GCOE Discussion Paper Series

Global COE Program

Human Behavior and Socioeconomic Dynamics

Discussion Paper No.186

Growth or Welfare State?
Optimal Composition of Government Expenditure?

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March 2011

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Growth or Welfare State? Optimal Composition of Government Expenditure*

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March 29, 2011
<Preliminary>

Abstract

In developing countries, a government must often simultaneously promote growth-enhancing and generational distribution policies. However, under most circumstances, it is difficult for any government to carry out both policies. Therefore, it is unavoidable to determine which policy should be carried out and then perform allocation of tax revenue through political compromise. This paper incorporates both a pay-as-you-go type pension and public investment into an overlapping generations model and shows how the contents of economic policy vary under a situation in which the policy determination is dependent on political issues.

Keywords: Commitment; Growth-enhancing vs. Redistribution Scheme; Overlapping Generations Model; Structure-induced Equilibrium.

JEL Classification: E61, H54, H55.

*The author is thankful to Kazuo Mino and Masatsugu Tsuji for their variable suggestions. Comments from Akihiko Kaneko, Gregory Gilpin, Hisakazu Kato, Nobuo Akai, Keigo Kameda, Hiroki Tanaka, Takashi Shimizu and the seminar participants at PET10 held at Boğaziçi University, Kansai Public Economics Seminar, The 16th Decentralization Conference, and JIPF meeting are also greatly acknowledged. The usual disclaimer applies.

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1 Introduction

Traditionally, for analyzing economic policies such as fiscal or monetary policy, economists have assumed that the government is a monolithic organization that is intended mainly to maximize social welfare or raise the economic growth rate. In this regard, the following points must be emphasized: First, various entities (voters, bureaucrats, representatives, organizations, etc.) that are involved in policy determination foster conflicts. Therefore, it is impossible for a government to organize a policy based on only one perspective or position. In other words, no government can avoid determining a policy that incorporates implications of numerous opinions. Second, even if a policy were derived that maximizes social welfare or the growth rate, carrying out such a policy with certainty would be difficult: it is difficult to commit to such a policy. Some conflicts exist even within actual governments. As one example, the Ministry of Finance and the Ministry of Health, Labour and Welfare in Japan engage in frequent conflicts. These two organizations have entirely different objectives: The former seeks to decrease the deficit or debt, although the latter is responsible for promoting the nation's social security system, even if it is very costly. Consequently, a contraposition of duties pertains between the two organizations. For that reason, we cannot regard actual governments as monolithic organizations, as many economists have done. Nevertheless, many studies have been made under the assumption that policy variables are one-dimensional. Given the existence of such conflicts, "political compromise" is unavoidable. It is difficult to maintain the government commitment entirely. As countermeasures for such a situation, it is apparent that it is necessary *ex post* to carry out some coordination policy to attenuate inefficiencies that result from discretionary policy. Then, as a source of such conflicts, we can point out that the policy determination is not necessarily one-dimensional but is often multi-dimensional. Therefore, it is necessary to expand the past analysis into multi-dimensional policy determination. This paper specifically presents consideration of the situation in which the government has policy options of two kinds: pension and public investment.

As described in this paper, we specifically consider a situation in which there exist two kinds of committee in the government: one related to public investment and one related to the pension system. Under such circumstances, how are policy contents determined? This paper describes an attempt to answer the question. This paper presents analysis of how the contents of the policy vary over time, assuming that policy is determined based on the results of voting.

Then, we consider the case of Japan. Figure 1 depicts the flow of the proportion of pension and public investment to gross domestic production[1)]. From this figure, it is apparent that the cost of pensions has been increasing, while the cost of public investment remains unchanged. One purpose of these analyses is to give an explanation of the fact that the cost of pensions has been increasing rather than public investment, as in Japan. Why is the cost of the one policy higher than the other? This paper describes that a pension is politically more favored than public

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1) Data Source:
Social Security: "the costs of social security" given by the System of National Accounts Statistics: SNA *National Institute of Population and Social Security Research.*
Public Investment (Government gross fixed capital formation): SNA

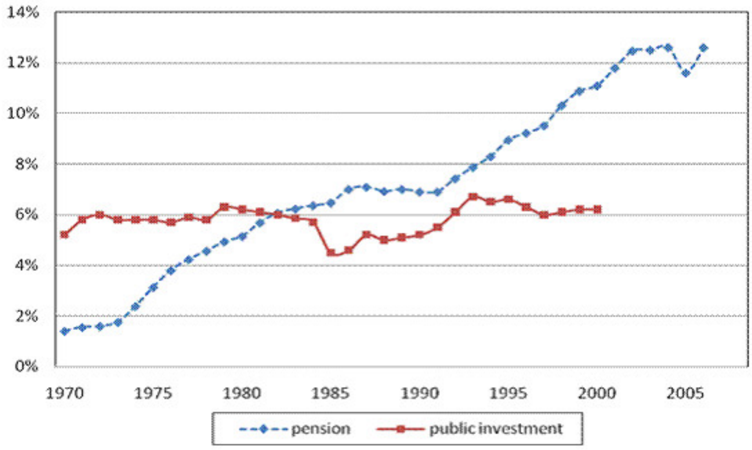

Figure 1 Flow of the proportion of pension and public investment to gross domestic production in JAPAN.

investment as a result of capital accumulation.

