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Optimal Degree of Commitment in a Tax Policy

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Abstract

In analyzing economic policies, a severe problem is the time-inconsistency problem. In this regard, the choice of commitment vs. discretion engenders a tradeoff of flexibility and credibility. Therefore, it might be necessary and acceptable to adopt a discretionary policy to some degree, but past studies do not clarify the degree to which a government exercises such a discretionary option. This paper clarifies the optimal degree of commitment using the framework of a repeated game reported by Chari and Kehoe (1990). We point out the relation between the optimal degree of commitment and the rate of time preference.

Keywords: Commitment vs. Flexibility; Degree of Commitment; Imperfect Public Monitoring; Repeated Games; Tax Policy.

JEL Classification: E61, E62, H11, H30.

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1 Introduction

Regarding economic policy, an important issue for any government is the decision of when a policy is to be implemented. For example, as is often the case in the Japanese economy, fiscal policy tends to be postponed because of pressure from the Liberal Democratic Party (LDP) and other interest groups. Policy is often altered shortsightedly. Changing a policy before its aim is achieved is often called *time-inconsistency*. The cause of this problem is related to the degree of a government's commitment to the original policy. In this paper, we analyze how a government should implement policy management. Analysis of that problem demands consideration of the *time-inconsistency* problem, which occurs under the following situation: Presuming that a government determines a policy at the beginning of a period. As time passes, the policy, which was originally optimal in the earlier time of the period, tends not to be optimal any longer. Nevertheless, the government cannot change the policy because of the commitment that was adopted and announced initially. Consequently, the original policy objective becomes inconsistent with the current situation.

As one example, in traditional dynamic optimal taxation theory, the *ex ante* optimal policy is that in which the tax rate on capital is zero or extremely low (see Chamley (1986), Judd (1985)). However, this result is based on the assumption that the government can commit to the policy forever; the result ignores the *time-inconsistency* problem. This problem is summarized as follows. Even if the government determines a tax structure according to optimal taxation theory, it remains unclear whether the government can continue its implementation exactly as it was determined initially. Therefore, apart from the commitment rule, some other rules governing policy changes should be imposed to avoid a *time-inconsistency* problem.

It is necessary to construct a disciplinary rule in economic policy to avoid such a problem. Broadly speaking, two methods exist: "rule" and "discretion"[1)] However, regarding the latter, the following faults exist. (1) A lag pertains between the point of policy-decision and that of policy-implementation, in addition to a lag between policy-implementation and the point until its effects become apparent. (2) Because the effect of the policy is uncertain, the policy is not always effective. (3) This mode of management is devoid of credibility. Therefore, it is apparent that it is necessary to reinforce commitment: to adopt a "rule". This method also has its faults. A salient fault is that adoption of a "rule" is based on the assumption that the economic environment does not change much or change quickly, or that the government can control such a transformation. However, it is not actually appropriate for

[1)] the term "discretion" differs from that used in the 1970s in the sense of a quick-fix policy. The term "discretion" used for this study is a kind of rule based on the assumption that the government cannot commit to the policy, or that the government uses the database only in the single period as an anchor for a policy decision.

us to consider that the government can control the source of uncertainty because the future economic environment is uncertain.

From the discussion presented above, it is apparently more realistic to adopt the commitment (in other words, rule) despite its faults. However, is it always right? Which management is more desirable, rule or discretion? As described in Taylor (1993), Lohmann (1992) or Amador, Werning and Angeletos (2006), there exists a tradeoff between these two ways of policy management. As explained in the latter, both *"rule"* and *"discretion"* include benefits and drawbacks[2]: in the case of a rule, a benefit is that credibility can be established: the reinforcement of a rule can engender higher credibility, although a disadvantage of this option is that a characteristic rigidity of policy-decision can result. In other words, it is difficult for the government to adapt when a change does occur. However, regarding discretion, the benefit is that flexibility is retained: the government can freely change the policy according to the circumstances, including the case in which the government coordinates the policy after the fact, although a disadvantage is decreased credibility and the possibility that frequent policy changes can engender ex ante moral hazard[3]

As explained above, because these two ways of management include advantages and disadvantages, neither an ironclad rule nor a pragmatic discretion is optimal. Therefore, it is considered that, to some degree, discretionary management is allowed. Moreover, because of asymmetric information or some contributing factors, the possibility exists that households cannot observe the government's behavior. The solution to the *time-inconsistency* problem therefore remains unclear. In that situation, we should analyze the extent to which the government should commit to the policy. This paper specifically describes such a problem, although it has not been analyzed in past studies. We intend to answer such a question using the method of a repeated games and numerical methods.

■Motivated Example It is illustrative to consider the following example[4] in which the government determines taxation to maximize the following function:

$$U(c,l,g) = l + \ln(\alpha + c) + \ln(\alpha + g),$$

where c, l, and g respectively denote consumption, labor supply, and public expenditure. The budget constraints of households and the government are written respectively as $c = (1-\tau)(1-l)$, and $g = \tau(1-l)$, where τ denotes the labor-income tax. Then, the Ramsey rule and the Nash rule are defined as follows: The Ramsey rule is a policy that the government determines in consideration of the households' response function. The Nash rule is a policy that the government re-determines under the

[2] the term "rule" and "discretion" respectively corresponds to "commitment" and "flexibility".

[3] "Ex ante moral hazard" means that there exist distortions in the households' ex ante choice because they expect that the government will change policies after the fact.

[4] This example is based on Ljungqvist and Sargent (2004, Ch.22).

assumption that households behave following the Ramsey rule.

As explained in greater detail, households determine their own response function (labor supply function), $l(\tau)$, and the government determines τ^* to maximize $U(c, l(\tau), g) = l(\tau) + \ln(\alpha + (1-\tau)(1-l(\tau))) + \ln(\alpha + \tau(1-l(\tau)))$, which is the Ramsey rule. In contrast, assume that households determine their own behavior taking the Ramsey rule above as given, and that the government determines taxation, τ^{**}, by maximizing utility: $U(c, l(\tau^*), g) = l(\tau^*) + \ln(\alpha + (1-\tau^*)(1-l(\tau^*))) + \ln(\alpha + \tau(1-l(\tau^*)))$. This is the Nash rule. First, the solutions of the Ramsey rule and the Nash rule do not coincide. Consequently, the *time-inconsistency* problem arises. Second, the utility under the Nash rule is higher than that achieved under the Ramsey rule. In other words, if the government deviates from the policy and they can cheat the households, then the government could thereby attain a higher level of utility than that attained under the Ramsey rule.

