Liquidity Preference and Persistent Unemployment with Dynamic Optimizing Agents: An Empirical Evidence

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Abstract

Standard money-in-utility dynamic models assume satiable liquidity preference, and thereby prove the existence of a full employment steady state. In the same framework it is known that under insatiable liquidity or wealth preference there is a case where a full employment steady state does not exist and then unemployment persistently occurs. Using both parametric and nonparametric methods this paper empirically finds that insatiable liquidity/wealth preference is more supported. Thus, without assuming any permanent distortion, we can analyze an effective demand shortage in a dynamic optimization framework.

Keywords: Persistent Unemployment, Dynamic Optimization, Insatiable Liquidity Preference.

JEL Classifications: E12, E24, E41

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1. Introduction

Standard money-in-utility dynamic models assume that the marginal utility of liquidity (or more precisely, the marginal rate of substitution of consumption for money) becomes zero for a sufficiently large amount of liquidity. Since under this assumption a full employment steady state is proven to exist,\(^1\) people do not bother to consider the possibility of permanent disequilibrium in this type of model. Moreover, they start the analysis by assuming that all markets are completely cleared at any point in time. When analyzing market disequilibrium on purpose (as in New Keynesian models), some economic distortions, such as monopoly power and imperfect information, are exogenously introduced.

In contrast, using the same model structure as standard money-in-utility dynamic models, Ono (1994, 2001) proves: when the marginal rate of substitution of consumption for money has a strictly positive lower bound, there is a case where a full employment steady state does not exist, and then a steady state with persistent unemployment obtains. Thus, without considering any market distortion, we can analyze persistent unemployment caused by an effective demand shortage in a dynamic optimization setting.\(^2\) In this steady state Keynesian implications hold, such that rapid wage/price adjustment reduces effective demand and that fiscal spending stimulates consumption.

One might then ask which hypothesis is more plausible, satiable or insatiable liquidity preference. In the literature, e.g. Feenstra (1986), it is insisted that the satiability of liquidity preference has a microeconomic foundation when liquidity is demanded only from the transaction motive. However, this is almost a tautology since they take into account only a motive that requires a finite amount of liquidity holding, although there are also other motives, such as wealth preference. Thus, to find which hypothesis is more plausible, empirical research on the (in)satiability of liquidity/wealth

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\(^1\) See Brock (1975), Obstfeld and Rogoff (1983, p.681), Blanchard and Fischer (1989, p.241), etc., for this proof.

\(^2\) In Chapter 17 of the General Theory (1936, p.239) Keynes concludes that under insatiable liquidity preference persistent unemployment may occur. However, since he had not clearly provided a model with rational behavior, economists formulated an ad hoc Keynesian model. Because of its ad hoc structure, the effective demand theory itself is nowadays negatively considered.
preference is required.

Using both parametric and nonparametric methods, we empirically investigate the (in)satiability of liquidity preference, and find insatiable liquidity preference to be more supported. Thus, a Keynesian effective demand shortage can be treated in a money-in-utility dynamic optimization model.

The plan of this paper is as follows. Sections 2 and 3 summarize Ono’s model. Particularly, section 3 shows that under insatiable liquidity preference there is a unique steady state in a perfect-foresight competitive economy and that unemployment persists in the steady state. By applying parametric and nonparametric methods respectively to two sets of data, section 4 empirically investigates if the marginal rate of substitution of consumption for money has a strictly positive lower bound. Consequently, it is found that both approaches support the hypothesis of insatiable liquidity preference more than satiable liquidity preference. Finally, section 5 summarizes the implication of this paper.

2. The Model Structure

Let us first summarize the structure of Ono’s model. For simplicity, we assume that the firm sector produces output \( y \) by using only labor \( l \) with a constant-returns-to-scale technology:

\[
y = \theta l, \tag{1}
\]

where input-output ratio \( \theta \) is constant.\(^3\) Therefore, given real wage \( w \), the firm sector’s demand for labor is represented by

\[
l = \begin{cases} 
0 & \text{if } w > \theta, \\
0 < l < \infty & \text{if } w = \theta, \\
l = \infty & \text{if } w < \theta.
\end{cases} \tag{2}
\]

A representative household owns asset \( a \) which consists of liquidity \( m \) and interest-bearing asset \( b \),\(^4\)

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\(^3\) Even if the marginal productivity of labor decreases as \( l \) increases (Ono, 1996), or even if firms use real capital as well as labor for production (Ono, 1994, Chap.12), the following argument is essentially the same.

\(^4\) There are no equities since the firm sector’s profits are zero under production function (1) and thus the firm value is zero. In the present setting there are only lending and borrowing between households.
\[ a = m + b, \]  
where \(a, m,\) and \(b\) are measured in real terms. She earns income from labor supply \(x\) and interests from \(b\). We assume her labor endowment to be 1, and therefore \(x\) represents the employment rate. Then, the flow budget equation is \(^5\)

\[ \dot{a} = ra + wx - c - Rm, \]  
where \(c\) is consumption, \(r\) the real interest rate, \(\pi\) the inflation rate, and \(R\) the nominal interest rate:

\[ R = r + \pi. \]

Subject to (3) and (4), the household maximizes

\[ U = \int_0^\infty U(c, m) \exp(-\rho t) dt, \]

where subjective discount rate \(\rho\) is assumed to be constant. The first-order optimal conditions are

\begin{align*}
U_c(c, m) &= \lambda, \quad (5) \\
U_m(c, m) &= \lambda R, \quad (6) \\
\dot{\lambda} &= (\rho - r)\lambda, \quad (7)
\end{align*}

where \(U_c\) (or \(U_m\)) implies the partial derivative of \(U\) with respect to \(c\) (or \(m\)). The transversality condition is

\[ \lim_{t \to \infty} \lambda, a, \exp(-\rho t) = 0. \]  

(8)

From (5), (6), and (7) we derive

\[ \rho = \frac{U_{cc} c}{U_c} \left( \frac{\dot{c}}{c} \right) - \frac{U_{cm} m}{U_c} \left( \frac{\dot{m}}{m} \right) + \pi = R = \frac{U_m(c, m)}{U_c(c, m)} (\equiv MRS(c, m)), \]

(9)

which implies equality among the three interest rates; that is, the time preference rate, the return rate of the interest-bearing asset, and the liquidity premium (viz. the marginal rate of substitution of \(c\) for \(m\)), all measured in monetary terms. \(^6\) The household decides the time paths of \(c\) and \(m\) so that they satisfy (9) at any point in time and (8) in the infinite future.

