R}}\frac{\partial \tilde{R}}{\partial b} + R \right\} \tag{28e}$$

From these equations, eliminating q and $\Omega(\cdot)$, we obtain

$$\frac{\partial \Omega}{\partial b} = q\left\{ -u'(c)\frac{\partial s}{\partial b} + \frac{1}{1+\rho}u'(d)\tilde{R}\frac{\partial s}{\partial b} \right\} + \beta \frac{\partial \Omega}{\partial b}\left\{ (\tilde{R} - R)\frac{\partial s}{\partial b} + R \right\}$$

$$\iff \quad q\left\{ -u'(c)\frac{\partial s}{\partial b} + \frac{1}{1+\rho}u'(d)\tilde{R}\frac{\partial s}{\partial b} \right\} + \frac{\partial \Omega}{\partial b}(1 - \beta R)(\frac{\partial s}{\partial b} - 1) = 0 \tag{29}$$

From Eq. (28c),

$$\frac{\partial \Omega}{\partial b} = -\frac{1}{\beta s}q\frac{1}{1+\rho}u'(d).$$

[29)] These equations can be derived through differentiation with respect to two state variables: b_t and $\tilde{w}_{t-1}$.

By substituting this equation into Eq. (29), we obtain

$$
\begin{aligned}
& \left\{-u'(c)+\frac{1}{1+\rho}u'(d)\tilde{R}\right\}\frac{\partial s}{\partial b}+\left\{-\frac{1}{\beta s}\frac{1}{1+\rho}u'(d)\right\}(1-\beta R)(\frac{\partial s}{\partial b}-1)=0 \\
\Longleftrightarrow \quad & \left\{-u'(c)+\frac{1}{1+\rho}u'(d)\tilde{R}-\frac{1}{\beta s}\frac{1}{1+\rho}u'(d)(1-\beta R)\right\}(-1)=\left\{-\frac{1}{\beta s}\frac{1}{1+\rho}u'(d)\right\}(1-\beta R) \\
\Longleftrightarrow \quad & u'(c)+\{\tilde{R}+\frac{1}{\beta s}(1-\beta R)\}\frac{1}{1+\rho}u'(d)=\left\{-\frac{1}{\beta s}\frac{1}{1+\rho}u'(d)\right\}(1-\beta R) \\
\Longleftrightarrow \quad & u'(c)+\tilde{R}\frac{1}{1+\rho}u'(d)=-2\left\{\frac{1}{\beta s}\frac{1}{1+\rho}u'(d)\right\}(1-\beta R)\neq 0.
\end{aligned}
$$

Thereby, we find that this equation does not necessarily coincide with Eq. (10g) because $\beta R=1$ is not satisfied. We find that Eq. (27) is not necessarily satisfied. This result means that the first-order conditions obtained using the Lagrange method do not coincide with those by DP: this proposition holds. This concludes the proof. ▮

■Proposition 1.2 We next proceed to prove proposition 1.2. First, we obtained paths of capital-income and labor-income taxes; $\{\tau_t\}_{t=0}^{\infty}$ and $\{\eta_t\}_{t=0}^{\infty}$ in the section 3.1. Now, let us derive taxation $\{\tau_t,\eta_t\}$ that maximizes the sum of the utility of younger and older generation by solving the following problem:

$$
\begin{aligned}
(\hat{\tau}_t,\hat{\eta}_t)=\max_{\tau_t,\eta_t}\ & [\theta\{u(c_t)+v(1-l_t)\}+h(g_t)+\{u(d_t)+h(g_t)\}] \\
& s.t.\ \ Eqs.(3),(1'),(7)
\end{aligned}
\tag{IP$_1$}
$$

In the first-term, the parameter θ is the relative weight that politicians attach to the well-being of the elderly. In the case of $\theta=1$, we can regard this allocation as that with intragenerational equal treatment. By solving the above problem with $\theta=1$, we can easily show $\hat{\tau}_t\neq\tau_t^*$, $\hat{\eta}_t\neq\eta_t^*$. ▮

A.2 Proof of Proposition 2

■Proposition 2.1 To prove this claim, we show that $\{\bar{g}_t,\bar{\tau}_t,\bar{\eta}_t\}$ meet conditions of politico-economic equilibrium (*i*, *ii*, and *iii*).