Next, we can consider the case in which the game described above is repeated. It is worth recalling that if a household behaves making allowance that the Nash rule is adopted, then the utility is lower than that attained under the Ramsey rule. Under the assumption that households can observe the behavior of the government because households can observe cheating or deviation by the government, they can punish the government in some form. Thereby, the government might seek to avoid such a punishment and not deviate. However, under the imperfect monitoring case, households cannot observe the behavior of the government, so the effect of the Nash rule might endure as effective for some periods. The government might deviate from the policy if the effect of the Nash rule is greater than that of the Ramsey rule. Then, another question arises. When or in what circumstances should the government commit to a policy or deviate from it? The analyses described in this paper are designed to answer that question, with particular emphasis on the relation between the effectiveness of the Nash rule and observability.

■Related Literature As described below, some studies have examined repeated games to elucidate some features of macro-economic policy[5)]. In the case of monetary policy, for instance, Athey, Atkeson and Kehoe (2005) investigate the monetary policy game under imperfect public monitoring and show that committing to a simple rule is best. Moreover, Rogoff (1985) established the idea of a "conservative" bank, which engenders the idea of independence of the central bank and showed that delegation of authority to a more conservative central bank than the government can eliminate an inflation bias and yield higher utility.

However, regarding fiscal policy, recently, some studies have begun to incorporate information asymmetry or the existence of an informational incompleteness[6)], as in the case of monetary pol-

[5)] See also Ljungqvist and Sargent (2004, Ch.22) or Mailath and Samuelson (2006, Ch.6,15) as a textbook.

[6)] For instance, Sleet (2004) and Albanesi and Sleet (2006) analyze the dynamic optimal taxation problem under a case in icy. Some studies analyze the optimal taxation problem in relation to the commitment problem using the framework of a repeated game.

The seminal work of this field is related to Chari and Kehoe (1990). This work is based on perfect monitoring, as developed by Abreu (1988). Households might punish the government if the government were to deviate from a policy that is once determined. This mechanism can be regarded as an implicit contract between the government and households. To be more precise, Chari and Kehoe establish a mechanism by which, if the government deviates from the policy, households might punish the government; because of this punishment, it is difficult for the government to deviate from the policy. Consequently, the government stays the course and remains committed to the policy. Chari and Kehoe emphasize that point and describe the reputation mechanism, which always forces the government to commit to the original policy, by extending the model of Fischer (1980) into an infinitely repeated game. They named such a punishment or reputation mechanism a "*sustainable plan*". In other words, Chari and Kehoe center upon a trigger strategy as a mechanism to retain the commitment[7)]. Recently, Phelan and Stacchetti (2001) describe a sustainable plan in a model of dynamic optimal taxation, using the methods of dynamic programming approach developed by Abreu, Pearce and Stacchetti (1990).

Although some studies including those of Benhabib and Rustichini (1997), Fernández-Villaverde and Tsyvinski (2002), and Reis (2006) investigate optimal fiscal policy without commitment, to the best of our knowledge, few studies have specifically addressed the degree of commitment in a tax policy. This paper specifically addresses the relation between the degree of precision of monitoring and the degree of commitment, thereby extending the analyses of Chari and Kehoe (1990). With more detail, the mechanism of creating reputation might be different between the case in which households can observe government behavior completely, and the case in which households can observe it incompletely. Regarding the former case, if the government deviates from a policy anticipated by households, then households can sanction the government in some manner, such that the government avoids deviation from a policy to prevent some reprisal. Regarding the latter case, the possibility exists that if the government deviates from the policy that households expect, they can obtain higher utility because households cannot observe the behavior of the government. Consequently, they might be cheated. Therefore, for the latter case, the possibility exists that discretion obtains a higher utility than a rule

which asymmetric information exists. Another report Kocherlakota (2006) presents a useful survey related to the recent issue of dynamic optimal taxation.

7) Abreu (1988) call this mechanism "stick and carrot punishment". See Stokey (1989, 1991), which presents a similar approach. Moreover, see Ireland (1997) or Kurozumi (2008) as a study which applies the idea of Chari and Kehoe (1990) to the monetary policy. Recently, Cho and Matsui (2005) analyze this problem under the game in which the moves of the government and households are altered in each period. Moreover, recently, Piguillem and Schneider (2008) investigates the optimal taxation problem in which neither government nor households have complete information related to one another's state or behavior.

(commitment) does. Chari and Kehoe emphasized only the former case. The present study investigates both cases by making allowance for the imperfect monitoring case, as developed by Abreu et al. (1990)[8)] and shows that the optimal degree of commitment might change depending on the degree of accuracy of monitoring.

The remainder of this paper is organized as follows. Section 2 presents a description of the model. Then, section 3 describes analyses of two cases. The first is the case in which households can observe the government behavior completely. The second is the case in which households can observe it incompletely. The final section (section 5) is the conclusion.

2 The Model

This section establishes the basic model used for the analyses described in this paper. The structure of our model follows Chari and Kehoe (1990), although our basic model is similar to that of Cooper (1999, Ch.7), which is a variant of Fischer (1980). The model extends the model of Cooper into an infinitely repeated game. The economy begins in period 0 and continues over time, extending indefinitely into the future $t = 1, 2 \ldots \infty$. All markets in this economy are perfectly competitive. In our setting, in each game, the model is structured as a Stackelberg game in which the government and household respectively constitute a leader and follower. In what follows, we explain the structure of the stage game.

2.1 Behavior of Each Agent

■Households This economy includes N identical households that exist for two periods. In our setting, the single term is divisible into two periods. The second period in t-th period and the first period in $t+1$-th period are not overlapped. Each agent is assumed to obtain utility from consumption and leisure. Therefore, the utility of an agent alive at the t-th period is written as $U(c_{1t}, c_{2t}, n - l_{2t})$, where c_{1t}, c_{2t}, and $n - l_{2t}$ respectively denote the consumption in the first and second stage, and leisure in the second stage. Here, we impose some assumptions on each utility function as follows:

(A 1.) $U(\cdot) : \Re^+ \rightarrow \Re$ are strictly concave and increasing in each variable.

(A 2.) $\lim_{c_1 \rightarrow \infty} \frac{\partial U(\cdot)}{\partial c_1} = \lim_{c_1 \rightarrow \infty} U_1(\cdot) = 0$

(A 3.) $\lim_{c_2 \rightarrow \infty} \frac{\partial U(\cdot)}{\partial c_2} = \lim_{c_2 \rightarrow \infty} U_2(\cdot) = 0$

8) For studies which analyze the repeated game under public monitoring, see Abreu (1988), Abreu et al. (1990), and Fudenberg, Levine and Maskin (1994), for instance. The former two studies characterize the equilibrium paths using dynamic programming. Fudenberg et al. (1994) proved the folk theorem under certain conditions under a public (including imperfect case) monitoring case. Moreover, an earlier report Pearce (1992) presents an excellent survey of repeated games.