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\(^5\) This equation is obtained from (3) and the following flow budget equation in nominal terms:

\[ \dot{A} = Wx + RB - Pc. \]

\(^6\) Note that liquidity premium \(U_c/U_m\) does not depend on whether it is measured in monetary or real terms since it is the marginal rate of substitution between \(m\) and \(c\) at the same point in time.
The market of liquidity is assumed to adjust perfectly, and hence at any time\(^7\)
\[
M/P = m, \quad (10)
\]
where \(M\) is the nominal liquidity stock. The adjustment of the commodity market is also assumed to be perfect, and therefore always
\[
\theta x = c. \quad (11)
\]

Finally, we assume that money wage \(W\) adjusts in a sluggish manner, dependent on the excess demand rate in the labor market, so that unemployment may occur.\(^8\) Since labor demand is \(l\) and the labor endowment is normalized to 1, the dynamics of \(W\) is given by
\[
\frac{\dot{W}}{W} = \alpha(l - 1). \quad (12)
\]
From (2), if \(W/P > \theta\), labor demand \(l\) is zero and therefore commodity supply is zero, which immediately makes \(P\) jump upward so that \(W/P = 0\). If \(W/P < 0\), then from (2) \(l\) is \(\infty\) and hence even under the sluggish money wage adjustment given by (12) \(W\) instantaneously rises to \(0P\). Thus, it is always satisfied that
\[
W/P = 0, \quad (13)
\]
from which
\[
\frac{\dot{W}}{W} = \frac{\dot{P}}{P}. \quad (14)
\]
Since (13) is valid, from (2) \(l\) can take any value. Since from (11) \(x\) equals \(c/\theta\),
\[
l = c/\theta.
\]
Therefore, from (12) and (14) we obtain the dynamics of \(P\):
\[
\frac{\dot{P}}{P} = \alpha(c/\theta - 1), \quad (15)
\]
which implies that \(P\) and \(W\) move in parallel in accordance with the gap between

\(^7\) Because of Walras’s law with respect to the asset markets, if the liquidity market is in equilibrium then the market of the interest-bearing asset also is.

\(^8\) Note that this assumption does not avoid the possibility of full employment to occur in steady state. In fact, we shall show that under satiable liquidity preference the sluggish adjustment of \(W\) defined by (12) eventually attains full employment. This assumption is imposed only for allowing the possibility of unemployment to exist. If perfect wage adjustment were assumed, the possibility of unemployment would tautologically be avoided from the beginning.
production capacity $\theta$ and effective demand $c$.\(^9\)

From (10) we derive

$$\frac{\dot{m}}{m} = -\pi \left(\equiv -\frac{\dot{P}}{P}\right).$$

By substituting (15) into this equation and (9), we obtain the dynamic equations of $m$ and $c$ respectively.

$$\frac{\dot{m}}{m} = -\alpha\left(\frac{c}{\theta} - 1\right)$$

(16)

$$\frac{\dot{c}}{c} = \frac{U_m}{U_c} - \alpha\left(\frac{c}{\theta} - 1\right) \frac{U_{cm}}{U_c} - \rho - \alpha\left(\frac{c}{\theta} - 1\right)$$

(17)

(16) and (17) formulate an autonomous dynamic system with respect to $m$ and $c$.

3. Steady States

The Full Employment Steady State

As long as full employment obtains in the steady state of the dynamics represented by (16) and (17), the steady state is the same as that of the standard money-in-utility model.\(^10\) In fact, in the steady state (16) implies

$$c = 0.$$  

(18)

From (17) and (18) commodity price $P$ satisfies

$$MRS(0, M/P) \equiv U_m(0, M/P)/U_c(0, M/P) = \rho,$$

(19)

where $MRS(c, m)$ naturally satisfies

$$\partial MRS(c, m)/\partial c > 0, \quad \partial MRS(c, m)/\partial m < 0.$$  

(20)

Does the steady state defined by (19) always exist? If $MRS(c, m)$ has a strictly positive lower bound as $m$ expands to infinity:

$$MRS(c, \infty) > 0,$$

(21)

\(^9\) Note that this equation is valid only when $c/\theta \leq 1$. If $c/\theta > 1$, demand exceeds supply in the commodity market since the maximum commodity supply is $\theta$, and because of the instantaneous adjustment of the commodity market $P$ immediately jumps upward so that $c/\theta = 1$. Thus, when considering the dynamics of this economy, we need to treat only the case where $c/\theta \leq 1$

\(^10\) The steady state given below is the same as that of the money-in-utility model without real capital or population growth. See Blanchard and Fischer (1989, pp.188-191) for it.
and if it rises high enough as \( c \) increases,\(^{11}\) there is a level of \( \theta \) that satisfies
\[
\rho < \text{MRS}(\theta, \infty) (< \text{MRS}(c, m) \text{ for } \forall m.)
\] (22)

If (22) is valid, \( P \) that satisfies (19) does not exist. This condition means that when consumption is determined to be large enough to attain full employment, liquidity premium \( \text{MRS}(\theta, \infty) \) exceeds subjective discount rate \( \rho \) for any value of \( m \). Obviously, the former implies the desire for saving whereas the latter the desire for consumption. Thus, consumption is set to be lower than the full employment level, causing a shortage of effective demand to occur. This tends to occur especially when per-capita production \( \theta \) is large. If \( \text{MRS}(\theta, \infty) \) is zero, as is usually assumed in the literature,\(^{12}\) (22) is not satisfied for any \( \theta \), and hence the full employment steady state given by (19) always exists.