■Relation with Past Studies First, we explain the position of the present paper in the literature. Since the seminal work in this area such as that of Barro (1990), Futagami, Morita and Shibata (1993) or Alesina and Rodrik (1994), many studies have analyzed the relation between public investment and economic growth[2)]. However, few studies have analyzed the relation in an overlapping generations (hereinafter, OLG) model. As starting studies, we can point out Pestieau (1974) or Yoshida (1986). Since their studies, some others have been conducted: Among others, for instance, Yakita (1994) and Burgess (2006) investigate the relation between the return of public investment and the discount factor. Glomm and Ravikumar (1997) introduce public investment into an OLG model in a framework of endogenous growth model. Recently, Kaas (2003) incorporates Majority Voting related to policy determination and shows the existence of cyclical equilibria as for the tax rate. However, policy determination of his model is one-dimensional. Moreover, Hung (2005) specifically examines the monetary aspect by introducing seigniorage. More recently, Yakita (2008) expands two-dimensional policy determination by incorporating not only public investment but also maintenance activity into an OLG model, yet he does not consider political issues related to the policy determination.

Regarding studies that analyze pension policy in an OLG model, there exist numerous efforts, but our survey is limited to analysis of a context of political economy[3)]. For instance, we can point out Casamatta, Cremer and Pestieau (2000), Wigger (1999), Razin, Sadka and Swagel (2002), Aíbo, Mahieu and Patxot (2004), Boldrin and Rustichini (2000), and so forth. What is common to these studies is that the policy determination is one-dimensional, although the analyses in those earlier papers resemble those of this paper in the respect that policy determination is conducted through voting.

As explained above, there exist many studies that analyze the effect of pension or public investment in an OLG model. However, to the best of our knowledge, few reports describe incorporation of both pensions and public

2) For a survey, see Irmen and Kühnel (2008).

3) For a survey, see Galasso and Profeta (2002).

investment into an OLG model[4]. As an exception, we can point out Maebayashi (2010) who introduces a pension into the model of Yakita (2008), but his result is limited to the corner solution. In fact, Maebayashi shows that the government allocates total tax revenue to public investment if they aim to maximize the growth rate. Regarding other studies, although Creedy, Li and Moslehi (2008) expands past studies into two-dimensional policies (pension and public goods provision), they limit the analysis to that of the balanced growth path.

In contrast to those studies, the features of this study are summarized as follows: First, we expand policy determination to two-dimensional policy determination by introducing a pension into the model of Kaas (2003), who incorporates public investment into an OLG model. Second, we regard public capital as a stock variable, whereas Kaas treats it as a flow variable. That is because we specifically examine the transitional path as well as balanced-growth-path. Third, this paper considers political issues related to policy determination, which differs from Yakita (2008). Additionally, we note that the model of this paper does not apply ordinal voting theory because policy determination in our model is two-dimensional. We resolve such a difficulty using the concept of structure-induced equilibrium developed by Shepsle (1979).

In summary, we demonstrate the following:

1. Under the case without political processes, the growth-rate-maximizing tax rate does not coincide with the welfare-maximizing one.
2. On the other hand, under the case with political processes, there are three cases, in which both pension and public investment survive or either of the two policies prevails.

The first result means that the result of Barro (1990) does not hold in an overlapping generations economy with public stock. More importantly, the second result means that we can show the possibility that only a pension or public investment is politically supported.

The remainder of this paper is structured as follows. In section 2, we set up the model; then we analyze optimal taxation in the case in which there is no political issue in the section 3. In section 4, we analyze the situation in which policy determination is dependent on the voting behavior. Final remarks are presented in section 5.

2 The Model

We employ the Diamond (1965)-type two-period overlapping generations model without a bequest motive in a closed economy. There is population growth, i.e. $N_{t+1} = (1+\mu)N_t$, by which μ can be both positive and negative. Time is discrete and goes to infinity. We incorporate social security policy into the model of Kaas (2003). Therefore, the government has policies of two kinds: pension and public investment. We introduce the heterogeneity of households: households vary depending on the labor productivity. We also consider political issue related to the policy determination as in Kaas (2003). The difference between Kaas (2003) and the analyses

[4] However, there exist some studies particularly addressing the combination of a pension and education. For instance, see Naito (2009), Boldrin and Montes (2005), Lambrecht, Michel and Vidal (2005), and Kaganovich and Zilcha (1999). These studies specifically examine the combination of Backward Intergenerational Goods (BIG) and Forward Intergenerational Goods (FIG) using the terminology presented in Rangel (2003).

presented in this paper is that we expand his analysis into two-dimensional policy determination. In what follows, we consider the case in which there is no political issue (voting behavior) in section 3 as a benchmark, while in section 4, we consider the case in which there are political issues.

