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A Theory of Multiperiod Financial Contracts that do not Balance the Budget

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Abstract

This paper seeks financial contracts that specify a repayment schedule that induces both the investor and the entrepreneur to contribute to the project when the entrepreneur privately observes the realizations. Any optimal contracts do not balance the budget, i.e., some amount of money must be transferred to or from a third party depending on the history of the output realizations. We specify multiperiod optimal contracts not only with budget feasibility constraint but also without it and explain venture capital contracts as an example of the contracts that expect a third party to break the budget without budget feasibility constraint.

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1 Introduction

In the case in which a project's output realizations can be observed only by an entrepreneur, a repayment must be independent of the realizations for the entrepreneur to truthfully report them. However, an investor and an entrepreneur must obtain payoffs that are dependent on the realizations to induce them to contribute to the project. Therefore any efficient contracts do not balance the budget, i.e., the amount the entrepreneur repays is different from the amount the investor obtains under some conditions. Players use a third party to differentiate between these two amounts. With budget feasibility constraint, i.e., the assumption that the amount the entrepreneur repays must be larger than the amount the investor obtains, the third party must receive some amount of money from the players to differentiate between two amounts. However in the real world, we do not need to assume budget feasibility; for example, venture capitalists and venture firms expect general investors to provide money for them if their projects succeed by buying venture firms' shares through initial public offerings.

This paper discusses financial contracts between an entrepreneur and an investor. The contract specifies a repayment schedule that depends on the history of the output realizations. Here, we assume that the entrepreneur can observe the realizations but the investor cannot do so. We also assume that neither the investor nor the entrepreneur can commit their contributions to the project every period. It is possible for the investor to decrease the quality his financial services or for the entrepreneur to lower her effort level, either of which would lower the probability that the project succeeds. Furthermore, since it is also possible for the entrepreneur to lie about the project's realization, the investor may be unwilling to contribute to the project.

We can find these kinds of financial contracts in the real world. First, one can deposit money in money management companies and later obtain a return based on performance. Second, under Mudarabah, one of the basic forms of Islamic banking, a bank and an entrepreneur share a project's profits or losses according to a predetermined ratio. In both cases, investors or depositors can choose the amount of money they contribute in each period and their continuous contributions are important to the borrowers' projects. Moreover, in neither case can investors or depositors directly observe the performance or output realization of projects, so borrowers may lie about them to increase their own payoffs.

Townsend (1979), Diamond (1984), and Gale and Hellwig (1985) analyze the situation that lenders cannot observe the project's outputs without paying a finite monitoring cost. To induce the borrower to be honest about the output realization, the lender must audit the borrower and deprive the borrower of all of the output realizations when the borrower reports poor performance. In contrast, we assume that the monitoring costs are infinitely high. Bolton and Scharfstein (1990) also analyze this case and propose that financial institutions must stop lending in the second period with a positive probability when the borrower reports poor performance at the end of the first period to induce them to truthfully send messages about a project's performance. However, they fail to give the borrower any incentive to be truthful in the last period.

This paper models the case where both entrepreneur's contribution and an investor's contribution are important to the project. The model is similar to the ones that study moral hazard in teams (Holmstrom(1982)) and double-sided moral hazard in franchise contracts (Bhattacharyya and Lafontaine(1995)). They assume that all players can observe project's output realizations but we analyze the situation that an entrepreneur can observe the realizations but an investor cannot do so. Holmstrom(1982) shows that noncooperative behavior by players always yields an inefficient result when joint output realization is fully shared among them and that they can make an efficient contract when the sum of the amounts shared among players can be more or less than the joint output realization, i.e., when a third party breaks the budget by seizing a part of the realization from players or by paying them a bonus.

Recently, MacLeod (2003) and Kambe (2006) discussed an agency contract when a principal privately observes an agent's output realizations. To induce the agent to exert effort and the principal to be truthful about the realizations, the wage the principal pays and that the agent receives must be different under some conditions. Since they assume budget feasibility, i.e., the amount the principal pays must be larger than the amount the agent obtains, the differentiated amount of money must be passed to a third party. Here, a third party again plays a role in breaking the budget. Fuchs (2007) proposes a method to reduce the amount to be passed to a third party in a repeated game setting. We apply their findings to a financial contract under which a principal's effort level also affects the project's output realizations. Although they assume budget feasibility constraint, we also seek optimal financial contracts without this constraint because financial contracts that count on a third party to pay exist in the real world.

For the entrepreneur to truthfully report output realizations to the investor, the amount she repays must be independent of the output realizations. However, to induce players to contribute to the project, the payoffs the investor and the entrepreneur obtain after the projects must be dependent on the output realizations. Since repayment must always be constant, it is the project's property that gives the entrepreneur an incentive to make a high level of effort. In other words, the project must be likely to generate high enough output realization when the entrepreneur exerts a high level of effort and low enough when she exerts a low level of effort. On the other hand, it is the transfer to or from the third party that gives the investor an incentive to provide good financial services in each period.

The advantage of multiperiod contract in this paper is to reduce the discounted expected amount of money to be transferred to or from the third party. When the entrepreneur hides information on the output realizations until the end of the relationship, the contracts that give players good incentives in the first period also can give them good incentives in the succeeding period. When we assume budget feasibility constraint, the optimal contract, which minimizes the discounted expected amount of the transfer to the third party, requires no amount of the transfer if the output realization in the first period is high regardless of the realization in the succeeding period.

Without budget feasibility constraint and with a rational third party, the optimal contract is the one that minimizes the discounted expected amount of the transfer from the third party and that requires no amount of the transfer if the realization in the first period is low regardless of the realization in the succeeding period. A venture capital contract, under which an investor provides funds and management advice to an entrepreneur by purchasing convertible securities issued by the entrepreneur, is a good example of this type of budget-breaking financial contracts in that an venture capitalist and a venture firm expect a third party to pay for them if their project succeeds.

The remainder of the paper is organized as follows. The next section presents the general model framework. In section 3, we analyze one-period financial contract with budget feasibility constraint and the inefficiency of asymmetric information on output realizations. The first half of Section 4 considers multiperiod contract with budget feasibility constraint and illustrate the way of reducing the inefficiency of budget-breaking. In the last half of the section, we seek multiperiod contract without budget feasibility constraint and explain venture capital contracts as an example of this type of budget-breaking contracts in the real world. Section 5 concludes. All the proofs are contained in the Appendix.

2 The Model

There are two risk-neutral players, an entrepreneur and an investor. The entrepreneur has a project that stochastically generates outputs, but she does not have funds for it. The entrepreneur has to finance the project from the investor, who can provide a variety of financial services in each period. The investor contributes to the project by providing sound financial and management assistance, but not all of it can be specified in a contract. The entrepreneur and the investor agree on a repayment schedule that depends on the project's output realization. The entrepreneur can observe the output realization, but the investor cannot. Therefore, the investor is worried that the entrepreneur may lie about the output realization. In turn, the entrepreneur is concerned that the investor may reduce his contribution to the project. The investor is also worried that the entrepreneur may shirk his responsibilities while carrying out the project because the investor cannot observe the entrepreneur's effort level.

