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Free Entry, Market Diffusion, and Social Inefficiency with Endogenously Growing Demand

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Free Entry, Market Diffusion, and Social Inefficiency with Endogenously Growing Demand*

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Abstract

This paper analyzes market diffusion in the presence of oligopolistic interaction among firms. Market demand is positively related to past market size because of consumer learning, networks, and bandwagon effects. Firms enter the market freely in each period with fixed costs and compete in quantities. We demonstrate that free entry leads to a socially inefficient number of firms over time, and that the nature of the inefficiency changes as the market grows: the number of firms is initially insufficient but eventually excessive. This is in contrast with previous findings in the theoretical literature.

JEL Classification Codes: D11, L11, L14.
Keywords: Free Entry; Market Diffusion; Intertemporal Externalities; Entry Regulation.

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1 Introduction

The number of suppliers changes over the lifetime of a product market. Gort and Klepper (1982) investigate 46 new products in the US from their initial introductions up to 1981.[1] They characterize the evolution of markets as having five stages. In Stage 1, the number of firms in the market is small. In Stage 2, the number drastically grows. In Stage 3, it reaches a maximum. In Stage 4, there is a shakeout of firms, and the number rapidly decreases. In Stage 5, the number stabilizes. The time pattern in the growth of the number of firms from Stage 1 to Stage 3 is characterized as "S-shaped diffusion."[2]

In the related theoretical literature, market diffusion from Stage 1 to Stage 3 is regarded as resulting from intertemporal externalities, such as learning by doing (Jovanovic and Lach (1989)), firms' learning of the market demand (Rob (1991)), or intertemporal consumption externalities (Vettas (2000) and Kitamura (2010)). In this literature, the number of producers under free entry is socially insufficient over time.

The aim of this paper is to investigate market diffusion with oligopolistic interaction among firms theoretically. The previous literature assumes that firms are atomistic price takers whose production levels are exogenously determined. Although this assumption is innocuous in explaining S-shaped diffusion, oligopolistic interaction may be a nonnegligible element when we discuss entry regulation policy for new industries.

To explore the importance of oligopolistic interaction, we construct a model of market diffusion following Kitamura (2010). In his model, the market grows because of intertemporal consumption externalities, and the market demand depends positively on the previous period's market size. His approach allows us to analyze the diffusion model with oligopolistic interaction by comparing free-entry diffusion, in which the number of firms is determined by the zero-profit condition, and socially optimal diffusion, in which the number of firms maxi-

[1] There are several papers on the growth of markets. See, for example, Klepper and Graddy (1990), Jovanovic and Macdonald (1994), and Klepper (1997).

[2] S-shaped diffusion is not an isolated phenomenon. Empirical evidence shows that the interfirm and intrafirm diffusion of new technology tends to be S-shaped: see the seminal work of Griliches (1957), Mansfield (1968), and Stoneman (2002) for a survey of technological diffusion. In addition, the S-shaped interhousehold diffusion is treated as a stylized fact in the marketing literature. For example, this phenomenon is observed in color televisions (Karshenas and Stoneman (1992)), fax machines (Economides and Himmelberg (1995)), clothes dryers (Krishnan, Bass, and Jain (1999)), and mobile phones (Gamboa and Otero (2009)).

mizes social welfare.[3] The novel dimension here is that not only the number of firms but also the output per firm is endogenously determined.[4]

We demonstrate that the existence of oligopolistic interaction among firms does not play an essential role in explaining S-shaped diffusion, however, more importantly, it is crucial in considering entry regulation policy in new industries. First, we find that S-shaped diffusion arises in free-entry equilibrium in our model with oligopolistic interaction. This result can be explained using the logic in the previous literature. Therefore, this finding implies that the assumption that firms are atomistic price takers does not play a crucial role in explaining the fundamental mechanism of S-shaped diffusion.

Second, we have an inefficiency result that differs from the previous literature, in which the number of firms under free entry is found to be inefficiently small over time. In our model, in contrast, the nature of the inefficiency (i.e., too few firms or too many) depends on the degree of market maturity: when the market is in the growing phase, the number of firms under free entry is insufficient. However, as the market enters the mature phase, the number of firms becomes excessive.

This result provides a new policy implication. According to the previous literature, entry into markets should be encouraged over time. Based on our result, however, entry regulations should be changed depending on the phase of market growth: entry should be initially encouraged but eventually restricted.

To understand this result, we begin by considering the case where the output level is exogenously determined, as in the previous literature. When the output level of each firm is exogenous, new entry only leads to the demand shift effect; that is, new entry today increases tomorrow's demand because of intertemporal consumption externalities. This effect can be regarded as the future benefit of increasing the current number of firms. The previous literature concludes that the number of firms under free entry is socially insufficient over time because firms under free entry do not internalize the future benefit from intertemporal

[3]One of the important elements to develop in a model of market diffusion is demand structure. In the models of Rob (1991) and Vettas (2000), the demand curve is a horizontal straight line, and we cannot examine the role of oligopolistic interaction. In contrast, the demand curve is downward sloping in Kitamura (2010). This allows us to analyze the model of diffusion with oligopolistic interaction.

[4]Bergemann and Välimäki (1997) analyze market diffusion with strategic behavior. In their model, firm output is endogenously determined, but the number of firms is fixed exogenously.

externalities when they enter the market.

In contrast, when oligopolistic interaction exists, as in our model, new entry today also leads to a "business stealing effect"; that is, new entry causes existing firms to reduce their output levels today. This effect can be regarded as the current loss of increasing the current number of firms. Therefore, in our model, the socially optimal number of firms depends on the magnitudes of the future benefit from demand shift and of the current loss from business stealing. Since S-shaped diffusion arises when the demand shift effect is initially stronger but eventually weaker, the number of firms under free entry is initially insufficient but eventually excessive.

Our findings suggest that although it is useful to express the fundamental mechanism of S-shaped diffusion via a framework in which firms are atomistic price takers, we need to take into account the oligopolistic interaction among firms when applying the model to entry regulation policy. Our analysis may especially apply to entry regulation policy when the government in a developing country encourages or regulates new entry in infant industries, which are already mature in developed countries and where an intertemporal consumption externality is most likely to be observed. When we discuss the entry regulation policy in such situations, the framework that ignores oligopolistic interaction may yield misleading predictions.

This paper is related to the literature concerned with the Excess Entry Theorem.[5] Mankiw and Whinston (1986) show that the number of firms under free entry is socially excessive because oligopolistic interaction leads to a business stealing effect. Our model can be interpreted as examining the social inefficiency of free entry from a dynamic perspective.