2.1 Behaviors

■Households Hereinafter, we call households created in t period as generation t. Generation t whose labor productivity is l_t^i solves the following problem:

$$\max_{s_t^i} \; U(c_t^{yi}, c_{t+1}^{oi}), \tag{1}$$

where c_t^y and c_{t+1}^o denote cosumption in young and old period, respectivley. Here, we impose some assumptions on the utility function, following Kaas (2003) or Kaas and von Thadden (2003). First, $U(\cdot)$ is twice differentiable with respect to each variable, homothetic, strictly increasing, and quasi-concave. Second, with respect to each variable, the utility function is homogeneous to the degree of $1-\gamma$, in which $\gamma \in [0,1]$ denotes the degrees of relative risk aversion ($\gamma \equiv -\dfrac{cu''}{u'}$). Third, consumption in the young period and old period are substitutes.

Taxes of two kinds exist: a pension tax (τ_t) and public investment tax (θ_t), which are imposed on both labor income and saving. Households formed at period t allocate after-tax labor income ($\tilde{w}_t$) to consumption (c_t^y) and savings (s_t), and consume (c_{t+1}^o) as after-tax savings and pension (d_t) when they are old. Consequently, the young-period and old-period budget constraints are represented respectively as

$$c_t^{yi} + s_t^i = \tilde{w}_t l_t^i, \quad c_{t+1}^{oi} = \tilde{R}_{t+1} s_t^i + d_{t+1}, \tag{2}$$

where $\tilde{w}_t \equiv (1-\tau_t-\theta_t)w_t$ and $\tilde{R}_{t+1} \equiv (1-\tau_{t+1}-\theta_{t+1})R_{t+1}$. Each household determines its own saving by solving the following problem.

$$\max_{s_t^i} \; U(-s_t^i + \tilde{w}_t l_t^i, \tilde{R}_{t+1} s_t^i + d_{t+1})$$

By solving the equation presented above, the saving rate ($s_t(\cdot)^i$) is dependent solely on $\tilde{R}_{t+1}$, and the saving function is denoted as the multiplicity that is separable as follows:[5].

$$s_t^i = s^i(\tilde{R}_{t+1})\tilde{w}_t l_t^i \tag{3}$$

By substituting this into the utility function, the indirect function is denoted as

$$V(\cdot) = U(1-s_t^i(\tilde{R}_t), s_t^i(\tilde{R}_t)\tilde{R}_{t+1})\tilde{w}_t^{1-\gamma}. \tag{4}$$

The following equation holds because consumption in the young and old period is assumed to be substituted as

$$\frac{\partial s(\cdot)}{\partial \tilde{R}_{t+1}} \geq 0. \tag{5}$$

We then assume that the economy is *dynamically efficient*, i.e. the following equation holds:

Assumption 1

$$R_t > 1 + \mu. \tag{6}$$

[5] Regarding the reason why the saving rate is dependent only on R_{t+1}, not w_t, see De La Croix and Michel (2002, pp.53–54). In the case in which the utility function is homothetic, the saving function can be denoted as multiplicity separable.

This equation means that the return of saving is larger than that of pension and the condition, which eliminates the trivial situation in which all households support a pension system.

■The Government Next, we examine government behavior. They have schemes of two kinds: a public investment scheme (growth-enhancing scheme) and the PAYG-type pension system (intergenerational redistribution scheme). They determine which policy should be carried out, or the allocation of tax revenue based on voting. The budget constraint of each scheme is balanced in each period. Tax revenue of public investment and pension respectively as E_t and M_t. The policy variables are θ_t (contribution to public investment) and τ_t (contribution to pension). Both schemes are covered with labor-income and capital-income taxes. In what follows, we normalize the total labor input as 1, i.e. $\sum_i l_t^i \equiv L_t = 1$.

Growth-enhancing scheme:

The government covers the cost of public investment with capital-income and labor-income taxes as in Barro (1990), although our case is a stock variable, not a flow variable. We assume that public investment has no congestion effect. Therefore, the budget constraint is written as presented below.

$$E_t = \theta_t Y_t = \sum_i \theta_t w_t l_t^i + \theta_t R_t s_{t-1} = \theta_t (w_t L_t + R_t s_{t-1}) = \theta_t (w_t + R_t s_{t-1}) \tag{7}$$

Therein, G_t and E_t respectively denote public capital stock and public investment. The evolution of public investment is written as

$$G_{t+1} - (1-\delta_G) G_t = E_t. \tag{8}$$

Arranging the above equation into per-capita terms, we have

$$(1+\mu) g_{t+1} = e_t + (1-\delta_G) g_t, \tag{9}$$

where δ_g denotes the depression rate of public capital.