The entrepreneur and the investor meet at the beginning of period 0, agree to cooperate on the project until the end of period $T \in \{0, 1\}$, and sign a contract that specifies the repayment schedule.

After signing the contract, the investor chooses his contribution to the project, $I_t \in \{I_H, I_L\}$, at the beginning of each period. Then the entrepreneur chooses her effort level, $e_t \in \{e_H, e_L\}$. We assume that contributing to the project costs them I_t and e_t, respectively. We also assume that $I_H > I_L$ and $e_H > e_L$. After the project generates output realization y_t, the entrepreneur sends a message about it $m_t(y_t)$. The message is observable and verifiable. Depending on the message, the entrepreneur repays part of the output realization, and the investor receives money as specified in the contract.

The output realization is also binary, $y_t \in \{y_H, y_L\}$, stochastic, and depends on both the amount of funds and the level of effort. We assume that the project generates y_H with probability $p(I_t, e_t)$ and y_L with probability $1 - p(I_t, e_t)$. We define the expected output levels as $\bar{y}_{sv} = E(y_t|I_t = I_s, e_t = e_v) = p_{sv}y_H + (1 - p_{sv})y_L$, where $p_{sv} \equiv p(I_t, e_t \mid I_t = I_s, e_t = e_v)$ for $s \in \{H, L\}$ and $v \in \{H, L\}$.

We assume that a riskless bond exists in the economy. When a player buys the riskless bond at the beginning of each period, he will get the same amount of the money at the end of the same period. We assume the interest rate of this bond is r; when player buys x worth of this bond in period 0, he will get money worth $(1 + r)x$ in period 1. Moreover, we have the following assumptions.

Assumptions

(a) $\bar{y}_{HH} > \bar{y}_{LH}$ and $\bar{y}_{HH} > \bar{y}_{HL}$

(b) $\bar{y}_{HH} - I_H - e_H > \bar{y}_{LH} - I_L - e_H > 0$

(c) $\bar{y}_{HH} - I_H - e_H > \bar{y}_{HL} - I_H - e_L$

(d) $\bar{y}_{LH} - I_L - e_H < \bar{y}_{LL} - I_L - e_L$

Assumption (a) implies that $p(I_t, e_t)$ is an increasing function of both I_t and e_t. In other words, the more funds the investor lends, the more likely the project will succeed.

Similarly, the more effort the entrepreneur exerts, the more likely the project will succeed. Assumption (b) means that, given that the entrepreneur exerts a high level of effort, more funds are more efficient than fewer funds, and fewer funds are more efficient than riskless bonds. Assumption (c) implies that, given that the investor lends a large amount of money, a high level of effort is more efficient than a low level. On the other hand, assumption (d) denotes that, given that the investor lends a small amount, a low level of effort is more efficient than a high level. It also means that the increase of the expected output realizations by intensifying the entrepreneur's effort level is lower than the increase of effort costs by doing so when the investor lends a small amount of money.

These assumptions imply that the project's expected output realization is maximized if and only if both players contribute to the project. We refer to a contract under which the project's expected output realizations are maximized as an *efficient contract*. In this paper, the entrepreneur offers a contract that maximizes her own expected payoff as well as maximizes the project's expected outputs. We call such a contract an *optimal contract*. An optimal contract is always efficient, but an efficient contract is not necessarily optimal. We also define the following expressions: $\Delta y \equiv y_H - y_L$, $\Delta I \equiv I_H - I_L$, $\Delta e \equiv e_H - e_L$, $\Delta p(I) \equiv p_{HH} - p_{LH}$, and $\Delta p(e) \equiv p_{HH} - p_{HL}$. Note that all of these expressions are nonnegative.

3 One-Period Contract

In this section, we discuss the case in which the project generates output only once: when $T = 0$. When the output realizations are observed by both players, the contracts can specify the repayment that depends on them. However, when the output realizations are only observed by the entrepreneur, the repayment can no longer depend on them.

3.1 Efficient and optimal contracts when output realizations are public information

To examine the effect of asymmetric information on output realizations, y_0, we first discuss efficient contracts when both the player and the third party observe the project's realizations [1]. In this case, we can write contracts that depend on the output realizations and assume that the entrepreneur repays $r^b \in \{r_H^b, r_L^b\}$, i.e., r_H^b if the output realization is y_H and r_L^b if it is y_L. Similarly, the investor receives $r^\ell \in \{r_H^\ell, r_L^\ell\}$, i.e., r_H^ℓ if the output realization is y_H and r_L^ℓ if it is y_L. In this section, we assume that $r_H^b \geq r_H^\ell$ and $r_L^b \geq r_L^\ell$. These constraints correspond to budget feasibility constraints in the context of mechanism design[2]. If $r_H^b = r_H^\ell$ and $r_L^b = r_L^\ell$, the amount the entrepreneur repays and the amount the investor receives are the same, and we say that the contract *balances the budget.* If either $r_H^b > r_H^\ell$ or $r_L^b > r_L^\ell$, we say that the contract *does not balance the*

[1] If players can observe the output realization but any third party cannot, the realization cannot be verifiable in court. Here we assume that output realizations are observable and verifiable.

[2] In section 4.2, we consider the case of $r_H^b \leq r_H^\ell$ and $r_L^b \leq r_L^\ell$, in which a third party provides money for players.

budget and interpret that the difference between the amount the entrepreneur repays and the amount the investor receives is passed to a third party. We postulate that both players are reluctant to transfer any hard-won value to a third party. We denote the difference of the amount the entrepreneur repays as $\Delta r^b \equiv r^b_H - r^b_L$ and the difference of the amount the investor receives as $\Delta r^\ell \equiv r^\ell_H - r^\ell_L$. Next, we discuss the constraints that efficient contracts must satisfy and the conditions under which efficient contracts exist.

First of all, any efficient contracts must guarantee expected payoffs that exceed the player's burdens, or guarantee nonnegative net expected payoffs. The entrepreneur writes and offers the contracts if and only if the expected payoff that she obtains by exerting effort e_H for the project is higher than or identical to her effort costs. Similarly, the investor signs the contract if and only if the expected payoff that he receives by investing I_H is higher than or identical to the payoff that he receives by investing the same amount of money into riskless bonds. Therefore, any efficient contracts must satisfy the following constraints:

$$E(r^\ell | I_0 = I_H, e_0 = e_H) \geq I_H, \tag{1}$$

$$E(y_0 - r^b | I_0 = I_H, e_0 = e_H) \geq e_H. \tag{2}$$

We refer to constraints (1) and (2) as *individual rationality constraints* for the investor and for the entrepreneur respectively. These constraints can be rewritten in the following way:

$$p_{HH}r_H^\ell + (1 - p_{HH})r_L^\ell \geq I_H, \tag{3}$$

$$p_{HH}(y_H - r_H^b) + (1 - p_{HH})(y_L - r_L^b) \geq e_H. \tag{4}$$

In addition to individual rationality constraints, any efficient contract must satisfy constraints that induce players to greatly contribute to the project:

$$E(r^\ell | I_0 = I_H, e_0 = e_H) - I_H \geq E(r^\ell | I_0 = I_L, e_0 = e_H) - I_L, \tag{5}$$

$$E(y_0 - r^b | I_0 = I_H, e_0 = e_H) - e_H \geq E(y_0 - r^b | I_0 = I_H, e_0 = e_L) - e_L. \tag{6}$$

The left-hand side of each inequity is each player's net expected payoff when he greatly contributes to the project, and the right-hand side is that when he do not so, given that the other player greatly contributes to the project. We refer to constraints (5) and (6) as *incentive compatibility constraints* for the investor and for the entrepreneur respectively. When both players can observe the project's output realizations, these constraints can be rewritten with difference expressions in the following way:

$$\Delta r^\ell \geq \frac{\Delta I}{\Delta p(I)} \tag{7}$$

$$\Delta r^b \leq \Delta y - \frac{\Delta e}{\Delta p(e)}. \tag{8}$$

Thus, we have the following lemma about the existence of efficient contracts when both players can observe the project's output realizations.