In the conventional model, only the cash-in-advance motive for liquidity holding is considered and hence liquidity preference is definitely satiable. However, if there is another motive, such as wealth-holding preference, (21) may be valid.\(^{13}\) In section 4 we shall empirically investigate if the wealth-holding preference satisfies it.

**The Unemployment Steady State**

What steady state obtains if (22) holds and therefore a steady state with full employment does not exist? In this subsection, for simplicity, we assume the utility function to be additive separable and thus
\[
U_{cm} = 0.
\] (23)

The case of a non-additive-separable utility function is discussed in the appendix.

If \( c \) is lower than \( \theta \), unemployment occurs and from (12) money wage \( W \) continues to decline. From (14), \( P \) follows \( W \), causing \( m \) to increase infinitely. Therefore, if \( c \) reaches the steady state level \( c_u \) that makes (17) zero, we have

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\(^{11}\) This implies a natural assumption that the marginal utility of consumption becomes small enough as \( c \) grows.

\(^{12}\) An additive-separable utility function, \( u(c) + v(m) \), is usually assumed (e.g., Obstfeld and Rogoff, 1983, and Blanchard and Fischer, 1989, pp.188-191) and in that case \( v'(\infty) \) is assumed to be zero instead of the present assumption.

\(^{13}\) Generally, each asset has both liquidity and profitability. In the present model \( m \) and \( b \) respectively summarize the liquidity and the profitability of all assets. In fact, we can obtain essentially the same result in a more general model where there are various assets \( a_i (i = 0, 1, \ldots, n) \) which constitute liquidity \( a_0 + m(a_1, \ldots, a_n) \), generate the utility of liquidity \( v(a_0 + m(a_1, \ldots, a_n)) \), and also yield returns \( (0, R_1, \ldots, R_n) \). Obviously, \( a_0 \) is cash in this model.
\[ MRS(c_u, \infty) = \rho + \alpha(c_u/\theta - 1) \]  \hspace{1cm} (24)

In figure 1 the \( \ell \) and \( \pi \) curves respectively show the left- and right-hand side of (24).

\[ \ell \text{ curve:} \quad R = MRS(c, \infty) \]
\[ \pi \text{ curve:} \quad R = \rho + \alpha(c/\theta - 1) \]  \hspace{1cm} (25)

Consumption \( c_u \) that satisfies (24) is given by \( A \), the intersection point of the two curves. Note that under (22) the \( \ell \) curve is located above the \( \pi \) curve when \( c = 0 \). Thus, for \( c_u \) to exist in the relevant range it must be satisfied that

\[ \rho > \alpha, \]  \hspace{1cm} (26)

as long as \( U_c(0, M/P) \) is very large and thus \( MRS(0, m) \) is very small.

In this state \( P \) continues to decline and expands \( m \) to infinity. Nevertheless, transversality condition (8) is satisfied, as proven below. From (16), (21) and (24) \( m \) satisfies

\[ \frac{\dot{m}}{m} = \rho - MRS(c_u, \infty) < \rho. \]  \hspace{1cm} (27)

From (27), transversality condition (8) is valid since the total stock of \( b \) is zero and thus \( a = m \).

The steady state obtained above has typical Keynesian features. For example, an increase in fiscal spending raises consumption. Specifically, in the presence of fiscal spending \( g \), the commodity market equilibrium condition (11) is replaced by

\[ \theta x = c + g. \]

Thus, in (25) the \( \pi \) curve changes to

\[ \pi \text{ curve:} \quad R = \rho + \alpha[(c + g)/\theta - 1], \]

while the \( \ell \) curve remains unchanged. An increase in fiscal spending \( g \) shifts only the \( \pi \) curve upward. In figure 2 it is represented by a movement of the \( \pi \) curve to the \( \pi' \) curve. Consequently, \( A \), the intersection point, moves to \( A' \), causing consumption \( c_u \) to rise.

Also, a rise in \( \alpha \), the adjustment speed of money wage \( W \), turns the \( \pi \) curve counterclockwise around \( B \) with the \( \ell \) curve unaffected, as is clear from (25). Consequently, in figure 2, the \( \pi \) curve turns to the \( \pi'' \) curve, and hence \( A \) moves to \( A'' \),

\[ ^{14} \text{Under (22) and (26) the dynamics given by (16) and (17) is proven to be saddle-path stable around the unemployment steady state defined by (24). See Ono (1994, 2001) for the proof of the stability.} \]
causing consumption $c_u$ to decline. This result is opposite to the neoclassical or Keynesian view that the more rapidly prices and wages adjust, the sooner an effective demand shortage disappears. It is more in conformity with Keynes’s own view (1936, Ch.19) that a rise in the wage adjustment speed tends to reduce effective demand.

Note that wage rigidity does not cause persistent unemployment in our model, while it does in Keynesian models. In fact, in the present steady state $P$ and $W$ continue to adjust and realize full-employment real wage $\theta$, yet persistent unemployment occurs. Even if we assume perfect adjustment of $W$, as in standard models, the full employment steady state does not exist. It is clear since condition (22) is unrelated to $\alpha$.