Intergenerational redistribution scheme:

Next, we move to the explanation of pension as a role of intergenerational redistribution. Here, we assume that the pension is pay-as-you-go type and Beveridgean, which means that the amount of pension received in old age is independent of the labor productivity[6]. In the aggregate, noting that the pension system is a pay-as-you-go type, the budget constraint of this scheme is written as shown below.

$$N_{t-1} d_t = \tau_t Y_t = \tau_t R_t s_{t-1} + \sum_i \tau_t w_t l_t^i = \tau_t (R_t s_{t-1} + w_t L_t) = \tau_t (R_t s_{t-1} + w_t) = M_t \tag{10}$$

Dividing both sides of the above equation with N_t, we obtain

$$\underbrace{d_t}_{\text{pension received by households}} = (1+\mu) \underbrace{\tau_t (w_t + R_t s_{t-1})}_{\text{contribution}} \equiv M_t. \tag{11}$$

Merging the two schemes, the budget constraint of the government is expressed as

$$E_t + M_t = (1+\mu)(\theta_t + \tau_t)(w_t L_t + R_t s_{t-1}) = (1+\mu)(\theta_t + \tau_t)(w_t + R_t s_{t-1}).$$

[6] There are pensions of two types: Bismarckian and Beveridgean. Regarding the difference between Bismarckian and Beveridgean types, see also Casamatta et al. (2000), or Conde-Ruiz and Profeta (2007).

■Firms We then describe the firms' behavior. We assume that factor markets are perfectly competitive and that firms maximize their profits. We herein specify the production function as follows:

$$Y_t = F(K_t, A_t L_t) = K_t^{\alpha}(A_t L_t)^{1-\alpha}.$$

Here, we specifically define A_t as

$$A_t \equiv a\left(\frac{K_t^{\beta} G_t^{1-\beta}}{L_t}\right), \ \beta \in [0,1] \tag{12}$$

where G_t denotes public capital. Presented as a narrative in Yakita (2008) and Kaas (2003), we can state the assumption that public investment has a positive externality in the sense that it takes up the marginal labor productivity. The model of Kaas (2003) corresponds to the case of $\beta = 0$ in eq. (12). Using this definition, the production function is rewritten as

$$Y_t = K_t^{\alpha+\beta(1-\alpha)} G_t^{(1-\beta)(1-\alpha)} = K_t^{\omega} G_t^{1-\omega}, \tag{13}$$

where $\omega \equiv \alpha + \beta(1-\alpha) \in (0,1)$. Firms are assumed to solve the following problem.

$$\max \ \Pi = F(K_t, A_t L_t) - R_t K_t - w_t L_t$$

Therein, K_t, L_t, and Y_t respectively denote capital, labor, and gross output. We define $k_t \equiv \frac{K_t}{A_t L_t}$, and $F(\frac{K_t}{A_t L_t}, 1) = f(k_t)$. Then, by solving the profit maximization problem presented above, we can obtain the following.

$$R_t = A_t f'(k_t) = \alpha\left(\frac{K_t}{G_t}\right)^{\omega-1} \equiv R(\theta_t) \tag{14a}$$

$$w_t = A_t\{\underbrace{f(k_t) - k_t f'(k_t)}_{w(k_t)}\} = (1-\alpha)\left(\frac{K_t}{G_t}\right)^{-\omega}\left(\frac{G_t}{L_t}\right) \equiv w(\theta_t), \tag{14b}$$

These two factor prices are functions of public investment tax because G_t is financed with θ_t. Taking eqs. (14b), (7), (12), and (8) into consideration, we have

$$w(k_t;\theta) = \frac{1}{a\theta_t}\left(\frac{G_t}{K_t}\right)^{\beta}. \tag{15}$$

Here, taking τ_t and θ_t as given, $F(\cdot)$ can be rewritten as

$$F(K_t, A_t L_t) = A_t L_t f(k_t) = \frac{f(k_t)}{k_t} K_t. \tag{16}$$

Therefore, we can rewrite the production function of $F(\cdot)$ in the form of an AK-type production function, taking the policy variables as given. Here, $\frac{f(k_t)}{k_t}$ is decreasing in k_t.

Moreover, net labor income can be written in the form of a linear function of capital:

$$w_t(1-\tau_t-\theta_t)L_t = \tilde{w}_t L_t \frac{w(k_t)}{k_t}(1-\tau_t-\theta_t)K_t = \Psi(\theta_t;\tau_t)(1-\tau_t-\theta_t)K_t, \tag{17}$$

where $\Psi(\theta_t;\tau_t) \equiv \dfrac{1}{a\tau_t k(\theta_t)}\left(\frac{G_t}{K_t}\right)^{\beta}$, and τ_t is given. Moreover, we assume that $\Psi(\cdot)$ satisfies the following:

Assumption 2 There exists only one solution, θ^* that maximizes $\Psi(\theta_t;\tau_t)$.

This assumption gives the condition that ensures the existence of the solution.

2.2 Market Equilibrium

Finally, we describe the equilibrium condition of each market.