In addition, this paper is related to the literature concerned with intertemporal consumption externalities. In a number of markets, demand may be positively related to past market size. There are several reasons for this phenomenon.[6] First, past market size may play an important role in the market in the presence of network externalities.[7] Second, past market

[5]There are a number of theoretical works on the Excess Entry Theorem. See, for example, Spence (1976), Dixit and Stiglitz (1977), and Suzumura and Kiyono (1987). For recent works, see Ghosh and Morita (2007), who analyze a vertical oligopoly model and show that free entry leads to a socially insufficient number of firms.

[6]One of the established literatures focuses on rational addiction, where a consumer's utility is positively related to the volume of his/her own past consumption (Becker and Murphy (1988)).

[7]See Katz and Shapiro (1994) for a survey. Goolsbee and Klenow (2002) empirically examine the impor-

size may be a signal of popularity, and consumers often desire a popular product because of the bandwagon effect, which is purely psychological.[8] The existence of media reports and advertisements informing consumers of past sales may support this hypothesis.[9] Finally, an increase in past market size may provide more information about product quality to consumers.[10]

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 introduces a free-entry equilibrium and a socially optimal equilibrium. Section 4 analyzes the social inefficiency of free-entry diffusion. Section 5 gives concluding remarks. The proofs of all results are provided in the Appendix.

2 Model

This section develops the model, which follows Kitamura (2010). The new dimension here is the oligopolistic interaction among firms: the output per firm is endogenously determined. This modeling strategy is designed to clarify the importance of oligopolistic interaction among firms.

We characterize the consumers' behavior in 2.1 and the firms' behavior under free entry in 2.2. Then, we introduce the timing of the game in 2.3. We assume that time is discrete and that the horizon is infinite. In this paper, it is also assumed that the market is a perishable goods market or a service market in which the service fee is charged in every period.

tance of learning and network externalities in the diffusion of home computers. They find that people are more likely to buy their first home computer in areas where a large fraction of households already own computers or when a large share of their friends and family already own computers.

[8]See Leibenstein (1950), a seminal work on the bandwagon effect. Becker (1991) studies restaurant pricing where consumer demand is positively related to market size. Biddle (1991) develops an empirical model of the bandwagon effect and shows that the current demand is positively related to past demand levels.

[9]See, for example, Monteiro and Gonzalez (2003).

[10]See Caminal and Vives (1996), who theoretically analyze the importance of past market share as a signal of product quality. Berndt, Pindyck, and Azoulay (2003) empirically examine the role of past sales in the demand for pharmaceuticals. They find empirical evidence that the past sales of a drug have a positive effect on both its value to consumers and its rate of diffusion at the brand level. See also Grinblatt, Keoharju, and Ikäheimo (2008), who find that the purchases of neighbors influence a consumer's purchases of automobiles.

2.1 Consumers

There are a number of mass unit consumers for all periods. Each consumer has a different preference for a product. Let θ be the type of consumer, which is stationary for all periods and is uniformly distributed on the interval $[0, 1]$. We also assume that the number of consumers is a/b, where $a > 0, b > 0$. The market size, defined as the number of consumers who purchase the product, at period t is denoted by Q_t. The consumers' willingness to pay depends on the previous period's market size because of the intertemporal consumption externality. We assume the following reservation price for consumers of type θ at periods $t = 1, 2, ..., u_t(\theta)$:

Assumption 1.

$$u_t(\theta) = U(\theta, Q_{t-1}) = a\theta + \sigma(Q_{t-1}), \tag{1}$$

where $\sigma(Q_{t-1}) > 0$ *represents the intertemporal consumption externality and has the following properties:* $\sigma(0) = 0, \sigma'(Q_{t-1}) > 0, \sigma''(Q_{t-1}) < 0, \sigma'''(Q_{t-1}) > 0, \lim_{Q_{t-1}\to 0} \sigma'(Q_{t-1}) = \infty$, *and* $\lim_{Q_{t-1}\to\infty} \sigma'(Q_{t-1}) = 0$.

From Assumption 1, it is easy to see that the intertemporal consumption externality has the following two properties: it is strictly increasing in the previous market size; however, its degree, or equivalently the benefit of the externality, is strictly decreasing. This assumption guarantees that the market size converges to a finite number.

A consumer of type θ pays p_t for the product and enjoys consumer surplus of $u_t(\theta) - p_t$. The consumer purchases the product if and only if the consumer surplus is nonnegative, i.e., $u_t(\theta) - p_t \geq 0$. Then, the inverse demand function at period t, $P(Q_{t-1}, Q_t)$, becomes:

$$P(Q_{t-1}, Q_t) = \begin{cases} a + \sigma(Q_{t-1}) - bQ_t & 0 \leq Q_t \leq \frac{a}{b}, \\ 0 & Q_t > \frac{a}{b}, \end{cases} \tag{2}$$

for all $t = 1, 2, ...,$ and $0 \leq Q_{t-1} \leq a/b$. It is easy to see that the inverse demand is a strictly increasing function of the previous period's market size but a strictly decreasing function of the current period's market size.[11]

[11] In the models of Rob (1991) and Vettas (2000), the demand is perfectly elastic: the demand curve is a horizontal straight line. This demand structure does not allow us to analyze the role of oligopolistic interaction, where firms compete in quantities. In contrast, the downward-sloping demand here allows us to introduce oligopolistic interaction to the model of market diffusion.

2.2 Firms under Free Entry

Firms under free entry are identical. In contrast to Kitamura (2010), firms in this paper compete in quantities, and the output per firm is endogenously determined. For every period, there are incumbents and a large (infinite) number of potential entrants. When potential entrants decide to enter the market, entry must incur a setup cost $f > 0$, which is the initial investment in purchases such as machines. We assume that machines are perfectly durable and can be operated in an environment of constant returns to scale for all periods. Let $c > 0$ be the marginal production cost, $i > 0$ be a constant interest rate, and $\beta \equiv 1/(1+i)$ denote the discount factor.

2.3 Timing

For each period, a period game consists of a two-stage game, as follows:

1. *Stage 1: Entry Decision*

   In Stage 1, potential entrants decide whether to enter the market or not.[12] If they enter the market with setup cost f, they compete with incumbents in Stage 2 and earn profits. If they do not, their profits for the period are zero. Let N_t be the number of incumbents at period t and let n_t be the number of new entrants in Stage 1 of period t. By definition, we have $n_t = N_t - N_{t-1}$ for all $t = 1, 2, \ldots$. Assuming that $N_0 = 0$, we have $N_t = \sum_{\tau=1}^{t} n_\tau$. If the demand is small and the fixed cost is high or the discount factor is low, then entry may not occur in the first period. The following assumption guarantees first-period entry:

   Assumption 2.
   $$\frac{(a-c)^2}{4b} > (1-\beta)f. \tag{3}$$

   Assumption 2 implies that the number of new entrants in the first period is at least 1, i.e., $N_1 \geq 1$. If inequality (3) does not hold, then first-period entry is not profitable and

[12] In the models of Rob (1991) and Vettas (2000), the incumbent decides whether or not to exit the market because of demand uncertainty. In contrast, exit never occurs here because there is no demand uncertainty, as in Kitamura (2010).

does not occur. Since this condition holds for all following periods, the entry never occurs.