4. Empirical Research on Insatiable Liquidity Preference

In the previous sections we have found that if $MRS(c, m)$ has a strictly positive lower bound then there is a case where persistent unemployment occurs in steady state. Then, the Keynesian effective demand shortage is derived not from an ad hoc model, such as the IS-LM analysis, but from a dynamic optimization framework. This section empirically investigates if $MRS(c, m)$ has a strictly positive lower bound.

To do so we use two different econometric methods. One is a parametric method in which we specify the utility function and test the hypothesis with the estimated parameters, while the other is a nonparametric method in which we do not specify any functional form. We apply these two methods to different data sets.

In the parametric model we use aggregate quarterly data in Japan. Its sample size is only $4 \times 27$ (1972-98), and hence it is too small to use a nonparametric method. In the nonparametric model we use the survey data called NIKKEI RADAR, whose sample size is large enough to apply a nonparametric method. If the same result is obtained from both of them, it should be regarded as a reliable one.

In the parametric model we estimate the first order conditions derived from the discrete version of the intertemporal utility maximization model developed in Section 2. In the nonparametric model the function to be estimated is given by the second equality of (9), which shows equality between the nominal interest rate and the liquidity premium.
Parametric Analysis with Aggregate Time Series Data

In the parametric analysis we use the following discrete version of the model given in section 2. It is given by

\[
\text{Max } E_t \left[ \sum_{\tau=t}^{\infty} \eta^{\tau-t} U(c_{\tau}, M_{\tau} / P_{\tau}) \right]
\]

subject to

\[
B_{\tau} = (1 + R_{\tau-1}) B_{\tau-1} + M_{\tau-1} + W_{\tau} x_{\tau} - P_{\tau} c_{\tau} - M_{\tau} \quad \text{for } \tau = t, t+1, t+2, \ldots,
\]

where \( \eta \) is the discount factor and stock variables \( B_{\tau} \) and \( M_{\tau} \) represent each value at the beginning of period \( \tau \).\(^{15}\) The two first order conditions are

\[
\frac{\partial U}{\partial c_{\tau}} = \eta E_t \left[ \frac{\partial U}{\partial c_{\tau+1}} (1 + R_{\tau}) \frac{P_{\tau}}{P_{\tau+1}} \right], \quad (28)
\]

\[
\frac{\partial U}{\partial M_{\tau} / P_{\tau}} = \eta E_t \left[ \frac{\partial U}{\partial c_{\tau+1}} R_{\tau} \frac{P_{\tau}}{P_{\tau+1}} \right]. \quad (29)
\]

Equation (28) is the familiar intertemporal first order condition between consumption in period \( t \) and consumption in period \( t+1 \). As for equation (29), the left-hand side represents the marginal utility of liquidity in period \( t \), while the right-hand side represents the period-\( t \) value of the marginal utility of consumption in period \( t+1 \) enabled by interest earning \( R_{\tau} \). Thus, (29) implies that the marginal benefit of holding liquidity during period \( t \) equals the marginal benefit of consumption enabled by the interest earnings received at the end of period \( t \). We shall estimate (28) and (29) and examine if (21) is valid.

In order to do so, we use the following utility function:

\[
U(c, m) = \frac{c^{1-\delta}}{1-\delta} + \beta m + \omega m^{1-\gamma}, \quad (30)
\]

where \( \delta \) and \( \gamma \) are both positive. It is reduced from an additive-separable and constant-relative-risk-aversion function.\(^{16}\) It is revised so that \( MRS(c, m) \) has a positive

\(^{15}\) This model has essentially the same structure as Holman (1998).

\(^{16}\) We also attempted the following estimation by assuming such a non-additive-separable utility function as presented in the appendix. Unfortunately, however, iterations did not converge although we set various starting values.
lower bound, as presented by (21). In fact, \( MRS(c, m) \) is

\[
MRS(c, m) = (\beta + \omega m^{-\gamma}) c^\delta,
\]

which satisfies

\[
MRS(c, \infty) = \beta c^\delta > 0 \quad \text{if} \quad \beta > 0.
\]

Note that it satisfies the standard Inada condition if \( \beta \) is zero.

By substituting (30) into (28) and (29) we obtain the following equations to be estimated:

\[
\eta \left( \frac{c_{t+1}}{c_t} \right)^{-\delta} (1 + R_t) \frac{P_t}{P_{t+1}} - 1 = \nu_{1,t+1},
\]

\[
\nu_{1,t+1} = \eta c_t^{-\delta} \frac{R_t}{P_{t+1}} \frac{P_t}{P_{t+1}} - \beta - \omega m_t^{-\gamma} = \nu_{2,t+1},
\]

where \( \nu_{1,t+1} \) and \( \nu_{2,t+1} \) are forecast errors uncorrelated with the variables in the consumer’s information set in period \( t \).

We use aggregate quarterly data from 1972 to 1998 to estimate (33) and (34). Consumption is expenditure on non-durables and services, which is taken from *Annual Report on National Accounts, Economic and Social Research Institute, Cabinet Office, Government of Japan*. Our measure of liquidity consists of cash, demand deposits, other deposits, postal savings, and trust held by the household sector, which is taken from *Flow of Funds Accounts, Bank of Japan*. All are measured on a per-capita basis and divided by the deflator of final consumption. The yield rate on 10-year interest-bearing government bond is used as a proxy of \( R \). The price variable is the deflator corresponding to consumption defined above. All the variables but the yield rate and population are seasonally adjusted by the X-12 program.