(1) Capital Market

In aggregate terms,

$$\sum_i s_t^i(\cdot)\tilde{w}_t l_t^i = K_{t+1} \tag{18}$$

From eq. (17), considering the saving rate $s_t(\tilde{R}_t)$, we have the following.

$$K_{t+1} = \sum_i s^i(R(\tau_{t+1}))\Psi(\theta_t;\tau_t)(1-\tau_t-\theta_t)K_t \tag{19}$$

Transforming the above equation into per-capita terms, we obtain

$$k_{t+1} = \bar{s}(R(\theta_{t+1}))\Psi(\theta_t;\tau_t)(1-\tau_t-\theta_t)k_t. \tag{20}$$

Here, $\bar{s}(\tilde{R}(\theta_{t+1}))$ denotes the average saving rate.

To summarize, the capital market clearing condition is written as

$$k_{t+1} = \frac{1}{1+\mu}\underbrace{s(\tilde{R}(\theta_{t+1})}_{\text{saving rate}}\tilde{w}(k_t)l_t^i. \tag{21}$$

(2) Goods Market

In the aggregate, we can state this condition as follows.

$$c_t^y N_t + c_t^o N_{t-1} + (K_{t+1} - K_t) + (G_{t+1} - (1-\delta_G)G_t) + M_t = Y_t$$

Dividing both sides of the above equation with N_t yields the expression shown below.

$$c_t^y + \frac{c_t^o}{1+\mu} + m_t + (1+\mu)(k_{t+1} + g_{t+1}) = R_t k_t + w_t l_t + (1-\delta_G)g_t \tag{22}$$

(3) Labor Market

Denoting labor demand as L_t, the condition is

$$N_t = L_t. \tag{23}$$

Here, we describe the definition of the competitive equilibrium.

Defnition 1 *Taking K_0 and G_0 as given, we define $\{c_t^y, c_t^o, s_t, k_t, l_t, R_t, w_t, g_t, \theta_t, \tau_t\}$ as a competitive equilibrium such that*

1. *For all t, taking $\{R_t, w_t, \theta_t, \tau_t\}$ as given, the condition of utility maximization for generation t holds.*
2. *For all t, taking $\{\tau_t, \theta_t\}$ as given, the condition of profit maximization holds.*
3. *Taking $\{c_t^y, c_t^o, s_t, k_t, l_t, R_t, w_t\}$ as given, $\{G_t, E_t, M_t, \theta_t, \tau_t\}$ meet the budget constraint of the government.*
4. *All markets clear.*

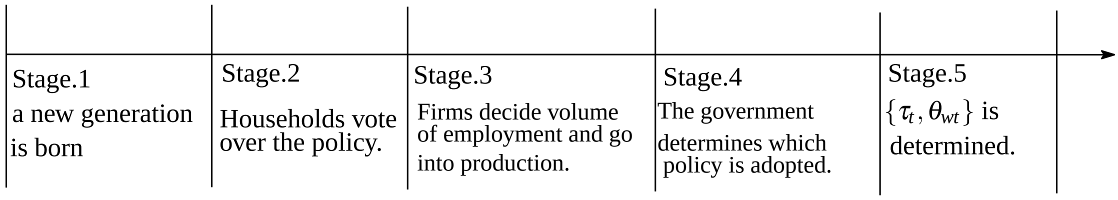

Figure 2 Sequence of Decision Making in the t-th period.

2.3 Timing of Decision Making

The timing of decision-making in t period is summarized as follows, which is also depicted in Fig. 2.

Stage 1. Households are born in t period.

Stage 2. Households vote over the two policy variables: contributions to the pension τ_t and that to public investment θ_t during the young period.

Stage 3. Firms produce.

Stage 4. Based on the voting results, the government determines the allocation of tax revenue (or which scheme the government admits).

Stage 5. A new generation $t+1$ is born in the next period.

3 Policy Determination without Political Process

In this section, we assume the case in which there is no political issue as a benchmark. In other words, we treat the policy determination as a solution of maximization problem. We consider the case in which the objective function of each scheme is the growth rate and social welfare.

3.1 Growth-Rate Maximizing Tax Rate

In what follows, we limit our analysis to the Balanced Growth Path. Here we seek the growth rate at a balanced growth path. The growth rate of each variable is the following:

Substituting the saving function, eq. (3) into the capital market-clearing condition, eq. (**??**), and using eqs. (14b), (8),(7), we have the following.

$$\frac{K_{t+1}}{K_t} = \frac{s(\tilde{R}(\tau_{t+1}))\Psi(\tau,\theta)(1-\tau-\theta)}{K_t} \tag{24a}$$

$$\frac{G_{t+1}}{G_t} = (1-\delta_G) + \frac{\tau_t w_t L_t}{G_t} \tag{24b}$$

Defining $\gamma = \frac{\frac{K_{t+1}}{N_{t+1}}}{\frac{K_t}{N_t}} = \frac{\frac{G_{t+1}}{N_{t+1}}}{\frac{G_t}{N_t}} = \frac{\frac{Y_{t+1}}{N_{t+1}}}{\frac{Y_t}{N_t}}$, then the intersection of the above two equations (24a) and (24b) is BGP.