(A 4.) $\lim_{n-l_2\to\infty}\frac{\partial U(\cdot)}{\partial n-l_2}=\lim_{n-l_2\to\infty}U_3(\cdot)=0.$

Taxes of two kinds exist: a tax on labor income (η) and a tax on saving (τ). Households that form at date t divide endowment (e_t) into consumption (c_{1t}) and saving (s_t) in the first period, and consume (c_{2t}) as after-tax savings and after-tax labor-income in the second period. Consequently, the first-period and second-period budget constraints are shown respectively as

$$c_{1t}+s_t=e_t,\quad c_{2t}=s_tR_t(1-\delta)+l_{2t}(1-\tau),$$

where s_t denotes saving. Given history h_t, explained later, each household determines its own behavior by solving the utility maximization problem, as denoted by $f_t=(f_{1t},f_{2t})=(s_t(h^t),l_{2t}(h^t))$, which is an allocation determined by households.

■Government The government determines taxation by maximizing the households' lifetime utility function. Under this setting, the budget constraint of the government is written as follows (balanced-budget rule):

$$g_t=\delta_t s_t R_t+\tau_t w_t l_{2t}. \tag{1}$$

We then assume that the government expenditure g_t is treated as given for households. The objective function of the government is written as $\sum_{t=0}^{\infty}\beta^t U(c_{1t},c_{2t},n-l_{2t})$, where the discount factor is $\beta\in(0,1]$. Moreover, we assume that the government expenditure does not affect the households' utility. The action profile of the government is denoted as $a^g=(\{\delta_t,\tau_t\}_{t=0}^{\infty})$. We define the history up to t-period as follows.

$h_t=\{a_s\}_{s=1}^{t-1}$: Path of tax rates which happened up to t-period.
$h^t=(h_0,h_1..,h_t)$: Path of events up to t-th period

The whole public history is recorded as $H=\bigcup_{t=1}^{\infty}h^t$, and the path of events from the initial point up to t period is denoted as $h^t=(h_1,h_2...,h_t)$. Then, the action of the government at t period can be written as $a_t^g=(\delta_t,\tau_t)$, and $h_t=a_t^g(h^{t-1})$. Moreover, the government strategy, σ^g is a mapping which satisfies

$$a_t^g=\sigma_t^g(h_{t-1}).$$

Incidentally, presuming circumstances in which the government must determine whether they continue the policy (in other words, commit to the policy) or change (deviate from) the policy, they signal a policy change at the beginning of the time (or each stage-game). Here, we designate such a public signal of policy-change (ω^g) as follows: For $t=1,2...\infty$,

$$\omega^g=(\omega_1^g,...,\omega_t^g)\in\Omega, \tag{2}$$

where $\Omega \in \Re$: the distributive function of ω is denoted as $f(\cdot)$. In addition, this public signal is assumed to be observable to all players (i.e. households) in the following subsection 3.2, which corresponds to the perfect monitoring case. In contrast, in subsection 3.3, the signal includes noise such that households cannot perfectly observe government behavior, which corresponds to the imperfect monitoring case.

Here, it is necessary to refer to an allocation rule. An allocation rule (hereinafter, denoted as "α") describes households' choices in each period, contingent on observation of the profile of government policy: α maps σ_t^g into f_t. Therefore, the decision-making of households at t-th period is a function of the history of government policy up to t.

■Firms We then describe firms' behavior. We assume that production process is achieved between the first and second period in each stage game[9], and factor markets are perfectly competitive and that firms maximize their profits. Labor and capital stock are used for production. They are functions of the history h^t; production technology yields constant returns to scale. Therefore, production functions are expressed as $y_t = F(k_t(h^t), l_t(h^t)) : \Re_+^2 \to \Re_+$, where k_t, l_t, and y_t respectively represent capital stock, labor demand, and output. The problem facing firms is the following.

$$\begin{aligned} \max \quad & y_t - R_t k_t(h^t) - w_t l_{2t}(h^t) \\ s.t. \quad & y_t \leq F(k_t(h^t), l_{2t}(h^t), h_t) \end{aligned}$$

In fact, maximization yields

$$F_k(h^t) = R_t(h^t), \text{ and } F_l(h^t) = w_t(h^t). \tag{3}$$

2.2 Market Equilibrium

Finally, we formulate equilibrium conditions for each market.

- Resource Constraint:
$$c_{1t}(h^t) + c_{2t}(h^t) + k(h^t) + g_t(h_t) \leq y_t(h^t) \tag{4a}$$
- Labor market:
It is assumed that full employment pertains in this economy, meaning that a supply–demand balance is attained. Denoting l_t^d as the labor demand, we have the following.
$$l_{2t} = l_{2t}^d. \tag{4b}$$
- Capital Market:
$$k_{1t} = s_{2t} \tag{4c}$$

[9] This assumption means that in each period, saving in the first period is used for production and its output is given as an endowment for the next generation.

2.3 Timing of the Stage Game

The timing of the t-th period stage game, given history h^{t-1}, is the following:

1. The government signals whether it will make a policy change or not.
2. Observing that signal, households determine their own behavior.
   a. In the case in which households expect that the government commits to the policy without a policy change,
      households trust the government's commitment and do not change their behavior. Therefore, the *time-inconsistency* problem does not occur
   b. In the case in which households expect that the government does not commit and will change the policy,
      households change their own decisions. Under a perfect monitoring case, households can adjust their behavior swiftly, although, under an imperfect monitoring case, it takes for households some time to adjust expectations. In this case, the effect of a discretionary policy can be transiently greater than that of the commitment policy.
3. The government determines the tax policy. In other words, whether to commit to the policy or not.
4. All markets clear.

Moreover, regarding step 2 explained above, it is noteworthy that if the government can deviate from the policy without being noticed by households, the government can attain higher utility. However, the level of utility will be lower than when the government does not change the policy if households come to a realization of policy change.

3 Analysis of Two Cases

This section presents analyses of the model under the two cases: perfect monitoring and imperfect monitoring. The difference between these two cases is explained as follows (see also Fig. 1.):[10)]

Case 1. Perfect Monitoring: All players (households) can observe the past action and the signal. In other words, households can observe the government's decision completely.

Case 2. Imperfect Monitoring: Only common signals based on the past action are visible. However,

[10)] Regarding the criticism for analysis with a framework of public monitoring, see, for instance, Matsushima (2002, section 5). He claimed that the analysis under public monitoring is not plausible for obtaining the policy implication. However, considering that households can observe the public behavior of the government to some degree using newspapers or the internet, we can presume that the analysis with a public monitoring framework corresponds to reality.

Action (NOT observable) $(a_1, a_2, \ldots, a_t)$ $\longrightarrow$ Signal (observable) $\omega \in \Omega$

(a) Perfect Monitoring

$$\omega = (a_1, a_2, \ldots, a_t)$$

(b) Imperfect Monitoring

ω is random variable

Figure 1 Difference between perfect monitoring and imperfect monitoring Case #1.

this signal includes noise; it is not necessarily accurate. Regarded in greater detail, households cannot capture the government's decision completely. Looking at this case, we consider the case in which households cannot observe the government's behavior for some periods. Subsequently, they finally take notice of a policy change[11].