2. *Stage 2: Production*

   In Stage 2, firms in the market compete in quantities to maximize operation profits for the period. We assume that the equilibrium in this stage is symmetric. Let q_t^i be the equilibrium output of firm i at period t. We also define p_t as the equilibrium price in period t and $\pi_t^i = [p_t - c]q_t^i$ as the equilibrium operation profits of firm i in period t.

3 Equilibrium

This section provides the characterization of free-entry equilibrium and socially optimal equilibrium. We first characterize the post-entry equilibrium at period t given the number of incumbents N_{t-1} and the number of new entrants n_t in 3.1. Then, we characterize free-entry equilibrium and show that free-entry diffusion becomes S-shaped in 3.2. Finally, we characterize socially optimal diffusion in 3.3.

3.1 Post-Entry Equilibrium

Given the number of firms in Stage 1, firms in Stage 2 compete in quantities and choose their output levels to maximize their profits for the period, π_t^i.[13] The post-entry equilibrium is determined by the market clearing condition, the firms' profit-maximizing behavior, and the symmetry property. Now, we define the post-entry equilibrium as follows:

Definition 1. *Given N_{t-1} and n_t, the post-entry equilibrium consists of sequences $\{Q_t, p_t, q_t^i\}$ that simultaneously satisfy the following conditions:*

1. *Firm output is symmetric for all $t = 1, 2, ...$:*

$$q_t^i = q_t \text{ for all } i. \tag{4}$$

[13]This implies that when firms choose their output levels, they do not take into account the entry of new firms in future periods. In other words, firms play the stage game Nash equilibrium at each period. Although it would be more realistic to assume that firms take future entry into account, the analysis would become considerably more complicated.

2. *The market clears for all* $t = 1, 2, ...$*:*

$$Q_t = N_t q_t. \tag{5}$$

3. *The market price is determined by the inverse demand for all* $t = 1, 2, ...$*:*

$$p_t = P\Big(Q_{t-1}, Q_t\Big). \tag{6}$$

4. *Each firm's output is the best responses to other firms' outputs for all* $t = 1, 2, ...$*:*

$$q_t^i = \arg\max_{q_t^i \geq 0} \Big[P\Big(N_{t-1}q_{t-1}, (N_t - 1)q_t + q_t^i\Big) - c\Big]q_t^i \ \ \textit{for all } i. \tag{7}$$

According to the above definition, we identify the properties of the post-entry stage equilibrium. From equation (2) and equilibrium conditions (4)–(7), the output per firm becomes:

$$q_t = \frac{1}{N_t + 1} \frac{a + \sigma(N_{t-1}q_{t-1}) - c}{b}. \tag{8}$$

Then, we obtain the post-entry equilibrium price and operation profits per firm, respectively:

$$p_t = c + \frac{(a + \sigma(N_{t-1}q_{t-1}) - c)}{N_t + 1}, \tag{9}$$

and

$$\pi_t = \frac{1}{(N_t + 1)^2} \frac{(a + \sigma(N_{t-1}q_{t-1}) - c)^2}{b}. \tag{10}$$

It is easy to see that the post-entry equilibrium has the standard properties of the Cournot–Nash equilibrium under linear demand and constant marginal cost. For the convenience of the analysis in the following sections, we summarize these properties of post-entry equilibrium as follows:

Lemma 1. *The post-entry equilibrium has the following properties:*

1. *Aggregate output is strictly increasing in the number of firms and is bounded:*

$$\frac{\partial N_t q_t}{\partial N_t} > 0 \ \textit{and} \ \lim_{N_t \to \infty} N_t q_t = \frac{a + \sigma(N_{t-1}q_{t-1}) - c}{b}. \tag{11}$$

2. *Output per firm is strictly decreasing in the number of firms and converges to zero:*

$$\frac{\partial q_t}{\partial N_t} < 0 \ \textit{and} \ \lim_{N_t \to \infty} q_t = 0. \tag{12}$$

3. *Equilibrium prices (operation profits per firm) are strictly decreasing in the number of firms and converge to the marginal cost (zero):*

$$\frac{\partial p_t}{\partial N_t} < 0, \lim_{N_t \to \infty} p_t = c \text{ and } \frac{\partial \pi_t}{\partial N_t} < 0, \lim_{N_t \to \infty} \pi_t = 0. \quad (13)$$

One of the significant properties of the post-entry equilibrium is that the entry generates an externality effect called a business stealing effect; that is, the new entry decreases the incumbents' output levels. As proved in Mankiw and Whinston (1986), this effect makes the free-entry equilibrium socially excessive in the static model. In the previous literature on market diffusion, the output per firm is exogenously determined under free entry. Therefore, the second property of Lemma 1 does not hold. This leads to different welfare implications for market diffusion.

3.2 Free-Entry Diffusion

In this subsection, we first characterize the free-entry diffusion in 3.2.1 given the post-entry equilibrium outcome derived in 3.1. Then, we examine the time pattern of free-entry diffusion and show that it becomes S-shaped by using the logic in the previous literature in 3.2.2.

3.2.1 Characterization of free-entry equilibrium

Let n_t^e be the free-entry equilibrium number of new entrants at period t, and let N_t^e be the free-entry equilibrium number of firms at period t. For simplicity, we treat N as a continuous variable. We define $R(N_{t-1}, n_t)$ as the discounted sum of future operation profits at period t, which is composed of the direct operation profits at period t and the discounted future operation profits,[14] i.e.:

$$R(N_{t-1}, n_t) = \pi_t + \beta R(N_t, n_{t+1}), \quad (14)$$

for all $t = 1, 2, ...$ In Stage 1 of each period, potential entrants enter the market as long as the present value of net profits is positive, i.e., $R(N_{t-1}, n_t) > f$. Therefore, in each period, the number of new entrants satisfies the zero-profit condition, defined as follows:[15]

[14]Because entrants and incumbents in Stage 2 are symmetric at each period, and the horizon is infinite, they have the same present value of their future revenue streams.