Lemma 1 *(i) If $\Delta y \geq \Delta I / \Delta p(I) + \Delta e / \Delta p(e)$, there exists an efficient contract that balances the budget. An efficient contract specifies a repayment schedule such that $r_H^b =$*

r^ℓ_H *and* $r^b_L = r^\ell_L$*; thus, no amount of money is transferred to the third party.*

(ii) If $\Delta y < \Delta I/\Delta p(I)+\Delta e/\Delta p(e)$ *and* $y_H \geq I_H+e_H+(1-p_{HH})\,(\Delta I/\Delta p(I) + \Delta e/\Delta p(e))$, *there are no efficient contracts that balance the budget, but there is an efficient contract that does not balance the budget. The efficient contract that minimizes the transfer to the third party specifies a repayment schedule such that* $r^b_H = r^\ell_H$ *and* $r^b_L = r^\ell_L+(\Delta I/\Delta p(I) + \Delta e/\Delta p(e) - \Delta y)$*. In the case of* $y_0 = y_L$*, players transfer* $\Delta I/\Delta p(I)+\Delta e/\Delta p(e) - \Delta y$ *to the third party, i.e., the ex ante expected transfer to the third party is* $(1 - p_{HH})(\Delta I/\Delta p(I) + \Delta e/\Delta p(e) - \Delta y)$.

(iii) If $\Delta y < \Delta I/\Delta p(I)+\Delta e/\Delta p(e)$ *and* $y_H < e_H+I_H+(1-p_{HH})\,(\Delta I/\Delta p(I) + \Delta e/\Delta p(e))$, *there are no efficient contracts.*

The case of (i) implies that, when the difference of output realizations between high and low is large enough, players can make efficient contracts under which they do not need to transfer any amount of money to a third party. When this difference is small, an efficient contract has to make a large enough difference by transferring some money to a third party. However, this transfer reduces the players' expected payoffs. Thus, when y_H is small, players cannot make contracts that satisfy the individual rationality constraints for them.

The efficient contract that minimizes the transfer to the third party requires the following rules. If the output realization is high, the amount of money paid by the entrepreneur is directly passed to the investor. However, if it is low, the amount of money paid by the entrepreneur must be divided into two parts; one is passed to the investor and the other to the third party.

It is impossible for these players to negotiate and share the amount to be transferred to a third party because this renegotiation increases the investor's payoff when the output realization is low and violates the incentive compatibility constraint for the investor.

The entrepreneur writes and offers contracts that not only satisfy all these constraints but also maximize her own expected payoff. We obtain an optimal contract without asymmetric information on output realizations as follows.

Proposition 1 *Suppose that both players and a third party can observe project's output realization* $y_0 \in \{y_H, y_L\}$ *and the contract specifies repayment schedule* $(r^b_H, r^b_L, r^\ell_H, r^\ell_L)$ *that depends on project's output realization* y_0.

(i) If $\Delta y > \Delta I/\Delta p(I) + \Delta e/\Delta p(e)$*, optimal contracts exist infinitely. Under any optimal contracts, the investor's expected payoff is* I_H*, and the entrepreneur's is* $\bar{y}_{HH} - I_H$*. The optimal contract that binds the incentive compatibility constraint for the investor specifies a repayment schedule such that*

$$r^b_H = r^\ell_H = I_H + (1 - p_{HH})\frac{\Delta I}{\Delta p(I)},$$
$$r^b_L = r^\ell_L = I_H - p_{HH}\frac{\Delta I}{\Delta p(I)}.$$

(ii) If $\Delta y \leq \Delta I/\Delta p(I) + \Delta e/\Delta p(e)$ *and* $y_H \geq I_H + e_H + (1 - p_{HH})\left(\Delta I/\Delta p(I) + \Delta e/\Delta p(e)\right)$*, an optimal contract exists and it is unique. The investor's expected payoff is* I_H*, and*

the entrepreneur's is $\bar{y}_{HH} - I_H - (1 - p_{HH})\,(\Delta I/\Delta p(I) + \Delta e/\Delta p(e) - \Delta y)$*. The optimal contract specifies a repayment schedule such that*

$$\begin{aligned} r_H^b = r_H^\ell &= I_H + (1 - p_{HH})\frac{\Delta I}{\Delta p(I)}, \\ r_L^b &= I_H + (1 - p_{HH})\frac{\Delta I}{\Delta p(I)} + \frac{\Delta e}{\Delta p(e)} - \Delta y, \\ r_L^\ell &= I_H - p_{HH}\frac{\Delta I}{\Delta p(I)}. \end{aligned}$$

For $y_0 = y_L$*, the amount of* $\Delta I/\Delta p(I) + \Delta e/\Delta p(e) - \Delta y$ *is transferred to a third party, i.e., the ex ante expected transfer to the third party is* $(1 - p_{HH})(\Delta I/\Delta p(I) + \Delta e/\Delta p(e) - \Delta y)$*.*

Proposition 1 provides two types of optimal contracts and the conditions under which they exist. When Δy is large, that is, when the difference between a large output realization and a low one is large, players can make optimal contracts that provide good incentives for them without transferring any money to a third party. Since players are risk-neutral, they are indifferent to contracts that give constant expected payoffs. Therefore, optimal contracts exist infinitely. Among these optimal contracts, the one that binds the incentive compatibility constraint for the investor is unique. When Δy is small but y_H is large, players can make a unique optimal contract; however, the budget won't balance.

The investor's expected payoff is always I_H. However, the entrepreneur's expected payoff varies depending on the project's property. When players must transfer money to a third party, the contract charges the entrepreneur this amount since the investor always receives, at most, the same amount of burden. When both Δy and y_H are small, players cannot make any efficient contracts by Lemma 1, nor can they make any optimal contracts.

The optimal contracts expressed in Proposition 1 depend on the assumption that both the players and the third party can observe the project's output realization. If neither the investor nor the third party can observe the output realizations, the entrepreneur may lie about the project's output realization to increase her own payoff. In the next section, we discuss efficient and optimal contracts when there is asymmetric information on the project's output realizations.