3.1 Equilibrium Concept

Before entering into analysis, we define the equilibrium concept (sustainable plans) following Chari and Kehoe (1990). An allocation rule (f_t) describes households' choices, $s_t(h^t)$ (saving), and $l_{2t}(h^t)$ (labor supply) in each period, contingent upon observed past government policy, given the continuation of the allocation rule after h^{t-1}.

First, we define the competitive equilibrium[12].

Defnition 1 *(competitive equilibrium) We define an equilibrium as a competitive equilibrium if it satisfies the following allocation.*

(i) Given the history up to $t-1$*;* h^{t-1}*, the condition for utility maximization, eq.* (6) *holds.*

(ii) The condition for firms' profit maximization holds.

(iii) Each market equilibrium condition holds: (4a)–(4c) .

(iv) The budget constraint of the government holds: eq.(1).

Here, it is necessary to define some terms. Given continuation of the allocation rule after h^{t-1}, the continuation of σ_t^g induces a continuation path (s_t, l_{2t}). This continuation path and an allocation rule then determine a sequence of aggregate actions (K_t, L_t). An allocation is competitive if and only if, for all t, the following condition holds. Given the history up to $t-1$-th period, the government

[11] Regarding this point, see section 4.

[12] For this definition, see also Chari, Kehoe and Prescott (1989).

and the allocation rule are competitive, the allocation after t-th period meets the proper definition of the competitive equilibrium. However, a government strategy σ_t^g is optimal if the continuation of σ^g maximizes the social planner's objective, given the continuation of the allocation rule after h^{t-1} and the condition presented above.

Now, we are ready to state the definition of Sustainable Plans.

Defnition 2 *(Sustainable Plan) We define the allocation rule and the government's strategy, which satisfy the following condition as Sustainable Plan.*
Given the history up to $t-1$*;* h^{t-1}*,*

(i) The government strategy σ_t^g *is admissible.*

(ii) An allocation rule, α*, meets the properties of competitive equilibrium.*

(iii) The government strategy of σ_t^g *meets the property of optimal allocation.*

(iv) Each market equilibrium condition: eqs.(4a)-(4c) *holds.*

The meaning of (i) $\sim$ (iii) is explainable in a narrative manner. First, the definition of "*admissible*" is explained as follows: Given the history up to $t-1$, h^{t-1}, then $\{h^s\}_{s=t}^{\infty}$ is defined so that $\{s_t, l_t, h_t\}_{t=0}^{\infty}$ is consistent with the conditions of competitive equilibrium (CE). A government strategy and an allocation rule are a sustainable plan (hereinafter, SP) if, after any history of government policies, (i) the continuation of the government strategy is optimal for the planner given the allocation rule, and (ii) the choices prescribed by the continuation of the allocation rule are optimal for each worker and investor, given the government strategy. In other words, an SP specifies behavior for the government and the private sector such that, after every contingency, the government acts optimally given households' behavior, and their behavior meets the property of the rational expectations equilibrium given the government policy. The condition (iii) requires both the private agents' decision rule and the monetary policy strategy to be sequentially rational. This condition implies the following situation. Let $V(\cdot;\sigma,\alpha|h^t)$ denote the payoff of the government, given history h^t. Then for all the continuation government's strategy, σ' satisfies the following constraint of

$$V(k_t;\sigma,\alpha|h^t) \geq V(k_t;\{\sigma\}_{t=0}^{t-1},\sigma_t',\alpha|h^t), \tag{5}$$

where "σ_t'" signifies the deviation by the government at t-period. This equation means that the one-time deviation of the government's strategy never engenders welfare improvement.

Here, regarding the property of SP, we state the following proposition.

Proposition 1 *Given any history* h^{t-1}*, if* $\{s_t, l_{2t}\}_{t=0}^{\infty}$ *is a sustainable plan, then* $\{s_s, l_{2s}\}_{s=t+1}^{\infty}$ *is also a sustainable plan.*

Proof *See A.2.* ∎

3.2 Benchmark: Case of Perfect Monitoring

This subsection presents analysis of the perfect monitoring scenario as a benchmark. This case is almost identical to that of Chari and Kehoe (1990). We continue the analysis based on the assumption that both players (government and households) can observe each other's behaviors. Then, we adopt the trigger strategy. What we should consider is under what mechanism the government will not deviate. Here, we borrow the idea penalty code proposed in Abreu (1988).

Under this setting, the government solves the following problem.

$$\max_{\{\delta,\tau\}_{t=s}^{\infty}} \sum_{t=s}^{\infty} \beta^{t-s} U(c_{1t}(h_t), c_{2t}(h_t), n - l_{2t}(h_t))$$

Incidentally, it is necessary to solve the stage-game in the t-th period. To obtain the Ramsey solution, we solve the stage game backward.

Second Stage Households solve the following maximization problem given the level of taxation.

$$\max \;\; U(e-s, sR(1-\delta) + l_2 w(1-\tau), n - l_{2t})$$

From this problem, we can obtain the following first-order conditions (response function):

$$\begin{cases} U_s = -U_1 + R(1-\delta)U_2 = 0 \\ U_l = (1-\tau)w_t U_2 - U_3 = 0. \end{cases} \tag{6}$$

First Stage Given $s(\delta^*, \tau^*)$, and $n(\delta^*, \tau^*)$, which are obtained by solving the two first-order conditions presented above, the government solves the following problem.

$$\begin{aligned} &\max \;\; U(c_{1t}, c_{2t}, n - l_{2t}) \\ &s.t. \;\; g_t = s_t(\delta) R_t \delta_t + l_{2t}(\tau) w_t \tau_t \\ &\text{and} \;\; eq.\ (6) \end{aligned}$$

Arranging the first-order condition of this problem yields the following:

$$U_2 sR = \lambda[sR + \delta s_k R + \tau l_k] \tag{7}$$

$$U_2 l_2 = \lambda[l_2 + R\delta s_l + \tau l_l], \tag{8}$$

where s_k, s_l, l_k,l_l, and λ respectively denote $\frac{\partial s}{\partial \delta}$, $\frac{\partial s}{\partial \tau}$, $\frac{\partial l_2}{\partial \delta}$, $\frac{\partial l_2}{\partial \tau}$, and the Lagrange multiplier. We designate the solutions as $\{\delta^*, \tau^*\}$, or $\{\delta_s^*, \tau_s^*\}_{s=0}^{\infty}$, which denotes the paths of solutions under the Ramsey rule. The level of utility under the Ramsey rule is denoted as $U^*(\cdot)$. Similarly, we can obtain the level of utility under the Nash rule and one-shot deviation, which are denoted respectively as $U^s(\cdot)$ and $U^d(\cdot)$. As portrayed in the A.1, the following condition holds[13].