[15]One of the important factors for the existence of market diffusion is that the horizon is infinite. When the horizon is infinite, the zero-profit condition holds with a positive number of entrants at each period. In contrast,

Definition 2. *Given the post-entry equilibrium outcome, the free-entry equilibrium consists of the sequence* $\{n_t^e\}_0^\infty$*, which satisfies the zero-profit condition for all periods, i.e.:*

$$f \geq R(N_{t-1}^e, n_t^e), \text{ with equality if } n_t^e > 0. \tag{15}$$

According to the above definition, the properties of the free-entry equilibrium under the transition process are identified. From the zero-profit condition (15), the operation profits in the free-entry equilibrium become $\pi_t^e = (1-\beta)f$ for each period. In addition, from equations (9) and (10), the price in the free-entry equilibrium becomes $p_t^e = c + \sqrt{(1-\beta)bf}$ for all periods. These properties imply that both the equilibrium profits and the equilibrium price are constant. By substituting equation (10) into the zero-profit condition (15), the market diffusion under free-entry equilibrium is summarized as follows:

Proposition 1. *Let* N^e *be the number of firms in the steady state under the free-entry equilibrium. Suppose that* $N_0^e = 0$*. Then, for all* $t = 1, 2, ...,$ *there exists a unique* $n_t^e > 0$ *that satisfies (15), while the free-entry equilibrium output per firm,* q_t^e*, and number of firms satisfy the following conditions:*

1. *The output per firm is constant over time:*

$$q_t^e = \sqrt{\frac{(1-\beta)f}{b}}, \text{ for all } t = 1, 2, ... \tag{16}$$

2. *The number of firms is an increasing function of the number in the previous period:*

$$N_t^e = \frac{a + \sigma\left(N_{t-1}^e \sqrt{\frac{(1-\beta)f}{b}}\right) - c}{\sqrt{(1-\beta)bf}} - 1, \text{ for all } t = 1, 2, ..., \tag{17}$$

and it satisfies $N_t^e \in [0, N^e]$ *for all* $t = 1, 2, ...,$ *and monotonicity,* $N_0^e = 0$ *and* $N_t^e \to N^e$ *as* $t \to \infty$.

Proof. See Appendix.

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if the horizon is finite, then incumbents and entrants are not symmetric. To hold the zero-profit condition, the entrants need to achieve higher profits than the incumbents, and the number of incumbents should decrease. Therefore, under a finite horizon, the zero-profit condition does not lead to a positive number of entrants at each period.

Q.E.D.

The dynamic system of equation (17) is summarized in Figure 1. Figure 1 shows that we have $\sigma'(N^e \sqrt{(1-\beta)f}/\sqrt{b})/b < 1$ in the steady state.[16] This indicates that the number of firms under free entry does not reach the steady state as long as the degree of consumption externality is strong enough.

The constant values of equilibrium output, profits per firm, and equilibrium price have several implications. First, the time pattern of the free-entry equilibrium number of firms coincides with that of the aggregate output. In addition, the constant equilibrium price implies that the firms under free entry act as if they were price takers whose output levels were exogenously determined. Therefore, the free-entry equilibrium, in this paper, basically has the same properties as in the previous literature on market diffusion.

3.2.2 S-shaped market diffusion

We now examine the time pattern of free-entry diffusion and show that it becomes S-shaped (initially convex and eventually concave) when the externality effect is initially strong enough. Note that the degree of consumption externality decreases as the market size increases. This makes the number of firms under free entry monotonically converge to the steady state, and the time pattern eventually becomes concave. Therefore, the time pattern of free-entry diffusion becomes S-shaped if and only if the number of new entrants initially increases.

Note that in free-entry diffusion, the numbers of firms in the first and second periods are $N_1^e = [a-c]/\sqrt{(1-\beta)bf} - 1$ and $N_2^e = [a+\sigma(N_1^e q_1^e)-c]/\sqrt{(1-\beta)bf} - 1$, respectively. While the former does not depend on the consumption externality, the latter does. This implies that the strong degree of consumption externality leads to the high market growth from Period 1 to Period 2 and the convex time pattern of market diffusion in the early periods:

Proposition 2. *The time pattern of free-entry diffusion becomes S-shaped if and only if:*

$$a - c - \sqrt{(1-\beta)bf} < \sigma\Big(\frac{a - c - \sqrt{(1-\beta)bf}}{b}\Big). \tag{18}$$

Proof. See Appendix.

[16]Note that by differentiating (17) with respect to N_{t-1}^e, we have $\partial N_t^e/\partial N_{t-1}^e = \sigma'(N_{t-1}^e \sqrt{(1-\beta)f}/\sqrt{b})/b$.

Q.E.D.

From inequality (18) and the properties of $\sigma(\cdot)$, it is easy to see that the initial convexity of the free-entry diffusion is observed in the environment of small values of initial market size. This follows from the low market demand, low discount factor, and high production and setup costs. In addition, given these parameters, the strong consumption externality effect contributes to the initial convexity of free-entry diffusion. From Figure 1, it is obvious that the free-entry diffusion has a convex time pattern provided that $\sigma'(N^e_{t-1}\sqrt{(1-\beta)f}/\sqrt{b})/b > 1$. Therefore, we conclude that the S-shaped diffusion is observed for low initial market size and a strong consumption externality effect.

The mechanism of the initial convexity of free-entry diffusion, explained above, is the same as that in Vettas (2000) and Kitamura (2010), in which firms are small atomistic price takers. Therefore, our result implies that the oligopolistic interaction does not play an essential role in explaining the fundamental mechanism of S-shaped diffusion.

3.3 Socially Optimal Diffusion

A social planner sets the number of firms to maximize the social welfare, given the post-entry equilibrium outcome derived in 3.1. Let n^o_t be the number of new entrants and N^o_t be the number of firms set by the planner at period t, and let q^o_t be the output per firm under socially optimal planning at period t. Now, we define the socially optimal equilibrium as follows:

Definition 3. *Given the post-entry equilibrium outcome, socially optimal planning satisfies the following Bellman equation:*

$$V(N^o_{t-1}) = \max_{n^o_t \geq 0} \int_0^{(N^o_{t-1}+n^o_t)q^o_t} [a+\sigma(N^o_{t-1}q^o_{t-1})-bQ]dQ-(N^o_{t-1}+n^o_t)q^o_t c-n^o_t f+\beta V(N^o_{t-1}+n^o_t), \quad (19)$$

subject to equation (8).

The interpretation of equation (19) is as follows. The present value of the sum of future welfare is current welfare plus the discounted next-period value of the sum of future welfare.

Note that the social planner is unable to control directly the output per firm in Stage 2 (post-entry stage) but is able to do so indirectly by controlling the number of firms.[17] We now characterize the socially optimal equilibrium as follows:

Proposition 3. *Suppose that $N_0^o = 0$. Then, the optimal diffusion path satisfies the following second-order difference equation:*

$$
\begin{aligned}
f = & R(N_{t-1}^o, n_t^o) \\
& + [p_t^o - c]N_t^o \frac{\partial q_t^o}{\partial n_t} \\
& + \beta\sigma'(N_t^o q_t^o)\Big[q_t^o + N_t^o \frac{\partial q_t^o}{\partial n_t}\Big] N_{t+1}^o q_{t+1}^o \\
& + \beta[p_{t+1}^o - c]N_{t+1}^o \Big[\frac{\partial q_{t+1}^o}{\partial N_t} - \frac{\partial q_{t+1}^o}{\partial n_{t+1}}\Big],
\end{aligned}
\tag{20}
$$

where $R(N_{t-1}, n_t) = \pi_t + \beta f$.