3.2 The efficient and optimal contracts when output realizations are private information

Suppose that neither an investor nor a third party can observe the output realizations, but the messages sent by the entrepreneur are observable and verifiable. In this case, the contract specifies a repayment schedule that depends on those messages. First we consider a case in which the contracts balance the budget. Then, if $r_H^b > r_L^b$, i.e., $\Delta r^b > 0$, the entrepreneur always sends message $m_0(y_0) = y_L$ and repays r_L^b regardless whether y_H or y_L is achieved. This is because the entrepreneur can increase her payoff by minimizing the repayment. However, if the investor anticipates the entrepreneur's behavior, he always lends I_L. The reason is that the investor's payoff is $r_L^b - I_H$ if he lends

I_H and $r^b_L - I_L$ if he lends I_L; thus, the latter is always larger than the former. Given that the entrepreneur always repays r^b_L, then the contract cannot satisfy the incentive compatibility constraint for the entrepreneur, since we have

$$\bar{y}_{HL} - r^b_L - e_H < \bar{y}_{LL} - r^b_L - e_L$$

by assumption (d).

In contrast, if $\Delta r^b < 0$, the entrepreneur always sends message $m_0(y_0) = y_H$, and the investor always lends I_L. As a result, the necessary and sufficient condition for the entrepreneur to be honest about the project's output realization is $\Delta r^b = 0$. This constraint is equivalent to $\Delta r^\ell = 0$ because we are now considering a contract that balances the budget. However, by constraint (7), $\Delta r^\ell = 0$ violates the incentive compatibility constraint for the investor. Therefore, no contract can balance the budget when there is asymmetric information on output realization. Consider the following proposition.

Proposition 2 *Suppose that neither the players nor the third party can observe project's output realization $y_0 \in \{y_H, y_L\}$ and the contract specifies repayment schedule $(r^b_H, r^b_L, r^\ell_H, r^\ell_L)$ that depends on message $m_0 \in \{y_H, y_L\}$.*

(i)No efficient contract balances the budget.

(ii)If $\bar{y}_{HH} \geq e_H + I_H + (1 - p_{HH})\Delta I/\Delta p(I)$ and $y_L \geq I_H + (1 - p_{HH})\Delta I/\Delta p(I)$, there is efficient contracts that do not balance the budget. The unique optimal contract

specifies a repayment schedule such that

$$r_H^b = r_L^b = r_H^\ell = I_H + (1 - p_{HH})\frac{\Delta I}{\Delta p(I)},$$
$$r_L^\ell = I_H - p_{HH}\frac{\Delta I}{\Delta p(I)}.$$

For $m_0 = y_L$, players transfer $\Delta I/\Delta p(I)$ to the third party, i.e., the ex ante expected amount of the transfer is $(1-p_{HH})\Delta I/\Delta p(I)$. The investor's expected payoff is I_H, and the entrepreneur's is $\bar{y}_{HH} - I_H - (1 - p_{HH})\Delta I/\Delta p(I)$.

(iii)If either $\bar{y}_{HH} < e_H + I_H + (1-p_{HH})\Delta I/\Delta p(I)$ or $y_L \geq I_H + (1-p_{HH})\Delta I/\Delta p(I)$, there are no efficient contracts.

Compared to Proposition 1, when output realizations are unobservable, players cannot make any efficient contract that balances the budget. The reason is that the entrepreneur will not truthfully send a message without requiring the amount she repays to be constant, and the investor will not lend a large amount of money without differentiating the amount he receives depending on the output realizations. The ex ante expected amount of money transferred to the third party is always $(1-p_{HH})\Delta I/\Delta p(I)$, which is larger than that in the case of Proposition 1 (ii) if $\Delta I/\Delta p(I) \geq \Delta I/\Delta p(I) + \Delta e/\Delta p(e) - \Delta y$, i.e., $\Delta y \geq \Delta e/\Delta p(e)$, which is always true by assumption (c).

For an efficient contract to exist, $\bar{y}_{HH}$ and y_L must be larger than the case of symmetric information. Unlike the case of Proposition 1 (ii), there is no requirement on Δy except for assumption (c).

The investor's expected payoff is the same as that in the case of symmetric information. In contrast, the entrepreneur's expected payoff is smaller than that in the case of symmetric information. The reason for this difference is that the expected amount the investor receives equals his burden regardless whether he can observe the output realizations, but the entrepreneur must repay a larger amount for asymmetric information.

4 Multiperiod Contract

In this section, we illustrate the gain of a multiperiod relationship between an investor and an entrepreneur. The previous section showed that no efficient contract that balances the budget exists in the case of asymmetric information on a project's output realizations. Any efficient contract has to specify the amount of money to be passed to a third party. Players minimize this amount so as to maximize their expected payoffs. In this section, we apply Fuchs's idea (2007) and seek an optimal contract when the business relationship continues for two periods.

Fuchs (2007)'s T-period principal-agent model reveals the condition that minimizes the amount of transfer to the third party when the principal privately observes the signal of an agent's effort[3]. He shows that, when the relationship continues for finite periods, players can minimize the expected amount of the reluctant transfer to a third party by forcing the principal to repay only once at the end of the last period instead of at the end of each period. He does not specify the bargaining power between the agent

[3]Fuchs (2007) also seeks efficient contracts in infinite horizon model.

and the principal; however, in this paper, we seek optimal contracts that maximize the entrepreneur's payoff, which corresponds to the principal's payoff in his model.

We assume that the business relationship continues for two periods. Let the discount factors of the players be identical and denote them by β. Moreover, we assume that the discounted factor is identical to the interest rate of the riskless bond, i.e., $\beta = r$. In Section 4.1, we seek optimal contracts with budget feasibility constraint, i.e., $r_s^b \geq r_s^\ell$, for $s \in \{H, L\}$, as we have done so far, and the conditions under which they exist. When the optimal contract specifies a repayment schedule such that $r_s^b > r_s^\ell$, the difference between the two values must be transferred to the third party. Section 4.2 relaxes the budget feasibility constraint, i.e., analyzes the case of $r_s^b \leq r_s^\ell$. In this case, when the optimal contract specifies a repayment schedule such that $r_s^b < r_s^\ell$, the difference between the two values must be transferred by the third party. An example of this type of contract is a venture capital contract since a venture capitalist expects to get money not only from a venture firm but also from a third party, such as general investors, through an initial public offering if a project succeeds.

4.1 The efficient and optimal contracts when the entrepreneur defers reporting output realizations

In the previous section, we analyzed a case in which an entrepreneur sends message $m_0(y_0) = y_s$ after the project ends in period 0 for $s \in \{H, L\}$. Fuchs (2007) allows the player who observes the output realizations to defer sending message m_0 until the end of the relationship. For $s \in \{H, L\}$, the entrepreneur is indifferent whether he should send message $m_0(y_0) = y_s$ and repay r_s^b at the end of period 0 or send message $m_0(y_0) = y_s$ and repay $\beta^{-1} r_s^b$ at the end of period 1. Similarly, for the investor to receive $m_0 = y_s$ and r_s^ℓ at the end of period 0 is equivalent to receiving $m_0 = y_s$ and $\beta^{-1} r_s^\ell$ at the end of period 1. Therefore, there exists a contract under which the entrepreneur defers sending a message until the end of a relationship; this contract is still efficient because the players are indifferent to an optimal contract under which the entrepreneur sends a message at the end of each period.