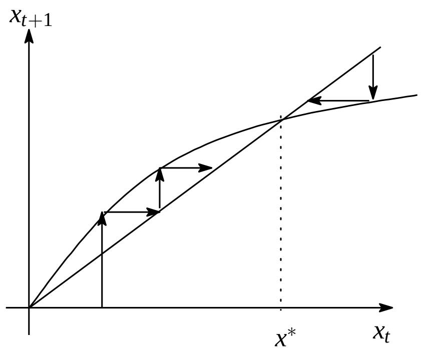

Figure 3 Determination of the Balanced Growth Path.

Letting x_t be $\dfrac{K_t}{G_t}$, we then must investigate the sign of $|\frac{dx_{t+1}}{dx_t}|$ and its slope. The two equations above yield

$$x_{t+1} = \frac{s(\tilde{R}(\tau_{t+1}))\Psi(\tau,\theta)(1-\tau-\theta)}{(1-\delta_G)+\frac{\tau_t w_t L_t}{x_t}}. \tag{25}$$

By differentiating with respect to x_t, we then have

$$\frac{dx_{t+1}}{dx_t} > 0,$$

and

$$\lim_{x_t\to\infty}\frac{dx_{t+1}}{dx_t} = \infty.$$

Therefore, we then can state this result in the form of a lemma.

Lemma 1 *There exists at least one BGP.*

Then, by differentiating BGP with respect to τ and $theta$, we can derive the growth-rate-maximizing tax. We define τ and θ as τ^* and θ^* such that

$$\frac{d\gamma}{d\tau} = 0 \tag{26}$$

$$\frac{d\gamma}{d\theta} = 0 \tag{27}$$

3.2 Welfare-Maximizing Tax Rate

Define the objective function as follows:

$$V^i(R_t) = s^i(\tilde{R}_{t+1})\tilde{w}_t^{1-\gamma} \tag{28}$$

$$\frac{\partial V(\cdot)^i}{\partial\theta} = 0 \tag{29}$$

$$\frac{\partial V(\cdot)^i}{\partial\tau} = 0 \tag{30}$$

By solving the above equations, we can derive the welfare-maximizing tax τ^{**}, and θ^{**}.

Comparison of Tax Rates Then, let us investigate the relation between the growth-rate maximizing and welfare-maximizing tax rate. The following proposition answers such a question:

Proposition 1 *The relation between the growth-rate-maximizing and social-welfare maximizing tax rate is given as shown below.*

$$\tau^* \neq \tau^{**}, \quad \theta^* \neq \theta^{**}$$

Proof *See Appendix.* ∎

Intuitively, the reason is explained as follows: They try to allocate the tax revenue to the greatest extent possible to public investment, which has a role of promoting economic growth if the government aims to maximize the growth rate. Therefore, the growth-rate maximizing tax is higher than the welfare-maximizing one.

This result differs from that of Barro (1990). In Barro (1990), he claimed that the tax rate that maximizes the growth rate equals that which maximizes social welfare. However, this result shows that his claim is not robust.

4 Policy Determination with Political Process

4.1 Equilibrium Concept and Some Assumptions

Given the discussion in the previous section, we advance the analysis by endogenizing policy determination. In this section, we introduce the political issue (i.e. voting behavior). Two policy variables exist in our model. Generally, *no* Condorcet winner exists in voting over multiple issues such as a combination of policy of two kinds, without imposing additional conditions on voter preference[7]. To avoid such a problem, following Conde-Ruiz and Galasso (2005), we adopt the concept of a structure-induced equilibrium developed by Shepsle (1979)[8].

We consider the following situation. There exist committees of two kinds: a committee that determines the contribution to pension and one that determines the contribution to public investment. The preferences to each policy of committee member are the same as those of voters. The policy determination itself is achieved independently, and that policy determination is based on the other policy determination. This situation can be regarded as the state in which there exist the following two reaction functions.

$$\begin{cases} \tau = \tau(\theta) & \text{: taking } \theta \text{ as given.} \\ \theta = \theta(\tau) & \text{: taking } \tau \text{ as given.} \end{cases}$$

We regard the intersection of the above two response functions as a (politico-economic) equilibrium.

We then consider the voting behavior related to determination of contributions to a pension and public investment. We assume here that

A 1. Voting is conducted in each period, which means issue-by-issue voting under direct democracy.

[7] Regarding this issue, see Persson and Tabellini (2000), for instance.

[8] Regarding studies of those who employ the structure-induced equilibrium, see Table 1. This paper differs from those studies in the sense that they specifically examine the combination of income redistribution schemes, whereas this paper specifically examines the combination of social security policy and other kinds of economic policy.

Table1 Past studies that use the concept of structure-induced equilibrium.