Equation (20) shows that the marginal expansion cost is equal to the marginal social benefit, which is composed of four elements. The first term on the right-hand side of equation (20) is the present value of future operation profits. The second term is welfare loss from the business stealing effect, which is captured as $\partial q_t^o / \partial n_t$: the new entry reduces the current output per firm. This term is regarded as the current loss: increasing current entry reduces the social welfare. The third term is the future benefit from the demand shift effect, in which an increase in the number of firms directly raises demand in the subsequent period. The last term is the future benefit following the business creating effect, $\partial q_{t+1}^o / \partial N_t - \partial q_{t+1}^o / \partial n_{t+1}$: the new entry indirectly raises the output per firm in the subsequent period through the demand shift effect.

The last three terms represent the intertemporal trade-off. The current loss gives the planner an incentive to restrict the number of firms. However, the planner has a competing incentive to raise the number of firms because of future benefits. The optimal planning is determined by the magnitudes of these losses and benefits.[18]

[17] Therefore, this equilibrium is interpreted as the second-best equilibrium rather than the first-best equilibrium, in which the planner would be able to control directly the output per firm. If that were the case, the first-best equilibrium number of firms would be 1 because of economies of scale.

[18] In socially optimal diffusion, in contrast to free-entry diffusion, the number of firms in the first period

Note that the new dimension here beyond the previous literature is the existence of the intertemporal trade-off in the optimal planning. This trade-off has not been addressed in the previous literature concerned with market diffusion. In past works, the business stealing effect and business creating effect are not considered, although there exists a benefit from internalizing intertemporal externalities, which are engines of the market growth.[19]

4 Social Inefficiency and Time Dependence

This section analyzes the social inefficiency of free-entry diffusion by comparison with socially optimal diffusion. Note that the only difference between free-entry diffusion and socially optimal diffusion is whether the intertemporal trade-off exists or not. We first explore the social inefficiency of free-entry diffusion when the market is in the growing phase in 4.1. Then, we examine the case when the market is in the mature phase in 4.2.

4.1 Social Inefficiency in the Growing Phase

To begin the analysis, we rewrite equation (20). Let the current loss of increasing the number of firms at period t be $\mu(N^o_{t-1}, N^o_t)$, and let the future benefit of increasing the number of firms at period t be $\lambda(N^o_{t-1}, N^o_t, N^o_{t+1})$. Then, we rewrite equation (20) with the linear demand function as follows:

$$(1-\beta)f = \pi(N^o_{t-1}, N^o_t) - \mu(N^o_{t-1}, N^o_t) + \lambda(N^o_{t-1}, N^o_t, N^o_{t+1}), \tag{21}$$

where

$$\pi(N_{t-1}, N_t) = \frac{1}{(N_t+1)^2}\frac{(a+\sigma(N_{t-1}q_{t-1})-c)^2}{b}, \tag{22}$$

$$\mu(N_{t-1}, N_t) = \frac{N_t}{N_t+1}\pi(N_{t-1}, N_t), \tag{23}$$

$$\lambda(N_{t-1}, N_t, N_{t+1}) = \frac{\beta\sigma'(N_tq_t)}{b}\frac{N_{t+1}q_{t+1}}{N_tq_t}\frac{N_{t+1}+2}{N_{t+1}+1}\mu(N_{t-1}, N_t). \tag{24}$$

---

depends on, and is positively related to, the degree of intertemporal consumption externality. Therefore, the strong externality effect leads to an initially large number of new entrants in socially optimal diffusion. This makes the S-shaped time pattern more difficult to obtain than in free-entry diffusion.

[19] Since the output per firm is exogenously determined in this literature, $\partial q^o_t/\partial n_t = 0$ and $\partial q^o_t/\partial N_{t-1} = 0$ for all $t = 1, 2, \ldots$. It is easy to see that the business stealing effect and the business creating effect in equation (20) are absent, but the future benefit from the demand shift effect still exists.

Note that the free-entry diffusion satisfies $(1-\beta)f = \pi(N^e_{t-1}, N^e_t)$ for all $t = 1, 2, ...$ Therefore, the difference in the number of firms depends on the magnitude of the current loss, $\mu(N^o_{t-1}, N^o_t)$, and the magnitude of the future benefit, $\lambda(N^o_{t-1}, N^o_t, N^o_{t+1})$, which depends on the demand shift effect and the business creating effect.

Furthermore, the future benefit at period t is determined by the degree of discounting, β, the degree of consumption externality, $\sigma'(N^o_t q^o_t)$, the growth rate of the market size, $[N^o_{t+1}q^o_{t+1} - N^o_t q^o_t]/N^o_t q^o_t$, and the number of firms at period $t+1$, N^o_{t+1}. By comparing equations (23) and (24), it is seen that future benefits are produced by a higher discount factor, a stronger degree of consumption externality, higher market growth, and a smaller number of firms. In this environment, free entry leads to a socially insufficient number of firms:

Proposition 4. *Suppose that* $\mu(N^o_{t-1}, N^o_t) \gtrless \lambda(N^o_{t-1}, N^o_t, N^o_{t+1})$, *i.e.:*

$$1 \gtrless \frac{\beta\sigma'(N^o_t q^o_t)}{b} \frac{N^o_{t+1}q^o_{t+1}}{N^o_t q^o_t} \frac{N^o_{t+1}+2}{N^o_{t+1}+1}. \tag{25}$$

Then, for all $N^e_{t-1} \gtreqless N^o_{t-1}$, *we have* $N^e_t \gtrless N^o_t$.

Proof. See Appendix.

Q.E.D.

Note that we have $N^e_0 = N^o_0 = 0$. Therefore, Proposition 4 implies that, at Period 1, the free-entry equilibrium number of firms becomes socially insufficient, $N^e_1 < N^o_1$, when the future benefit is larger than the current loss, $\mu(N^o_0, N^o_1) < \lambda(N^o_0, N^o_1, N^o_2)$.

In addition, by interpreting Proposition 4 differently, we see that the free-entry equilibrium number of firms is more likely to be socially insufficient at early periods. At early periods, (a) the degree of consumption externality is strong,[20] (b) the growth rate of the market size is high,[21] and (c) the number of firms is small. Therefore, we conclude that the free-entry equilibrium number of firms tends to be socially insufficient when the market is in the growing phase.

[20]Note that the right-hand side of (25) is larger than 1 as long as $\beta\sigma'(N^o_t q^o_t)/b \geq 1$.

[21]Note that insufficient entrants appear even if $\beta\sigma'(N^o_t q^o_t)/b < 1$. This occurs if the growth rate of the market is high enough and the number of firms is small enough.