The first order conditions are estimated by the GMM technique. The instrument variables are a constant, once-lagged to four-times-lagged consumption, nominal interest rates, GDP deflator and liquidity. The estimation results of (33) and (34) with and without a positive lower bound of \( MRS \) are summarized in table 1.\(^{17}\)

When we assume the non-existence of a positive lower bound of \( MRS \) (i.e., \( \beta = 0 \)),

\(^{17}\) In the course of estimation (34) is divided by coefficient \( \omega \) to reduce the number of parameters to be estimated. That is, we estimate (33) and (34) by GMM to obtain the parameter estimates of \( \eta, \delta, \eta/\omega, \beta/\omega \) and \( \gamma \). Then the original parameters of \( \omega \) and \( \beta \) are retrieved from the estimated parameters.
all the parameter estimates are significant and satisfy the sign conditions that the theory imposes upon. However, judging from the p-value in this case (see table 1), the overidentifying restrictions are decisively rejected by the J-statistics. In the presence of a positive lower bound of \( MRS (\beta > 0) \) all the estimates are also statistically significant at the standard significance level and satisfy the sign conditions that the theory imposes upon. Furthermore, the overidentifying restrictions are not rejected at the 10% significance level, as shown in table 1. The estimate of relative risk aversion \( \delta \) is 0.2638, which is also of reasonable magnitude. Indeed, it is quite comparable with those in Hamori (1996).\(^{18}\)

More importantly, the estimate of \( \beta \) is significantly positive, which implies that \( MRS \) has a strictly positive lower bound, as shown by (32). The likelihood-ratio statistics of \( \beta = 0 \) is 25.2711, which indicates that the null hypothesis of \( \beta = 0 \) is decisively rejected.

Furthermore, from (31),

\[
\frac{d(m/c)}{dm} \bigg|_{MRS(c, m) = \text{const.}} = 1 - \frac{\gamma_0}{\delta (\beta m^\gamma + \omega)}.
\]

From the parameter values shown in table 1 we find this value to be positive\(^{19}\) – i.e., the Marshallian \( k (= m/c) \) tends to rise as \( m \) expands. It may explain the upward trend of the Marshallian \( k \) that is clearly observed especially after the bubble period.

### Nonparametric Analysis with NIKKEI RADAR

In the nonparametric analysis we do not specify the utility function. We employ data from NIKKEI RADAR 1994, which surveys 5000 people from 25-year-old to 69-year-old, living in Tokyo, Kanagawa, Chiba, or Saitama Prefecture. The liquidity measure that we use consists of bank deposits and trusts.\(^{20}\) Consumption is calculated as the difference between annual income and saving. After excluding the households

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\(^{18}\) The estimate of relative risk aversion \( \delta \) ranges from 0.14 to 0.269 in Hamori (1996).

\(^{19}\) It is 0.341 if we apply the average \( m \) of the present data.

\(^{20}\) The measure consists of bank deposits, post-office deposits, loan trusts (called BIG), money trusts (called HIT) and the long-term credit banks’ bonds (called WIDE). The BIG, HIT and WIDE are regarded as assets as safe as bank deposits and we can anytime withdraw them in principle. Thus, we include them in our liquidity measure. We need not distinguish the real and nominal data of liquidity and consumption since we use only one-year data.
that do not report all of income, savings, and money related to the liquidity measure, we eventually have 2013 samples.

We represent the implicit function of $c$ and $m$ given by the second equation of (9):\(^{21}\)

$$R = MRS(m, c)$$

(35)

as

$$1/c = q(1/m).$$

(36)

Since we use the cross-section survey data of one year (1994) and everybody faces the same interest rate $R$ for the year, we treat $R$ as a constant.\(^{22}\) Defining $X_i$ and $Y_i$ as the inverse of household $i$'s observed per-capita liquidity and that of per-capita consumption respectively, and $v_i$ as a random error, we have the following stochastic model:

$$Y_i = q(X_i) + v_i, \quad i = 1, \ldots, N.$$  

(37)

The errors are assumed to be i.i.d.

We consider the following null and alternative hypotheses:

$$H_0 : q(0) = 0,$$

$$H_1 : q(0) > 0.$$  

If the null is rejected, consumption $c$ in (36) is bounded to be finite for any level of $m$. From (35) this implies that the lower bound of $MRS$ is strictly positive. We here employ a nonparametric method, namely, local polynomial regression methods, to estimate $q(0)$ and evaluate its 90% and 95% one-sided confidence intervals with bootstrap.\(^{23}\) Since we do not specify $q(\cdot)$, the results are robust to its functional form.

Because we focus on the value of the conditional mean of $Y$ on $X = 0$, which is the left boundary of the support of $X$ defined on $(0, \infty)$, we have to deal with the boundary bias of the estimate. Nadaraya-Watson kernel estimates are known to have a large bias near the boundary of $X$. On the other hand, it is known that local polynomial

\(^{21}\) Note that (35) is valid whether the utility function is additive-separable or not.

\(^{22}\) One may consider that the rich and the poor face different rates of return. However, in 1994 the rates of return from loan trusts, money trusts and the long-term credit banks' bond were almost the same for any amount of deposits as long as being over the required minimum amounts.

\(^{23}\) For the detail of this method, see chapter 5 of Simonoff (1996).
regression methods automatically correct the boundary bias. Thus, we employ three types of the regression methods, namely, local-linear, local-quadratic and local-cubic regression methods.

A local polynomial regression estimate is defined as one of the minimizer, \( \theta_0 \), of the objective function given \( x \) as follows:

\[
\sum_{i=1}^{n} \left( Y_i - \theta_0 - \theta_1(x - x_i) - \cdots - \theta_k(x - x_i)^k \right)^2 K \left( \frac{x - x_i}{h} \right),
\]

where \( K(\cdot) \) is a kernel function, \( h \) a bandwidth and \( n \) the sample size. The local-linear, local-quadratic and local-cubic estimates are defined when \( k = 1, 2 \) and 3, respectively, while the Nadaraya-Watson kernel estimate is regarded as a special case when \( k = 0 \).