Conde-Ruiz and Galasso (2005)	Class of Social Security
	PAYG-type pension vs. Redistribution Policy
Poutvaara (2006)	Class of Social Security
	Pension vs. Public Education
Conde-Ruiz and Profeta (2007)	Type of Pension
	Bismarckian vs. Beveridgean
Konishi (2008)	Financial Resource of Social Security
	Consumption Tax vs. Labor-income Tax
Bethencourt and Galasso (2008)	Class of Social Security
	Pension vs. Health Investment
Kinai (2008)	Class of Social Security
	PAYG-type Pension vs. Unemployment Insurance
This Paper	Class of Economic Policy
	Public Investment vs. PAYG-type Pension

A 2. Voting on contributions to pension and public investment takes place *simultaneously*.

A 3. Voters are young and old people who co-exist in the same period.

A 4. Policy determination is based on the median voter theorem.

A 5. Voting is repeated among successive generations of voters.

Before entering into the analyses, we must present the following lemma:

Lemma 2 *Defining $V(\cdot)$ as the indirect utility function, the following equations hold:*

$$\frac{\partial^2 V(\cdot)}{\partial \theta^2} < 0, \qquad \frac{\partial^2 \gamma}{\partial \tau^2} < 0$$

.

Proof *As presented in* (4)*, we can obtain the indirect utility function as follows.*

$$\begin{aligned} V(\cdot) &= U(1 - s_t(R_t), s_t(R_t)\tilde{R}_{t+1})I_t^{\gamma-1} \\ &= s^i(R_{t+1})\tilde{w}_t \end{aligned} \tag{31}$$

From this equation, by differentiating twice, we obtain

$$\frac{\partial^2 V(\cdot)}{\partial \tau^2} < 0.$$

∎

This lemma shows that the indirect utility function is concave. Therefore, we can employee the median voter theorem relative to policy determination.

■Determination of the contribution of public investment

- We employ the Median Voter Theorem[9].
- As in the previous analysis, the indirect utility function is derived as follows.

$$\begin{aligned} V(\cdot) &= U(1-s_t(R_t), s_t(R_t)\tilde{R}_{t+1})I_t^{\gamma-1} \\ &= s^i(R_{t+1})\tilde{w}_t \end{aligned} \tag{32}$$

Applying the Median Voter Theorem to the policy determination of contribution to public investment, we obtain

$$\frac{\partial V(\cdot)}{\partial \tau} = 0. \tag{33}$$

From this equation, this relation can be written as shown below.

$$\tau^{med} = \tau^{med}(\theta)$$

■Determination of Contribution to Pension System We also employ the Median Voter Theorem. The indirect utility function is written as

$$\begin{aligned} V(\cdot) &= U(1-s_t(R_t), s_t(R_t)\tilde{R}_{t+1})I_t^{\gamma-1} \\ &= s^i(R_{t+1})\tilde{w}_t. \end{aligned} \tag{34}$$

Differentiating with respect to θ, we obtain

$$\frac{\partial V(\cdot)}{\partial \theta} = 0$$

The median voter preference is

$$\frac{\partial V(R_t)^{med}}{\partial \theta} = 0. \tag{35}$$

Solving this equation can be stated in the following form:

$$\theta^{med} = \theta(\tau). \tag{36}$$

Here, we can show that the two reaction functions are downward-sloping by total differentiation.

4.2 The Case with Commitment

First, we consider the case with commitment. We assume the tax rate is constant over time: $\theta = \theta_t = \theta_{t+1}$, $\tau = \tau_t = \tau_{t+1}$. Here the shapes of the two response functions $\tau = \tau(\theta)$ and $\theta = \theta(\tau)$ are down-sloping. Then, there are three plausible cases as depicted in Figs. 4–6: To summarize the discussion presented above,

Proposition 2 *depending on the parameters, three plausible cases exist:*

1. *Both policies (pension and public investment) survive.*
2. *Only the pension survives.*
3. *Only public investment survives.*

[9] In this respect, our approach resembles that of Alesina and Rodrik (1994).

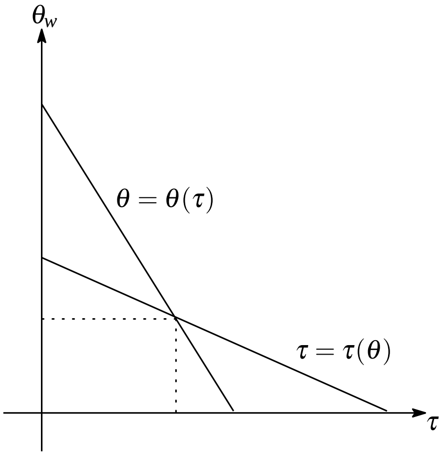

Figure 4 Case 1..

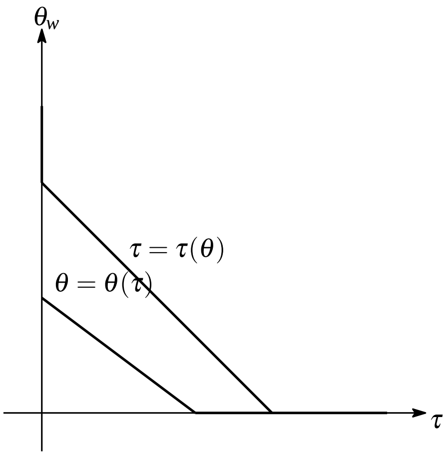

Figure 5 Case 2..