(i) Because $\{\bar{g}_t,\bar{\tau}_t,\bar{\eta}_t\}$ are also a solution of both generations, this solution meets the first condition.
(ii) Because this solution is an aggregation, the second condition holds.
(iii) Clearly, each market-clearing condition holds.

Therefore, we find that this solution meets the three conditions. ∎

■Proposition 2.2 First, for public goods, let us define H_g as

$$H_g \equiv \frac{\partial \Phi_1}{\partial g_t} = \alpha \cdot \frac{h'(g_t) - (v'(1-l_t) + u'(c_t))}{U(c_t, d_{t+1}, 1-l_t, g_t, g_{t+1})} + (1-\alpha) \cdot \frac{h'(g_t) - u'(d_t)}{\{u(d_t) + h(g_t)\}} = 0. \tag{30}$$

By applying the implicit function theorem, we obtain the following:

$$\frac{\partial \bar{g}_t}{\partial \alpha} = -\left.\frac{\frac{\partial H_g}{\partial \alpha}}{\frac{\partial H_g}{\partial g_t}}\right|_{\text{other variables constant}} > 0.$$

Next, let us turn to the bargaining problem with respect to taxation. Define H_τ and H_η as

$$H_\tau \equiv \frac{\partial \Phi_2}{\partial \tau_t} = \alpha \frac{-u'(d_t) + R_{t+1} s_t h'(g_t)}{U(c_t, d_{t+1}, 1-l_t, g_t, g_{t+1})} \cdot \frac{\partial R_t}{\partial \tilde{R}_t} + (1-\alpha) \frac{R_{t+1} s_t h'(g_t)}{\{u(d_t) + h(g_t)\}} \cdot \frac{\partial R_t}{\partial \tilde{R}_t} = 0 \tag{31a}$$

$$H_\eta \equiv \frac{\partial \Phi_2}{\partial \eta_t} = \alpha \frac{-u'(c_t) + w_t l_t h'(g_t)}{U(c_t, d_{t+1}, 1-l_t, g_t, g_{t+1})} \cdot \frac{\partial w_t}{\partial \tilde{w}_t} + (1-\alpha) \frac{w_t l_t h'(g_t)}{u(d_t) + h(g_t)} \cdot \frac{\partial w_t}{\partial \tilde{w}_t} = 0. \tag{31b}$$

From these equations, by applying the implicit function theorem, we then have the following result:

$$\frac{\partial \bar{\tau}_t}{\partial \alpha} = -\left.\frac{\frac{\partial H_\tau}{\partial \alpha}}{\frac{\partial H_\tau}{\partial \tau}}\right|_{\text{other variables constant}} > 0,$$

$$\frac{\partial \bar{\eta}_t}{\partial \alpha} = -\left.\frac{\frac{\partial H_\eta}{\partial \alpha}}{\frac{\partial H_\eta}{\partial \eta}}\right|_{\text{other variables constant}} < 0.$$

∎

A.3 Proof of Proposition 3

We then show that the solutions derived by intergenerational bargaining coincide with those obtained by dynamic programming, depending on the value of α. Let us compare Eqs. (30)–(31) with Eqs. (25a)–(25e). Then, let us interlock Eq. (30) and Eq. (25c) and define that interlocked equation as $F(\alpha; k_t, k_{t+1})$. That is,

$$F(\alpha; k_t, k_{t+1}) \equiv \text{the R.H.S. of eq.(30)} - \text{the R.H.S. of eq. (25c)}.$$

We then obtain the following using the assumption that α is within the interval, $(0,1)$. Thereby, we have

$$F(0)\cdot F(1) < 0. \tag{32}$$

Using the Intermediate Value Theorem, we then find that this equation has at least one solution within the interval, $\alpha \in (0,1)$. Therefore, this policy through bargaining is *time-consistent*, depending on the value of α. Regarding the tax burden, $\{\bar{\tau}_t, \bar{\eta}_t\}$, we can obtain the same conclusion similarly. ∎

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