After repeating the project twice, four types of histories emerged: (y_H, y_H), (y_H, y_L), (y_L, y_H), and (y_L, y_L). The entrepreneur sends one of them as message $m_1(y_0, y_1)$ at the end of period 1. The amounts the investor receives are denoted by $z^\ell \in \{z_{HH}, z_{HL}, z_{LH}, z_{LL}\}$ depending on each message. We relate the amount the investor receives to each history rather than to each period-output realization. As in the single-period model, to give the entrepreneur an incentive to send an honest message, the amount to be repaid by the entrepreneur must be constant regardless of the message. We denote this amount by z^b. The entrepreneur offers the investor an optimal contract that minimizes z^b subject to the incentive compatibility constraints and the individual rationality constraints for both players.

First, we consider the constraints that an optimal contract must satisfy to prevent the investor from lending a small amount of money. Given that the investor lends a large amount in period 1, the constraint that induces him to lend a large amount in period 0 is

$$\beta\left\{E(z^\ell|I_0=I_1=I_H)-E(z^\ell|I_0=I_L,I_1=I_H)\right\}\geq \Delta I. \tag{IC0}$$

Similarly, given that he lends a large amount in period 0, the constraint that induces him to lend a large amount in period 1 is

$$\beta\left\{E(z^\ell|I_0=I_1=I_H)-E(z^\ell|I_0=I_H,I_1=I_L)\right\}\geq \beta\Delta I. \tag{IC1}$$

Finally, the constraint that prevents him from lending a small amount in both periods is

$$\beta\left\{E(z^\ell|I_0=I_1=I_H)-E(z^\ell|I_0=I_1=I_L)\right\}\geq (1+\beta)\Delta I. \tag{IC2}$$

Next, we consider the constraint for the entrepreneur to exert an effort both times. Given that the amount to be repaid is constant, the constraints that prevent any possibility of shirking responsibilities only in period 0, only in period 1, or in both periods are as follows:

$$E(y_0+\beta y_1|e_0=e_1=e_H)-E(y_0+\beta y_1|e_0=e_L,e_1=e_H)\geq \Delta e,$$

$$E(y_0+\beta y_1|e_0=e_1=e_H)-E(y_0+\beta y_1|e_0=e_H,e_1=e_L)\geq \beta\Delta e,$$

$$E(y_0+\beta y_1|e_0=e_1=e_H)-E(y_0+\beta y_1|e_0=e_1=e_L)\geq (1+\beta)\Delta e.$$

We also consider the individual rationality constraints for each player. Given that both players greatly contribute to the project, these constraints are

$$\beta E(z^\ell|I_0=I_1=I_H,e_0=e_1=e_H)\geq (1+\beta)I_H,$$

$$E(y_0+\beta y_1-z^b|I_0=I_1=I_H,e_0=e_1=e_H)\geq (1+\beta)e_H.$$

They lead to another proposition that deals with optimal contracts in a case involving a relationship that continues for two periods.

Proposition 3 *The project continues for two periods, 0 and 1, and generates output realizations at the end of each period. The entrepreneur privately observes output realizations* y_0 *and* y_1*. Assume that she sends a message about the project output only once and repays at the end of period 1. Denote the amounts of the transfer to the third party depending on each history by* $a \equiv z_{HH} - z_{HL}$*,* $b \equiv z_{HH} - z_{LH}$*, and* $c \equiv z_{HH} - z_{LL}$*.*

(i) If $\bar{y}_{HH} \geq I_H + e_H + (1-p_{HH})\Delta I/(1+\beta)\Delta p(I)$ *and* $y_L \geq I_H + (1-p_{HH})\Delta I/(1+\beta)\Delta p(I)$*, there is a set of optimal contracts that does not balance the budget. Under optimal contracts, the amounts that must be transferred to the third party are as follows:*

$$a = 0,$$
$$b \in \left[0, \frac{1-\beta}{\beta}\frac{\Delta I}{\Delta p(I)}\right],$$
$$c = \frac{1}{\beta(1-p_{HH})}\frac{\Delta I}{\Delta p(I)} - \frac{p_{HH}}{1-p_{HH}}b.$$

Therefore, an optimal repayment schedule $(z^b, z_{HH}, z_{HL}, z_{LH}, z_{LL})$ *is*

$$z^b = z_{HH} = z_{HL} = \frac{1+\beta}{\beta}I_H + \frac{1-p_{HH}}{\beta}\frac{\Delta I}{\Delta p(I)},$$
$$z_{LH} = z^b - b,$$
$$z_{LL} = z^b - c.$$

The discounted expected amount of the transfer from a third party is $(1-p_{HH})\Delta I/\Delta p(I)$*.*

The investor's discounted expected payoff is $(1+\beta)I_H$ *and the entrepreneur's is* $(1+\beta)\{\bar{y}_{HH} - I_H - (1-p_{HH})\Delta I/(1+\beta)\Delta p(I)\}$.

(ii) If either $\bar{y}_{HH} < I_H + e_H + (1-p_{HH})\Delta I/(1+\beta)\Delta p(I)$ *or* $y_L < I_H + (1-p_{HH})\Delta I/(1+\beta)\Delta p(I)$, *there are no efficient contracts.*

The efficient and optimal contracts infinitely exist if and only if $\bar{y}_{HH}$ and y_L are sufficiently large. The optimal contract specifies a repayment schedule such that $a = 0$, i.e., $z_{HH} = z_{HL}$. This means that no money has to be transferred to a third party to induce players to contribute significantly to the project in period 1 if the output realization is large in period 0. Note that under such a contract, the investor will not have any incentive to lend a large amount of money in period 1 as long as he knows that $y_0 = y_H$ has been realized at the end of period 0. The optimal contract provides good incentive to the investor by hiding information on y_0. Thus, an optimal contract minimizes the discounted expected amount of money that must be transferred to a third party; the minimum achieved value is $(1-p_{HH})\Delta I/\Delta p(I)$, which is the amount that, at most, gives the investor a good incentive in period 0.

Since the discounted expected amount of the transfer is $(1+\beta)(1-p_{HH})\Delta I/\Delta p(I)$ when the entrepreneur sends a message in each period by the result of Proposition 2 (ii), the amount of transfer to a third party is reduced by not sending a message at the end of period 0. Since players can reduce a reluctant transfer to a third party and keep the investor's expected discounted payoff identical to his discounted burden, the entrepreneur's expected discounted payoff increases. In addition, players can agree to optimal contracts under smaller $\bar{y}_{HH}$ and y_L here than when the entrepreneur sends a message in each period[4].

4.2 The mutiperiod optimal financial contract without budget feasibility constraint

The characteristics of venture capital contracts are similar to those of optimal contracts presented in Proposition 3, except that in the former contracts a third party breaks the budget by providing money for players but in the latter contracts he does so by receiving money from players. Venture capitalists often invest in new firms by purchasing convertible securities (Sahlman (1990)). If firms are profitable, venture capitalists often recoup their investments through initial public offerings (Black and Gilson (1998) and Gompers and Lerner (2004)). Venture capitalists contribute to projects not only by financial assistance but also by management assistance, as do firms that contribute to projects through their efforts. However, not all of their contributions can be specified in a contract. Venture capitalists contribute to each project several times (stage financing) but obtain a return only once: at the end of the relationship. If a venture firm is profitable, the venture capitalist exercises a conversion option from debt to equity and

[4] When the entrepreneur sends a message and repays in each period, the efficient and optimal contacts exist if and only if $\bar{y}_{HH} \geq e_H + I_H + (1 - p_{HH})\Delta I/\Delta p(I)$ and $y_L \geq I_H + (1 - p_{HH})\Delta I/\Delta p(I)$ as I sought in Proposition 2.

obtains the right to take the firm's surplus. If the venture firm is not profitable, the venture capitalist requires the firm to pay back the debt or liquidate the firm.