The local polynomial regression estimate, \( \hat{\phi}(x) = \hat{\theta}_0(x) \), is explicitly obtained as

\[
\hat{\phi}(x) = e' (X_x' W_x X_x)^{-1} X_x' W_x Y,
\]

where \( e = (1 \ 0 \ \ldots \ 0)' \) and \( Y = (Y_1 \ \ldots \ Y_n)' \) are \( n \times 1 \) vectors, \( X_x \) is an \( n \times (k + 1) \) matrix:

\[
X_x = \begin{pmatrix}
1 & x - x_1 & \cdots & (x - x_1)^k \\
\vdots & \vdots & \ddots & \vdots \\
1 & x - x_n & \cdots & (x - x_n)^k
\end{pmatrix},
\]

and \( W_x \) is an \( n \times n \) diagonal matrix:

\[
W_x = h^{-1} \text{diag} \left[ K \left( \frac{x - x_1}{h} \right), \ldots, K \left( \frac{x - x_n}{h} \right) \right].
\]

It is known that the boundary bias of the estimate depends on the \((k+1)\)'th derivative of the true conditional mean function and the asymptotic order of \( h^{k+1} \) so that the bias is likely to decrease much more with a higher order local polynomial regression estimate. However, the variance of the estimate increases as \( k \) being larger. This implies that we cannot reduce the boundary bias without paying the cost of a larger variance.

Table 2 summarizes the distributional features of the household size, household-based and per-capita money and consumption, and their inverted values, which imply \( Y_i \) and \( X_i \) in (37). The average household size is 2.91. The means of household-based and per-capita money A0 and A1 in table 2 are 77.03 and 32.99 hundred thousand yen, while their medians are 38.03 and 14.4, respectively. This means that the distributions of these variables have long and thick right-hand-side tails.
Household-based and per-capita consumption C0 and C1 in table 2 also have this property. The inverted values of household-based and per-capita money 1/A0*10 and 1/A1*10 also have nearly the same skewed distributions, though they have weaker skewness than the original values do. Fifty percent samples of the inverted values of the household-based and per-capita money are less than 0.2632 and 0.6944 and their minimums are 0.0054 and 0.0080, respectively.

The estimation results are shown by table 3. We use the half of the samples closer to zero since we focus on the values of $\phi(x)$ near zero and samples far from zero do not seriously affect the estimations. We employ the Gaussian kernel and select bandwidth $h$ with cross-validation by changing $h$ from 0.01 to 0.10 at intervals of 0.01 so that we find the value that minimizes aggregated prediction errors. In the case where the optimal value is 0.01, we again change $h$ from 0.004 to 0.01 at intervals of 0.002.

The estimations are conducted for both household-based and per-capita-based data. N.W., L.L., L.Q. and L.C in the table stand for the Nadaraya-Watson, local-linear, local-quadratic and local-cubic regressions, respectively. The selected bandwidths are nearly the same for all estimation methods in both the household-based and per-capita cases.

$\hat{\phi}(0)$ and $\hat{\phi}(x_{\text{min}})$, the estimated conditional mean at zero and that at the minimum of $x$, are shown in the second and third columns of the table. The N.W. estimates of $\hat{\phi}(0)$ and $\hat{\phi}(x_{\text{min}})$ are apparently bigger than the other estimates. $\hat{\phi}(0)$ is smaller under L.Q. than under L.L. and smallest under L.C. It is positive under all of them in the household-based case whereas it is negative only under L.C. in the per-capita case. $\hat{\phi}(x_{\text{min}})$ has the same tendency as $\hat{\phi}(0)$, though it is positive under all estimation methods.

Now we turn to the estimated critical points of one-sided 90% and 95% confidence intervals with bootstrap, shown in columns 4 to 7 of the table. We resample the data one hundred times to estimate the critical points of 90% and 95% confidence intervals. In the household-based case the estimated critical points of both 90% and 95% intervals are positive under N.W., L.L. and L.Q. but negative under L.C. In the per-capita case they are both positive under N.W. and L.L. but negative under L.Q. and L.C.
Figure 3 plots observations and estimated conditional means $\hat{\phi}(x)$ obtained by N.W., L.L., L.Q. and L.C. near zero in the household-based case. The slope of the N.W. estimate is less inclined than the other local polynomial estimates. All these estimates lie close to each other around $1/Money = 0.025$ but diverge as they approach zero. In particular, the N.W. estimate is not so much influenced by the observations near zero as the other estimates are. The N.W. regression line is located above almost all observations where $x$ is less than 0.01 and its 95% critical point of $x_{\text{min}}$, which equals 0.1035, is far over them. Thus, the estimate seems to have a serious boundary bias.

The local-linear and local-quadratic regression estimates are nearly the same, while the local-cubic regression estimate bends downward much more than the other estimates as $x$ approaches zero. The point estimates and the 90%- and 95%-critical points of $\phi(0)$ are all positive in the local-linear and local-quadratic regressions, while the point estimate is positive but the 90%- and 95%-critical points are negative in the local-cubic regression. Then, how should we statistically infer the hypothesis, $\phi(0) = 0$, with these results?