4.2 Social Inefficiency in the Mature Phase

Next, we turn to the analysis of the mature phase. Let N^o denote the steady-state number of firms in the socially optimal diffusion that satisfies $N_t^o = N_{t+1}^o = N^o$. Then, in the steady state, we can rewrite equations (21)–(24) as follows:

$$(1-\beta)f = \pi(N^o) - \mu(N^o) + \lambda(N^o), \tag{26}$$

where

$$\pi(N) = \frac{(a + \sigma(Nq) - c)^2}{(N+1)^2 b}, \tag{27}$$

$$\mu(N) = \frac{N}{N+1}\pi(N), \tag{28}$$

$$\lambda(N) = \frac{\beta\sigma'(Nq)}{b}\frac{N+2}{N+1}\mu(N). \tag{29}$$

Note that $(1-\beta)f = \pi(N^e)$ holds in free-entry diffusion. Thus, as in the analysis of the growing phase, the difference in the number of firms depends on the magnitudes of the current loss, $\mu(N^o)$, and future benefit, $\lambda(N^o)$. As the market becomes mature, the degree of consumption externality and the growth rate of the market size become lower, and the number of firms operating in the market increases. Compared with the growing phase, these changes in the market environment lower the magnitude of the future benefit relative to the current loss, i.e., $\lambda(N_{t-1}^o, N_t^o, N_{t+1}^o)/\mu(N_{t-1}^o, N_t^o) \geq \lambda(N^o)/\mu(N^o)$.[22] More importantly, the following lemma shows that the future benefit becomes smaller than the current loss in the steady state:

Lemma 2. *In the steady state, the future benefit becomes smaller than the current loss, i.e.,* $\mu(N^o) > \lambda(N^o)$. *More precisely, we have:*

$$1 > \frac{\beta\sigma'(N^o q^o)}{b}\frac{N^o+2}{N^o+1}. \tag{31}$$

Proof. See Appendix.

[22] By comparing equations (24) and (29), it is easy to see that:

$$\frac{\lambda(N_{t-1}^o, N_t^o, N_{t+1}^o)}{\mu(N_{t-1}^o, N_t^o)} = \frac{\beta\sigma'(N_t^o q_t^o)}{b}\frac{N_{t+1}^o q_{t+1}^o}{N_t^o q_t^o}\frac{N_{t+1}^o + 2}{N_{t+1}^o + 1} \geq \frac{\beta\sigma'(N^o q^o)}{b}\frac{N^o+2}{N^o+1} = \frac{\lambda(N^o)}{\mu(N^o)}, \tag{30}$$

because $\sigma''(\cdot) < 0$, $\partial N_t q_t/\partial N_t > 0$, and $N_t^o \leq N_{t+1}^o \leq N^o$.

Q.E.D.

Lemma 2 implies that, in the steady state, the social benefit of increasing the number of firms, $\pi(N) - \mu(N) + \lambda(N)$, becomes smaller than the private benefit of firms entering the market, $\pi(N)$. Therefore, it is optimal for the social planner to slow down the rate of new firms entering as the market becomes mature. The following proposition shows that, in the steady state, the socially optimal number of firms is smaller than the free-entry equilibrium number of firms:

Proposition 5. *In the mature phase (steady state), the free-entry equilibrium number of firms is socially excessive, i.e.,* $N^e > N^o$.

Proof. See Appendix.

Q.E.D.

From Propositions 4 and 5, we conclude that the free-entry equilibrium number of firms tends to be initially insufficient but eventually excessive. This is in contrast to the results of previous studies, where the number of firms in the free-entry diffusion is socially insufficient over time. Table 1 and Figure 2 present a numerical example in which the number of firms under free entry is initially insufficient but eventually excessive.[23]

The result in this section implies that oligopolistic interaction between firms can be an important factor for the discussion of entry regulation policy in new industries. According to the previous literature, since the number of firms in free-entry diffusion is socially insufficient over time, encouraging entry is a desirable policy regardless of the phase of market growth. In contrast, this paper provides a different policy implication. The entry regulation policy should be changed depending on the phase of market growth: entry should be initially encouraged but eventually discouraged.

5 Concluding Remarks

This paper models market diffusion in the presence of oligopolistic interaction. In contrast to the previous literature on market diffusion, not only the number of new entrants but also the

[23]In this example, the steady state of the socially optimal diffusion path is locally saddle stable.

output per firm is endogenously determined by the oligopolistic interaction. In this setting, we explore the social inefficiency of free-entry diffusion.

The major result reported here is that the social inefficiency of free-entry diffusion depends on the time period. The free-entry equilibrium number of firms tends to be initially insufficient but eventually excessive. This result is in contrast to the previous literature not only on market diffusion, where the free-entry equilibrium is found to be socially insufficient over time, but also on the Excess Entry Theorem, where free entry is found to be socially excessive in the static model.

These results provide important policy implications for entry regulations in new industries. The entry regulation policy needs to be changed depending on the time and the degree of market growth: entry should be initially encouraged but eventually discouraged. It is possible to improve social welfare by giving subsidies to early entrants but taxing late entrants without violating intertemporally balanced budget constraints.[24]

There are several issues that require future research. First, as the number of firms becomes stable, there is a possibility of market restructuring. A stable number of firms may generate collusion between firms. Another instance of market restructuring is horizontal mergers. If there exist cost synergies, market maturity may lead to horizontal mergers, which reduce the number of firms and improve welfare (see Davidson and Mukherjee (2007)). This may explain the shakeout of firms corresponding to Stage 3 to Stage 5 in Gort and Klepper (1982). Second, there is concern about whether or not our results hold under other intertemporal externalities. Although we use only intertemporal consumption externalities in our model, we predict that our results would hold even in the presence of other intertemporal externalities. Finally, there is concern about the generality of our results. Our results are obtained in terms of a parametric example; however, they may apply in a more general setting. We hope that our study helps researchers to address these issues.

[24]The optimal subsidy or tax is equal to the difference between the right-hand side of equation (15) and the right-hand side of equation (20), that is, equal to the last three terms of equation (20). If these terms are positive (negative), then subsides (tax) should be given (imposed).

Appendix

Proof of Proposition 1

We first guess that $n_t^e > 0$ for all $N_{t-1}^e \in [0, N^e)$. Then, $R(N_{t-1}^e, n_t^e) = f$ for all $N_{t-1}^e \in [0, N^e)$. By solving $(1-\beta)f = \pi(N_{t-1}^e, N_t^e)$ with respect to N_t^e, we have:

$$N_t^e = N(N_{t-1}^e) = \frac{a + \sigma(N_{t-1}^e q_{t-1}^e) - c}{\sqrt{(1-\beta)bf}} - 1, \tag{32}$$

for all $t = 1, 2, \ldots$. Together with equation (8), we obtain equations (16) and (17). From the properties of $\sigma(\cdot)$, we have $N_1^e = N(N_0^e) = [a-c]/\sqrt{(1-\beta)bf} - 1 > 0$, $N'(N_{t-1}^e) > 0$, $N''(N_{t-1}^e) < 0$, $\lim_{N_{t-1}^e \to 0} N'(N_{t-1}^e) = \infty$, and $\lim_{N_{t-1}^e \to \infty} N'(N_{t-1}^e) = 0$. Therefore, $N(N_{t-1}^e)$ crosses the $N_t^e = N_{t-1}^e$ line only once, and there is a unique steady state, N^e. We finally verify that $n_t^e > 0$. From Figure 1, it is easy to see that we have $n_t^e > 0$ for all $N_{t-1}^e \in [0, N^e)$.