Suppose that a venture capital contract requires that a firm pay z^b as dividends when he sends a good message and z^b as liquidated values when he sends a bad message. The firm is then indifferent to which messages he sends. Thus, firms always honestly report the history of the output realizations. If the history is good, venture capitalists can obtain capital gains through initial public offerings in addition to dividends. Therefore, in this case, the amounts obtained by venture capitalists differ depending on the history, but the amounts repaid by firms are constant. A third party or general outside investors contribute funds to the players if the history is good, which is in contrast to the results of Proposition 3, where the third party receives money from these players if the history is bad. Here we assume $z^b \leq z^\ell$ for $z^\ell \in \{z_{HH}, z_{HL}, z_{LH}, z_{LL}\}$ and denote the amount of the transfer from the third party depending on each history as $a' \equiv z_{LH} - z_{LL}$, $b' \equiv z_{HL} - z_{LL}$, and $c' \equiv z_{HH} - z_{LL}$.

If third party or general outside investors are too generous and buy a venture firm's share through initial public offerings for enough money to satisfy the individual rationality constraint for the venture capitalist, the venture firm, which minimizes the repayment, never repays anything to the venture capitalist. In Proposition 4, we seek an optimal venture contract when outside investors are rational and are not so generous to minimize the amount they pay when buying the venture firm's share through an initial public offering.

Proposition 4 *The project continues for two periods and generates output realization at the end of each period. The entrepreneur privately observes output realizations* y_0 *and* y_1*. She sends a message about the history of the output realizations and repays only once at the end of the last period. We assume that markets for an initial public offering are developed and a third party is willing to contribute a minimum fund that ensures high contributions by both the entrepreneur and the investor.*

(i) If $y_L \geq I_H - p_{HH}\Delta I/(1+\beta)\Delta p(I)$*, there is a set of optimal contracts that does not balance the budget.*[5] *Under optimal contracts, minimum values that must be transferred from a third party are as follows:*

$$\begin{aligned} a' &= 0, \\ b' &\in \left[0, \frac{1-\beta}{\beta}\frac{\Delta I}{\Delta p(I)}\right], \\ c' &= \frac{1}{\beta p_{HH}}\frac{\Delta I}{\Delta p(I)} - \frac{1-p_{HH}}{p_{HH}}b'. \end{aligned}$$

Therefore, an optimal repayment schedule $(z^b, z_{HH}, z_{HL}, z_{LH}, z_{LL})$ *is*

$$\begin{aligned} z^b = z_{LL} = z_{LH} = &\frac{1+\beta}{\beta}I_H - \frac{p_{HH}}{\beta}\frac{\Delta I}{\Delta p(I)}, \\ z_{HL} = &z^b + b', \\ z_{HH} = &z^b + c'. \end{aligned}$$

The discounted expected amount of the transfer from a third party is $p_{HH}\Delta I/\Delta p(I)$*.*

[5]In fact, there is a set of optimal contracts if and only if $\bar{y}_{HH} \geq e_H + I_H - p_{HH}\Delta I/(1+\beta)\Delta p(I)$ and $y_L \geq I_H - p_{HH}\Delta I/(1+\beta)\Delta p(I)$. The condition on $\bar{y}_{HH}$ is always true because of assumption (b).

The investor's discounted expected payoff is $(1+\beta)I_H$ *and the entrepreneur's is* $(1+\beta)\{\bar{y}_{HH} - I_H + p_{HH}\Delta I/(1+\beta)\Delta p(I)\}$.

(ii) If $y_L < I_H - p_{HH}\Delta I/(1+\beta)\Delta p(I)$, *there are no efficient contracts.*

The optimal contract presented in Proposition 4 provides the sufficient conditions under which such contracts exist, the minimum contribution by outside investors, and the amounts the venture firm repays and the venture capitalist obtains. The optimal contract specifies a repayment schedule such that $a' = 0$, i.e., $z_{LH} = z_{LL}$. This means that no money must be transferred by a third party if the output realization is small in period 0. As in the case of Proposition 3, players minimize the amount of the transfer from the third party by hiding information on the output realization in period 0. If this type of outside investors and a market for initial public offering exist, an entrepreneur can offer venture capital contracts to an investor. There is no requirement on $\bar{y}_{HH}$ under assumption (b) and the expected payoff for the entrepreneur is larger than that in the case of Proposition 3; the entrepreneurs are more likely to offer efficient contracts to investors than in the case of Proposition 3.

Even if players do not expect outside investors to pay sufficiently large amounts for them or the markets for initial public offerings are underdeveloped or sluggish, the entrepreneur can offer a contract such as that presented in Proposition 3 and pursue efficient results. They are free to pay money to a third party, such as a charity.

5 conclusion

This paper seeks financial contracts that specify a repayment schedule that induces both an investor and an entrepreneur to contribute to a project in every period and that maximizes the entrepreneur's expected payoff when the entrepreneur privately observes project's output realizations. To induce the entrepreneur to truthfully report output realizations, repayments must be identical regardless of the history of the output realizations. However, to induce both players to greatly contribute to the project, their payoffs must reflect the history. Therefore, one type of optimal contract requires players to directly transfer repayment from the entrepreneur to the investor if the history is good and to pass some part of the money repaid by the entrepreneur to a third party and give the rest to the investor if it is bad. Another type of optimal contract requires players to directly transfer repayment from the entrepreneur to the investor if the history is bad and to give money to the investor, money that the entrepreneur repays and that the third party provides, if it is good.

When the project continues for two periods, contracts that require the entrepreneur to deter reporting on output realizations and repaying the loan until the end of the last period can reduce the amount of money that must be passed to or be provided by the third party. When the entrepreneur hides information on the output realizations until the end of the relationship, the contracts that give players good incentives in the first period also can give them good incentives in the succeeding period. An example of the budget-breaking financial contracts that expect a third party to pay money for players depending on project's performance is a venture capital contract, which expects general investors to purchase venture firm's shares if the project succeeds.

References

Bhattacharyya, Sugato and Francine Lafontaine (1995), "Double-Sided Moral Hazard and the Nature of Share Contracts," *RAND Journal of Economics*, Vol.26, No.4, pp.761-781.

Black, Bernard S. and Ronald J. Gilson (1998),"Venture capital and the structure of capital markets: banks versus stock markets ," *Journal of Financial Economics*, Vol.47, No.3, pp.243-277.

Bolton, Patrick and David S. Scharfstein (1990), "A Theory of Predation Based on Agency Problems in Financial Contracting," *American Economic Review*, Vol.80, No.1, pp.93-106.

Diamond, Douglas W. (1984), "Financial Intermediation and Delegated Monitoring," *Review of Economic Studies*, Vol.51, No.3, pp.393-414.