Firstly, the confidence interval of the local-cubic regression estimate cannot be used for the test since its variance is so large that it easily tends to reject the null hypothesis. Secondly, the local-linear and local-quadratic regression estimates may have a little bit upward bias compared with the local-cubic regression estimate. However, even if we correct the possible bias using the difference at $x_{\text{min}}$ between the local-quadratic and local-cubic estimates, 0.0247 ($= 0.0987 - 0.0740$), both the corrected 95%- and 90%-critical points at zero are still positive for the local-quadratic estimate. For the local-linear estimate the bias-corrected 90%-critical point is positive. Lastly, since the point estimate of the local-cubic regression gives the smallest bias, the local-cubic regression estimate, 0.0081, which is positive, is the most reliable as the point estimate of $\phi(0)$. In total, we conclude that the null hypothesis, $\phi(0) = 0$, is to be

24 It is known that odd polynomial orders have clear advantages over even orders (1 versus 0, 3 versus 2, and so on) in the asymptotic sense (see e.g. Simonoff, 1996). In particular, within each pair of odd and even orders given above the odd order has a smaller asymptotic bias than the even order in the boundary region. In the finite–sample case, however, we should not rely on this property. Simonoff (1996) discusses this issue in ‘5.2.4 Choosing the degree of the polynomial fit’. He uses two types of data set, electricity usage data and vineyard harvest data, and applies local-quadratic and local-cubic regressions. He finds that the variances of the local-cubic estimates near the boundary are much larger than the local-quadratic estimates, although the local-cubic estimates are not so much different from the local-quadratic estimates. Therefore, he concludes that he adopts the local-quadratic regression estimates.
rejected against $\varphi(0) > 0$ in the household-based case.

In the per-capita case we do not reach the same conclusion. The 95%- and 90%-critical points at zero of the local-quadratic estimate are negative. When we correct the possible bias of the local-linear estimate in the same way as in the household-based case, both the 90%- and 95%-critical points of $\varphi(0)$ are negative. The point estimate at zero of the local-cubic regression is also negative. Thus we cannot reject the null hypothesis in this case.

5. Conclusion

If the marginal rate of substitution of consumption for liquidity ($MRS$) reaches zero as the amount of liquidity infinitely increases, as assumed in standard money-in-utility dynamic models, a steady state with full employment necessarily exists. In this state neoclassical implications, such as the crowding-out effect of fiscal spending on private expenditure, would hold. If liquidity preference is insatiable, by contrast, and hence $MRS$ has a positive lower bound, as assumed in Ono (1994, 2001), there is a case where a steady state with full employment does not exist and then persistent unemployment occurs. In this state various Keynesian implications hold, e.g., fiscal spending raises private consumption.

Using two data sets, time series and individual, and applying parametric and nonparametric methods to each of them respectively, we empirically investigate which hypothesis is more plausible. Consequently, we find that the existence of a strictly positive lower bound of $MRS$ is supported more than the neoclassical assumption that $MRS$ reaches zero.

The possibility of persistent unemployment has so far been ignored in the neoclassical dynamic optimization framework. It has been treated only in either the ad hoc IS-LM analysis or static general equilibrium models with some permanent distortions. However, since the present empirical analysis shows insatiable liquidity preference to be more plausible, we can deal with persistent unemployment in the standard money-in-utility dynamic framework without assuming permanent market distortions.
Appendix

In the full-employment steady state (19) must be valid regardless of the form of the utility function. Thus, whether the utility function is additive-separable or not, if (22) is valid, the full-employment steady state does not exist.

In the case of a non-additive-separable utility function, however, $U_{cm}$ is not zero. Therefore, from (17), the stagnation steady state must satisfy

$$\lim_{m \to \infty} \left[ \frac{U_c}{U_m} - \frac{U_{cm} m}{U_c} \alpha(c/\theta - 1) \right] = \rho + \alpha(c/\theta - 1). \quad (A1)$$

Even in this case, however, as long as the left-hand side is an increasing function with respect to $c$, the discussion of the text is still valid.

For example, if the utility function has the following non-additive-separable form:\textsuperscript{25}

$$U(c, m) = \frac{\left( \frac{c^{1-\varepsilon}}{1-\varepsilon} + \frac{\beta}{\nu} m^{1-\nu} \right)^{1-\sigma}}{1-\sigma} - 1, \quad (A2)$$

$MRS(c, \infty)$ is

$$MRS(c, \infty) = (\beta/\nu)c^\varepsilon.$$

Therefore, as long as

$$(\beta/\nu)\theta^\varepsilon > \rho, \quad (A3)$$

there is no full-employment steady state. Obviously, if $\beta$ is strictly positive in (A2), $MRS(c, m)$ has a positive lower bound and thus (A3) is valid if $\theta$ is large enough. Moreover, since (A1) reduces to

$$MRS(c, \infty) = (\beta/\nu)c^\varepsilon + \sigma\alpha(c/\theta - 1) = \rho + \alpha(c/\theta - 1),$$

the stagnation steady state has essentially the same properties as discussed in the text.

Therefore, using (A2) we attempted the same estimation as in the parametric analysis of section 4. Unfortunately, however, iterations did not converge although we set various starting values.

\textsuperscript{25} This function for the case where $\beta$ is zero is frequently assumed in the empirical literature of money-in-utility functions. For example, see Finn et. all (1990) and Holman(1998).
References


Table 1: Empirical Result on Equations (33) and (34) by the GMM Technique

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\eta$</th>
<th>$\omega$</th>
<th>$\gamma$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4390</td>
<td>0.9953</td>
<td>$0.3954 \times 10^9$</td>
<td>7.8280</td>
<td>(0)</td>
</tr>
<tr>
<td>(3.35)</td>
<td>(1176.99)</td>
<td>(4.98)</td>
<td>(394.66)</td>
<td></td>
</tr>
<tr>
<td>J-statistics: 47.6021</td>
<td>p-value: 0.006</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\eta$</th>
<th>$\omega$</th>
<th>$\gamma$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2638</td>
<td>0.9944</td>
<td>$0.5008 \times 10^9$</td>
<td>8.1493</td>
<td>0.0066</td>
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<tr>
<td>(1.98)</td>
<td>(1205.55)</td>
<td>(5.77)</td>
<td>(302.91)</td>
<td>(4.91)</td>
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<tr>
<td>J-statistics: 32.7035</td>
<td>p-value: 0.110</td>
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<td></td>
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</table>

*) $t$-values in parentheses.