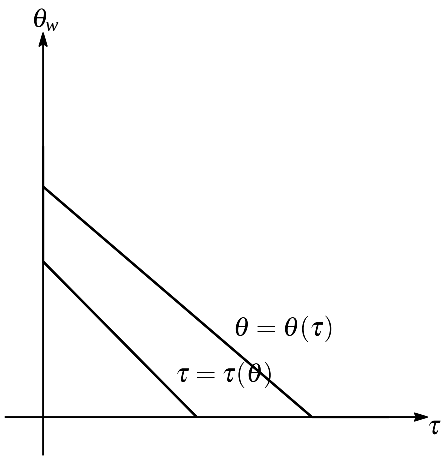

Figure 6 Case 3..

4.3 Case without Commitment

Next we examine analysis under no commitment. Before entering into analysis, let us define the game structure of our model. We then formally define the voting game. The public history of the game at t period, $h_t = \{(\tau_0, \theta_{w0}), (\tau_1, \theta_1), ..., (\tau_{t-1}, \theta_{t-1})\} \in H_t$ is the sequence of social security system (pension and public investment). Actually, H_t is the set of all possible history at time t. An action profile for those who support the pension is, $\{\tau_t, b_t\} \in [0,1] \times [0,1]$. Analogously, an action for unemployed individuals at time t is $\{\tau_t, b_t\} \in [0,1] \times [0,1]$.

A strategy for those who support the pension at t period is a mapping from the history of the game into the action space, i.e. $\sigma^{pension} : h_t \to \{\tau_t, \theta_t\}$. Analogously, a strategy for those who support public investment is at t period is $\sigma^{pu} : h_t \to \{\tau_t, \theta_{wt}\}$. The strategy profile played by both individuals at t period is denoted by $\sigma_t \equiv \sigma_t^e \cup \sigma_t^u$.

At t periods, the objective function for young each player is

$$V_t^i(\sigma_0^i, \sigma_1^i, ...\sigma_t^i, \sigma_{t+1}^i...) = V_t^i(\tau_t, \theta_{wt}, \tau_{t+1}, \theta_{w,t+1}).$$

Regarding agents, those who support public investment,

$$V_t(\sigma_0, \sigma_1, ...\sigma_t, \sigma_{t+1}...) = V_t^i(\tau_t, \theta_{wt}).$$

These solutions describe the relation between the policy at t period and that at $t+1$ period.

We describe the definition of equilibrium.

Defnition 2 *(The Definition of Markovian Structure-Induced Equilibrium)*

1. *σ meets the property of Markov perfect equilibrium.*
2. *For all t, at t period, the equilibrium outcome associated to σ_t is a structure-induced equilibrium of the static game with commitment.*

As contrasted with the analysis in the previous subsection, we assume in this subsection that the government has no commitment to technology. Then, let us define the history of the game H_t as

$$H_t^0 \equiv \{h_t \in H_t | \theta_t = \theta_w*, \;\; t \in \{0, 1, ...\}\},$$

and

$$H_t^\sigma \equiv \{h_t \in H_t | \theta_k = 0, \; k = 0, 1, ...t_0, \; and \; \theta_{wt} = 0, \; t \geq t_0.\}.$$

Moreover, the strategy profiles of people those who support pension and public investment are denoted respectively as σ_t^e and σ_t^u. We then investigate whether each player has an incentive to deviate from the solution under full commitment, as discussed in the previous subsection. Under this setting, we first verify that unemployed people have no incentive to deviate from the strategy. We assume that unemployed people adopt the following strategy: $\theta_{t0}^{deviate} > \theta_w^*$ and $\tau_{t0}^* < \tau_t^{deviate}$. However, employed people do not obtain an additional payoff by deviation because they punish others by reducing the payment of contributions to the pension system, τ, which exerts negative effects on the welfare of both agents. Therefore, it is apparent that unemployed people have no incentive to deviate from the commitment solution.

Regarding people who support public investment, presuming that those who support public investment deviate from equilibrium, i.e. they avoid paying contributions to pensions, then the workers will punish others by not paying contributions to public investment. They would pay contributions to pensions to avoid being punished. Therefore, it is apparent that they have no incentive to deviate. To summarize, neither those who support pensions nor those who support public investment have an incentive to deviate.

From the discussion, we have:

Proposition 3

Policies discussed in the previous subsection (with commitment case) coincide with those without commitment. In other words, the strategies with commitment are time-consistent.

5 Concluding Remarks

As described in this paper, we consider the case in which the government has policies of two kinds: a pension system and public investment. In our setting, policy determination is based on majority voting. The government has social security policy mechanisms of two kinds: pension and public investment. Under this setting, we show how the contents of economic policy vary.

The extension of this research is to increase policy variables. For instance, taking pensions as an example, there are at least three variables: pension benefits, pension contributions, and retirement age. It is necessary to consider these variables to obtain policy implications.

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