Fuchs, William (2007), "Contracting with Repeated Moral Hazard and Private Evaluations," *American Economic Review*, Vol.97, No.4, pp.1432-1448.

Gale, Douglas and Martin Hellwig (1985), "Incentive-Compatible Debt Contracts: The One-Period Problem," *Review of Economic Studies*, Vol.52, No.4, pp.647-63.

Gompers, Paul A. and Lerner Josh (2006), *The Venture Capital Cycle*, 2nd Edition, The MIT Press.

Holmstrom, Bengt (1982), "Moral Hazard in Teams," *Bell Journal of Economics*, Vol.13, No.2, pp.324-340

Kambe, Shinsuke (2006), "Subjective Evaluation In Agency Contracts," *Japanese Economic Review*,Vol. 57, No.1, pp.121-140.

MacLeod, W. Bentley (2003), "Optimal Contracting with Subjective Evaluation," *American Economic Review*, Vol.93, No.1, pp.216-240.

Sahlman, William A. (1990), "The structure and governance of venture-capital organizations," *Journal of Financial Economics*, Vol.27, No.2, pp.473-521.

Townsend, Robert M. (1979), "Optimal contracts and competitive markets with costly state verification," *Journal of Economic Theory*, Vol.21, No.2, pp.265-293.

Appendix

Proof of Lemma 1

(i) Suppose $\Delta y \geq \Delta I/\Delta p(I) + \Delta e/\Delta p(e)$. Then, there exists Δr^b such that $\Delta y - \Delta e/\Delta p(e) \geq \Delta r^b \geq \Delta I/\Delta p(I)$. When the budget balances , the schedule satisfies constraints (7) and (8) simultaneously. Constraints (3) and (4) imply

$$\bar{y}_{HH} \geq I_H + e_H + p_{HH}(r_H^b - r_H^\ell) + (1 - p_{HH})(r_L^b - r_L^\ell). \tag{9}$$

When the budget balances, this constraint is $\bar{y}_{HH} \geq I_H + e_H$ and always satisfied by assumption (b).

(ii) Suppose $\Delta y < \Delta I/\Delta p(I) + \Delta e/\Delta p(e)$ and Δr^b satisfies (8). Then, there is no Δr^ℓ which satisfies (7) when the budget balances. If Δr^ℓ satisfies (7) and Δr^b satisfies (8), i.e., $r_L^\ell \leq r_H^\ell - \Delta I/\Delta p(I)$ and $r_L^b \geq r_H^b - (\Delta y - \Delta e/\Delta p(e))$, we have

$$\begin{aligned} r_L^b - r_L^\ell &\geq \left\{ r_H^b - \left(\Delta y - \frac{\Delta e}{\Delta p(e)} \right) \right\} - \left\{ r_H^\ell - \left(\frac{\Delta I}{\Delta p(I)} \right) \right\} \\ &= (r_H^b - r_H^\ell) + \frac{\Delta I}{\Delta p(I)} + \frac{\Delta e}{\Delta p(e)} - \Delta y. \end{aligned}$$

To minimize the reluctant transfer to the third party, the repayment schedule satisfies $r_H^b = r_H^\ell$ and $r_L^b = r_L^\ell + \Delta I/\Delta p(I) + \Delta e/\Delta p(e) - \Delta y$. Considering this transfer, constraint (9) can be written as $\bar{y}_{HH} \geq I_H + e_H + (1 - p_{HH}) (\Delta I/\Delta p(I) + \Delta e/\Delta p(e) - \Delta y)$, i.e., $y_H \geq I_H + e_H + (1 - p_{HH}) (\Delta I/\Delta p(I) + \Delta e/\Delta p(e))$.

(iii) Suppose $\Delta y < \Delta I/\Delta p(I) + \Delta e/\Delta p(e)$. By the proof of Lemma 1(ii), if $y_H < I_H + e_H + (1 - p_{HH}) (\Delta I/\Delta p(I) + \Delta e/\Delta p(e))$ holds, players cannot make any efficient contracts.

Proof of Proposition 1

(i) Suppose $\Delta y \geq \Delta I/\Delta p(I) + \Delta e/\Delta p(e)$. Then, efficient contracts are those that Lemma 1(i) specifies. The optimal contracts maximize the entrepreneur's expected payoff; thus, the individual rationality constraint for investor (3) binds, i.e., $p_{HH} r_H^\ell + (1 - p_{HH}) r_L^\ell = I_H$. The pair (r_H^ℓ, r_L^ℓ) that satisfies this constraint and constraint (7) exists infinitely.

Therefore, the optimal contracts exist infinitely.

If we assume that constraint (7) binds, i.e., $r_H^\ell = r_L^\ell + \Delta I/\Delta p(I)$, constraint (3) is $p_{HH}\left(r_L^\ell - \Delta I/\Delta p(I)\right) + (1-p_{HH})r_L^\ell = I_H$. Then, we have $r_L^\ell = I_H - p_{HH}\Delta I/\Delta p(I)$, and therefore, $r_H^\ell = I_H + (1-p_{HH})\Delta I/\Delta p(I)$. Since the repayment balances the budget, we have the results.

(ii) Suppose $\Delta y < \Delta I/\Delta p(I) + \Delta e/\Delta p(e)$. Then, efficient contracts are those that Lemma 1(ii) specifies. Since the optimal contract binds (3) and (7), the repayment schedule is $r_L^\ell = I_H - p_{HH}\Delta I/\Delta p(I)$ and $r_H^\ell = I_H + (1-p_{HH})\Delta I/\Delta p(I)$. By the result of Lemma 1(ii), we have $r_H^b = r_H^\ell$ and

$$\begin{aligned} r_L^b &= r_L^\ell + \left(\frac{\Delta I}{\Delta p(I)} + \frac{\Delta e}{\Delta p(e)} - \Delta y\right) \\ &= I_H + (1-p_{HH})\frac{\Delta I}{\Delta p(I)} + \frac{\Delta e}{\Delta p(e)} - \Delta y. \end{aligned}$$

Then, the entrepreneur's expected payoff is

$$\begin{aligned} E(y_0 - r^b | I_0 = I_H, e_0 = e_H) &= \bar{y}_{HH} - p_{HH} r_H^b - (1-p_{HH}) r_L^b \\ &= \bar{y}_{HH} - p_{HH}\left(I_H + (1-p_{HH})\frac{\Delta I}{\Delta p(I)}\right) \\ &\quad - (1-p_{HH})\left(I_H + (1-p_{HH})\frac{\Delta I}{\Delta p(I)} + \frac{\Delta e}{\Delta p(e)} - \Delta y\right) \\ &= \bar{y}_{HH} - I_H - (1-p_{HH})\left(\frac{\Delta I}{\Delta p(I)} + \frac{\Delta e}{\Delta p(e)} - \Delta y\right). \end{aligned}$$

Considering the transfer to the third party, constraint (9) can be written as $y_H \geq I_H + e_H + (1-p_{HH})\left(\Delta I/\Delta p(I) + \Delta e/\Delta p(e)\right)$.

Proof of Proposition 2

(i) To induce the entrepreneur to be honest about project output, players must set $\Delta r^b = 0$; in addition, to induce the investor to lend a large amount of money, they must set $\Delta r^\ell > 0$. These constraints do not hold simultaneously if the repayment schedule balances the budget.