Lemma 1

$$U^d(\cdot) > U^*(\cdot) > U^s(\cdot).$$

Then, presuming a situation in which the government should determine whether to commit to the policy or not at s period, the government should determine the option depending on the magnitude relation in the following equation.

$$\sum_{t=s}^{\infty} \beta^{t-s} U^*(c_1, c_2, 1-l_1) \gtreqless U^d(\cdot) + \sum_{t=s+1}^{\infty} \beta^{t-s-1} U^s(\cdot) \tag{9}$$

The left side of the equation shown above defines the utility obtained by committing to the policy. However, the first term of the right hand of the equation denotes the utility obtained when the government deviates at s-th period. Moreover, the second term denotes the mean utility that households obtain, taking the policy change into consideration *rationally* after the $s+1$-th period. From another perspective, $U^s(\cdot)$ can be interpreted as a result that the government is *punished* by households.

Using lemma 1, we are now ready to describe the following proposition.

Proposition 2

1. *For all s, the following condition holds.*

$$\sum_{t=s}^{\infty} \beta^{t-s} U^*(c_1, c_2, 1-l_1) \geq U^d(\cdot) + \frac{\beta}{1-\beta} U^s(\cdot) \tag{10}$$

This equation can be rewritten as

$$\frac{\beta}{1-\beta} [U^*(\cdot) - U^s(\cdot)] \geq [U^d(\cdot) - U^*(\cdot)]. \tag{11}$$

The government should commit to the policy when this condition holds.

2. *There exists a lower value of β; $\underline{\beta} \in [0,1]$ which satisfies the equation presented above.*

Proof *See appendix A.3.* ∎

This proposition says that no room exists for discretion under this case depending on the interval of β. Under the perfect monitoring case, the effect of deviation lasts only one period. Under the imperfect monitoring case, the effect of deviation might last for some periods, depending on the monitoring ability. In the next subsection, we investigate how this result varies when we incorporate the imperfect monitoring case.

[13] This lemma is only described as a narrative in Cooper (1999, Ch.7); it is not proved formally.

3.3 Case of Imperfect Monitoring

The case in which both players can observe the other's behavior, as in the previous subsection, is less than realistic. We then consider the following situation.

- Neither government nor households can observe the other's behavior. In other words, neither player can observe which action is taken (or not taken) for sure.
- It is assumed that households can obtain information related to government behavior: *public signal*.
- However, this public signal includes noise: this signal is not entirely correct. Therefore, we assume that this is an unobservable stochastic variable: (i.i.d., independent and identically distributed), whose marginal distribution is denoted as $f(\omega_t|\sigma^g)$. Moreover, we assume that the signal has a cumulative distribution function (c.d.f.) ; $F(\omega_t|\sigma^g)$. [14)]

Regarding the signal, we impose some assumptions:

(A 5.) Ω is a finite subset of $\Re$.
(A 6.) $f(\omega_t|\sigma^g) > o$ holds for all $\omega \in \Omega$ and $f(\omega_t|\sigma^g)$ is continuous in σ^g.

As in the preceding subsection, we adopt the trigger strategy, which means that the players who deviate are sanctioned. We should consider how the government should behave so that it will not deviate from the policy. In this imperfect monitoring case, we designate a public signal as ω^g, and consider $\{\delta_{t-1}, \tau_{t-1}\}$ as a public strategy. Consequently, the expected payoff of each player (government and households) at the t-th period is written as $E_t[U(c_{1t}, c_{2t}, n-l_{2t})|\Omega] = \sum_{\omega\in\Omega} f(\omega_1, \omega_2....|\sigma^g)U(c_{1t}, c_{2t}, n-l_{2t})$. Therefore, each objective function is shown as the following.

$$\sum_{t=s}^{\infty} \beta^{t-s} E_t[U(c_{1t}, c_{2t}, n-l_{2t})|\Omega] = \sum_{t=s}^{\infty}\sum_{\omega\in\Omega} f(\omega|\sigma^g)\beta^{t-s}U(c_{1t}, c_{2t}, n-l_{2t}) \tag{12}$$

$$\sum_{t=s}^{\infty} \beta^{t-s} E_t[U(c_{1t}, c_{2t}, n-l_{2t})|\Omega] = \sum_{t=s}^{\infty}\sum_{\omega\in\Omega} f(\omega|\sigma^g)\beta^{t-s}U(c_{1t}, c_{2t}, n-l_{2t})) \tag{13}$$

[14)] Strictly speaking, this signal includes signals of two kinds:
- ω^g: A public signal, which is observable by all players.
- ω^p: A private signal, which is different in each household.

However, for simplicity, we ignore the private signal in this paper.

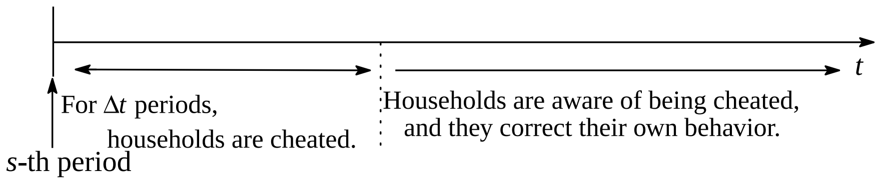

Figure 2 Imperfect Monitoring Case #1.

Under this setting, eq. (9) under the imperfect monitoring case is rewritten as

$$\overbrace{\sum_{t=s}^{\infty}\beta^{t-s}U(c_1,c_2,1-l_1)}^{\text{the utility without policy-change}} \gtreqless U^d(\cdot)+\left[\sum_{t=s+1}^{s+\Delta t}\sum_{\omega\in\Omega}\beta^{t-s-1}f(\omega|\sigma^g)U^d(\cdot)\right]+\sum_{t=s+\Delta t}^{\infty}\beta^{t-(s+\Delta t)}U^s(\cdot). \quad (14)$$

The left side of the equation presented above is the same as the perfect monitoring case. The right side of the equation presented above consists of two terms. The first is the utility obtained using the Nash rule: one-shot deviation. In other words, this effect is that the government deviates from its chosen policy at s period. The second term is that of the expected utility, which consists of two factors. The first factor is the utility for which the Nash rule is effective when households cannot discern the government behavior. The second factor is the effect whereby households expect the government *rationally* to adopt the Nash rule. In the case of perfect monitoring, the left side of the equation presented above is more than, or at least equal to that of the right side, so that the government should commit to the original policy. However, in the imperfect monitoring case, the possibility exists that the right side of the equation presented above is larger than the left-side as long as households are cheated by the government. If this is the case, then the government should deviate, or take the discretionary option, rather than commit to the policy. Figure 2 depicts the situation described above.