Q.E.D.

Proof of Proposition 2

From Figure 1, the time pattern of free entry eventually becomes concave. Therefore, it becomes S-shaped if and only if $2N_1^e < N_2^e$. From equation (17), we obtain inequality (26).

Q.E.D.

Proof of Proposition 4

We prove the first case. Let $1 > \frac{\beta\sigma'(N_t^o q_t^o)}{b} \frac{N_{t+1}^o q_{t+1}^o}{N_t^o q_t^o} \frac{N_{t+1}^o + 2}{N_{t+1}^o + 1}$ and $N_{t-1}^e \geq N_{t-1}^o$. Suppose in negation that $N_t^e \leq N_t^o$. Then, from the properties of $\pi(N_{t-1}, N_t)$, we would have the following inequalities:

$$\pi(N_{t-1}^e, N_t^e) \geq \pi(N_{t-1}^e, N_t^o) \geq \pi(N_{t-1}^o, N_t^o). \tag{33}$$

Because $\pi(N_{t-1}^e, N_t^e) = (1-\beta)f$ in the free-entry equilibrium, we have $\pi(N_{t-1}^o, N_t^o) \leq (1-\beta)f$. This is a contradiction to equation (20) because $\mu(N_{t-1}^o, N_t^o) > \lambda(N_{t-1}^o, N_t^o, N_{t+1}^o)$. In the same way, we can prove the second case.

Q.E.D.

Proof of Lemma 2

Suppose in negation that:

$$1 \leq \frac{\beta\sigma'(N^o q^o)}{b}\frac{N^o+2}{N^o+1}. \tag{34}$$

Then, we would have the following inequalities:

$$N^o q^o \leq \frac{\beta\sigma'(N^o q^o)N^o q^o}{b}\frac{N^o+2}{N^o+1} < \frac{\sigma(N^o q^o)}{b}\frac{N^o+2}{N^o+1}, \tag{35}$$

where the last inequality follows from the properties of $\sigma(\cdot)$. By substituting $q^o = (a + \sigma(N^o q^o) - c)/(b(N^o+1))$ into inequality (35), we have

$$N^o q^o < \frac{\sigma(N^o q^o)}{b}\frac{N^o+2}{N^o+1} \Leftrightarrow N^o\left[\frac{a+\sigma(N^o q^o)-c}{b(N^o+1)}\right] < \frac{\sigma(N^o q^o)}{b}\frac{N^o+2}{N^o+1} \tag{36}$$

$$\Leftrightarrow \frac{N^o[a-c]}{2} < \sigma(N^o q^o). \tag{37}$$

By using inequality (37), we have:

$$\pi(N^o) = \frac{(a+\sigma(N^o q^o)-c)^2}{b(N^o+1)^2} > \frac{(a+\frac{N^o[a-c]}{2}-c)^2}{b(N^o+1)^2} = \left[\frac{N^o+2}{N^o+1}\right]^2\frac{(a-c)^2}{4b} > \frac{(a-c)^2}{4b}. \tag{38}$$

Note that inequality (34) implies that $\mu(N^o) \leq \lambda(N^o)$. Then, together with inequality (38) and Assumption 2 (inequality (3)), we have:

$$\pi(N^o) - \mu(N^o) + \lambda(N^o) > \frac{(a-c)^2}{4b} > (1-\beta)f. \tag{39}$$

However, this contradicts the equilibrium condition (26). Therefore, inequality (31) holds.

Q.E.D.

Proof of Proposition 5

As mentioned in Subsection 3.2, we have $\sigma'(N^e q^e)/b < 1$ in the steady state of free-entry diffusion. Hence, we assume that $N^e > \hat{N}$, where $\hat{N}$ is such that $\sigma'(\hat{N}\hat{q})/b = 1$. To begin the proof of Proposition 5, we have the following lemma:

Lemma 3. *Let $Q(N) = Nq$. Then, for $N \in (\hat{N}, \infty)$,*

1. *$Q'(N) \to 0$ and $Q(N) \to m \in (0, \infty)$ as $N \to \infty$.*

2. *$\pi(N)$ is strictly decreasing in N and approaches zero as N becomes larger: $\pi'(N) < 0$ and $\lim_{N\to\infty}\pi(N) = 0$.*

Proof of Lemma 3

1. By differentiating $Q(N)$ with respect to N, we have:
$$Q'(N) = \frac{[1 - \frac{N}{N+1}]q(N)}{1 - \frac{N}{N+1}\frac{\sigma'(Q(N))}{b}} > 0, \tag{40}$$
for $N \in (\hat{N}, \infty)$. It is easy to see that $Q'(N) \to 0$ as $N \to \infty$. Then, by using L'Hôpital's rule, we obtain:
$$\begin{aligned}\lim_{N\to\infty} Q(N) &= \lim_{N\to\infty}\Big\{\frac{a + \sigma(Q(N)) - c}{b} + \frac{\sigma'(Q(N))NQ'(N)}{b}\Big\}\\ &= \lim_{N\to\infty}\Big\{\frac{a + \sigma(Q(N)) - c}{b}\Big\} + \lim_{N\to\infty}\Big\{\frac{\sigma'(Q(N))}{b}\frac{[1 - \frac{N}{N+1}]Q(N)}{1 - \frac{N}{N+1}\frac{\sigma'(Q(N))}{b}}\Big\}\\ &= \lim_{N\to\infty}\Big\{\Big[\frac{a + \sigma(Q(N)) - c}{b}\Big]\Big[1 + \frac{\sigma'(Q(N))}{b}\frac{[1 - \frac{N}{N+1}]\frac{N}{N+1}}{1 - \frac{N}{N+1}\frac{\sigma'(Q(N))}{b}}\Big]\Big\}\\ &= \lim_{N\to\infty}\Big\{\frac{a + \sigma(Q(N)) - c}{b}\Big\}.\end{aligned} \tag{41}$$
Since $\sigma(\cdot)$ satisfies the Inada condition, and $a > b$, there exists a unique $m \in (0, \infty)$ such that $m = [a + \sigma(m) - c]/b$.