(ii) There do not exist any efficient contracts which balance the budget. Then, the optimal contract binds (3) and (7), and r_H^ℓ and r_L^ℓ are thus the as those in Proposition 1(ii). Since $\Delta r^b = 0$, constraint (8) can be rewritten by $\Delta y \geq \Delta e/\Delta p(e)$. $\Delta r^b = 0$ also means that the transfer to the third party is $\Delta I/\Delta p(I)$ when the message is $m_0 = y_L$. The entrepreneur always repays r_H^ℓ, i.e., $r_H^b = r_L^b = I_H + (1 - p_{HH})\Delta I/\Delta p(I)$. Then, $y_L \geq I_H + (1 - p_{HH})\Delta I/\Delta p(I)$ must hold and the entrepreneur's expected payoff is $\bar{y}_{HH} - I_H - (1 - p_{HH})\Delta I/\Delta p(I)$. The individual rationality constraint for the entrepreneur requires $\bar{y}_{HH} \geq I_H + e_H + (1 - p_{HH})\Delta I/\Delta p(I)$.

(iii) By the proof of Proposition 2(ii), if either $\bar{y}_{HH} < I_H + e_H + (1 - p_{HH})\Delta I/\Delta p(I)$ or $y_L < I_H + (1 - p_{HH})\Delta I/\Delta p(I)$, players cannot make any efficient contract.

Proof of Proposition 3

First, we seek optimal contracts which prevent the investor from deviating to lend a small amount of money in either period 0 or period 1. Later, we show that this contract is also the solution to the problem with the possibility to deviate twice by the investor.

Constraints (IC0) and (IC1) are rewritten as follows:

$$\beta\{p_{HH}(z_{HH} - z_{LH}) + (1 - p_{HH})(z_{HL} - z_{LL})\} \geq \frac{\Delta I}{\Delta p(I)}, \tag{IC0$'$}$$

$$\beta\{p_{HH}(z_{HH} - z_{HL}) + (1 - p_{HH})(z_{LH} - z_{LL})\} \geq \beta\frac{\Delta I}{\Delta p(I)}. \tag{IC1$'$}$$

These constraints mean the discounted expected amount that must be transferred to the third party. Using the definitions of $a \equiv z_{HH} - z_{HL}$, $b \equiv z_{HH} - z_{LH}$, and $c \equiv z_{HH} - z_{LL}$, those constraints can be

$$p_{HH}(a + b) + (1 - p_{HH})c \geq \frac{\Delta I}{\beta\Delta p(I)} + a, \tag{IC0$''$}$$

$$p_{HH}(a + b) + (1 - p_{HH})c \geq \frac{\Delta I}{\Delta p(I)} + b. \tag{IC1$''$}$$

The individual rationality constraint for the investor is

$$\beta\{(p_{HH})^2 z_{HH} + p_{HH}(1 - p_{HH})(z_{HH} - a + z_{HH} - b) + (1 - p_{HH})^2(z_{HH} - c)\}$$
$$\geq (1 + \beta)I_H.$$

This constraint can be

$$z_{HH} \geq \frac{1 + \beta}{\beta} I_H + (1 - p_{HH})\{p_{HH}(a + b) + (1 - p_{HH})c\}. \tag{10}$$

The entrepreneur who wants to maximize her discounted expected payoffs minimizes z_{HH} subject to the constraints (IC0$''$), (IC1$''$), and (10); therefore, he minimizes the expected amount of the transfer to the third party subject to these constraints.

Note that $\Delta I/\beta\Delta p(I) \geq \Delta I/\Delta p(I)$. To set $a = 0$ and $b \in [0, (1 - \beta)\Delta I/\beta\Delta p(I)]$ minimizes the amount of $p_{HH}(a + b) + (1 - p_{HH})c$ required by constraints (IC0$''$) and

(IC1$''$). Constraint (IC0$''$), which prevents period 0 deviation, must bind, but constraint (IC1$''$), which prevents period 1 deviation, does not have to bind. c will be determined by $c = \Delta I/\beta(1-p_{HH})\Delta p(I) - p_{HH}b/(1-p_{HH})$.

The minimum amount of $p_{HH}(a+b) + (1-p_{HH})c$ equals $\Delta I/\beta\Delta p(I)$; thus, the optimal z_{HH} is given as

$$z_{HH} = \frac{1+\beta}{\beta} I_H + (1-p_{HH}) \cdot \frac{\Delta I}{\beta \Delta p(I)} = \frac{1+\beta}{\beta} I_H + \frac{1-p_{HH}}{\beta} \frac{\Delta I}{\Delta p(I)}.$$

Next, we show that the optimal contracts we have sought are optimal when we add the possibility of deviating twice by the investor. Consider the constraint which prevents the investor from deviating in period 1 given that he has deviated in period 0. This constraint is

$$\beta \left\{ E(z|I_0 = I_L, I_1 = I_H) - E(z|I_0 = I_1 = I_L) \right\} \geq \beta \Delta I. \tag{IC3}$$

Like other constraints, it can be transformed as

$$p_{LH}(z_{HH} - z_{HL}) + (1-p_{LH})(z_{LH} - z_{LL}) \geq \frac{\Delta I}{\Delta p(I)}, \tag{IC3$'$}$$

i.e.,

$$p_{LH}a + (1-p_{LH})(c-b) \geq \frac{\Delta I}{\Delta p(I)}. \tag{IC3$''$}$$

Constraint (IC1), which prevents period 1 deviation given that the investor has not deviated in period 0, can be rewritten similarly as

$$p_{HH}a + (1-p_{HH})(c-b) \geq \frac{\Delta I}{\Delta p(I)}. \tag{IC1$'''$}$$

Note that $p_{HH} > p_{LH}$ and $a = 0$, and any pair (b,c) which satisfies (IC1‴) satisfies (IC3″). It follows that any repayment schedule $(z^b, z_{HH}, z_{HL}, z_{LH}, z_{LL})$ which satisfies (IC1) satisfies (IC3). Combining (IC0) and (IC3), we have constraint (IC2). Therefore, any repayment schedule $(z^b, z_{HH}, z_{HL}, z_{LH}, z_{LL})$ which satisfies (IC0) and (IC1) satisfies (IC2), and the optimal repayment schedule under constraints (IC0) and (IC1) is then also optimal under constraints (IC0), (IC1), and (IC2).

Since the entrepreneur always repays z_{HH}, the project's property must satisfy $(1+\beta)y_L/\beta \geq z_{HH}$, i.e., $y_L \geq I_H + (1-p_{HH})\Delta I/(1+\beta)\Delta p(I)$. The entrepreneur's discounted expected payoff is $(1+\beta)(\bar{y}_{HH} - z_{HH}) = (1+\beta)\left\{\bar{y}_{HH} - I_H - (1-p_{HH})\Delta I/(1+\beta)\Delta p(I)\right\}$ and therefore the individual rationality constraint for her implies $\bar{y}_{HH} \geq I_H + e_H + (1 - p_{HH})\Delta I/(1+\beta)\Delta p(I)$.

Proof of Proposition 4

We will obtain the results in the same way as the proof of Proposition 3.