The equation in which the commitment policy is sustainable under the imperfect monitoring case is written as[15)]

$$\frac{\beta}{1-\beta}\left[U^*(\cdot)-U^s(\cdot)\right] \geq \left[U^d(\cdot)-U^*(\cdot)\right]+E_t\sum_{\omega\in\Omega}\sum_{t=s+1}^{\infty}\beta^{t-s-1}[\{U^d(\cdot)|f(\omega_t|\sigma^g)\}. \qquad (15)$$

Under such a situation, the distributive function of the public signal is an important consideration. Here, the following propositions hold:

Proposition 3 *For such an interval, $\beta\in[\underline{\beta'},1]$, it is better for the government not to deviate from the policy. In other words, the commitment policy is sustainable.*

Proof *See appendix A.4.* ∎

15) We use the fact that the last term of RHS of the equation presented above can be written as $\frac{\beta}{1-\beta}U^s(\cdot)$.

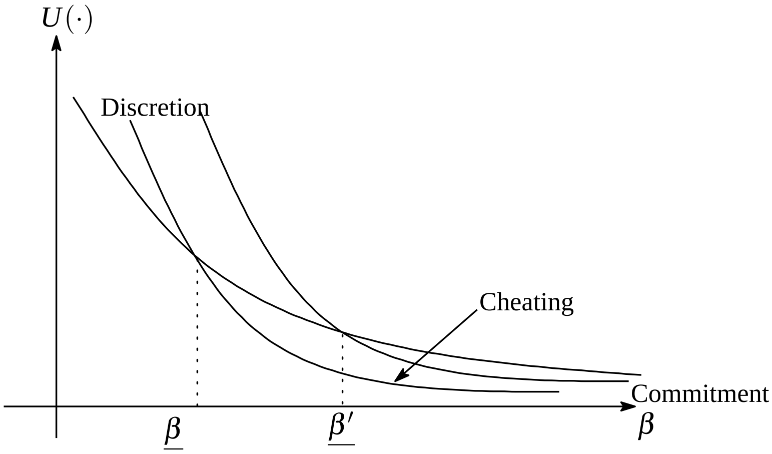

Figure 3　Relation between the value of β and the choice of policy management.

Proposition 4 *Comparing $\underline{\beta}$ with $\underline{\beta'}$, the following equation holds:*

$$\underline{\beta'} > \underline{\beta}.$$

Proof *See appendix A.5.* ∎

This proposition means that the interval of β is narrower when the commitment policy is *sustainable* than in the perfect monitoring case. In other words, this suggests that if private agents are more patient than in the perfect monitoring case, then the commitment policy is attainable even in the absence of a commitment device and the imperfect monitoring case. Actually, Fig. 3 displays the above-referenced explanation in intuitive terms. Moreover, if β is less than $\underline{\beta'}$, then the possibility exists that it is better for the government to deviate from the policy. In other words, the commitment policy is not *sustainable*.

■Meaning of Δt　Here, we investigate the optimal degree of commitment, first by seeking the value of Δt^* which satisfies the following equation.

$$\sum_{t=0}^{\infty} \beta^t U^*(c_{1t}, c_{2t}, n - l_{2t}) = U^d(\cdot) + [\sum_{\omega \in \Omega} \sum_{t=s+1}^{s+\Delta t} f(\omega|\sigma^g)\beta^{t-s-1}U^d(\cdot) + \frac{\beta}{1-\beta}U^s(\cdot)] \tag{16}$$

Solving this equation, we obtain[16)]

$$\Delta t^* = \log_\beta (U^*(\cdot) - U^s(\cdot)).$$

The discretionary policy is optimal if the government can cheat households for more than Δt^* periods. To summarize, the following can be said.

Proposition 5 *The government can cheat households for more than Δt^* period, defined in eq.* (16). *It is better for the government to take discretionary management.*

Remark. Under the imperfect monitoring case, households are not aware of a policy change (in other words, they are cheated by the government) for Δt periods. The value of Δt varies depending on the accuracy of monitoring, the possibility exists that discretionary policy is optimal. From another angle, Δt can be interpreted as the *degree of commitment*. The case of $\Delta t = 1$ corresponds to the full commitment[17)], and $\Delta t = \infty$ means full discretion.

Based on this result, by differentiating Δt^* with respect to β, we obtain the following:

Corollary 1 *As the value of β increases, the value of Δt^* decreases.*

This corollary means that the degree of commitment is dependent on the value of β. The government must cheat households longer as households become more patient.

4 Discussion

Some readers might note that the analysis in the previous subsection is a case of information structure, not a case of accuracy of monitoring. Then, in this section, we consider another version of imperfect monitoring.

We then consider another version of the imperfect monitoring case. We consider the situation described in the preceding subsection.

16) This can be derived as follows:

$$\begin{aligned}
&\frac{\beta}{1-\beta}(U^*(\cdot) - U^s(\cdot)) = (1+\beta^{s-1}(\omega_1 + \omega_2\beta + \cdot + \omega_{\Delta_t}\beta^{\Delta t})U^d(\cdot) \\
&\frac{\beta}{1-\beta}(U^*(\cdot) - U^s(\cdot)) = (\beta + \beta^2 + \cdots + \beta^{\Delta t})U^d(\cdot) \\
\iff &\frac{\beta}{1-\beta}(U^*(\cdot) - U^s(\cdot)) = \frac{1+\beta(1-\beta^{\Delta t})}{1-\beta}U^d(\cdot) \\
\iff &\Delta t^* = \log_\beta (U^*(\cdot) - U^s(\cdot)).
\end{aligned} \tag{17}$$

17) See the subsection 3.2.

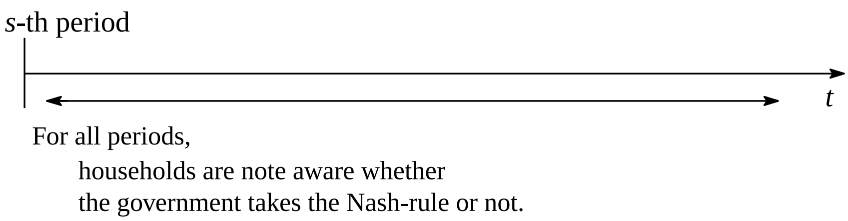

Figure 4 Imperfect Monitoring Case #2.

The main difference between the previous subsection (Case 1 of imperfect monitoring) is that we assume the incomplete awareness of government of household behavior *forever*, although the government is assumed to be aware of household behavior at some point in the previous subsection. Regarding this point, see Fig. 4.