Instrument variables: a constant, and once-lagged to four-times-lagged consumption, nominal interest rates, GDP deflator and liquidity.
Table 2: Data Description

<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>s.d.</th>
<th>Min</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>90%</th>
<th>95%</th>
<th>97.5%</th>
<th>99%</th>
<th>max</th>
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<tbody>
<tr>
<td>Household size</td>
<td>2.91</td>
<td>1.32</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
<td>6</td>
</tr>
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<td>Household Money (A0)</td>
<td>77.03</td>
<td>130.70</td>
<td>1.0</td>
<td>16.0</td>
<td>38.0</td>
<td>81.8</td>
<td>175.0</td>
<td>282.1</td>
<td>381.4</td>
<td>621.8</td>
<td>1856.0</td>
</tr>
<tr>
<td>Household Consumption (C0)</td>
<td>64.90</td>
<td>39.48</td>
<td>7.0</td>
<td>40.0</td>
<td>55.0</td>
<td>80.0</td>
<td>105.0</td>
<td>126.0</td>
<td>160.0</td>
<td>218.7</td>
<td>400.0</td>
</tr>
<tr>
<td>Per-capita Money (A1)</td>
<td>32.99</td>
<td>62.65</td>
<td>0.2</td>
<td>6.0</td>
<td>14.4</td>
<td>33.9</td>
<td>72.9</td>
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<td>1245.0</td>
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<tr>
<td>Per-capita Consumption (C1)</td>
<td>25.55</td>
<td>17.35</td>
<td>3.8</td>
<td>15.0</td>
<td>21.0</td>
<td>31.0</td>
<td>45.0</td>
<td>57.2</td>
<td>73.3</td>
<td>82.5</td>
<td>200.0</td>
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<table>
<thead>
<tr>
<th></th>
<th>mean</th>
<th>s.d.</th>
<th>Min</th>
<th>1%</th>
<th>2.5%</th>
<th>5%</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverted Household Money (1/A0*10)</td>
<td>0.6027</td>
<td>1.1121</td>
<td>0.0054</td>
<td>0.0157</td>
<td>0.0260</td>
<td>0.0351</td>
<td>0.0569</td>
<td>0.1220</td>
<td>0.2632</td>
<td>0.6250</td>
<td>10.0000</td>
</tr>
<tr>
<td>Inverted Household Consumption (1/C0*10)</td>
<td>0.2121</td>
<td>0.1447</td>
<td>0.0250</td>
<td>0.0455</td>
<td>0.0625</td>
<td>0.0791</td>
<td>0.0952</td>
<td>0.1250</td>
<td>0.1818</td>
<td>0.2500</td>
<td>1.4286</td>
</tr>
<tr>
<td>Inverted Per-capita Money (1/A1*10)</td>
<td>1.6817</td>
<td>3.5073</td>
<td>0.0080</td>
<td>0.0313</td>
<td>0.0551</td>
<td>0.0767</td>
<td>0.1370</td>
<td>0.2941</td>
<td>0.6944</td>
<td>1.6667</td>
<td>60.0000</td>
</tr>
<tr>
<td>Inverted Per-capita Consumption (1/C1*10)</td>
<td>0.5373</td>
<td>0.3109</td>
<td>0.0500</td>
<td>0.1206</td>
<td>0.1364</td>
<td>0.1739</td>
<td>0.2222</td>
<td>0.3226</td>
<td>0.4762</td>
<td>0.6667</td>
<td>2.6667</td>
</tr>
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</table>

Note: The units of money and consumption are both 100 thousand yen.
Table 3: Estimation Results

<table>
<thead>
<tr>
<th>Selected bandwidth</th>
<th>Estimated conditional mean</th>
<th>Bootstrapped 90% critical point</th>
<th>Bootstrapped 95% critical point</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( h )</td>
<td>( \hat{\phi}(0) )</td>
<td>( \hat{\phi}(x_{\text{min}}) )</td>
</tr>
<tr>
<td>Household-based</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N.W.</td>
<td>0.008</td>
<td>0.1151</td>
<td>0.1216</td>
</tr>
<tr>
<td>L.L.</td>
<td>0.01</td>
<td>0.0640</td>
<td>0.0970</td>
</tr>
<tr>
<td>L.Q.</td>
<td>0.02</td>
<td>0.0616</td>
<td>0.0987</td>
</tr>
<tr>
<td>L.C.</td>
<td>0.02</td>
<td>0.0081</td>
<td>0.0740</td>
</tr>
<tr>
<td>Per-capita-based</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N.W.</td>
<td>0.03</td>
<td>0.3286</td>
<td>0.3343</td>
</tr>
<tr>
<td>L.L.</td>
<td>0.02</td>
<td>0.1156</td>
<td>0.2050</td>
</tr>
<tr>
<td>L.Q.</td>
<td>0.03</td>
<td>0.0276</td>
<td>0.1603</td>
</tr>
<tr>
<td>L.C.</td>
<td>0.04</td>
<td>-0.0650</td>
<td>0.1151</td>
</tr>
</tbody>
</table>

Note: N.W., L.L., L.Q. and L. C. imply Nadaraya-Watson, local-linear, local-quadratic and local-cubic estimates, respectively. The Gaussian kernel is used for the estimations. The half of the samples that are closer to zero are used. Data are one hundred times resampled for evaluating 90% and 95% critical points. The optimal bandwidth is selected with cross-validation by changing \( h \) from 0.01 to 0.10 by 0.01. In the case where 0.01 is optimal, we repeat the same procedure from 0.004 to 0.01 by 0.002. \( x_{\text{min}} \) is the minimum of \( x \).
Figure 3: Estimated Conditional Mean Function by NW, LL, LQ and LC: Household-based Case

Note: In order to make the performance of the regression lines clear, we set the upper limit of the y-axis to be 0.3. We do not plot four observations where the values of 1/consumption are between 0.3 and 0.8.