2. By the properties of $Q(N)$, it is easy to see that $\pi(N) \to 0$ as $N \to \infty$. Next, we show that $\pi(N)$ is strictly decreasing in N. By differentiating $\pi(N)$, we have:
$$\pi'(N) = -\frac{2\pi(N)[b - \sigma'(Nq)]}{N[b - \sigma'(Nq)] + b} < 0, \tag{42}$$
for $N \in (\hat{N}, \infty)$. Thus, $\pi(N)$ is strictly decreasing in N.

This completes the proof of Lemma 3.

Q.E.D.

Now, we turn to the proof of Proposition 5. Suppose in negation that $N^e \leq N^o$. Then, because $\pi(N)$ is strictly decreasing in $N \in (\hat{N}, \infty)$ by Lemma 3, we have the following inequality:
$$\pi(N^e) \geq \pi(N^o). \tag{43}$$
Since N^e satisfies $\pi(N^e) = (1-\beta)f$, we have $\pi(N^o) \leq (1-\beta)f$. However, this is a contradiction to (26) because $\mu(N^o) > \lambda(N^o)$ by Lemma 2. Therefore, we have $N^e > N^o$.

Q.E.D.

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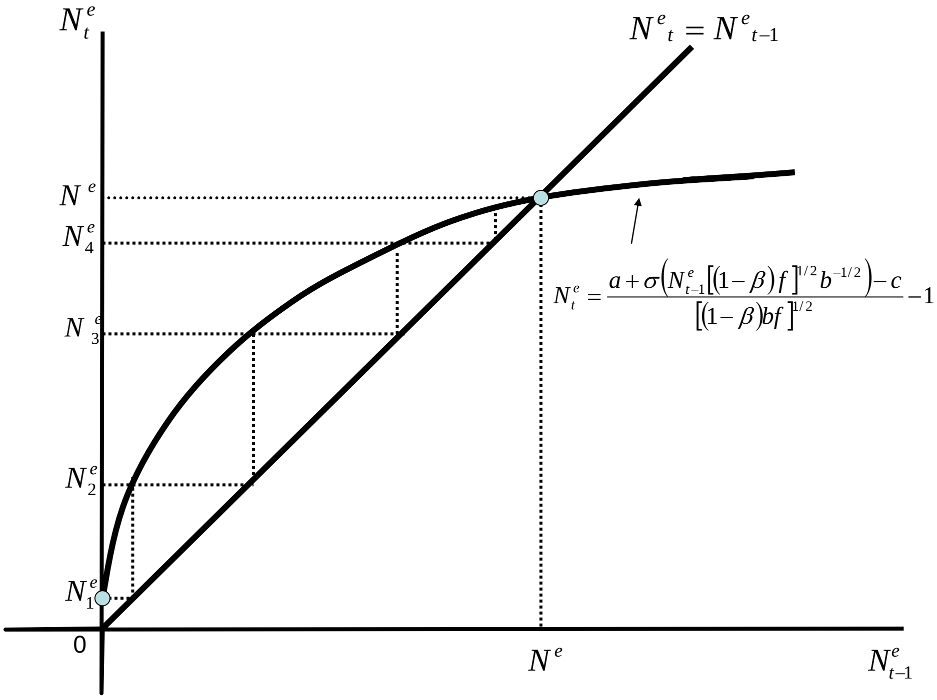

N_t^e
$N^e{}_t = N^e{}_{t-1}$
N^e
N_4^e
N_3^e
N_2^e
N_1^e
0
N^e
N_{t-1}^e
$N_t^e = \frac{a + \sigma\left(N_{t-1}^e\left[(1-\beta)f\right]^{1/2} b^{-1/2}\right) - c}{\left[(1-\beta)bf\right]^{1/2}} - 1$

Figure 1

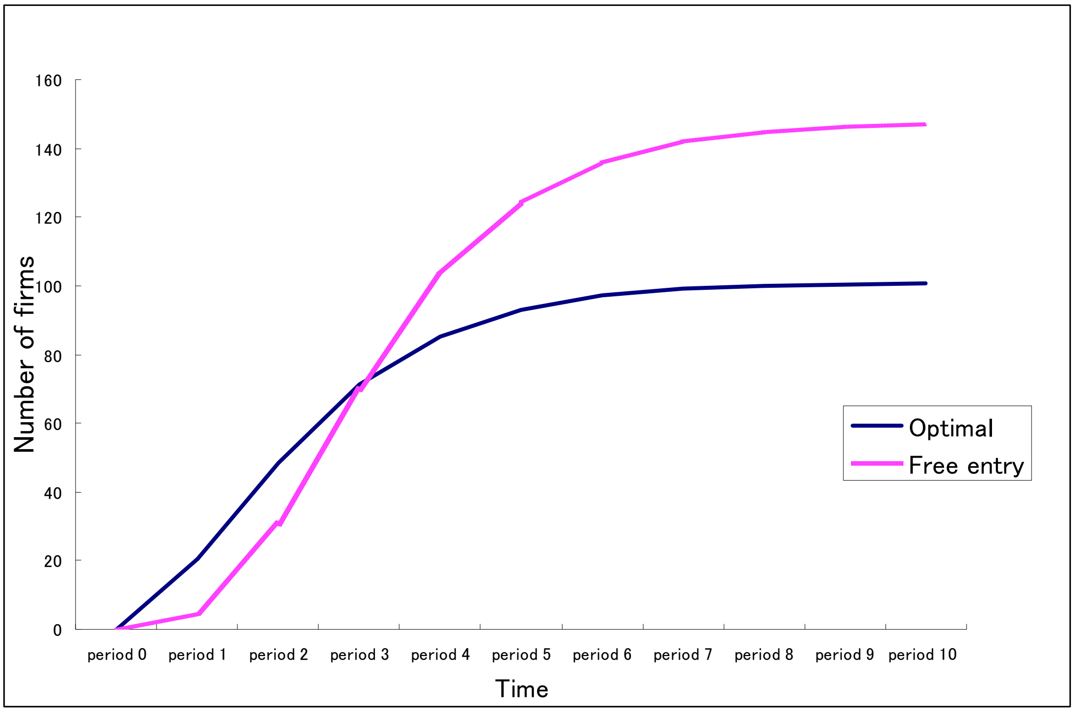

Figure 2

The optimal and the free-entry equilibrium paths for

$a = 8,\ b = 0.5,\ c = 5,\ f = 18,\ \beta = 0.97,\ \sigma(Q_{t-1}) = 6(Q_{t-1})^{0.5}$

Time	Optimal Path	Free-entry Path
period 0	0	0
period 1	20.68957495	4.773502692
period 2	48.58768294	30.49189952
period 3	71.5955845	69.77412988
period 4	86.02806271	103.1004422
period 5	93.94248156	124.2976272
period 6	98.00723622	136.0106333
period 7	100.0294918	142.054915
period 8	101.0202103	145.0721357
period 9	101.5019707	146.5542658
period 10	101.7353969	147.2766772
period 11	101.8483019	147.6274666

Table 1