Then, the expected utility of households and the government are shown respectively as shown below.

$$\sum_{t=s}^{\infty} \beta^{t-s} E_t[U(c_{1t}, c_{2t}, n-l_{2t})|\Omega] = \sum_{t=s}^{\infty} \sum_{\omega \in \Omega} f(\omega|\sigma^g) \beta^{t-s} U(c_{1t}, c_{2t}, n-l_{2t}) \tag{18a}$$

$$\sum_{t=s}^{\infty} \beta^{t-s} E_t[U(c_{1t}, c_{2t}, n-l_{2t})|\Omega] = \sum_{t=s}^{\infty} \sum_{\omega \in \Omega} f(\omega|\sigma^g) \beta^{t-s} U(c_{1t}, c_{2t}, n-l_{2t})) \tag{18b}$$

Under this setting, eq. (9) under the imperfect monitoring case can be rewritten as presented below.

$$\overbrace{\sum_{t=s}^{\infty} \beta^{t-s} U(c_1, c_2, 1-l_1)}^{\text{the utility without policy-change}} \gtreqless U^d(\cdot) + \left[\sum_{t=s+1}^{\infty} \sum_{\omega \in \Omega} \beta^{t-s-1} f(\omega|\sigma^g) \{U^d(\cdot) + U^s(\cdot)\} \right] \tag{19}$$

The left side of the equation presented above is the same as that in the perfect monitoring case. The right side of the equation presented above comprises two terms. The first term is the utility obtained using the Nash rule: one-shot deviation. In other words, this effect is that the government deviates from its chosen policy at s period. The difference from the previous subsection is the second term of the right side of the equation presented above. The second term is that of the expected utility for which the Nash rule is effective when households cannot discern government behavior. In the case of perfect monitoring, the left-side of the equation presented above is more than, or at least equal to that of the right-side, so that the government should commit to the original policy. However, in the case of imperfect monitoring, the possibility exists that the right-side of the equation presented above is larger than the left-side as long as households are cheated by the government. If such is the case, then the government should deviate, or take the discretionary option, rather than commit to the policy.

The government should take the discretionary option if the RHS of eq. (19) is larger than LHS. However, another question arises. Under which case should the government deviate? The following proposition answers such a question.

Proposition 6 *There exists a lower value, $\underline{\beta''} \in (0,1)$ that satisfies the condition for which the commitment solution is sustainable.*

Proof *See appendix A.6.* ▮.

5 Concluding Remarks

This paper presented analysis of the optimal degree of commitment using the method of a repeated game. What we showed can be summarized as follows:

1. In the case of perfect monitoring, the government should not deviate from its policy, depending on the value of the household's patience.
2. However, in the case of imperfect monitoring, the possibility exists that the government deviates from its chosen policy if households are more patient than in the perfect monitoring case.

Finally, we conclude this paper by showing the direction of extension. The plausible extension is to consider private monitoring in which the signals differ among households.[18] It is natural to consider that the signals that the households receive mutually differ. Although the study of private monitoring is not imported into the analysis of the macro-economic policy, this extension might be important.

Appendix A

A.1 Proof of Lemma 1

In this appendix, we first describe the model of Cooper (1999, Ch.7). Each household lives two periods and maximizes the following utility: $U(c_1, c_2, n-l_2; g_t)$. Herein, $c_t (t = 1,2)$, l_2, $n-l_2$, and e respectively denote the consumption in t period, labor supply, leisure, and endowment. Moreover, g_t denotes the government expenditure, which does not affect the households' utility. Actually, $U(\cdot)$ is assumed to be strictly increasing and concave in each variable. As described in section 2, the budget constraints of the first and second period are written as

$$c_1 + s = e, \quad c_2 = sR(1-\delta) + l_2 w(1-\tau).$$

However, the government determines the labor-income and capital-income tax so that they maximize the social welfare to raise the public expenditure; g_2. Then the budget constraint of the government is the following:

$$g_2 = sR\delta + l_2 w\tau. \tag{20}$$

[18] See, for instance, Matsushima (2004).

First, considering the commitment case, we presume circumstances in which the government determines the tax rates. Subsequently, the government cannot change the policy). Denoting the life-time utility as $V(\delta,\tau)$, the government strategy (or policy) is set as $\sigma_t^g = (\delta_t,\tau_t)$ to maximize $V(\delta,\tau)$.

$$\max \;\; U(e-s, sR(1-\delta)+l_2w(1-\tau), n-l_2; g_t)$$

From this, we obtain the following first-order conditions,

$$\begin{cases} U_s = -U_1 + R(1-\delta)U_2 = 0 \\ U_n = (1-\tau)wU_2 - U_3 = 0, \end{cases} \tag{21}$$

Which are interpreted as the households' response function. Given $s(\delta,\tau)$、$n(\delta,\tau)$, which is obtainable by solving these equations, the government solves the following problem.

$$\begin{gathered} \max \; U(c_1,c_2,n-l_2;g_t) \\ s.t. \;\; g_t = s(\delta)R\delta + l(\tau)w\tau \end{gathered}$$

Now, the Lagrange function is the following:

$$\begin{aligned} \mathscr{L} &= U(c_1,c_2,n-l_2,g_t) + \lambda\{g_t - sR\delta - l_2w\tau\} \\ &= U(e-s(\delta,\tau), s(\delta,\tau)R(1-\delta)+n(\delta,\tau)(1-\tau), L-n(\delta,\tau), s(\delta,\tau)R\delta+n(\delta,\tau)\tau) + \lambda\{g_t - sR\delta - l_2w\tau\} \end{aligned}$$

$$\begin{aligned} \frac{\partial \mathscr{L}}{\partial \delta} &= -s_kU_1 + (Rs_k(1-\delta) - Rs(\tau) + l_k(1-\tau))U_2 - l_kU_3 + \lambda\{-Rs_k\delta - Rs(\tau) - l_k\tau\} = 0 \\ \frac{\partial \mathscr{L}}{\partial \tau} &= -s_lU_1 + (Rs_n(1-\delta) - l_l(1-\tau) + l_2(\tau))U_2 - l_lU_3 + \lambda\{-Rs_l\delta - l_l\tau - l_2(\tau)\} = 0, \end{aligned}$$

where $\frac{\partial s}{\delta} \equiv s_k$, $\frac{\partial s}{\tau} \equiv s_l$, $\frac{\partial l_2}{\delta} \equiv l_k$, and $\frac{\partial l_2}{\tau} \equiv l_l$. Considering eq. (21), from the first-order conditions, we obtain

$$U_2sR = \lambda[sR + \delta s_kR + \tau l_k] \tag{22}$$

$$U_2l_2 = \lambda[l_2 + R\delta s_l + \tau l_l]. \tag{23}$$

Next, consider the case without commitment. We consider the following extensive game in which (1) households determine the amount of saving, and (2) the government determines the tax rates, and (3) the government determines the amount of labor supply. The outcome under this case is called called Kydland–Prescott outcome.

Now, we consider the following two-stage game:

1) The government determines the tax rate.
2) Subsequently, households determine the amount of labor supply.

Under this regime, we solve this game *backward*. At the second stage of this game, the government sets the tax rates to maximize $V(\delta,\tau)$. Compared with the commitment case, the decision of household s's saving does not respond to the policy-change. Consequently, the government will impose more capital-income taxes.

$$\max \; U(e-s, sR(1-\delta)+l_2w(1-\tau), n-l_2; g_t)$$

From this first-order condition, we obtain

$$U_l = (1-\tau)wU_2 - U_3 = 0.$$

Given $l = l(\tau)$ obtained from the equation presented above, the government solves the following problem.

$$\begin{aligned}\mathscr{L} &= U(c_1, c_2, n-l_2, g_t) + \lambda\{g_t - sR\delta - l_2w\tau\} \\ &= U(e-s(\delta,\tau), s(\delta)R(1-\delta)+n(\tau)(1-\tau), n-l_2(\tau)) + \lambda\{g_2 - sR\delta - l_2w\tau\}\end{aligned}$$

Solving this problem yields the following.

$$U_2sR = \lambda[sR + \tau l_k] \tag{24}$$

$$U_2l_2 = \lambda[l_2 + \tau l_l] \tag{25}$$

Now, we derive $U^d(\cdot)$ and $U^s(\cdot)$. Presuming circumstances under which the government deviates (i.e., change the capital-income tax rate) at some point. Under this setting, households solve the following problem.

$$\begin{aligned}&U^d(\cdot) \equiv \max_{\{c_{1t}, c_{2t}, l_{2t}\}} \; U(e-s, sR(1-\delta)+l_2(1-\tau)w, n-l_2) \\ &s.t. \;\; c_{2t} = s_tR_t(1-\delta)+l_{2t}w(1-\tau) \\ &\qquad U_s = -U_1 + R(1-\delta)U_2 = 0\end{aligned}$$

By adopting this policy, they can obtain higher utility than that obtained under the Ramsey equilibrium in the short run, which is denoted as $U^d(\cdot)$ in this paper.

Moreover, $U^s(\cdot)$ is the same as "without commitment" case discussed in the above. Under this situation, the government has an incentive to impose more taxes on capital income. Therefore, the utility level of this situation is lower than that of "commitment case".

Under this setting, we have the following lemma 1:

Lemma 1

$$U^d(\cdot) > U^*(\cdot) > U^s(\cdot).$$

Proof: The second part of the inequality is attributable to the fact that the government has an incentive to impose more taxes on capital-income after the fact and households decrease the amount of their saving expecting such government behavior. Consequently, the level of utility without commitment is higher than that with commitment. ∎

A.2 Proof of Prop.1

Given any history up to t, h^t, a sustainable plan attains the outcomes $h_t = \sigma(h^{t-1})$, and $(s_t, l_{2t}) = f_t(h^{t-1})$. Given these outcomes, we define the competitive outcomes as follows:

$$CE_h^t = \{\{h_t\}_{t=0}^{\infty}, \text{which meets the conditions of the competitive equilibrium.}\}$$

Then, the government's strategy σ given the history h^{t-1} is also admissible. Given $h^{t-1} \in CE_h^t$, σ^g induces the continuation outcome path, $\{s_t, l_{2t}, h_t\}$, which is a part of competitive equilibrium. Then, α given the history h^{t-1} also meets the property of competitiveness. Finally, $\{s_s, l_{2s}, h_s\}_{s=t+1}^{\infty}$ must meet the property of competitive equilibrium. ∎

A.3 Proof of Prop.2

1. As in Chari and Kehoe (1990), the right side of eq. (10) is positive (See Lemma 1) although the left-side of that equation is increasing concomitantly with increasing β. Therefore, eq. (10) holds. ∎

2. Equation (11) can be rewritten as follows.

$$\frac{\beta}{1-\beta} \geq \frac{U^d(\cdot) - U^*(\cdot)}{U^*(\cdot) - U^s(\cdot)}$$

   As $\beta \to 1$, the left side of the equation presented above converges to ∞. Therefore, there exists a lower bound $\underline{\beta}$ such that the equation presented above holds.

A.4 Proof of Prop.3

The second term of the right-side of equation, (14) in the case of uniform distribution can be rewritten as follows.

$$\begin{aligned}
&E_t \sum_{t=s+1}^{\infty} \sum_{\omega\in\Omega} \beta^t [U^d(\cdot)|\pi(\omega_t) + \frac{\beta}{1-\beta} U^s(\cdot)] \\
&= E_t \sum_{t=s+1}^{\infty} \sum_{\omega\in\Omega} \beta^t [U^d(\cdot)|\pi(\omega_t)] + \frac{\beta}{1-\beta} U^s(\cdot) \\
&= [\omega_{s+1} + \ldots + \omega_{s+1+\Delta t}] U^d(\cdot) + \frac{\beta}{1-\beta} U^s(\cdot) \\
&\geq \text{(left-side of the equation, (14))}
\end{aligned}$$

Then for $\beta \in [\underline{\beta'}, 1]$ $(\underline{\beta'} \in [0,1]$, we have

$$\overbrace{\sum_{t=0}^{\infty} \beta^t U(c_1, c_2, 1-l_1)}^{\text{the utility without policy-change}} \leq U^d(\cdot) + E_t \sum_{\omega \in \Omega} \sum_{t=s+1}^{\infty} \beta^t [\{U^d(\cdot)|\pi(\omega_t)\} + \frac{\beta}{1-\beta} U^s(\cdot)], \tag{26}$$

which means that the discretionary option is better. ∎

A.5 Proof of Prop.5

Comparison of the following two equations shows the following: the first and second equations respectively represent the conditions for which the commitment policy is sustainable under perfect and imperfect monitoring cases.

$$\frac{\beta}{1-\beta} [U^*(\cdot) - U^s(\cdot)] \geq U^d(\cdot) - U^*(\cdot) \tag{27a}$$

$$\frac{\beta}{1-\beta} [U^*(\cdot) - U^s(\cdot)] \geq \left[U^d(\cdot) - U^*(\cdot)\right] + E_t \sum_{\omega \in \Omega} \sum_{t=s+1}^{\infty} \beta^t [\{U^d(\cdot)| f(\omega_t \sigma^g)\}, \tag{27b}$$

In eq. (27b), the second term of the right side exists, which does not exist in the first equation. Therefore, the second equation is expected to hold under the upper value of β. We have $\underline{\beta'} > \underline{\beta}$. ∎

A.6 Proof of Prop.6

Compare the following two equations: the first and second equations are those conditions for which the commitment policy are, respectively, sustainable under perfect and imperfect monitoring cases.

$$\frac{\beta}{1-\beta} [U^*(\cdot) - U^s(\cdot)] \geq U^d(\cdot) - U^*(\cdot) \tag{28a}$$

$$\frac{\beta}{1-\beta} [U^*(\cdot) - U^s(\cdot)] \geq \left[U^d(\cdot) - U^*(\cdot)\right] + E_t \sum_{\omega \in \Omega} \sum_{t=s+1}^{\infty} \beta^t [\{U^d(\cdot)| f(\omega_t \sigma^g)\}, \tag{28b}$$

In eq. (28b), the second term of the right side exists, which does not exist in the first equation. Therefore, the second equation is expected to hold under the upper value of β. Consequently, we have $\underline{\beta'} > \underline{\beta}$. ∎

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